Determination of the endpoint of the first order deconfiniement phase transition in the heavy quark region of QCD

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WHOT-QCD Collaboration

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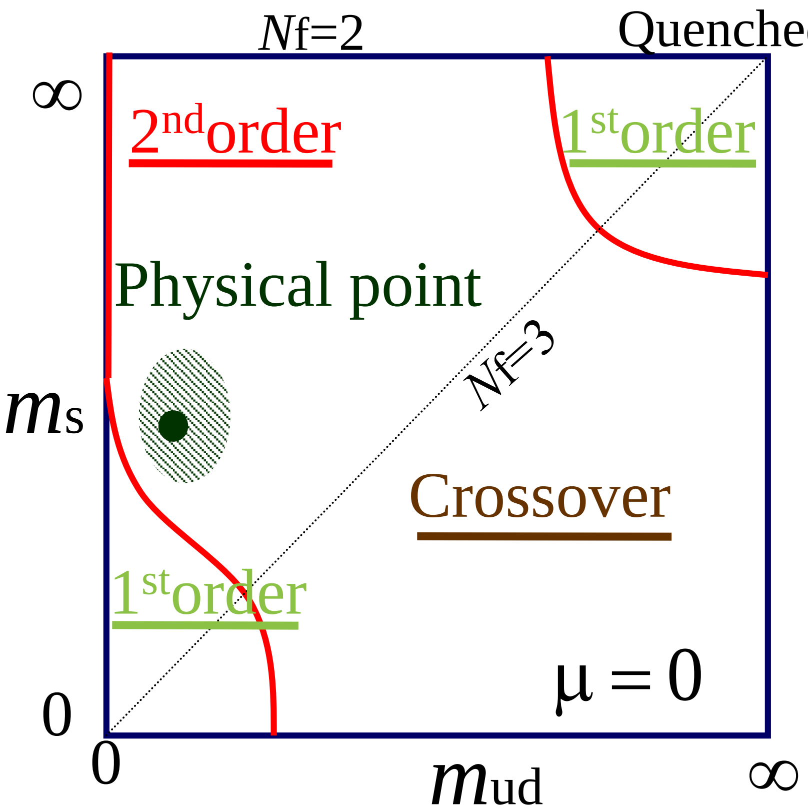
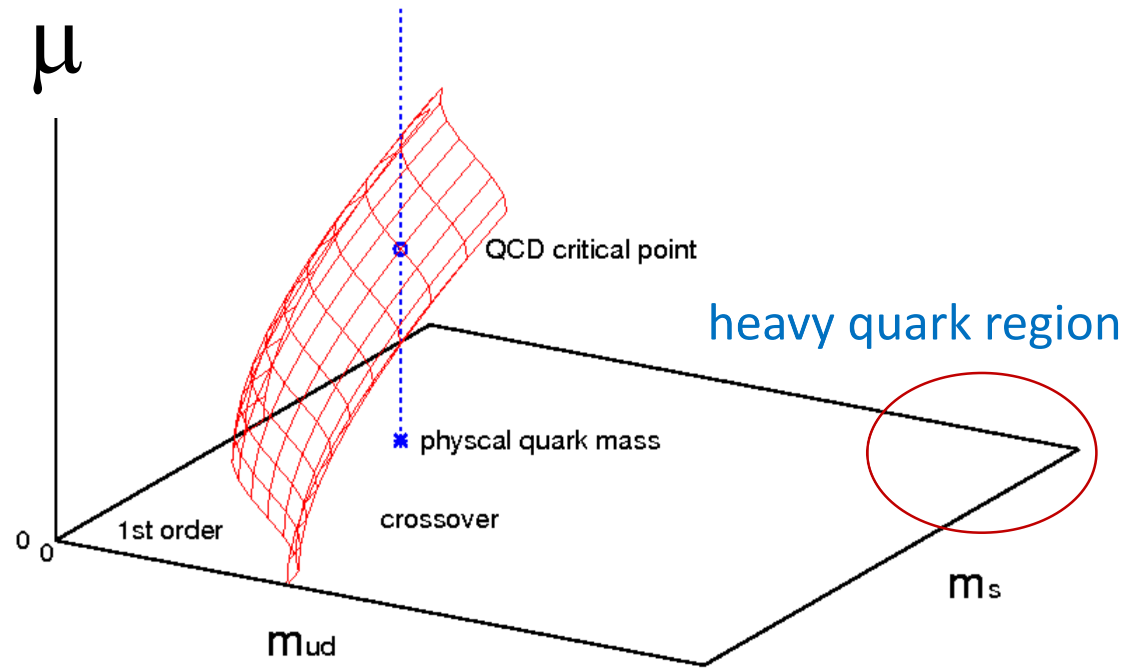
Quark Mass dependence of QCD phase transition





- The determination of the boundary of 1st order region: important.
- On the line of physical mass, the crossover at low density

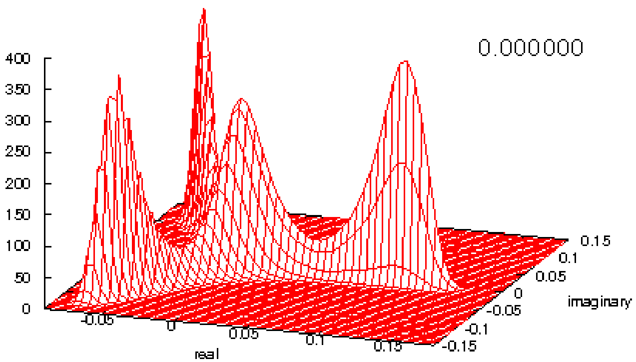
  ➡ 1st order transition at high density (?)
- We study the boundary in the heavy quark region.

Polyakov loop distribution at β_c in the complex plane

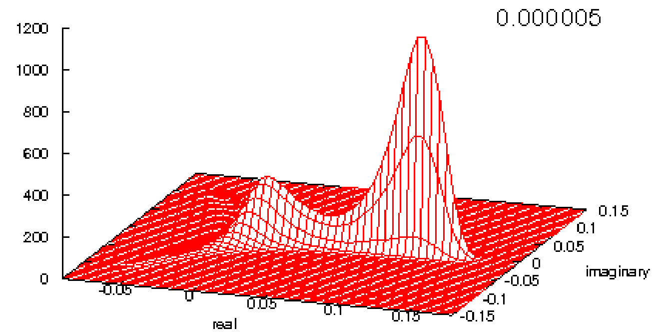
(2-flavor, $24^3 \times 4$ lattice, Phys.Rev.D89, 034507(2014))

Quenched QCD

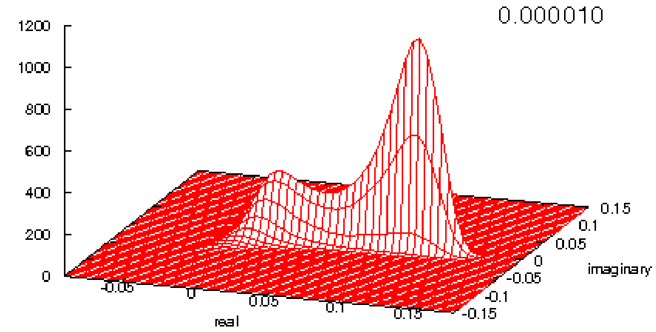
$K^4 = 0.0$ Z(3) symmetric



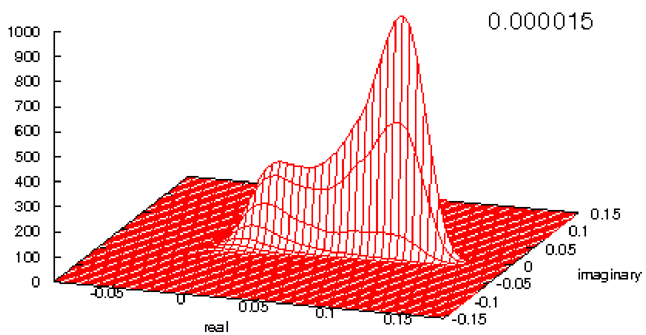
$K^4 = 5.0 \times 10^{-6}$



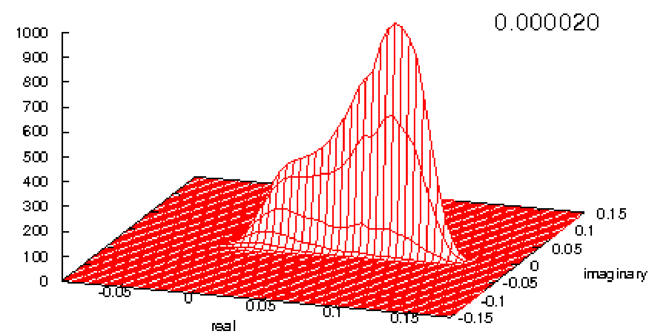
$K^4 = 1.0 \times 10^{-5}$



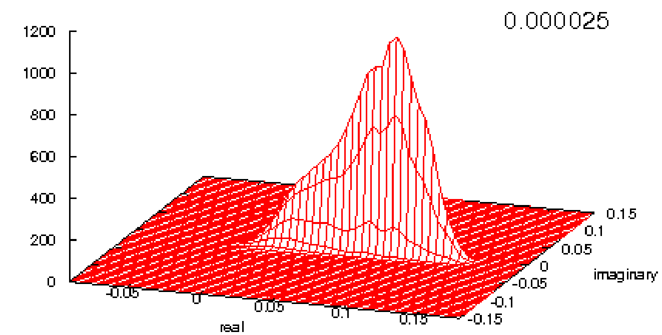
$K^4 = 1.5 \times 10^{-5}$



$K^4 = 2.0 \times 10^{-5}$



$K^4 = 2.5 \times 10^{-5}$



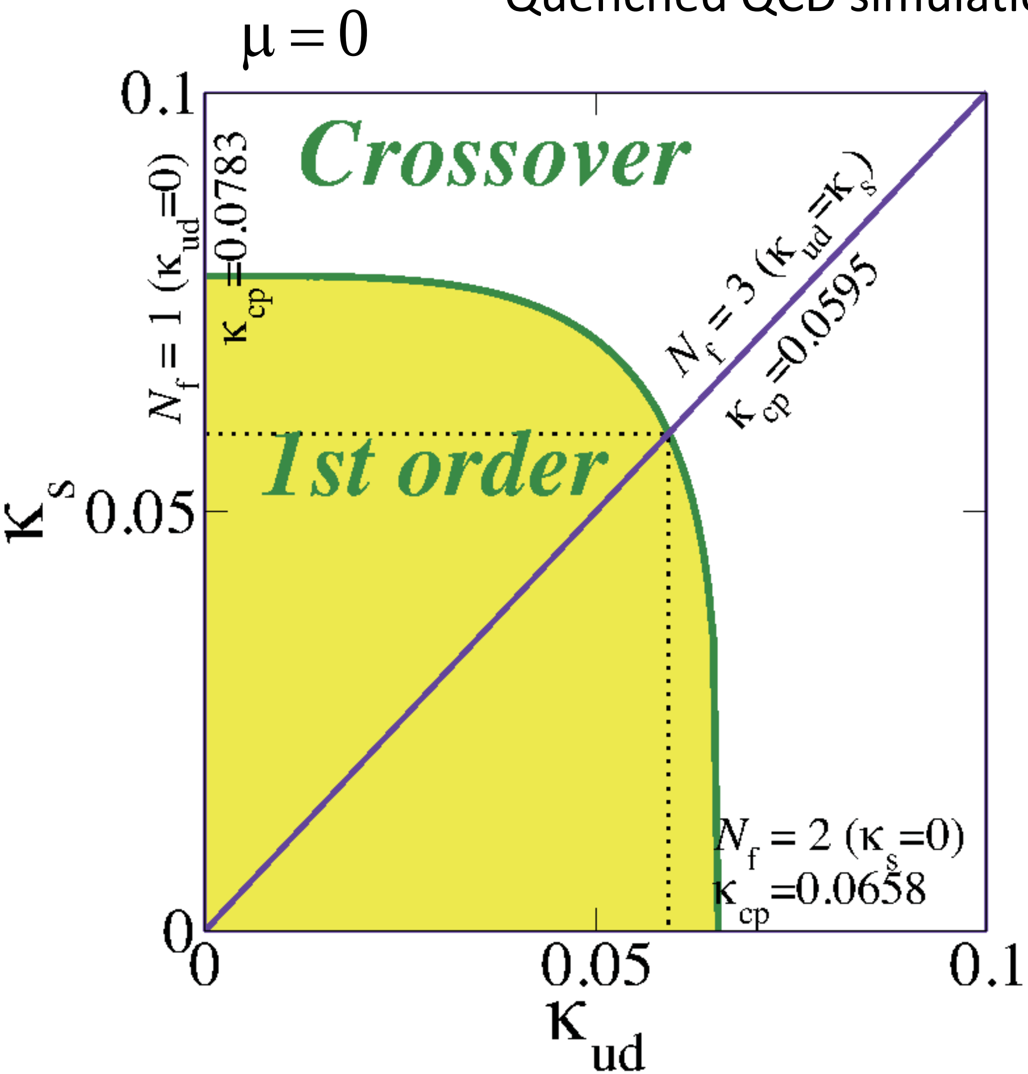
critical point

K: hopping parameter ~ 1/(mass)

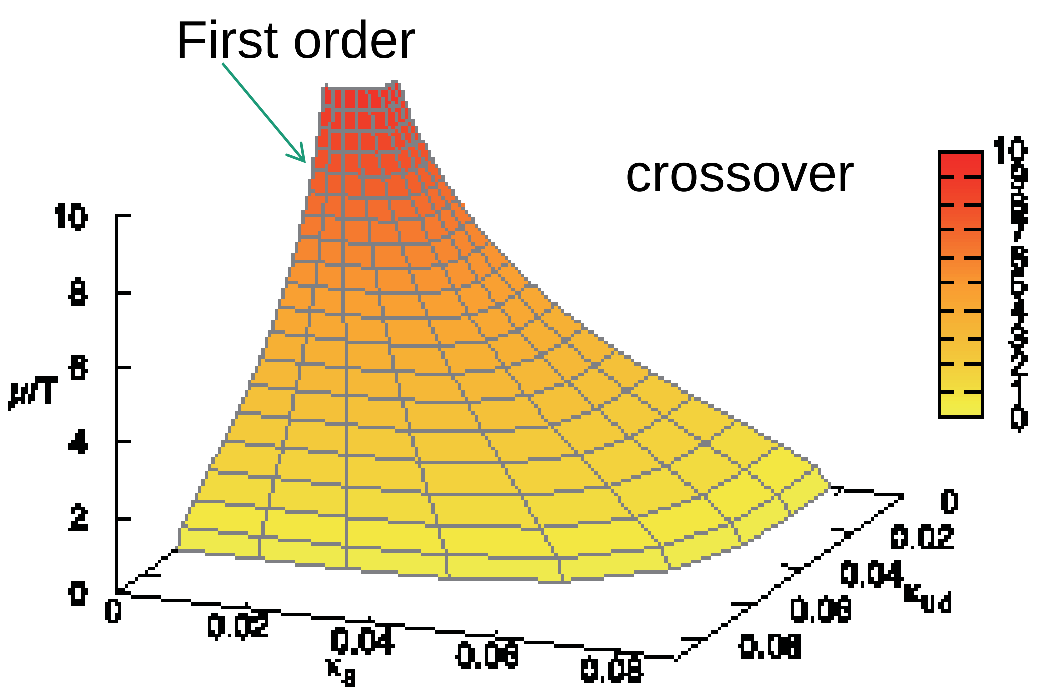
Critical surface in the heavy quark region of (2+1)-flavor QCD [Phys.Rev.D84, 054502(2011); Phys.Rev.D89, 034507(2014)]

Quenched QCD simulations + reweighting

$\left(24^3 \times 4 \text{ lattice}\right)$



Critical surface at finite density



$$\frac{m_{PS}}{T_c} \approx 16.$$ at κ_{cp} for 2-flavor

This talk

- Calculation of Kc on lattices of N_t=6 and 8
- Limitation of this analysis by the shape of the histogram
  - Lattice spacing dependence
    → truncation error of hopping parameter expansion
  - Spatial volume dependence → Overlap problem

- New approach
  - Simulations with a Polyakov loop term
  - Finite volume scaling analysis

Histogram method

- Probability distribution function (Histogram)

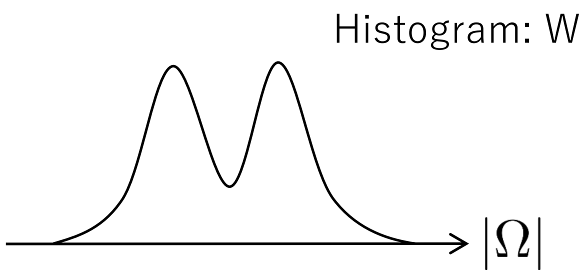
  Ω: Polyakov loop (order parameter)

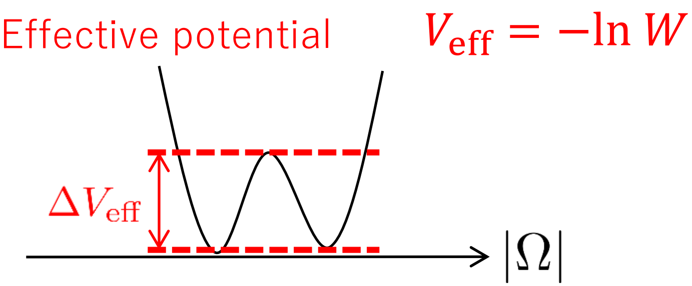
$$W(\Omega;\beta,K) \equiv \frac{1}{Z}\int DU\ \delta(\Omega-\hat{\Omega})\prod_{f=1}^{N_f}\det M(K)\, e^{-S_g}$$

(S_g: gauge action, M: quark matrix)

- Effective potential $V_{\text{eff}} = -\ln W$

First order transition





- Critical point of K: $\Delta V_{\text{eff}} = 0$

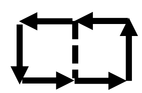
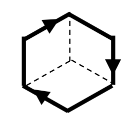
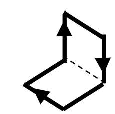
Reweighting method in the heavy quark region

- Quenched QCD simulations + reweighting

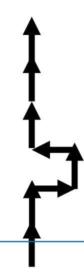
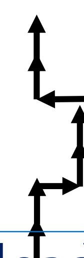
$$W(\Omega;\beta,K) \equiv \frac{1}{Z}\int DU\ \delta(\Omega-\hat{\Omega})\prod_{f=1}^{N_f}\det M(K)\, e^{-S_g}$$ Histogram

$$= \frac{\left\langle \delta(\Omega-\hat{\Omega})\prod_f \det M(K)\right\rangle_{\text{quench}}}{\left\langle \prod_f \det M(K)\right\rangle_{\text{quench}}}$$

- Multi-point (β) reweighting method is used.
- Hopping parameter expansion $(K \sim 1/(ma))$

$$\ln(\det M(K)) = 288 N_{\text{site}} K^4 P + [768\, N_{\text{site}} K^6 (3$$  $$+$$  $$+6$$  $$)]+\cdots$$

plaquette

$$+12\times 2^{N_t}\times N_s^3\, [K^{N_t}\mathrm{Re}\Omega + 6N_t K^{N_t+2}$$  $$+6N_t K^{N_t+2}$$  $$+3N_t K^{N_t+2}$$  $$]$$

Polyakov loop

Leading term

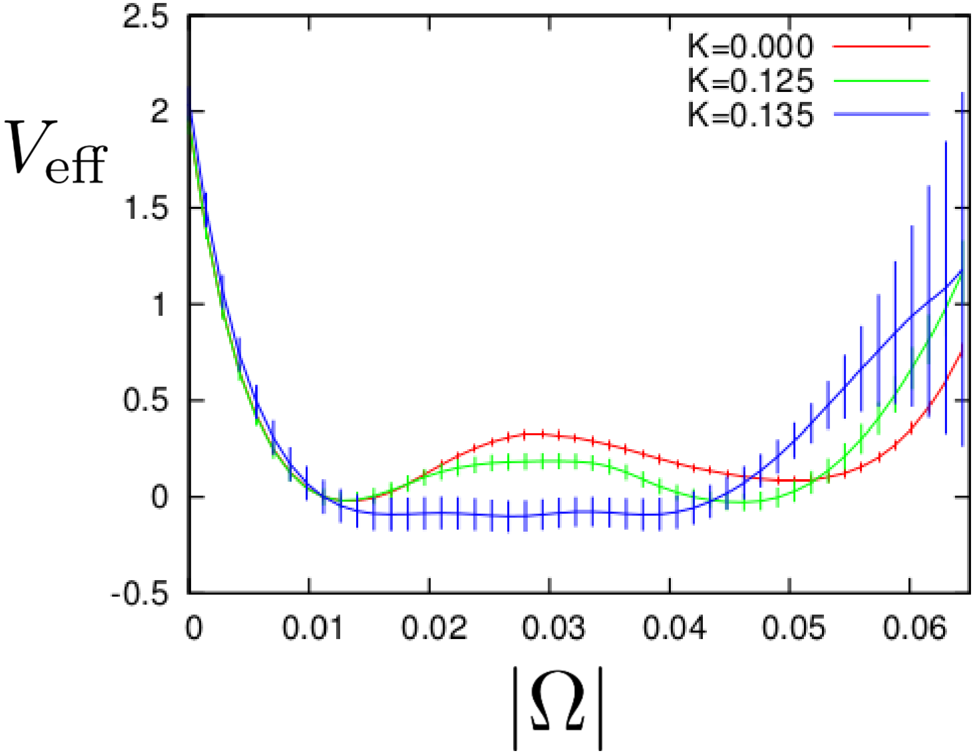
Next to leading terms

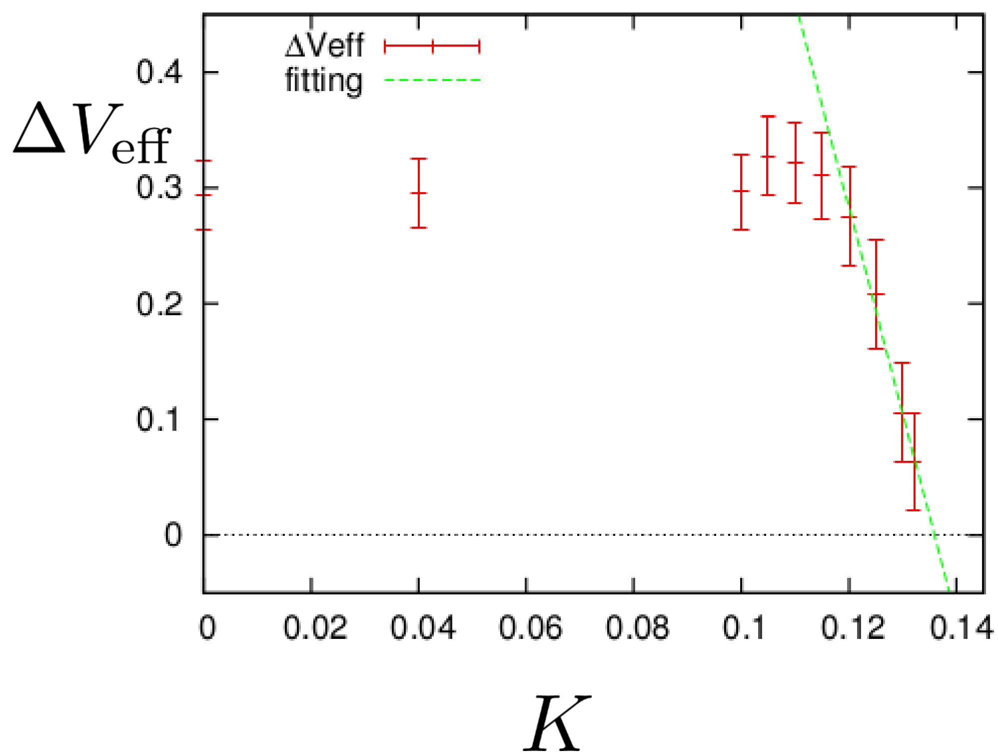
* plaquette and 6-step Wilson loop can be absolved into the gauge action.

Determination of Kc (leading order calculation)

$N_t = 6$ lattice (24^3x6)

Effective potential





K_c=0.1359(30)

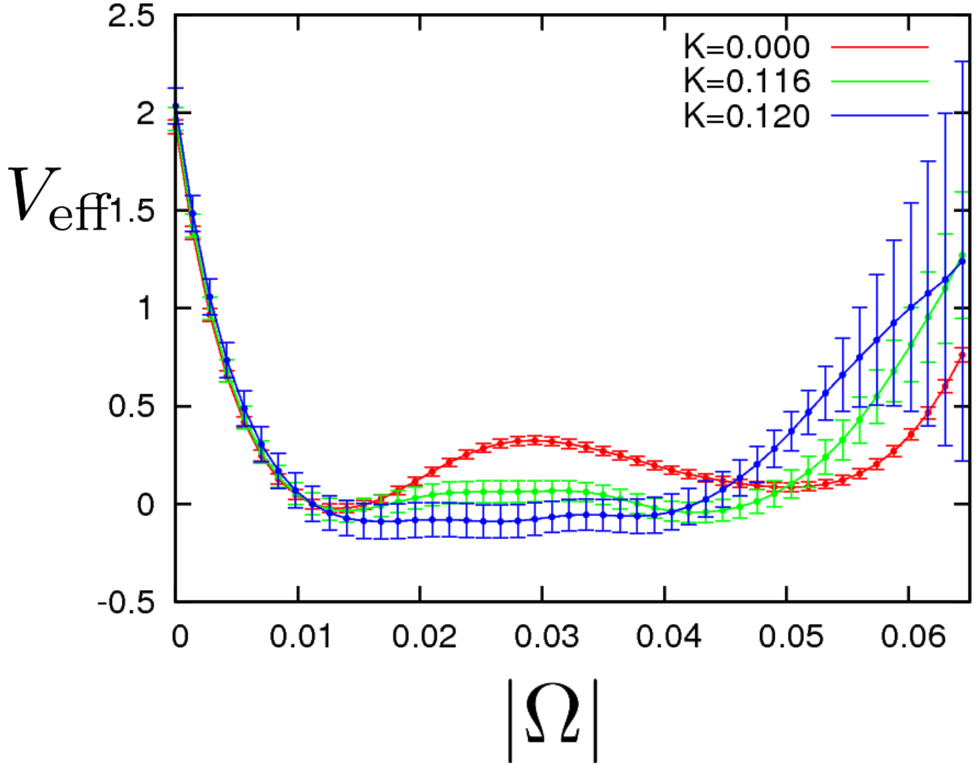
Lattice	Configurations
24^3x6	676,190
32^3x6	1,172,000
24^3x8	342,821

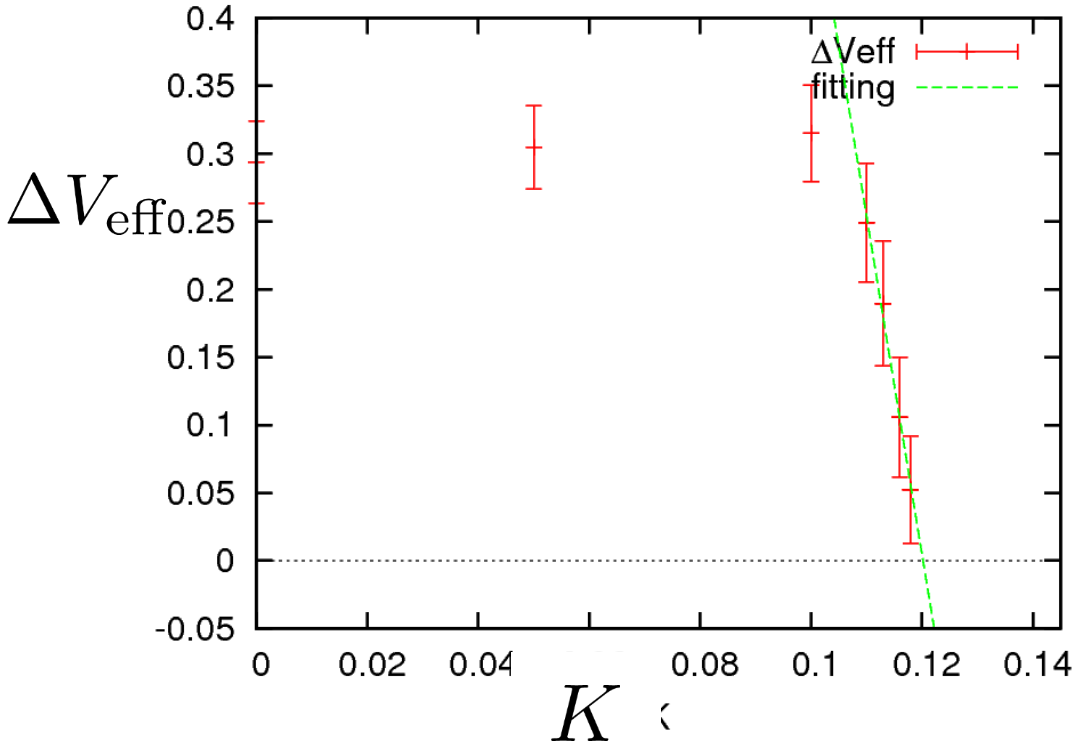
Determination of K_c (Next to leading order calculation)

To estimate the truncation error of the hopping parameter, the next to leading contribution is computed.

$N_t = 6$ lattice (24^3x6)

Effective potential





K_c=0.1202(19)

Lattice	Configurations
24^3x6	676,190
32^3x6	1,172,000
24^3x8	342,821

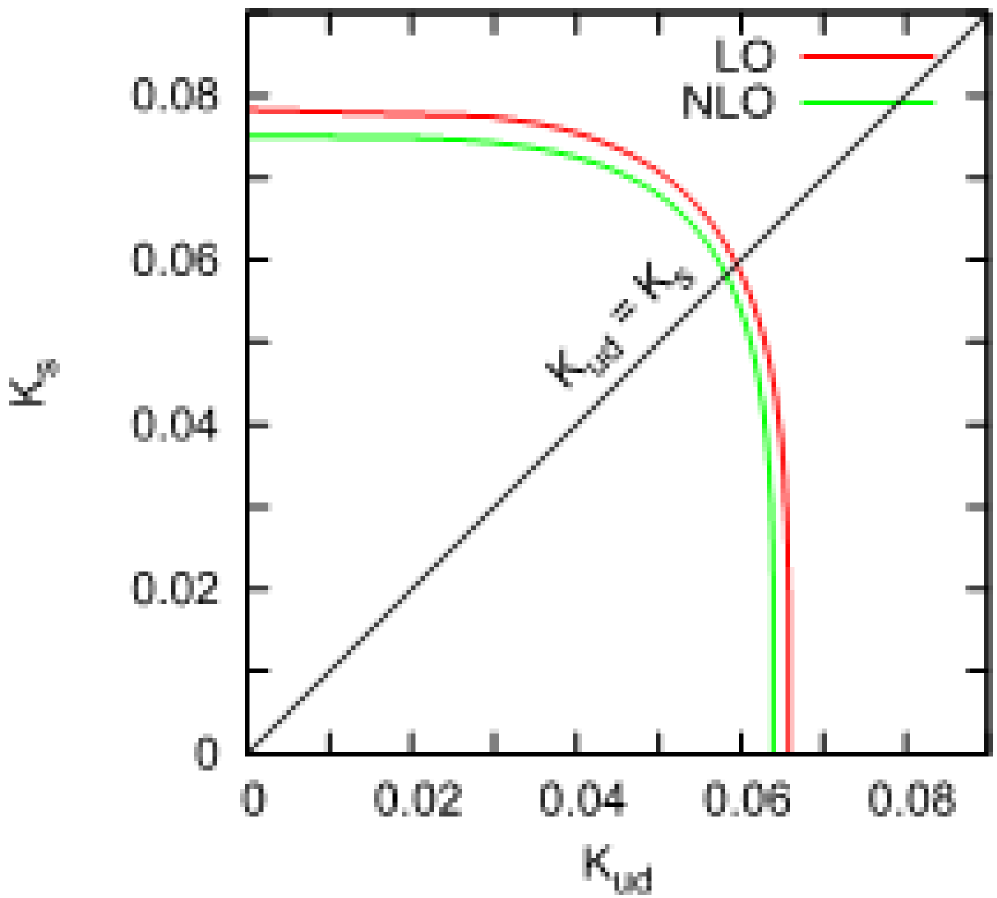
Determination of Critical K for 2-flavor QCD

	Leading order		Next to leading order	
Lattice	K_c	m_{PS}/T_c	K_c	m_{PS}/T_c
24^3x4	0.0658(10)	15.47(14)	0.0640(10)	15.73(14)
24^3x6	0.1359(30)	7.43(78)	0.1202(19)	11.15(42)
24^3x8	> 0.18	> (chiral limit)		

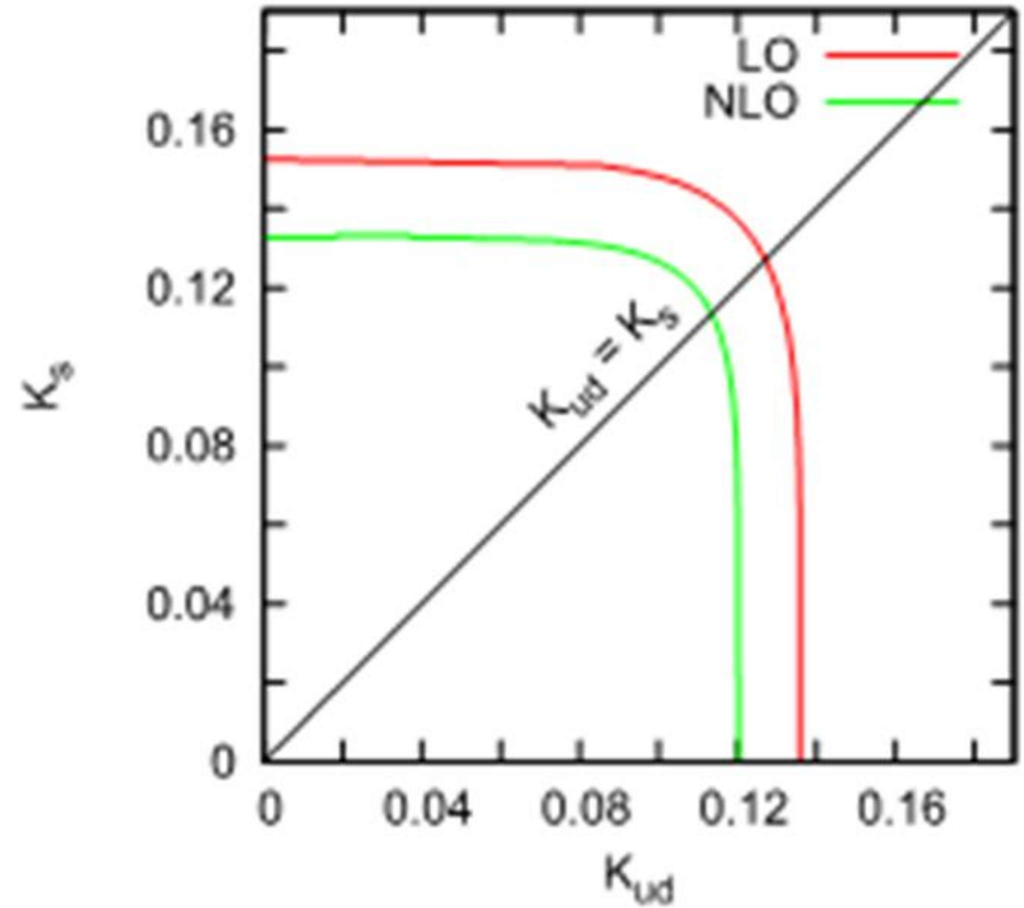
- In the previous study on a 24^3x4 lattice, the truncation error of the hopping parameter expansion is negligible.
- The truncation error is visible for the 24^3x6 lattice.
- Pseudo-scalar meson mass m_{PS} measured by T=0 full QCD simulations at K_c for N_t=6 is smaller than that for N_t=4.
- This analysis is not applicable for N_t=8.
- Higher order terms are necessity for large N_t.

Determination of Kc in 2+1 flavor QCD

24^3x4 lattice



24^3x6 lattice



LO: leading order NLO: next to leading order

- Difference between the leading order and next to leading calculations are sizeable for the 24^3x6 lattice.

Volume dependence of Critical K

Lattice	K_c
24^3x6	0.1359(30)
32^3x6	0.1286(40)
36^3x6	[overlap problem]

- K_c becomes smaller as the volume increases.
- 36^3x6 lattice: overlap problem arises before K_c.
- Reweighting form quenched simulation dose not work for large volume.

  --> Simulations with a Polyakov loop term.

Simulations with the Polyakov loop term Ω

- Simulations with an effective action

$$S_{\text{eff}} = -6N_s^3\beta P + N_s^3\lambda\text{Re}\Omega$$

  - The Polyakov loop term corresponds to the leading order contribution of the hopping parameter expansion of det M.

$$\lambda = 384\, K^4 \quad \text{(for 2-flavor, } N\text{t=4)}$$

- Heat bath algorithm is applicable.

  → computational cost is small.

- We include the next leading contribution of the hopping parameter expansion by the reweighting.
- Overlap problem can be avoided.
- Simulation parameter
  - Lattice size: Nt=4, Ns=32, 36, 40, 48

Binder cumulant

$$B_4 = \frac{\langle(\Omega - \langle\Omega\rangle)^4\rangle}{\langle(\Omega - \langle\Omega\rangle)^2\rangle^2}$$

- No volume dependence at the critical point λ_{cp}
- 3D Ising universality crass: B_4=1.604 at λ_{cp} and ν=0.63.

Fit result Nt=4

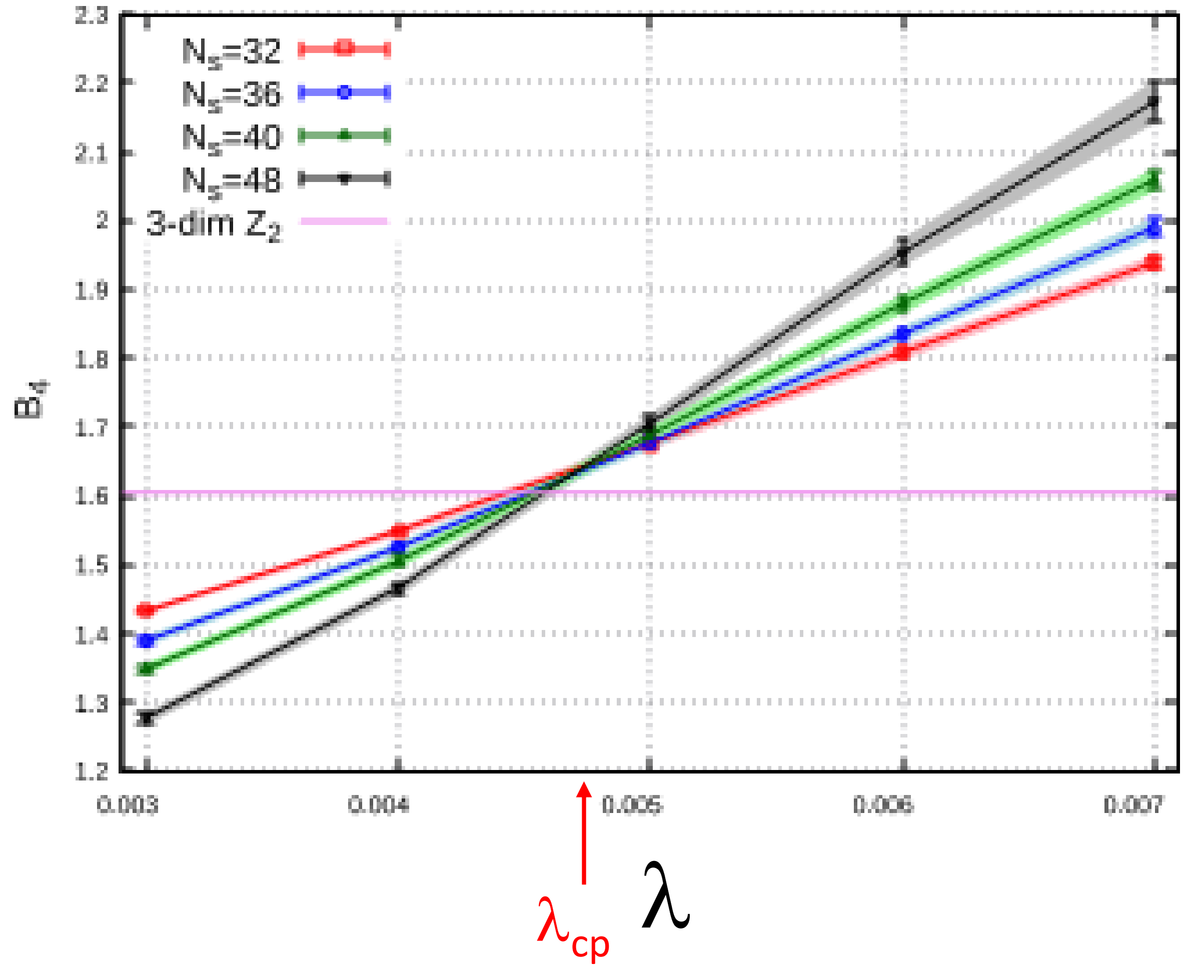
$$B_4(N_s, \lambda) \equiv B_{\rm cp} + A N_s^{1/\nu}(\lambda - \lambda_{\rm cp})$$

Ns=32, 36, 40, 48

- λ_{cp}=0.004754(84)
- B_{cp}=1.644(13)
- ν=0.65(8)

Ns=36, 40, 48

- λ_{cp}=0.00468(11)
- B_{cp}=1.630(20)
- ν=0.65(11)



λ_{cp}=0.00475 → Kc=0.0539 is smaller than the result by histogram method: 0.0640(10)

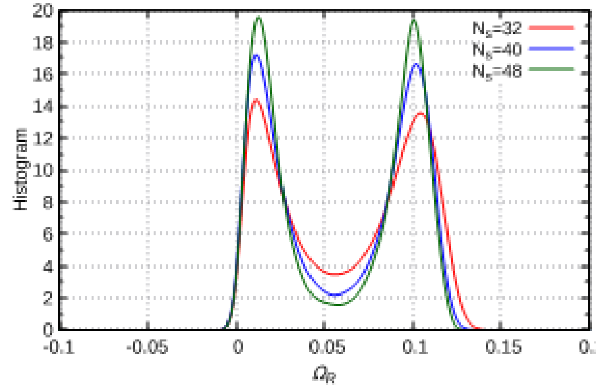
(24^3x4 lattice)

Volume dependence of the histogram

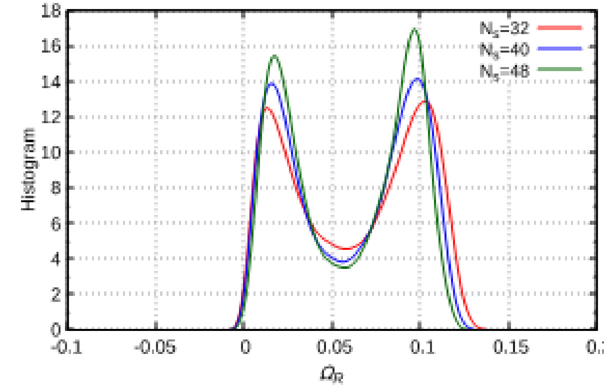
- $\lambda<\lambda_{cp}$, The middle dent in the histogram gets deeper as the volume increases.
- $\lambda>\lambda_{cp}$, the middle dent becomes shallower and disappears.
- λ_{cp} is the boundary that divides one peak or two peaks in the volume infinity limit.

λ_{cp}=0.004754(84)

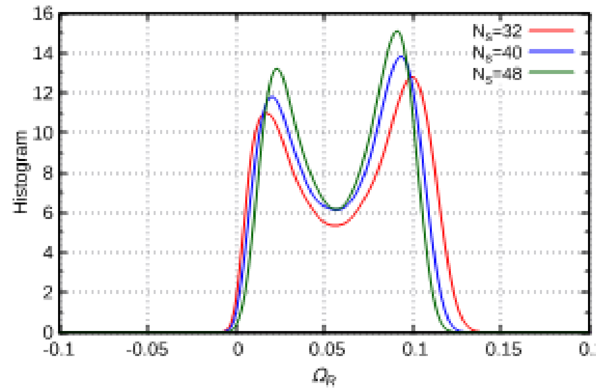
λ=0.003



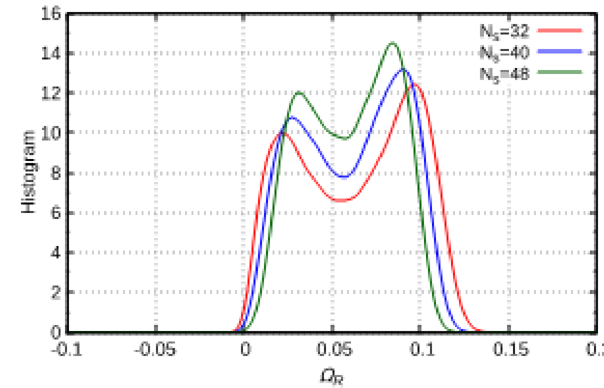
λ=0.004



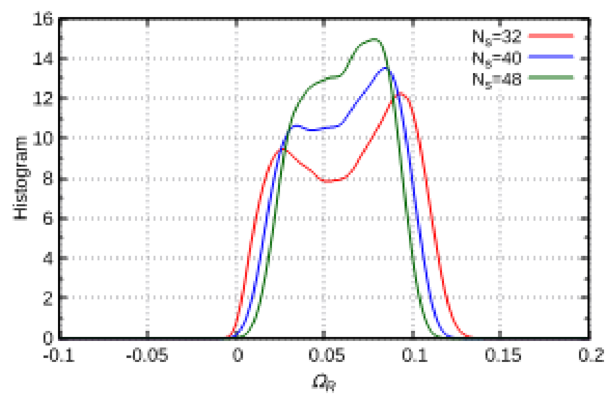
λ=0.005



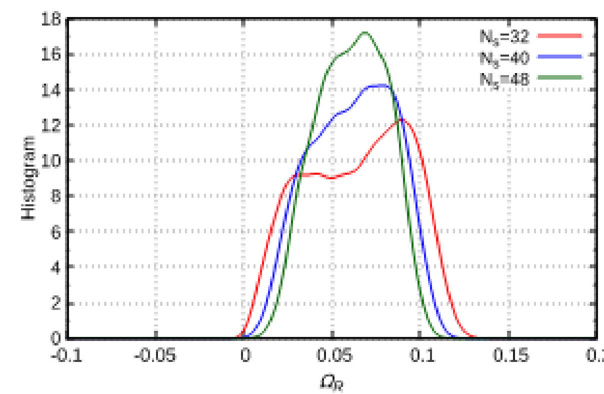
λ=0.006



λ=0.007



λ=0.008

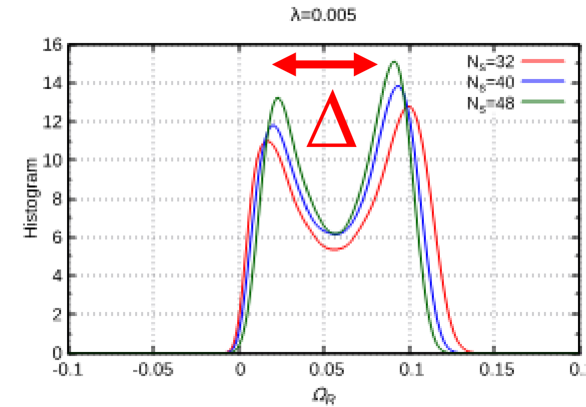


Finite volume Scaling of the gap

- Assuming the gap Δ corresponds to (magnetization)/(2×volume) of 3D ising,

$$\Delta \propto N_s^{-0.519}$$

is expected at λ_{cp}.



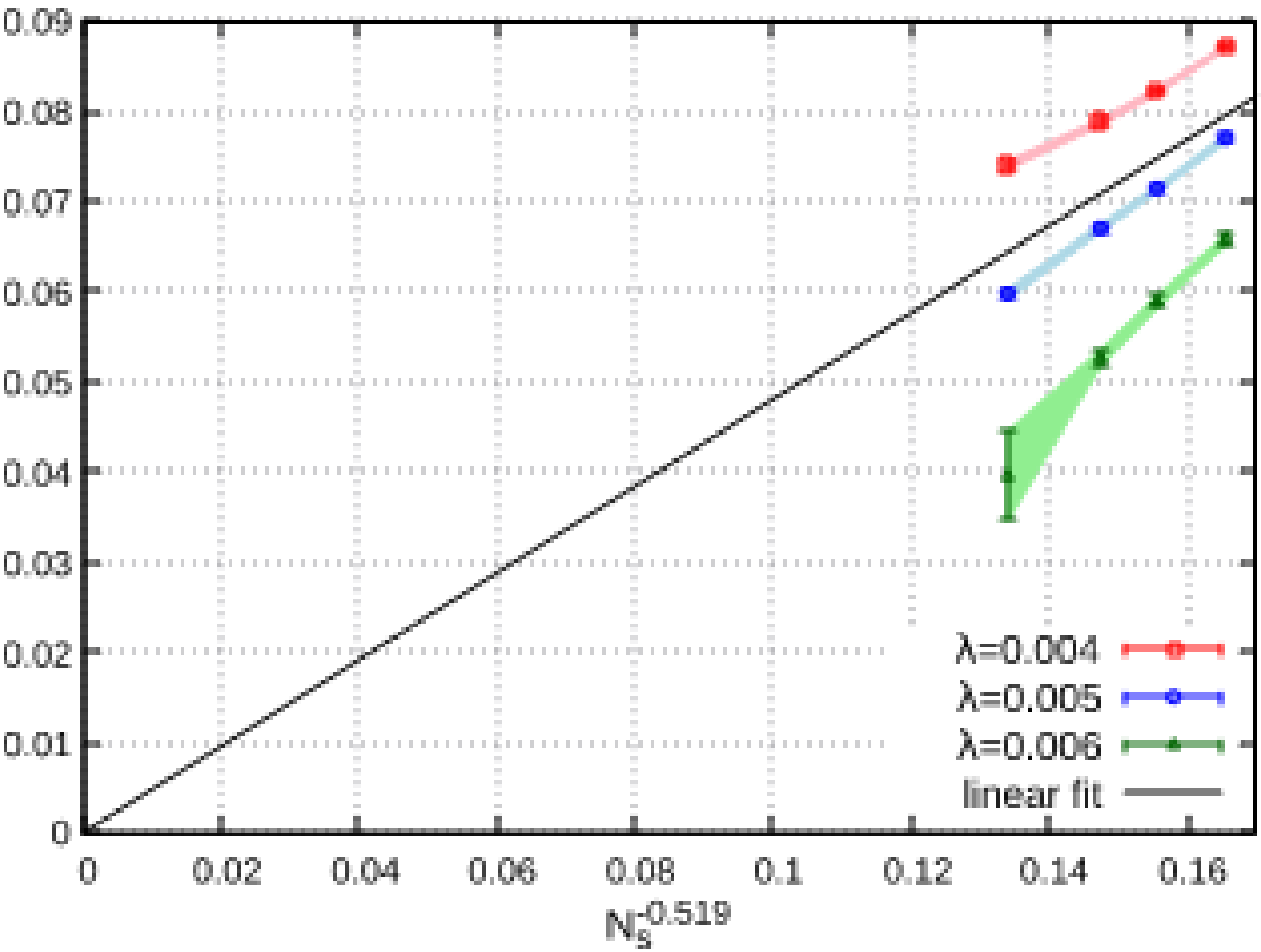
In $N_s \to \infty$,

- λ=0.005:

Linearly decrease

- λ=0.006:

Δ vanishes at finite N_S



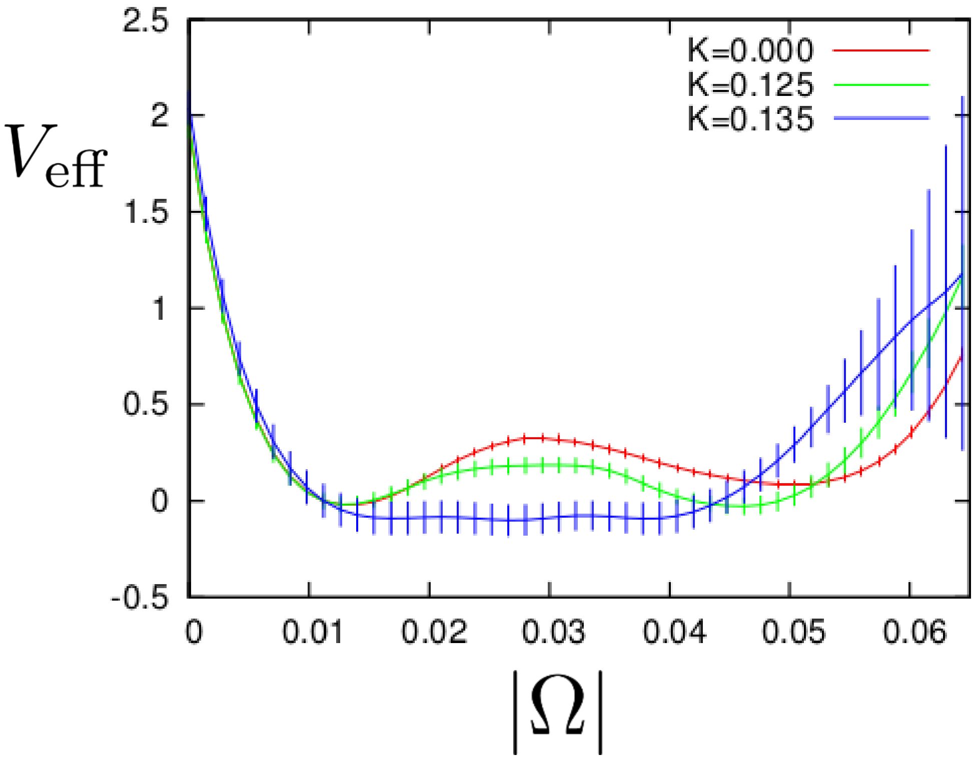
Summary and out look

- We study the location of critical point at which the first order phase transition changes to crossover in the heavy quark region by investigating the histogram of the Polyakov loop and applying the finite-size scaling analysis.
  - simulations of quenched QCD
  - reweighting method
  - quark determinant: hopping parameter expansion
- Truncation error of the hopping parameter expansion: Visible for $N_t \geq 6$. → Higher order terms are needed.
- Overlap problem arises for large volume.
- To reduce the overlap problem, we introduce an external source term of the Polyakov loop in the simulation.
  - Scaling behavior at λ_{cp} is consistent with 3D ising model.

Determination of Kc (leading order calculation)

$N_t = 6$ lattice (24^3x6)

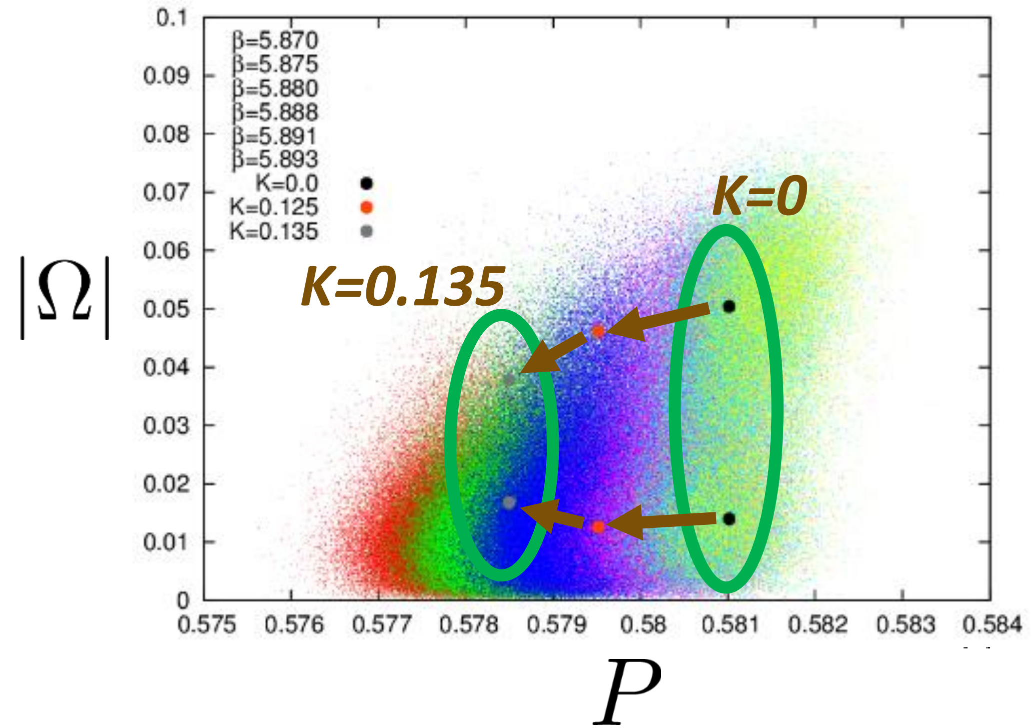
Effective potential



K_c=0.1359(30)

Distribution of P and Ω

Peak positions of the histogram



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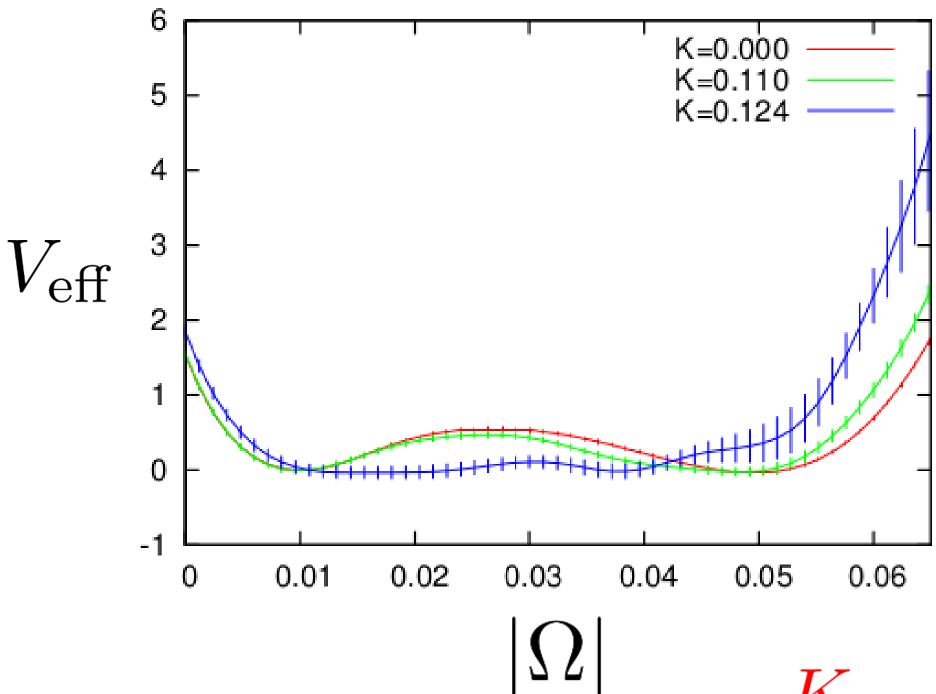
24^3x8	342,821

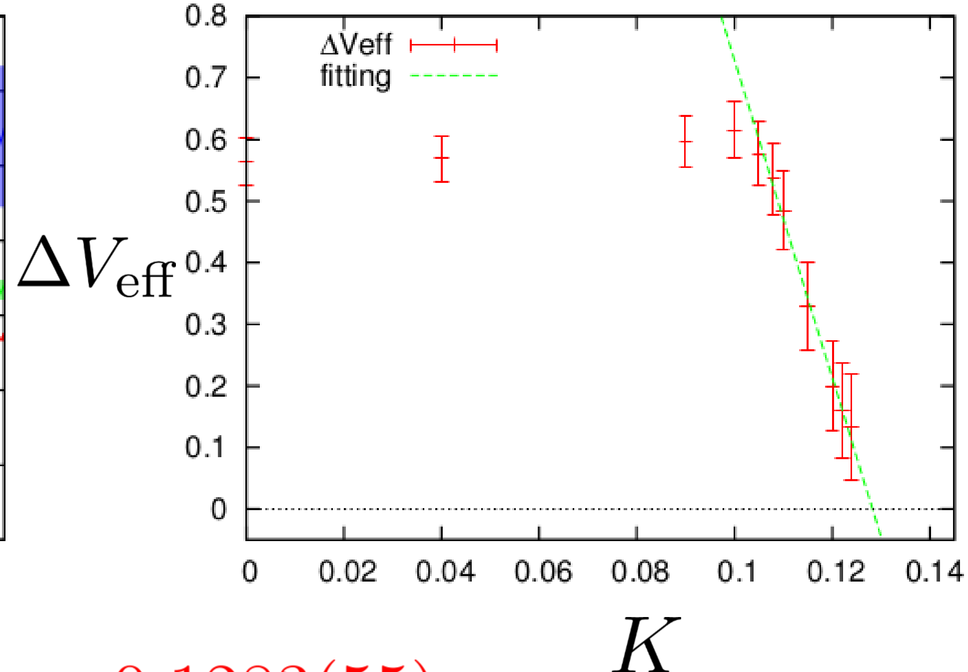
Determination of Kc (leading order calculation)

$N_t = 6$ latice (32^3x6)

Effective potential



$\Delta V_{\rm eff}$の変化

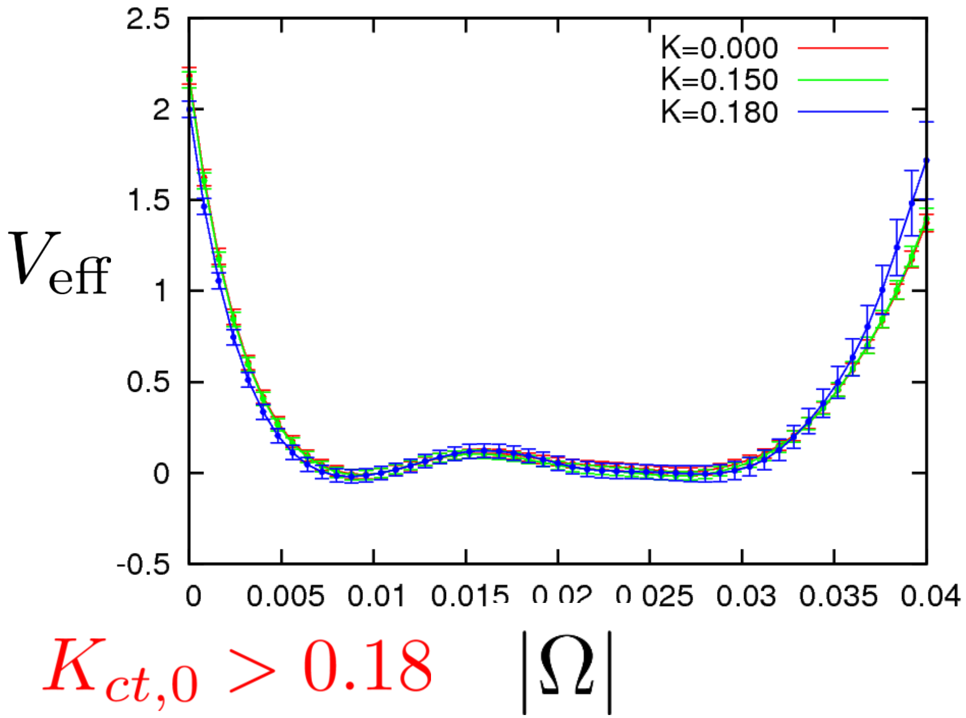


$K_{ct,0} = 0.1283(55)$

Determination of Kc (leading order calculation)

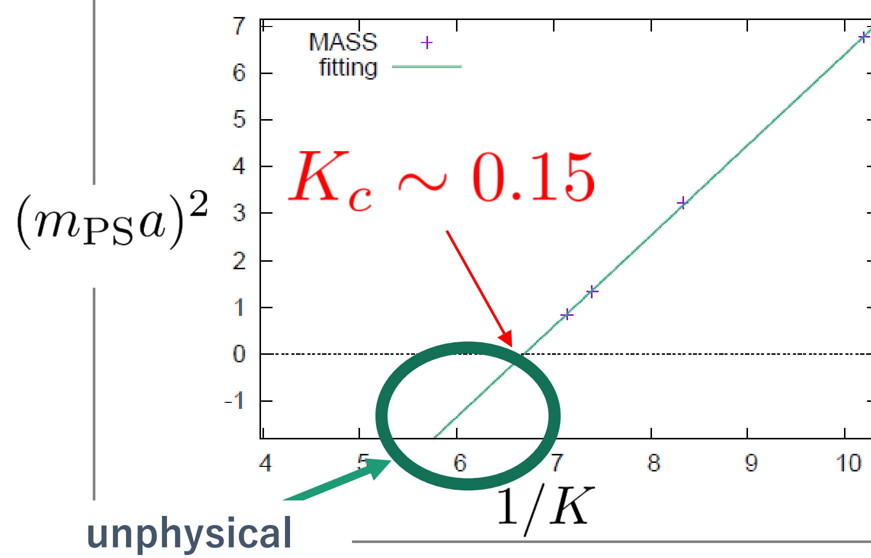
$N_t = 8$ lattice (24^3x8)

Effective potential



$K_{ct,0} > 0.18$

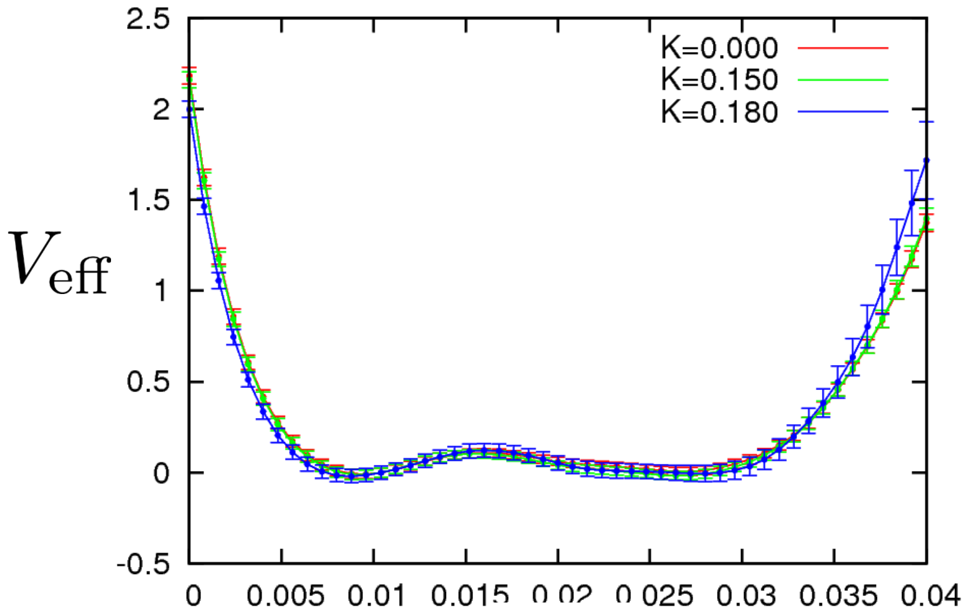
- The shape of the effective potential does not change even when Kc>0.15



Determination of Kc (leading order calculation)

$N_t = 8$ lattice (24^3x8)

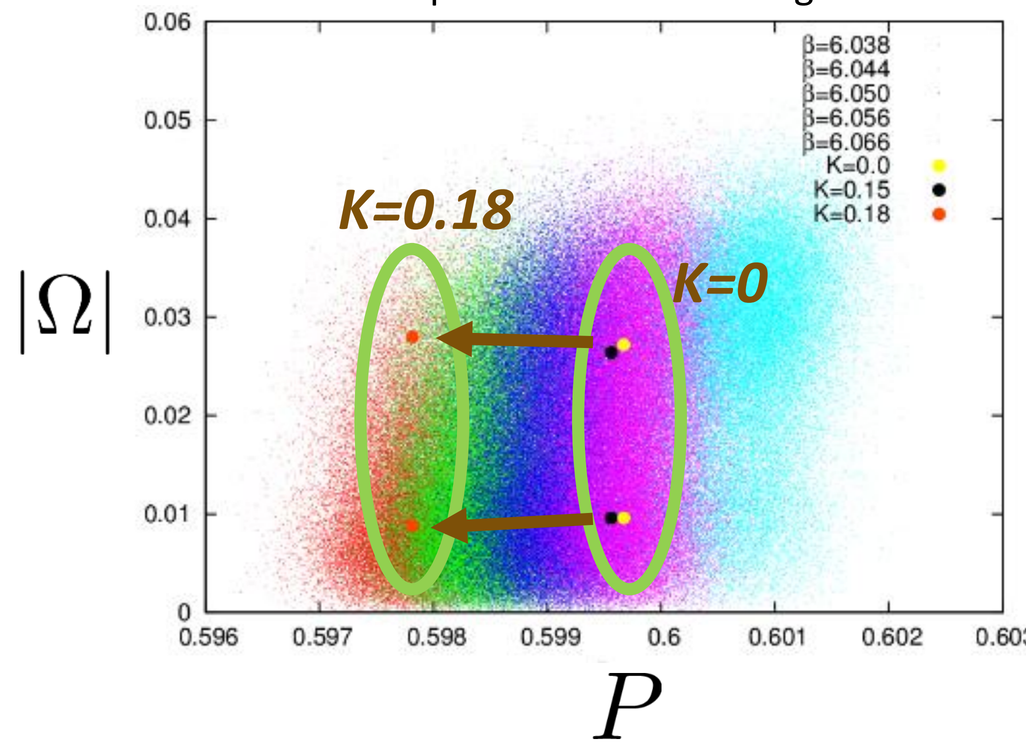
Effective potential



$K_{ct,0} > 0.18$

Distribution of P and Ω

Peak positions of the histogram

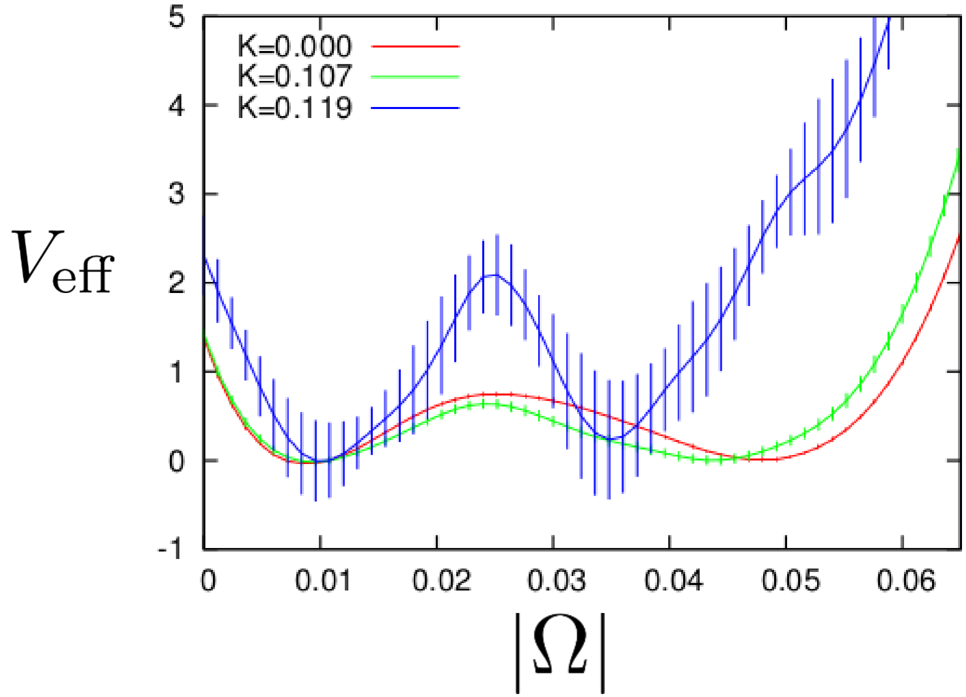


No overlap problem. Truncation error of hopping parameter expansion: large for Nt=8

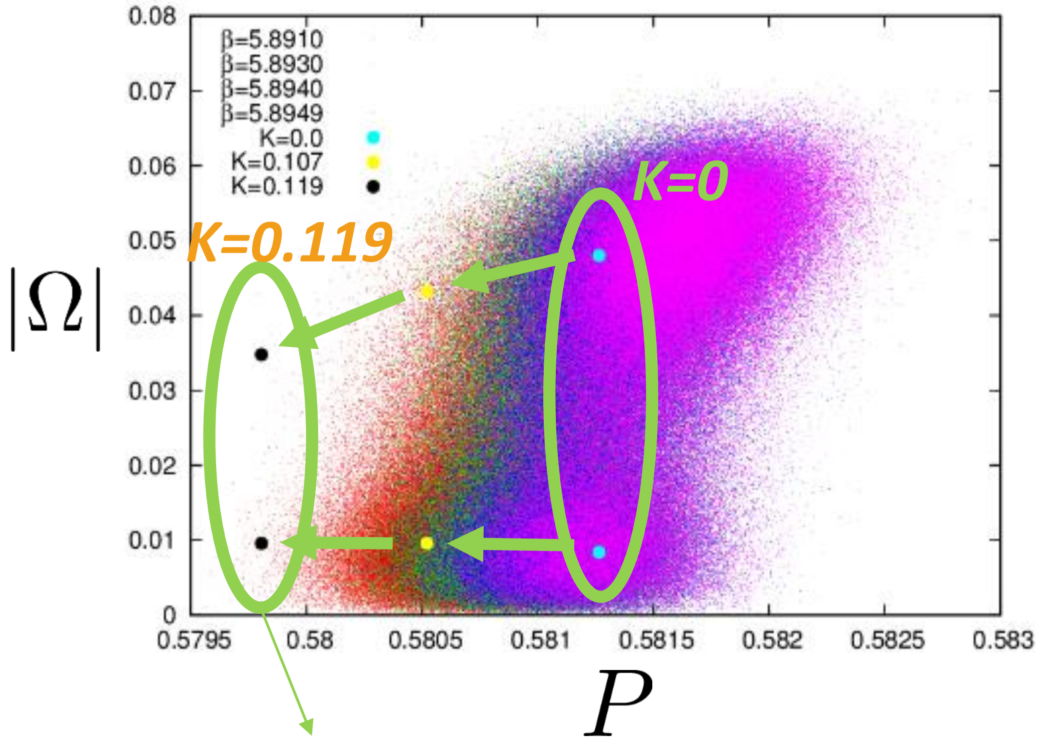
Determination of Kc (leading order calculation)

$N_t = 6$ lattice (36^3x6)

Effective potential



Distribution of P and Ω
Peak positions of the histogram



Over lap problem arises.