Conserved charge fluctuations with smaller-than-physical quark masses

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for

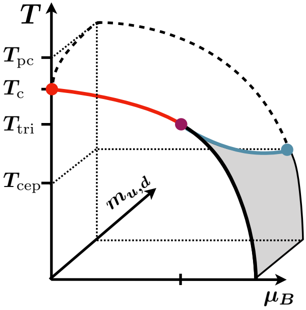
HotQCD Collaboration

UNIVERSITÄT BIELEFELD
Fakultät für Physik

Lattice 19

CRC–TR 211
Strong-interaction matter under extreme conditions

Chiral phase transition in QCD phase diagram

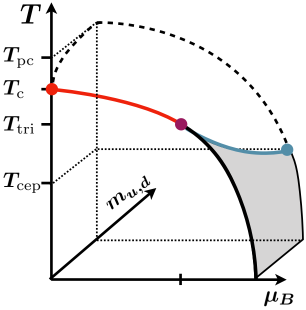


[F. Karsch, arxiv:1905.03936]

The chiral phase transition in QCD in the limit of vanishing up and down quark masses is expected to belong to the universality class of $3d\ O(4)$ spin model. [R.D. Pisarski, F. Wilczek, PRD 29 338 (1984)]

Question : At non-zero quark masses, how much is the thermodynamics near the chiral crossover governed by the $O(4)$ universality class scaling laws?

Chiral phase transition in QCD phase diagram



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Question : At non-zero quark masses, how much is the thermodynamics near the chiral crossover governed by the $O(4)$ universality class scaling laws?

Relevant for experimental measurements of conserved charge fluctuations at RHIC and LHC near vanishing baryon density.

Critical behavior of thermodynamic quantities

Free energy density near the chiral phase transition at T_c and $m_l \equiv m_u = m_d = 0$:

$$f(T, \vec{\mu}, m_l)/T^4 = h^{(2-\alpha)/\beta\delta} f_f(z) + f_r(T, \vec{\mu}, m_l), \quad z \equiv t/h^{1/\beta\delta}$$

where scaling variables are

$$t = \frac{1}{t_0}\left(\frac{T - T_c}{T_c} + \kappa_2^X \left(\frac{\mu_X}{T}\right)^2\right), \quad h = \frac{1}{h_0}\frac{m_l}{m_s}, \quad X = B, Q, S$$

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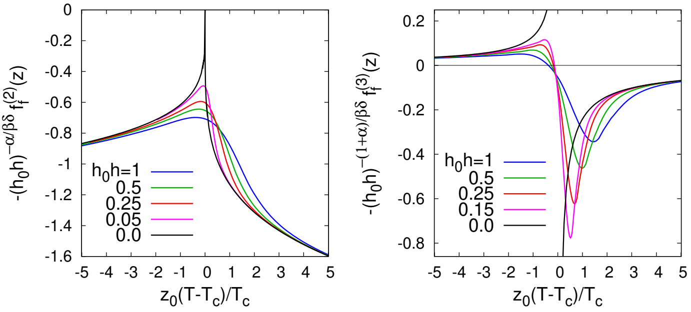
Conserved charge fluctuations at $\boldsymbol{\mu = 0}$ (Singular part) :

$$\chi_{2n}^X = -\left.\frac{\partial^{2n} f/T^4}{\partial(\mu_X/T)^{2n}}\right|_{\mu_X=0} \sim \quad -\left(2\kappa_2^X\right)^n \; h^{(2-\alpha-n)/\beta\delta} f_f^{(n)}(z)$$

Scaling Expectation : $\frac{\partial}{\partial T} \sim \kappa_2^X \frac{\partial^2}{\partial \mu^2} \Rightarrow \chi_2 \sim$ Energy density, $\chi_4 \sim$ Specific heat

Derivatives of the singular part for $3d\ O(4)$ universality class

$h^{0.116}\, f_f^{(2)}$ $\qquad$ $h^{-0.429}\, f_f^{(3)}$



[B. Friman, F. Karsch, K. Redlich,V. Skokov, Eur. Phys. J. C (2011) 71:1694]

$\sim \chi_4$ $\qquad$ $\sim \chi_6$

Lattice Setup

- Gauge ensembles generated with HISQ fermion discretization and Symanzik-improved gauge action, used in chiral T_c determination [HotQCD, arxiv:1905.11610].
- Ensembles for smaller-than-physical quark (up, down) masses $m_l = ms/27, m_s/40, m_s/80, m_s/160$, keeping strange quark mass m_s fixed at physical value.
- Correspond to pion masses : 140 MeV, 110 MeV, 80 MeV, 55 MeV
- Measurement of various traces uses Thick-Restarted Lanczos algorithm for deflation.
- As of now, measurements for lower masses have been done at the largest available volumes.

Scaling expectation for χ_2^Q

- Scaling expectation if singular part dominates : $\frac{\partial}{\partial T} \sim \kappa \frac{\partial^2}{\partial \mu^2}$.

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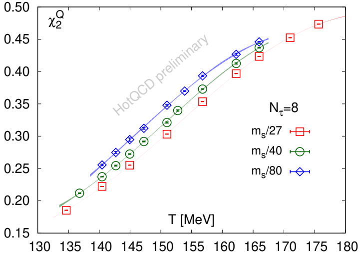

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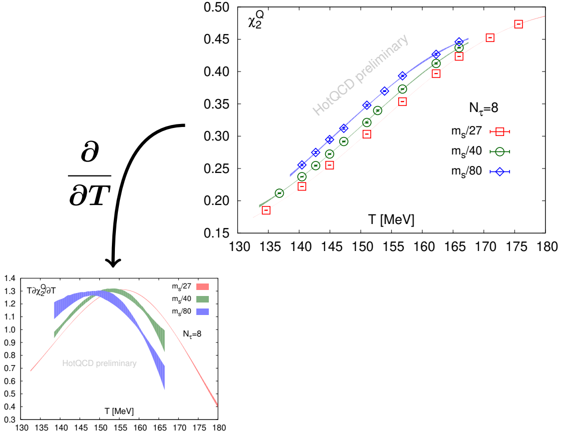

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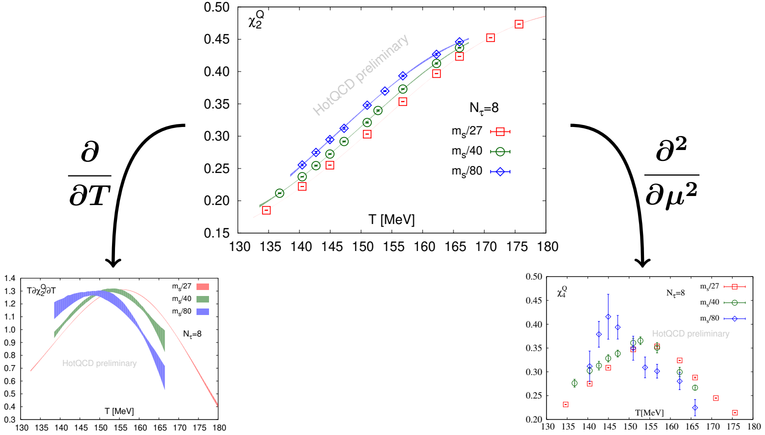

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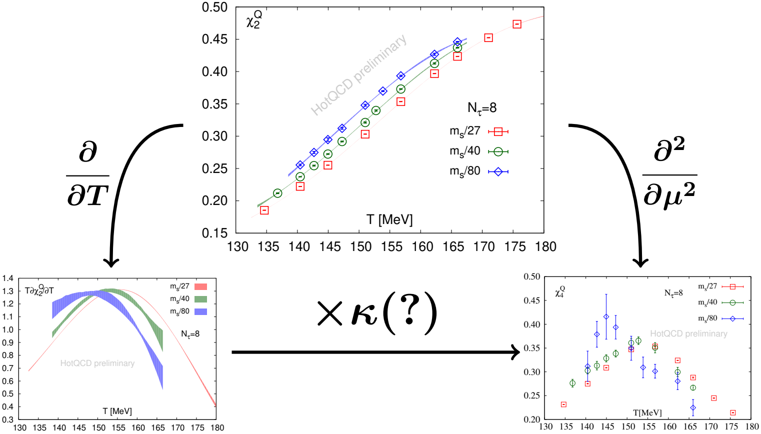

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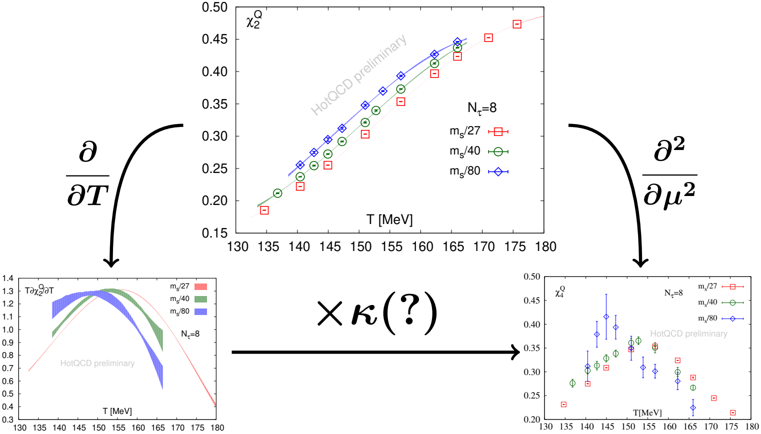

Singular term in χ_2 (energy-like) is not dominant
unlike in order parameter

Estimation of the singular contribution to χ_2^B

$$\chi_2^X(T_c, m_l) \sim -\kappa_2^X h^{(1-\alpha)/\beta\delta} f_f^{(1)}(0) + \text{const. reg. term} + \mathcal{O}(m_l^2)$$

Linear in H plot

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- T_δ is a good approximation for T_c [HotQCD, arxiv:1905.11610]

$$\frac{H\chi_M(T_\delta,H,L)}{M(T_\delta,H,L)} = \frac{1}{\delta}$$

(see talk by Christian Schmidt)

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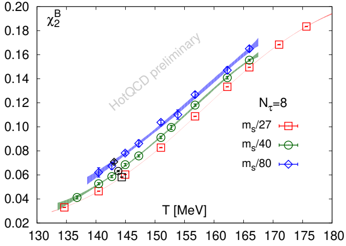
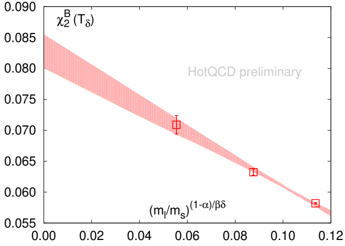


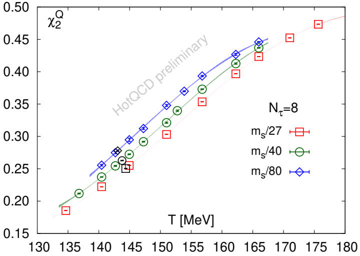


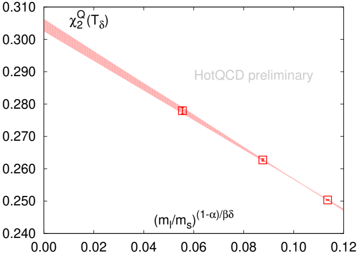
$$\chi_2^X(T_\delta, m_l = 0) - \chi_2^X(T_\delta, m_l = m_s/27) = \text{Singular part of } \chi_2^X$$

▸▸ Linear in H plot

Scaling behavior of χ_2^Q

$$\chi_2^X(T_c, m_l) \sim -\kappa_2^X h^{(1-\alpha)/\beta\delta} f_f^{(1)}(0) + \text{const. reg. term} + \mathcal{O}(m_l^2)$$





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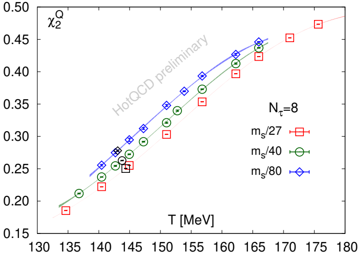
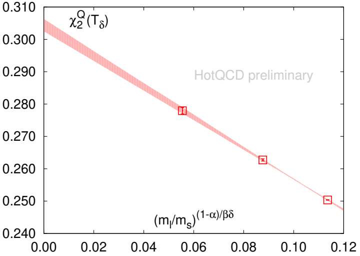




$$\frac{\text{Singular part of } \chi_2^Q}{\text{Singular part of } \chi_2^B} = \frac{\kappa_2^Q}{\kappa_2^B} \sim 2$$

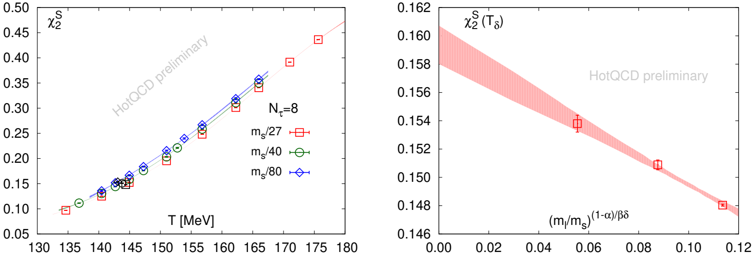
Similar to result for physical mass : 1.81 ± 0.40

[HotQCD, Phys. Lett. B 795 (2019) 15]

Linear in H plot

Scaling behavior of χ_2^S

$$\chi_2^X(T_c, m_l) \sim -\kappa_2^X h^{(1-\alpha)/\beta\delta} f_f^{(1)}(0) + \text{const. reg. term} + \mathcal{O}(m_l^2)$$



Singular contributions in χ_2^B and χ_2^S are nearly same.

Consistent with previous result for physical mass

[HotQCD, Phys. Lett. B 795 (2019) 15]

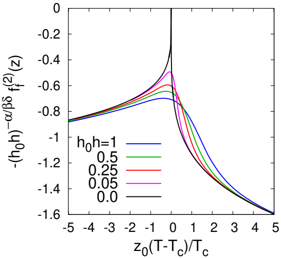
Scaling behavior of χ_4^Q

Singular part :

$$\chi_4^Q \sim h^{-\alpha/\beta\delta} f_f^{(2)}(z)$$

$$O(4) : -\alpha/\beta\delta = 0.116$$

Not divergent



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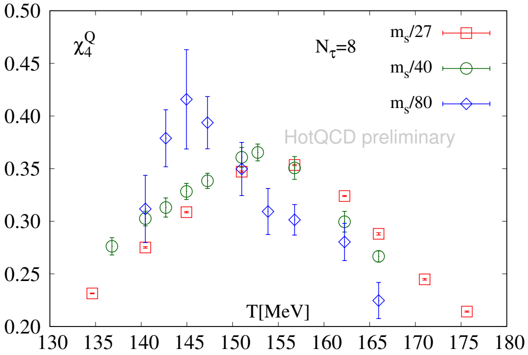

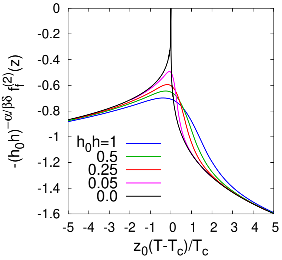

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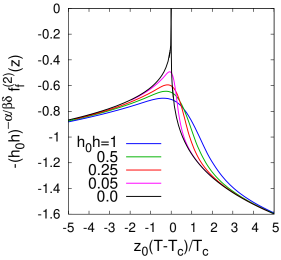
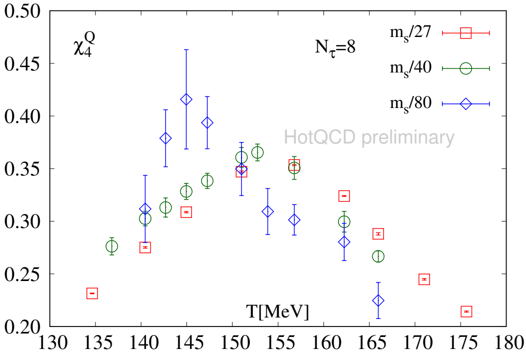


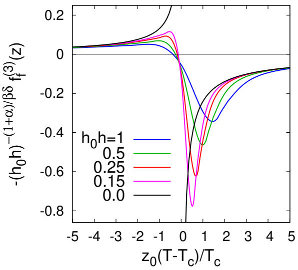


- Expected features are apparent for $m_l = m_s/27, m_s/40$
- $m_s/80$ requires more statistics

Singular part :

$$\chi_6^Q \sim h^{-(1+\alpha)/\beta\delta} f_f^{(3)}(z)$$
$$\sim h^{-0.429}$$

Divergent



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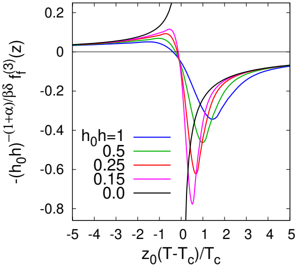

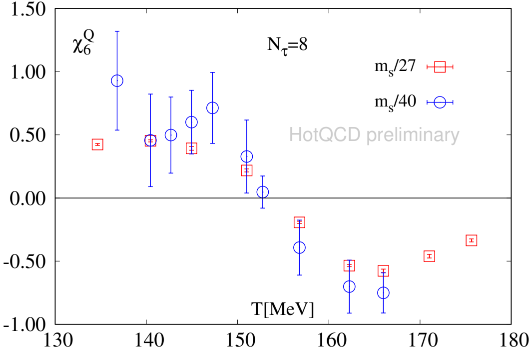

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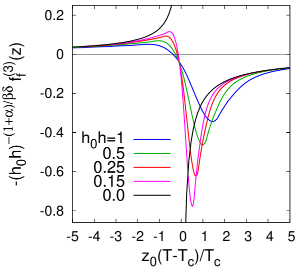

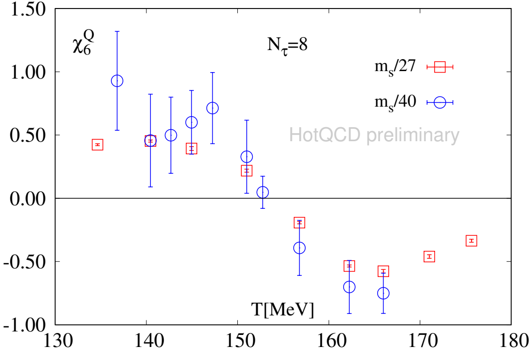

Ratio of peak heights expected from scaling : $(\chi_6^Q)_{1/40}^{max}/(\chi_6^Q)_{1/27}^{max} \sim 1.18$

Conclusions and Outlook

- Fluctuations of conserved charges seem consistent with chiral phase transition belonging to $O(4)$ universality class.
- χ_2^X behaves like energy density w.r.t chiral phase transition.
- Singular part can be extracted from χ_2^X and may be used to determine the curvature coefficients of the chiral critical line.
- Strangeness susceptibility seems to be sensitive to the singular part of the QCD partition function.
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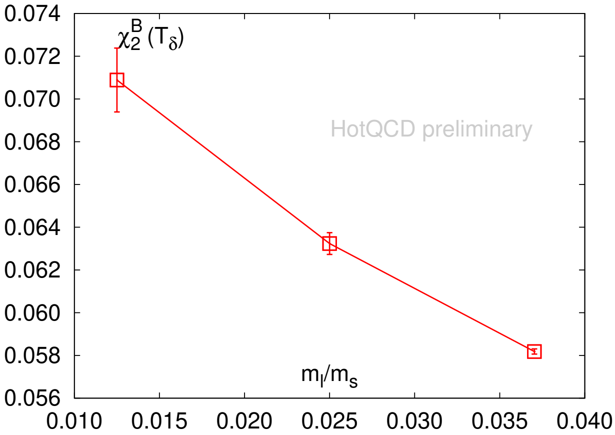

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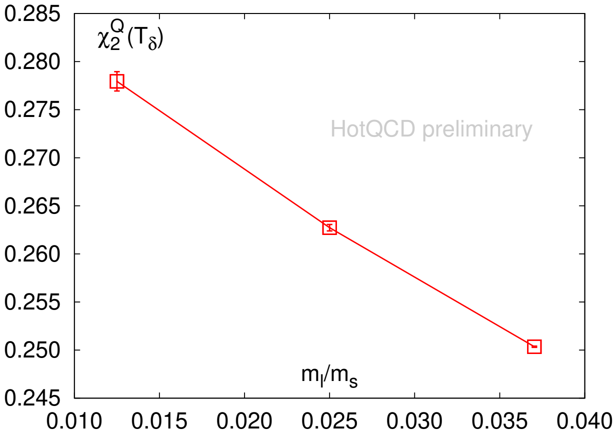

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