

Mesonic correlators at non-zero baryon chemical potential

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on behalf of FASTSUM collaboration

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Taylor expansion of the meson correlator

Let's consider $\vec{p} = 0$ non-singlet meson correlator:

$$G(\tau) = \frac{1}{V} \int d^3\vec{x} \langle J(\tau, \vec{x}) \bar{J}(0, \vec{0}) \rangle, \quad \text{where } J = \bar{\psi}_u \Gamma \psi_d$$

The idea is to perform the Taylor expansion of the correlator itself (firstly was proposed by QCD-TARO Collab. in hep-lat/0107002):

$$\frac{1}{T} G = \frac{1}{T} G|_{\mu=0} + \frac{\mu}{T} \frac{\partial G}{\partial \mu} \Big|_{\mu=0} + \frac{T}{2} \left(\frac{\mu}{T} \right)^2 \frac{\partial^2 G}{\partial \mu^2} \Big|_{\mu=0} + O\left(\frac{\mu^3}{T^3}\right)$$

- Expansion of both determinant and correlator
- $\dot{G}_\mu|_{\mu=0} = 0$, thus $G(\tau, \mu) = G(\tau)|_{\mu=0} + (\mu^2/2) \ddot{G}_\mu(\tau)|_{\mu=0}$
- \ddot{G}_μ has to be computed
- \ddot{G}_μ contains several noisy disconnected terms

- Meson correlator has simple form: $G(x) = \langle g(x) \rangle$, where
 $g(x) = \text{Tr} \left[S_{x;0}^{(u)} \Gamma S_{0;x}^{(d)} \Gamma^\dagger \right]$, $S_{x;0}^{(f)} = D_{x,0}^{-1}(\mu_f, m_f)$
 $\langle O \rangle = (1/Z) \int dU O \det D_{u,d} \det D_s \exp[-S_G]$
- We consider symmetric μ set: $\mu_u = \mu_d$

Second derivative in chemical potential:

$$\begin{aligned} \ddot{G}(x) = & \langle g''_\mu(x) \rangle \\ & + \left\langle \frac{\det''_\mu}{\det} g(x) \right\rangle - \left\langle \frac{\det''_\mu}{\det} \right\rangle \langle g(x) \rangle \\ & + 2 \left\langle \frac{\det'_\mu}{\det} g'_\mu(x) \right\rangle - 2 \left\langle \frac{\det'_\mu}{\det} \right\rangle \langle g'_\mu(x) \rangle \\ & - 2 \left(\left\langle \frac{\det'_\mu}{\det} g(x) \right\rangle - \left\langle \frac{\det'_\mu}{\det} \right\rangle \langle g(x) \rangle \right) \left\langle \frac{\det'_\mu}{\det} \right\rangle \end{aligned}$$

\ddot{G} structure: connected and disconnected terms

$$\begin{aligned}\ddot{G}|_{\mu=0} = & -2\text{Re} \left\langle \text{Tr} \left[\gamma_5 \Gamma^\dagger \left(D^{-1} \dot{D} D^{-1} \right)_{n,0} \Gamma \gamma_5 \left(D^{-1} \dot{D} D^{-1} \right)_{n,0}^\dagger \right] \right\rangle \\ & + 4\text{Re} \left\langle \text{Tr} \left[\gamma_5 \Gamma^\dagger \left(D^{-1} \dot{D} D^{-1} \dot{D} D^{-1} \right)_{n,0} \Gamma \gamma_5 \left(D^{-1} \right)_{n,0}^\dagger \right] \right\rangle \\ & - 2\text{Re} \left\langle \text{Tr} \left[\gamma_5 \Gamma^\dagger \left(D^{-1} \ddot{D} D^{-1} \right)_{n,0} \Gamma \gamma_5 \left(D^{-1} \right)_{n,0}^\dagger \right] \right\rangle \\ & - 8 \left\langle \text{Im Tr} \left[\gamma_5 \Gamma^\dagger \left(D^{-1} \dot{D} D^{-1} \right)_{n,0} \Gamma \gamma_5 \left(D^{-1} \right)_{n,0}^\dagger \right] \text{Im Tr} \left[\dot{D} D^{-1} \right] \right\rangle \\ & + 2 \left\langle \text{Tr} \left[\gamma_5 \Gamma^\dagger \left(D^{-1} \right)_{n,0} \Gamma \gamma_5 \left(D^{-1} \right)_{n,0}^\dagger \right] \right. \\ & \quad \times \left. \left(2\text{Tr} \left[\dot{D} D^{-1} \right]^2 + \text{Tr Tr} \left[\left(\dot{D} D^{-1} \right)^2 \right] \right) \right\rangle \\ & + 2 \left\langle \text{Tr} \left[\gamma_5 \Gamma^\dagger \left(D^{-1} \right)_{n,0} \Gamma \gamma_5 \left(D^{-1} \right)_{n,0}^\dagger \right] \right\rangle \\ & \quad \times \left\langle \left(2\text{Tr} \left[\dot{D} D^{-1} \right]^2 + \text{Tr} \left[\ddot{D} D^{-1} \right] - \text{Tr} \left[\left(\dot{D} D^{-1} \right)^2 \right] \right) \right\rangle\end{aligned}$$

For free massless quarks μ^2 -correction may be written analytically:

$$T^2 \ddot{G}_\Gamma(\tau) \Big|_{\mu=0} = \frac{N_c T^3}{\pi^2} \left[a_\Gamma^{(1)} + a_\Gamma^{(2)} - \frac{1}{12} (a_\Gamma^{(1)} - a_\Gamma^{(2)}) h(u) \right]$$

where

$$h(u) = [3u(\pi^2 - u^2 - 2) + u(\pi^2 - u^2 + 6) \cos(2u) - 2(\pi^2 - 3u^2) \sin(2u)] / \sin^3(u)$$

and $u = 2\pi T(\tau - 1/2T)$, $-\pi < u < \pi$

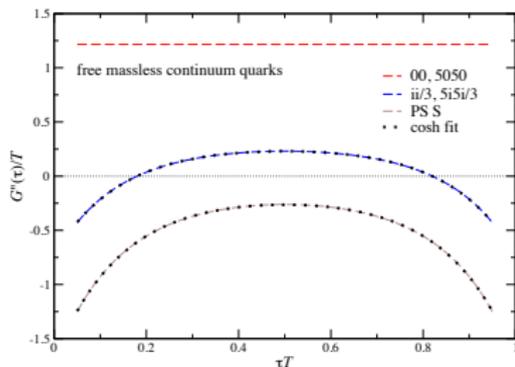
- τ -dependence is contained in $h(u)$
- Coefficients $a_\Gamma^{(j)}$ depend on the channel

Free analytical expression above provides qualitative check

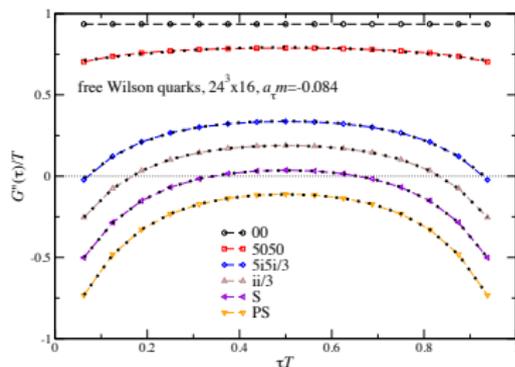
- \ddot{G} is well described by one-cosh fit

$$\ddot{G}(\tau)/T = c_0 + c_1 \cosh[c_2(\tau T - 1/2)]$$

both in continuum and on lattice



continuum, $c_2 \sim 7$



lattice, $c_2 \sim 7.4$

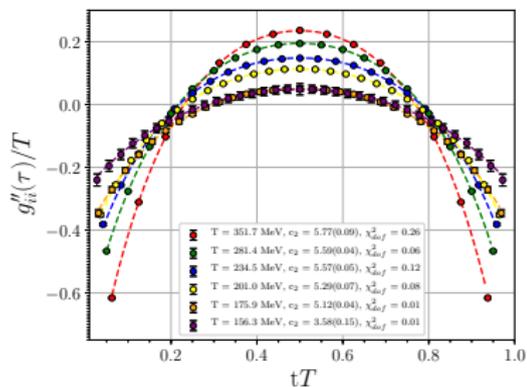
Lattice parameters (Gen2 ensemble)

- $N_f = 2 + 1$ clover-improved Wilson fermions
tree-level improved anisotropic gauge action, mean-field improved Wilson clover fermion action with stout-smear links
- Anisotropic lattice:
 $a_s = 0.1227(8)$ fm, $a_\tau = 0.0350(2)$ fm, $\xi = a_s/a_\tau = 3.5$
- $L_s = 24$, $m_\pi = 384(4)$ MeV, $m_\pi L_s = 5.7$
- L_τ is varied to vary temperature (see the table below)
- Tuning and ensembles at the lowest temperatures have been provided by HadSpec

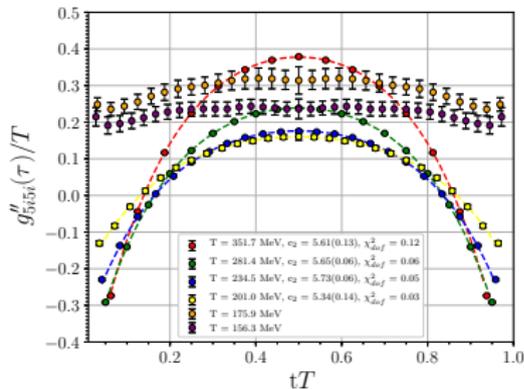
L_τ	128	40	36	32	28	24	20	16
T [MeV]	44	141	156	176	201	235	281	352
T/T_c	0.24	0.76	0.84	0.95	1.09	1.27	1.52	1.90
N_{cfg}	139	501	501	1000	1001	1001	1000	1001

\ddot{G} : connected part only

- For $\gamma_5\gamma_i$ one may see drastic change in behaviour at $T < 180$ MeV



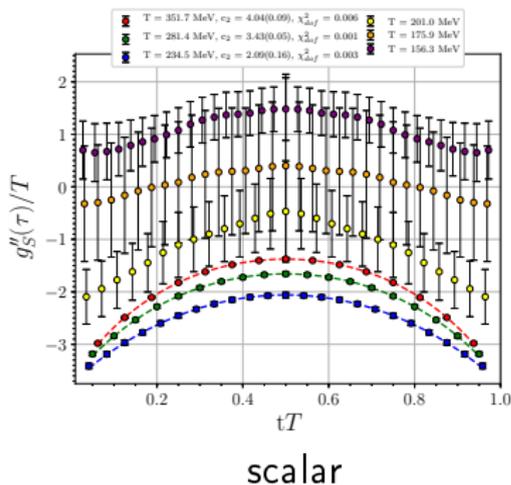
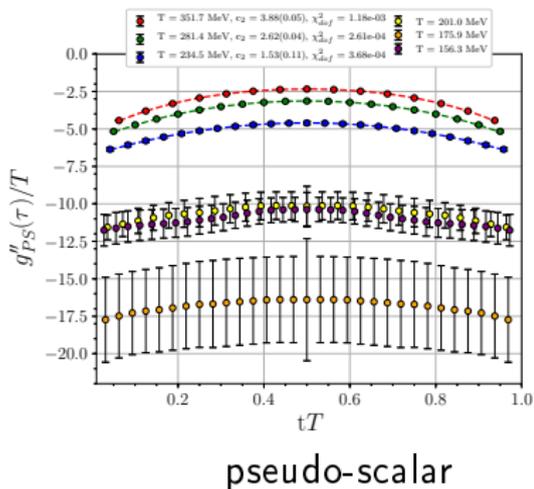
vector



axial-vector

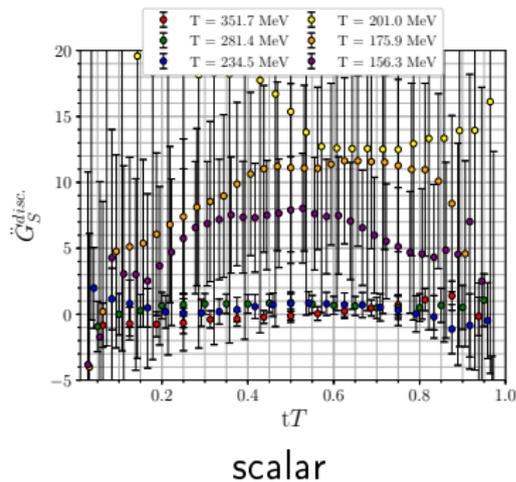
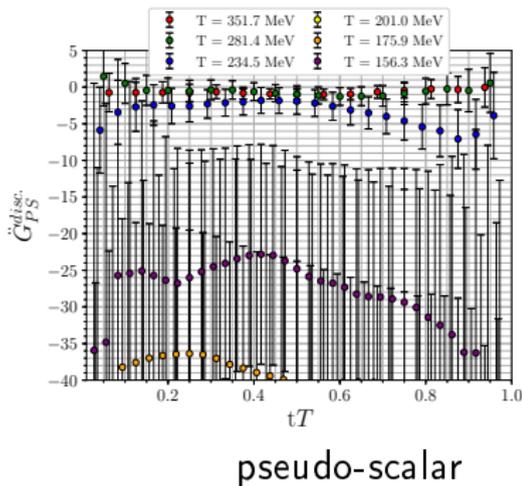
\ddot{G} : connected part only

- For γ_5 and I behaviour changes at $T \propto 200$ MeV



\ddot{G} : disconnected part

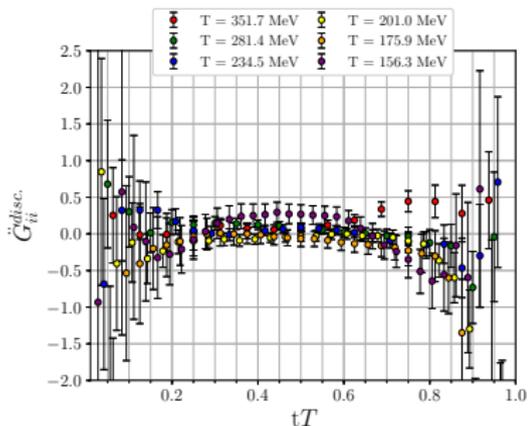
- Disconnected parts vanish in the absence of interactions
- Very noisy despite $N_{st.} = 10000$ Gaussian random vectors



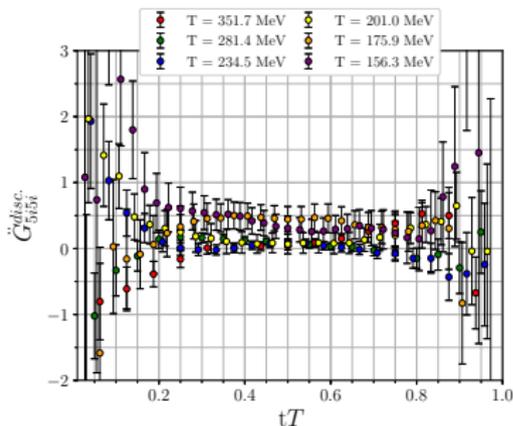
No signal at low T for pseudoscalar and scalar mesons

\ddot{G} : disconnected part

- Disconnected parts vanish in absence of interactions
- Very noisy despite $N_{st.} = 10000$ Gaussian random vectors



vector



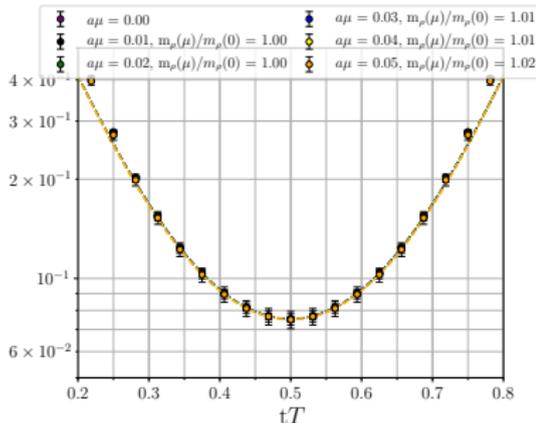
axial-vector

For ρ -meson the signal is the best compared to other channels

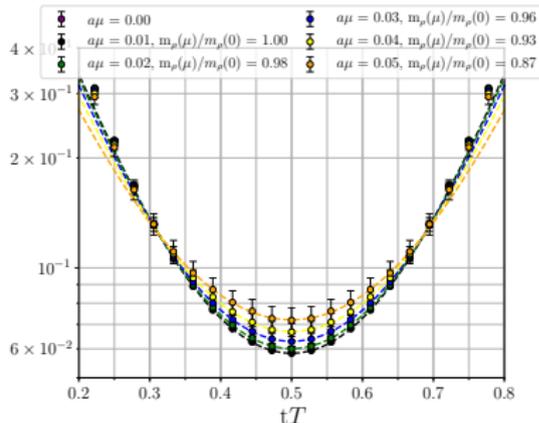
$G(\tau, \mu)$ for ρ -meson

$$\frac{1}{T} G(\tau, \mu) = \frac{1}{T} G(\tau)|_{\mu=0} + \frac{T}{2} \left(\frac{\mu}{T}\right)^2 \ddot{G}_\mu(\tau)|_{\mu=0} \quad \text{is plotted below}$$

- At $T = 156$ MeV μ^2 -correction to the correlator is noticeable
- Mass may be extracted from single cosh fit



$T = 176$ MeV



$T = 156$ MeV

Conclusions

- High temperatures: good agreement with non-interacting theory
- Low temperatures: results are very noisy
- \ddot{G} in some channels seems to be sensitive to confinement-deconfinement transition
- Noise reduction techniques for disconnected \ddot{G} contributions are needed
- Smearred sources for $G + \mu^2 \ddot{G}$ spectroscopy may not be implemented directly because of \ddot{G} , where there is a mix of connected and disconnected terms

Backup slides

Backup slides: \ddot{G} for ρ -meson

- $T < T_c$ is suitable region for spectroscopy
- Unfortunately at $N_t = 40$ (141 MeV) $\ddot{G} \approx 0$ within the errorbars

