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Lattice 2019
Wuhan

Study of 2+1 flavor finite-temperature QCD using improved Wilson quarks at the physical point with the gradient flow

K. Kanaya, A. Baba, S. Ejiri, M. Kitazawa, A. Suzuki,
H. Suzuki, Y. Taniguchi, T. Umeda
(WHOT-QCD Collaboration)

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Tomonaga Center for the History of the Universe

Application of GF to thermodynamics of QCD

Gradient flow

Narayanan-Neuberger (2006), Lüscher (2009-)

Flowed operators are free from UV divergences and short-distance singularities. Lüscher-Weisz (2011)

Suzuki's general method based on GF to correctly calculate any renormalized observables on the lattice: H. Suzuki, PTEP 2013, 083B03 (2013) [E: 2015, 079201]

Define renormalized operators in the continuum, and evaluate their flowed ones on the lattice.
Effects of the flow can be removed by taking $t \to 0$.

Applicable also to observables whose base symmetry is broken on the lattice (Poincaré inv., chiral sym. ...).

⇨ We apply to **QCD with Wilson-type quarks**, to cope with the problems due to the chiral violation.

Our first study: **2+1 flavor QCD with heavy u,d quarks on a fine lattice**

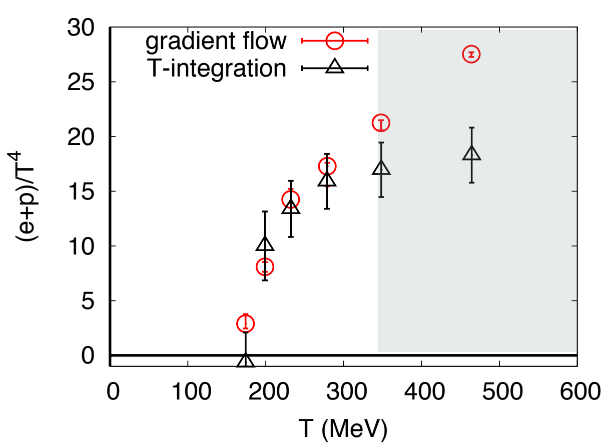
Taniguchi et al.(WHOT-QCD), Phys.Rev. D 95, 054502 (2017); D 96, 014509 (2017)

$m_{PS}/m_V \approx 0.63$
$a \approx 0.07$fm

- RG-improved Iwasaki gauge + NP O(a)-improved Wilson quarks
- CP-PACS+JLQCD's $T = 0$ config. (β=2.05, 28^3x56, a≈0.07fm), heavy u,d and ≈physical s
- $T > 0$ by fixed-scale approach, (32^3xNt, Nt = 4, 6, ... , 14, 16): T≈174--697MeV
- EoS by T-integration method available (WHOT-QCD, PRD85)
- gauge meas. at every config., quark meas. every 10 config's.

The method seems to work well:

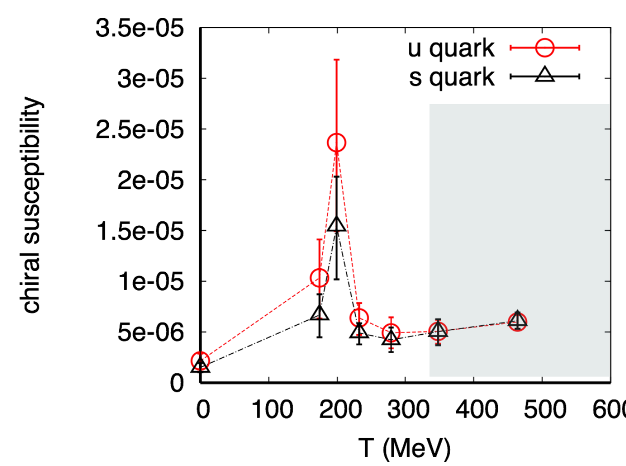
➤ **EoS from En.Mom.Tensor**



☆ EoS by GF agrees well with conventional integral method at $T \lesssim 300$ MeV ($Nt \geq 10$).

☆ Disagreement at $T \gtrsim 350$ MeV may be attributed to O($(aT)^2 = 1/Nt^2$) artifacts at $Nt \leq 8$.

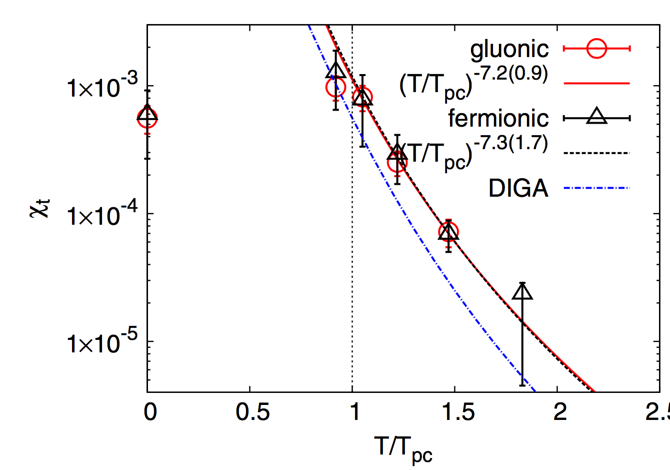
➤ **Chiral susceptibility** (disconnected)



☆ Clear peak at $T_{pc} \approx 190$ MeV, as suggested by Polyakov loop etc.

☆ Peak higher with decreasing m_q.

➤ **Topological susceptibility**



☆ Gluonic and fermionic definitions agree with each other, already at $a \neq 0$ with Wilson-type quarks!

☆ Power-low behavior consistent with DIGA.

this talk

1. An extension of the study to

   2+1 flavor QCD with physical u,d,s quarks.

used in Suzuki's method

2. Test of **2-loop matching coefficients** for EMT
   recently calculated by
   R.V. Harlander, Y. Kluth, F. Lange, EPJC 78:944 (2018)

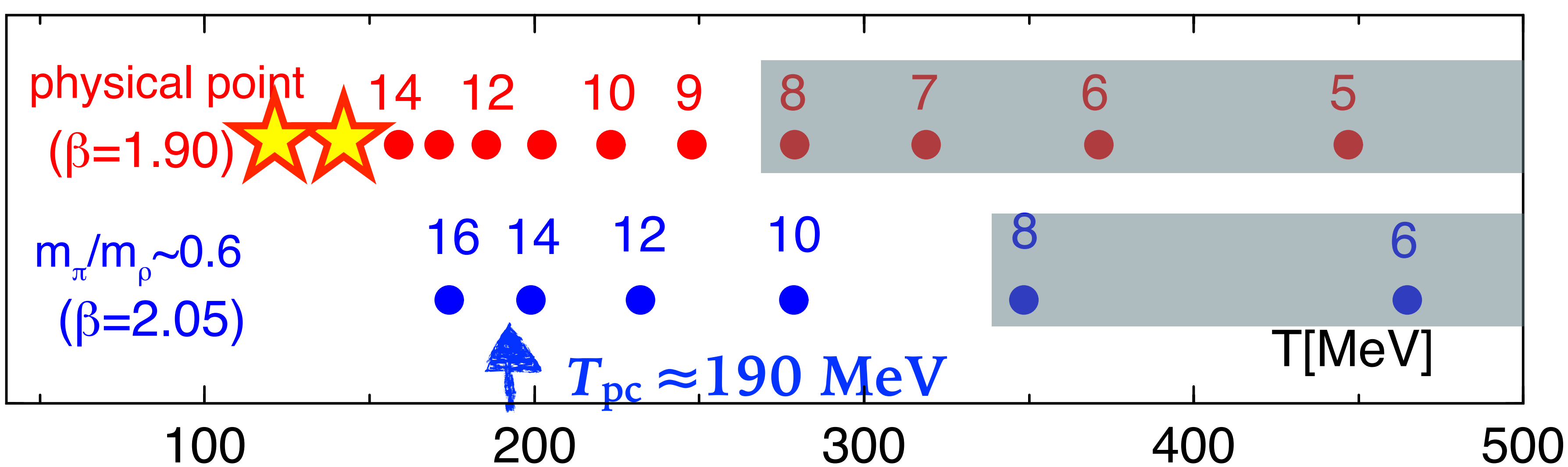
   revisiting 2+1 flavor QCD with heavy u,d quarks.

(2+1)-flavor **phys.pt.** QCD

WHOT-QCD, EPJ Conf. 175, 07023 (2018)

+ New data at $T\approx$122 and 137 MeV (prelim.)

- RG-improved Iwasaki gauge + NP O(a)-improved Wilson quarks
- ***T*=0** configs. of PACS-CS (ß=1.9, 32^3x64, ***a*≈0.09fm**) [Phys.Rev.D79, 034503 (2009)] 80 configs.
  All quark masses fine-tuned to the phys.pt. by reweighting [Phys.Rev.D81, 074503 (2010)] using m_π, m_K, m_Ω inputs.
- ***T*>0** by fixed-scale approach, (32^3xNt, Nt = 4, 5, ... , **16**, **18**): $T\approx$**122, 137** – 549 MeV.
  Odd Nt too, to have a finer T-resolution.
  Generated directly at the phys.pt. w/o reweighting [ß=1.9, Kud=0.13779625, Ks=0.13663377].
- Gauge meas. every 5 tau, quark meas. every 50 tau.



T[MeV]	T/T_{pc}	N_t	$t_{1/2}$	gauge confs.	fermion confs.
0	0	64	32	80	80
122		18	10.125	260	260
129		17	9.03125		
137		16	8	212	212
146		15	7.03125	42	42
157		14	6.125	650	65
169		13	5.28125	550	55
183		12	4.5	610	61
199		11	3.78125	890	89
219		10	3.125	690	69
244		9	2.53125	780	78
274		8	2	680	68
313		7	1.53125	220	22
366		6	1.125	280	280
439		5	0.78125	130	130
548		4	0.5	70	70

on-going

- Where is T_{pc} for physical m_q? Expect T_{pc}^{phys} < 190 MeV.
- Lattice slightly coarser than the heavy QCD case ($a\approx$0.07fm).
- Expect lattice artifacts of O($(aT)^2=1/Nt^2$) at $Nt \le 8$ ($T\ge$274 MeV)

Gauge and Quark Flows

We adopt the simplest flow by Lüscher:

Lüscher, JHEP 1008, 071 (2010); 1304, 123 (2013)

Gauge flow: standard Wilson flow

original gauge field at $t = 0$

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(t=0,x) = A_\mu(x)$$

$$G_{\mu\nu}(t,x) = \partial_\mu B_\nu(t,x) - \partial_\nu B_\mu(t,x) + [B_\mu(t,x), B_\nu(t,x)],$$

$$D_\nu G_{\nu\mu}(t,x) = \partial_\nu G_{\nu\mu}(t,x) + [B_\nu(t,x), G_{\nu\mu}(t,x)],$$

Quark flow: as suggested by Lüscher

original quark field at $t = 0$

$$\partial_t \chi_f(t,x) = \Delta \chi_f(t,x), \qquad \chi_f(t=0,x) = \psi_f(x),$$

$$\partial_t \bar{\chi}_f(t,x) = \bar{\chi}_f(t,x) \overleftarrow{\Delta}, \qquad \bar{\chi}_f(t=0,x) = \bar{\psi}_f(x),$$

$$\Delta \chi_f(t,x) \equiv D_\mu D_\mu \chi_f(t,x), \qquad D_\mu \chi_f(t,x) \equiv [\partial_\mu + B_\mu(t,x)]\, \chi_f(t,x),$$

$$\bar{\chi}_f(t,x) \overleftarrow{\Delta} \equiv \bar{\chi}_f(t,x) \overleftarrow{D}_\mu \overleftarrow{D}_\mu, \qquad \bar{\chi}_f(t,x) \overleftarrow{D}_\mu \equiv \bar{\chi}_f(t,x) \left[\overleftarrow{\partial}_\mu - B_\mu(t,x)\right]$$

only gauge fields involved

Quark field renormalization:

Makino-Suzuki, PTEP 2014, 063B02 (2014)

$$\chi_R(t,x) = Z_\chi \chi_0(t,x) \quad Z_\chi = \sqrt{\varphi(t)} \qquad \varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}_f(t,x) \overleftrightarrow{\not{D}} \chi_f(t,x) \right\rangle_0}.$$

VEV (T=0)

No more renormalizations needed for any composite op's.

Full QCD En.Mom.Tensor by GF

Measure operators which can mix with EMT at $t \neq 0$: Makino-Suzuki, PTEP 2014, 063B02 [E: 2015. 079202]

$$\tilde{O}_{1\mu\nu}(t,x) \equiv G^a_{\mu\rho}(t,x)G^a_{\nu\rho}(t,x),$$
$$\tilde{O}_{2\mu\nu}(t,x) \equiv \delta_{\mu\nu}G^a_{\rho\sigma}(t,x)G^a_{\rho\sigma}(t,x),$$
$$\tilde{O}^f_{3\mu\nu}(t,x) \equiv \varphi_f(t)\bar{\chi}_f(t,x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\chi_f(t,x),$$
$$\tilde{O}^f_{4\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x),$$
$$\tilde{O}^f_{5\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\chi_f(t,x),$$

and combine them

$$T_{\mu\nu}(x) = \lim_{t\to 0}\Big\{c_1(t)\left[\tilde{O}_{1\mu\nu}(t,x) - \frac{1}{4}\tilde{O}_{2\mu\nu}(t,x)\right] + c_2(t)\left[\tilde{O}_{2\mu\nu}(t,x) - \left\langle\tilde{O}_{2\mu\nu}(t,x)\right\rangle_0\right] + c_3(t)\sum_{f=u,d,s}\left[\tilde{O}^f_{3\mu\nu}(t,x) - 2\tilde{O}^f_{4\mu\nu}(t,x) - \left\langle\tilde{O}^f_{3\mu\nu}(t,x) - 2\tilde{O}^f_{4\mu\nu}(t,x)\right\rangle_0\right] + c_4(t)\sum_{f=u,d,s}\left[\tilde{O}^f_{4\mu\nu}(t,x) - \left\langle\tilde{O}^f_{4\mu\nu}(t,x)\right\rangle_0\right] + \sum_{f=u,d,s} c^f_5(t)\left[\tilde{O}^f_{5\mu\nu}(t,x) - \left\langle\tilde{O}^f_{5\mu\nu}(t,x)\right\rangle_0\right]\Big\},$$

Physical EMT extracted by $t \to 0$ extrapolation

$$c_1(t) = \frac{1}{\bar{g}(1/\sqrt{8t})^2} - \frac{1}{(4\pi)^2}\left[9(\gamma - 2\ln 2) + \frac{19}{4}\right],$$
$$c_2(t) = \frac{1}{(4\pi)^2}\frac{33}{16},$$
$$c_3(t) = \frac{1}{4}\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[2 + \frac{4}{3}\ln(432)\right]\right\},$$
$$c_4(t) = \frac{1}{(4\pi)^2}\bar{g}(1/\sqrt{8t})^2,$$
$$c^f_5(t) = -\bar{m}_f(1/\sqrt{8t})\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[4(\gamma - 2\ln 2) + \frac{14}{3} + \frac{4}{3}\ln(432)\right]\right\}$$

with conventional renormalization scale $\mu = \mu_d(t) \equiv \frac{1}{\sqrt{8t}}$

using **matching coefficients** by Makino-Suzuki calculated in **1-loop** PT:

- to make $t \to 0$ smoother by removing known small-t mixings & t-dep. in the continuum
- to match the renormalization schemes when the observable is scheme-dependent

Higher-order errors may affect at small $t \neq 0$.

News: 2-loop coefficients for EMT available now: R.V. Harlander, Y. Kluth, F. Lange, EPJC 78:944 (2018)

We discuss it later.

(2+1)-flavor **phys.pt.** QCD with GF

Preliminary

We adopt the renormalization scale $\mu = \mu_0(t) \equiv \dfrac{1}{\sqrt{2e^{\gamma_E} t}}$ suggested by HKL, in the matching coefficients.

=> wider linear windows => reduction of uncertainties due to the $t \to 0$ extrapolation on coarse lattices

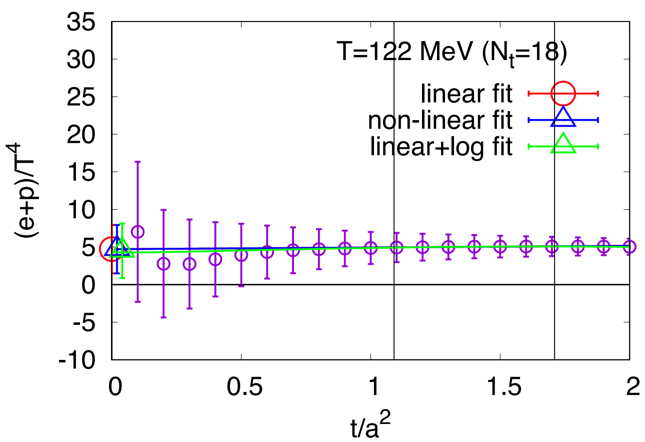
[t with $\mu=\mu_0$] > [t with conventional $\mu=1/\sqrt{(8t)}$]

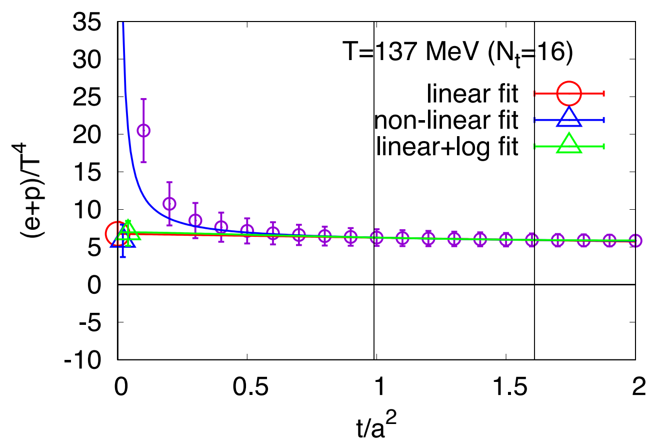
=> μ_0 extends the perturbative region towards larger t

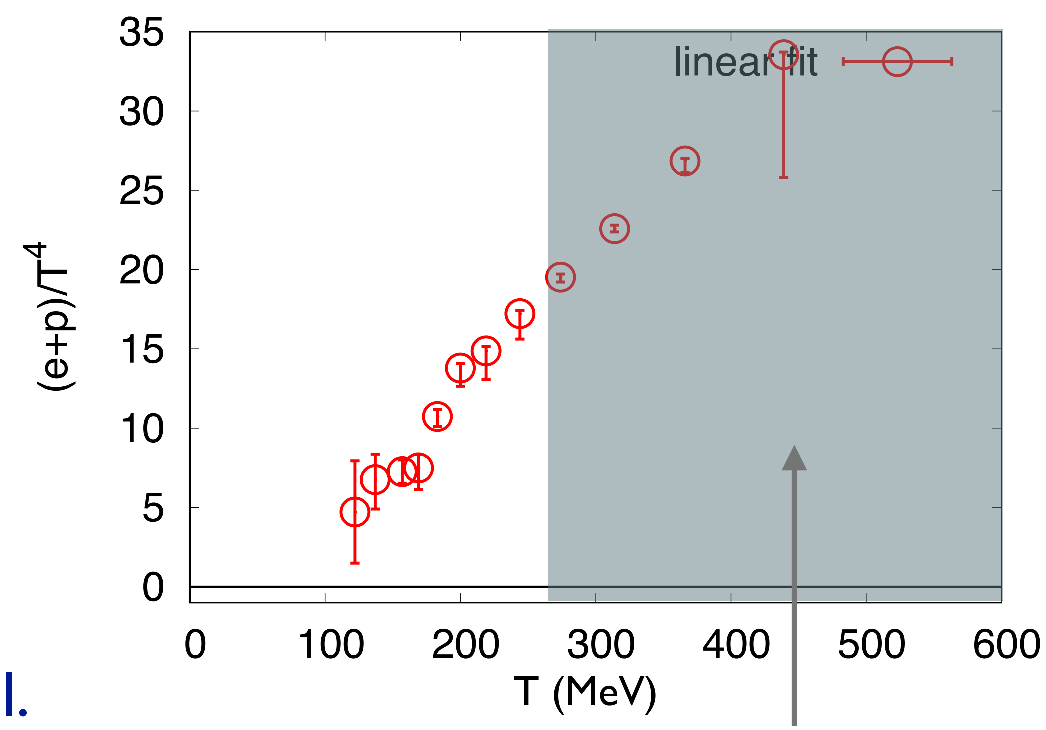
➤ EoS: $(e+p)/T^4$

$$\epsilon = -\langle T_{00} \rangle, \; p = \frac{1}{3} \sum_i \langle T_{ii} \rangle$$

With μ_0



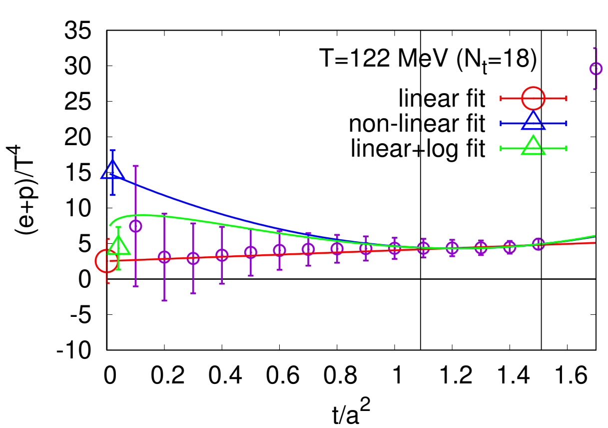


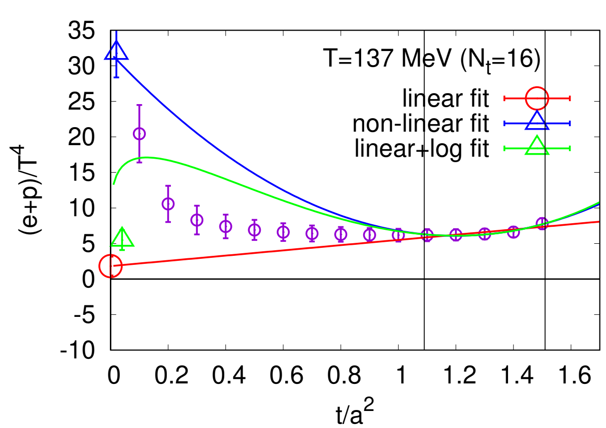


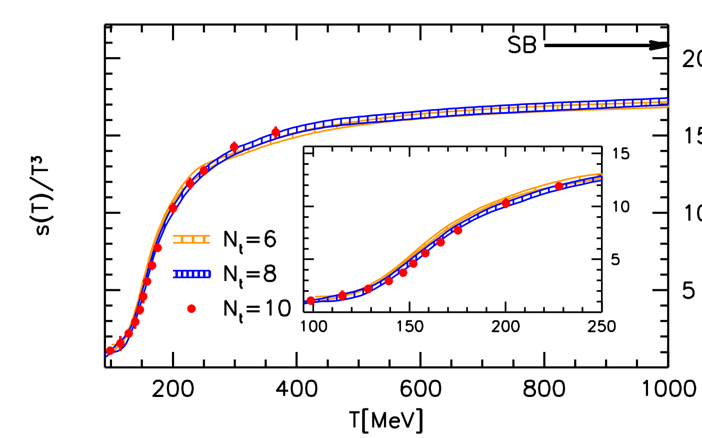
O((aT)^2) lattice artifacts at $Nt \leq 8$

☑ Similar to the heavy QCD case. The method works well.

(cf.) With conventional $\mu=1/\sqrt{(8t)}$, $t/a^2 > 1.5$ out of pert. region at $a \approx 0.09$fm.







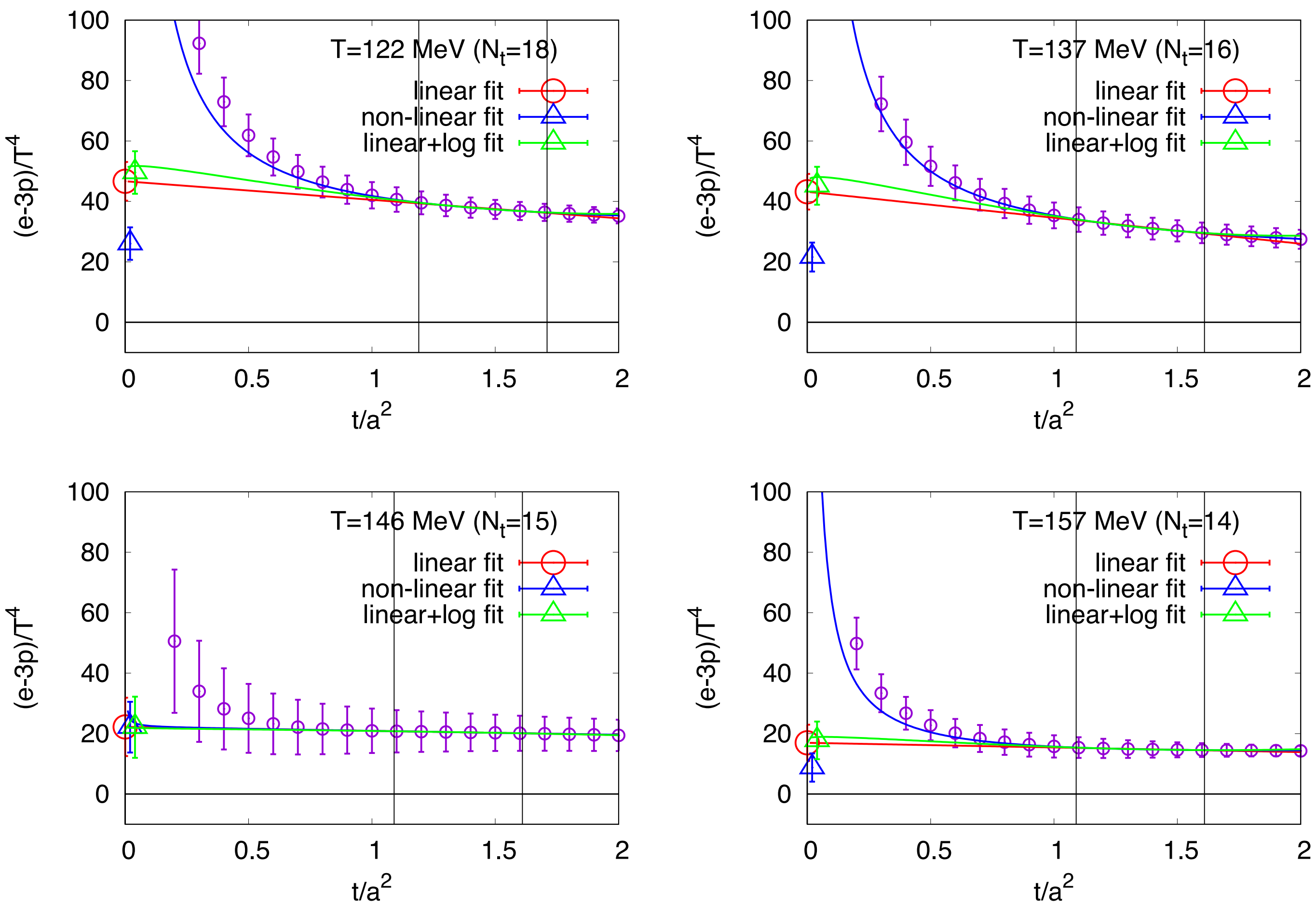
Borsany et al., JHEP 1011, 077 (2010), KS, cont. lim.

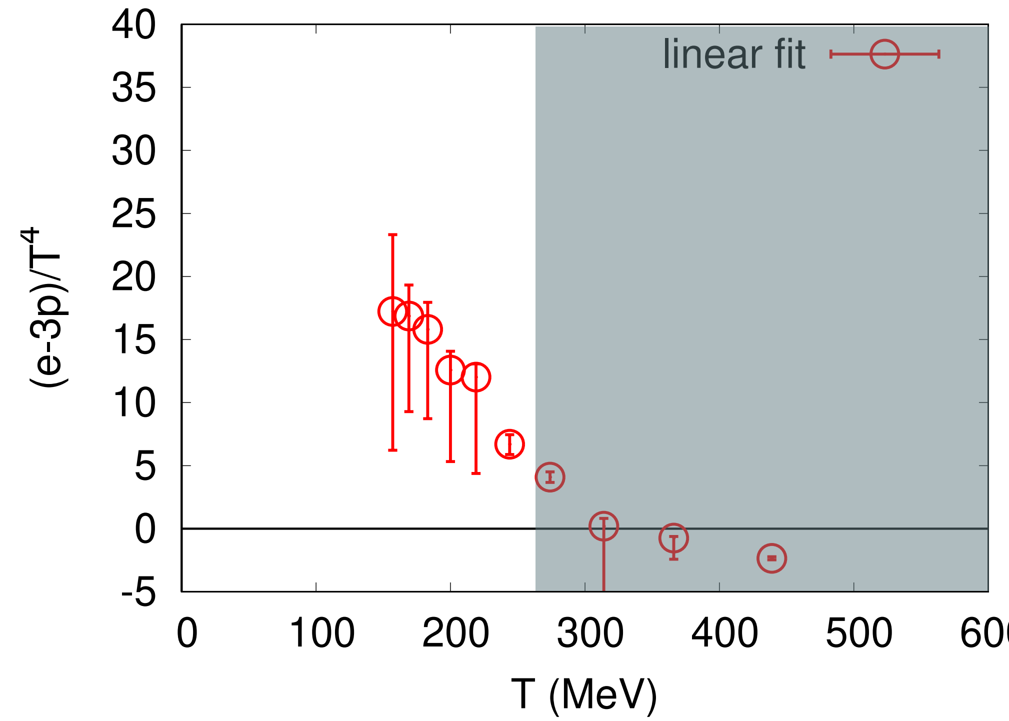
(2+1)-flavor **phys.pt.** QCD with GF

Preliminary

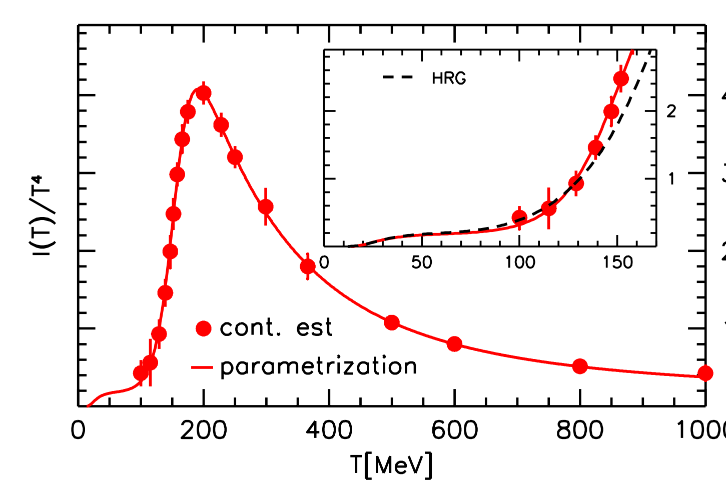
➤ EoS: $(e-3p)/T^4$

$$\epsilon = -\langle T_{00}\rangle,\ p = \frac{1}{3}\sum_i \langle T_{ii}\rangle$$





- ☑ Similar to the heavy QCD case.
  The method seems to work well.
- ☑ $T\approx$122-137MeV in the transition region ??
- ☑ Need more statistics/data at low T 's



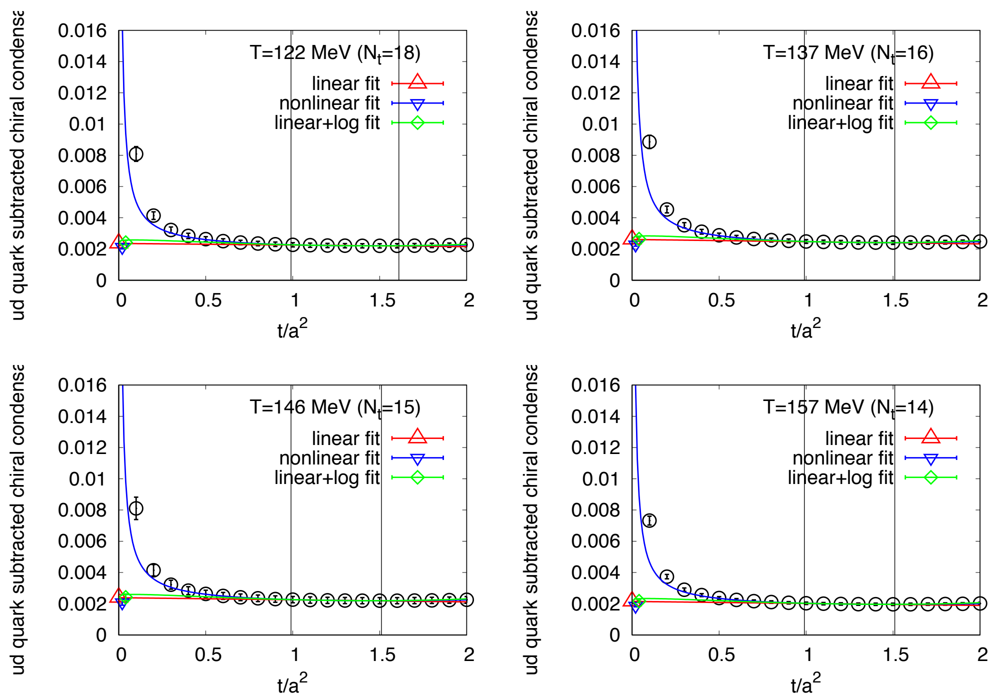
Borsany et al., JHEP 1011, 077 (2010), KS, cont. lim.

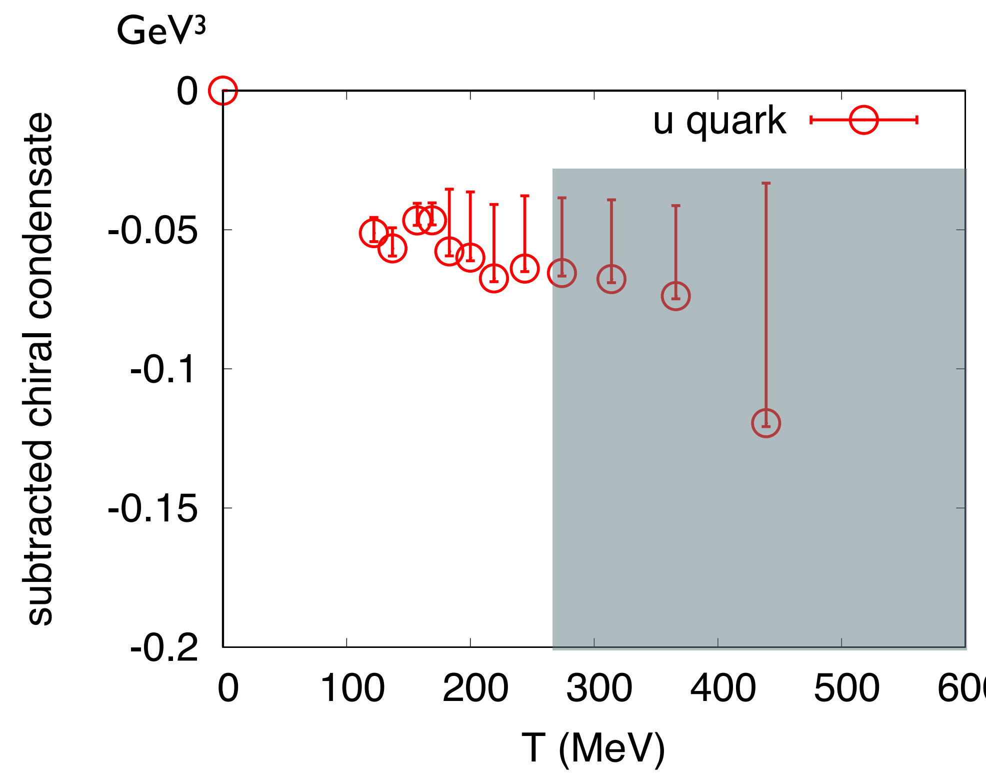
(2+1)-flavor **phys.pt.** QCD with GF

Preliminary

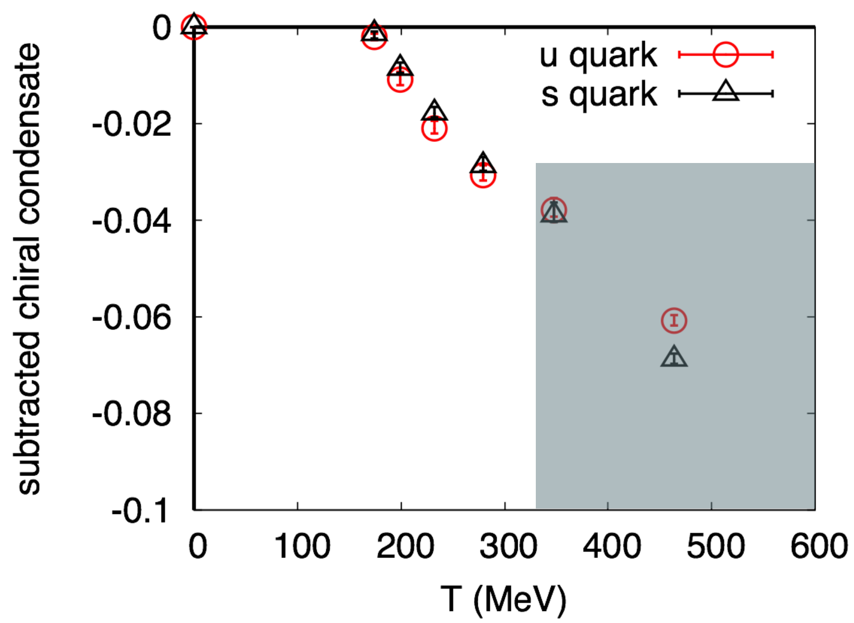
- chiral cond. (VEV-subtracted, μ=2GeV)

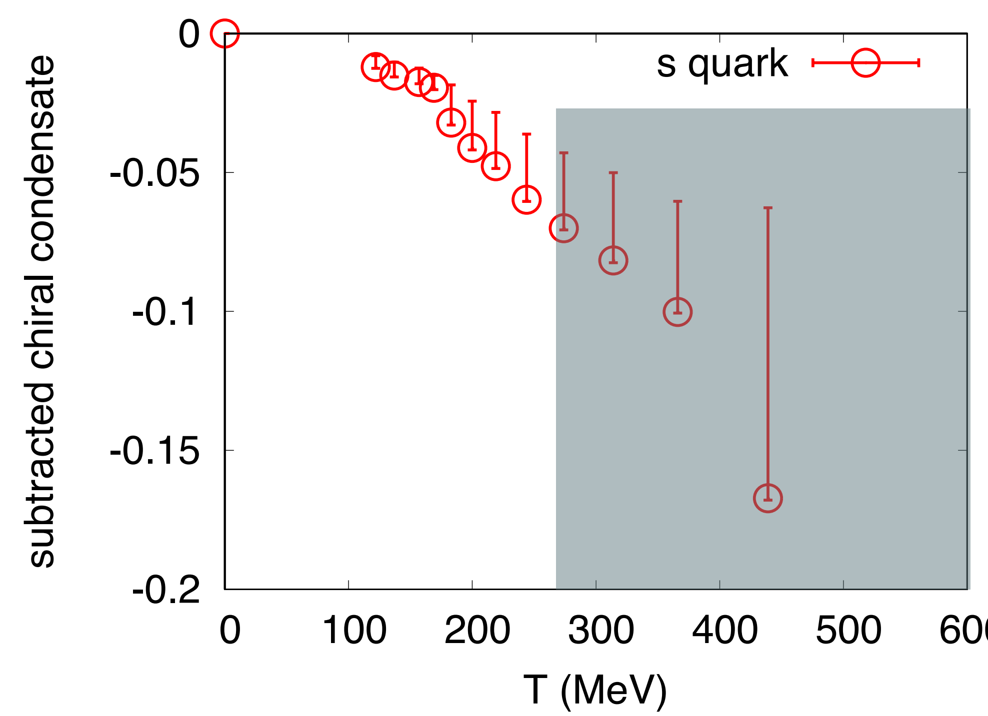
u, d





heavy QCD





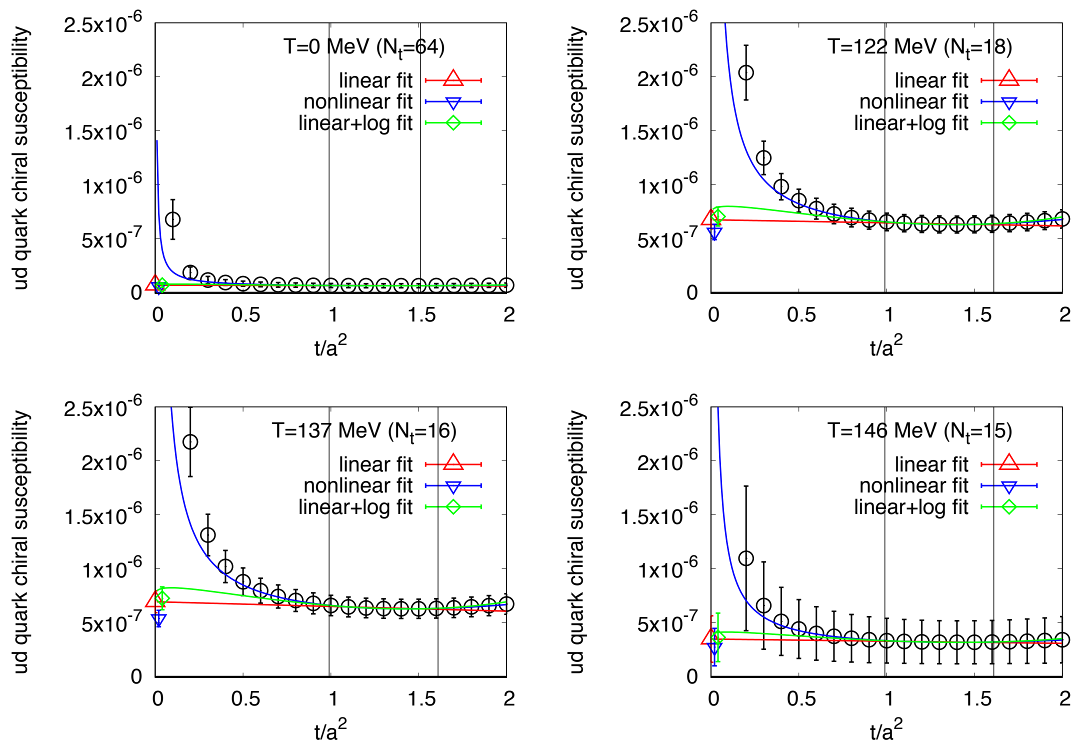
☑ Lighter quarks show a sharper transition/crossover

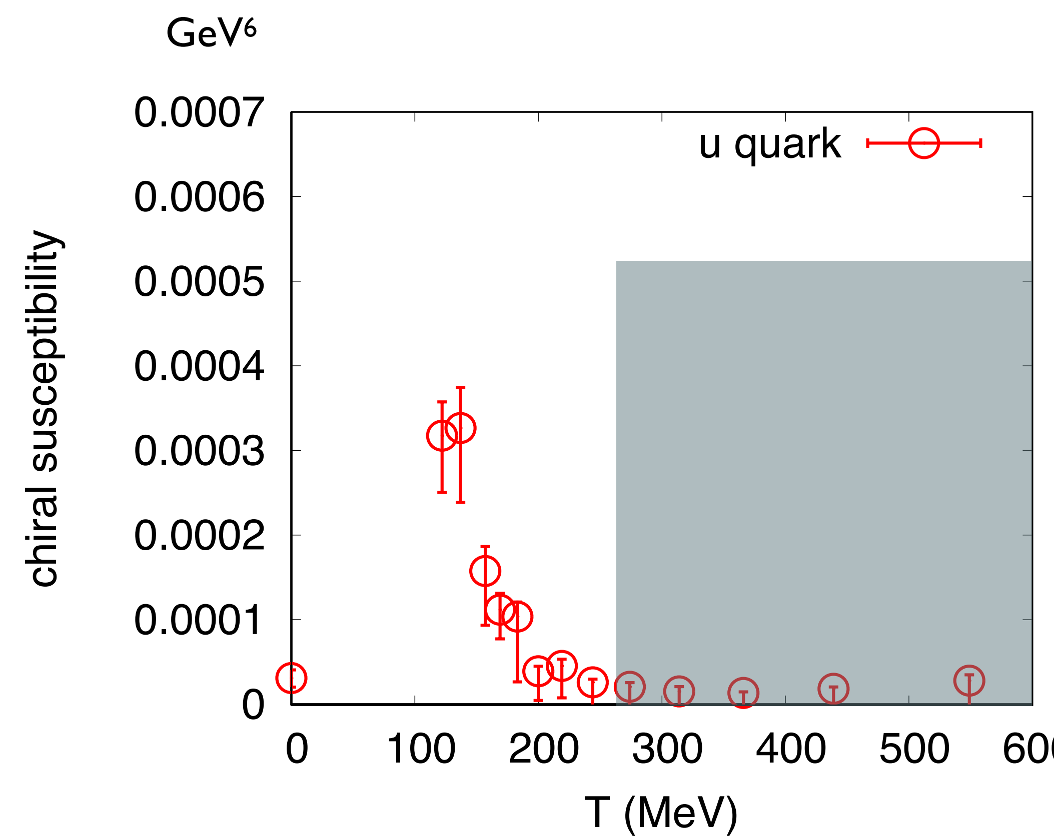
(2+1)-flavor **phys.pt.** QCD with GF

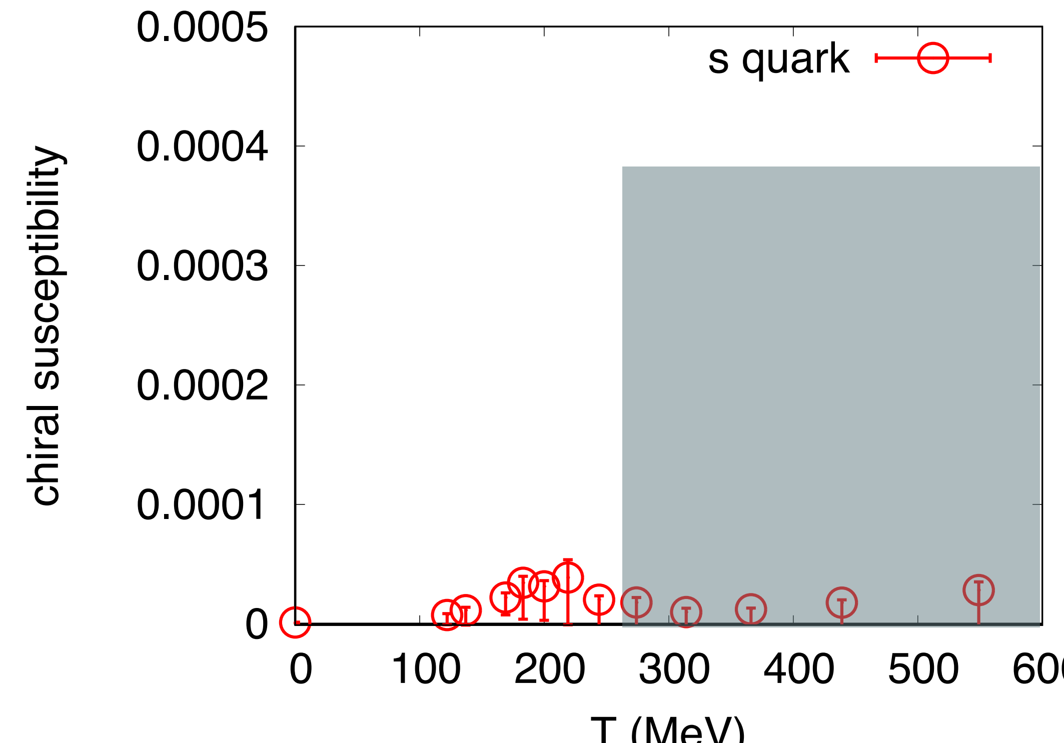
Preliminary

- **chiral suscept.** (disconnected, μ=2GeV)

u, d







- ☑ Similar to the heavy QCD case.
  The method seems to work well.
- ☑ T_{pc}^{phys} < 157 MeV (T≈122-137MeV in the transition region ??)
- ☑ Need more statistics/data at low T 's

this talk

1. An extension of the study to

   2+1 flavor QCD with physical u,d,s quarks.

2. Test of **2-loop matching coefficients** for EMT
   recently calculated by
   R.V. Harlander, Y. Kluth, F. Lange, EPJC 78:944 (2018)

   revisiting 2+1 flavor QCD with heavy u,d quarks.

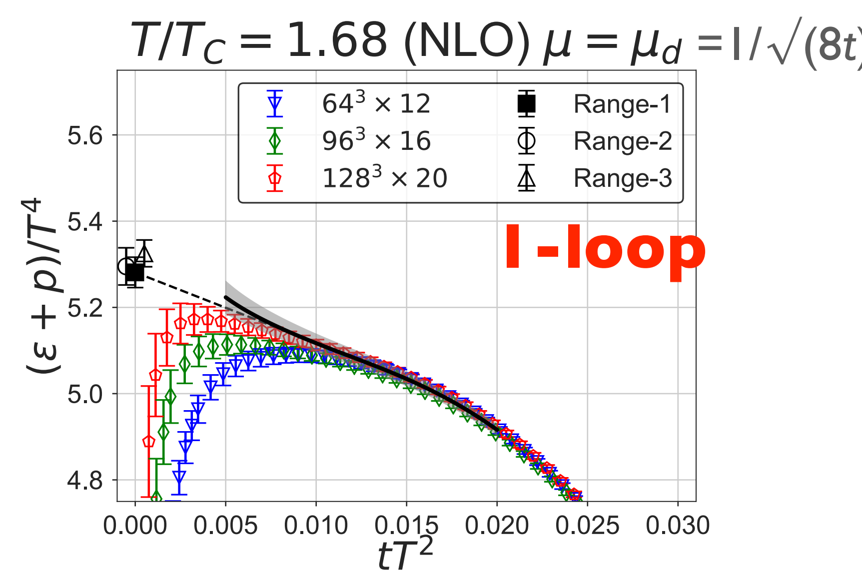
2-loop coefficients for EMT

$$
\begin{aligned}
c_1(t) = \frac{1}{g^2}\Bigg\{1 &+ \frac{g^2}{(4\pi)^2}\left[-\frac{7}{3}C_A + \frac{3}{2}T_F - \beta_0\, L(\mu,t)\right] \\
&+ \frac{g^4}{(4\pi)^4}\Bigg[-\beta_1\, L(\mu,t) + C_A^2\left(-\frac{14482}{405} - \frac{16546}{135}\ln 2 + \frac{1187}{10}\ln 3\right) + C_A T_F\left(\frac{59}{9}\mathrm{Li}_2\left(\frac{1}{4}\right) + \frac{10873}{810} + \frac{73}{54}\pi^2\right. \\
&\left. - \frac{2773}{135}\ln 2 + \frac{302}{45}\ln 3\right) + C_F T_F \left(-\frac{256}{9}\mathrm{Li}_2\left(\frac{1}{4}\right) + \frac{2587}{108} - \frac{7}{9}\pi^2 - \frac{106}{9}\ln 2 - \frac{161}{18}\ln 3\right)\Bigg] + \mathcal{O}(g^6)\Bigg\},
\end{aligned}
$$

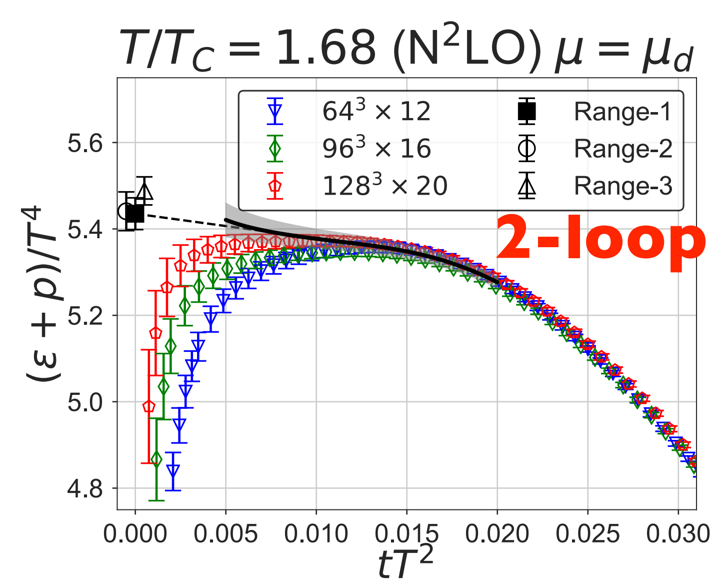
Harlander-Kluth-Lange, EPJC 78:944 (2018)

etc. with $L(\mu,t) \equiv \ln\left(2\mu^2 t\right) + \gamma_{\mathrm{E}}$

Removing known small-t properties further, we may expect a milder t-dep. at small t.



(b)



- first test in quenched QCD

Iritani-Kitazawa-Suzuki-Takaura, PTEP 2019, 023B02 (2019) [arXiv:1812.06444]

- ☑ Results of EoS with 1- and 2-loop coefficients are consistent with each other.
- ☑ With 2-loop coefficients, t-dep. is milder.
- ☑ Thus, 2-loop coefficients reduce systematic errors from the $t \to 0$ extrapolation.

(2+1)-flavor QCD with 2-loop coefficients

➤ coefficients for full QCD EMT

Harlander-Kluth-Lange, EPJC 78:944 (2018)

Harlander *et al.* used the **equation of motion (EoM) in the continuum**

$$0 = \mathcal{O}_{4,\mu\nu}(x) + 2\,\mathcal{O}_{5,\mu\nu}(x)$$

$$\mathcal{O}_{4_f,\mu\nu}(x) \equiv \delta_{\mu\nu}\bar{\psi}_f(x)\overleftrightarrow{\not{D}}\psi_f(x),$$
$$\mathcal{O}_{5_f,\mu\nu}(x) \equiv \delta_{\mu\nu}m_{f,0}\bar{\psi}_f(x)\psi_f(x)$$

to reduce the number of independent operators/coefficients,
assuming that the EMT operators are isolated.

This should be OK when we take the continuum limit.

However, **EoM gets corrections** at $a \neq 0$ on the lattice.
=> May introduce another source of errors.

(Note 1) EoM not used in the quenched coefficients.

(Note 2) EoM affects the trace-part of EMT only.

(2+1)-flavor **heavy** QCD with **2-loop** coefficients

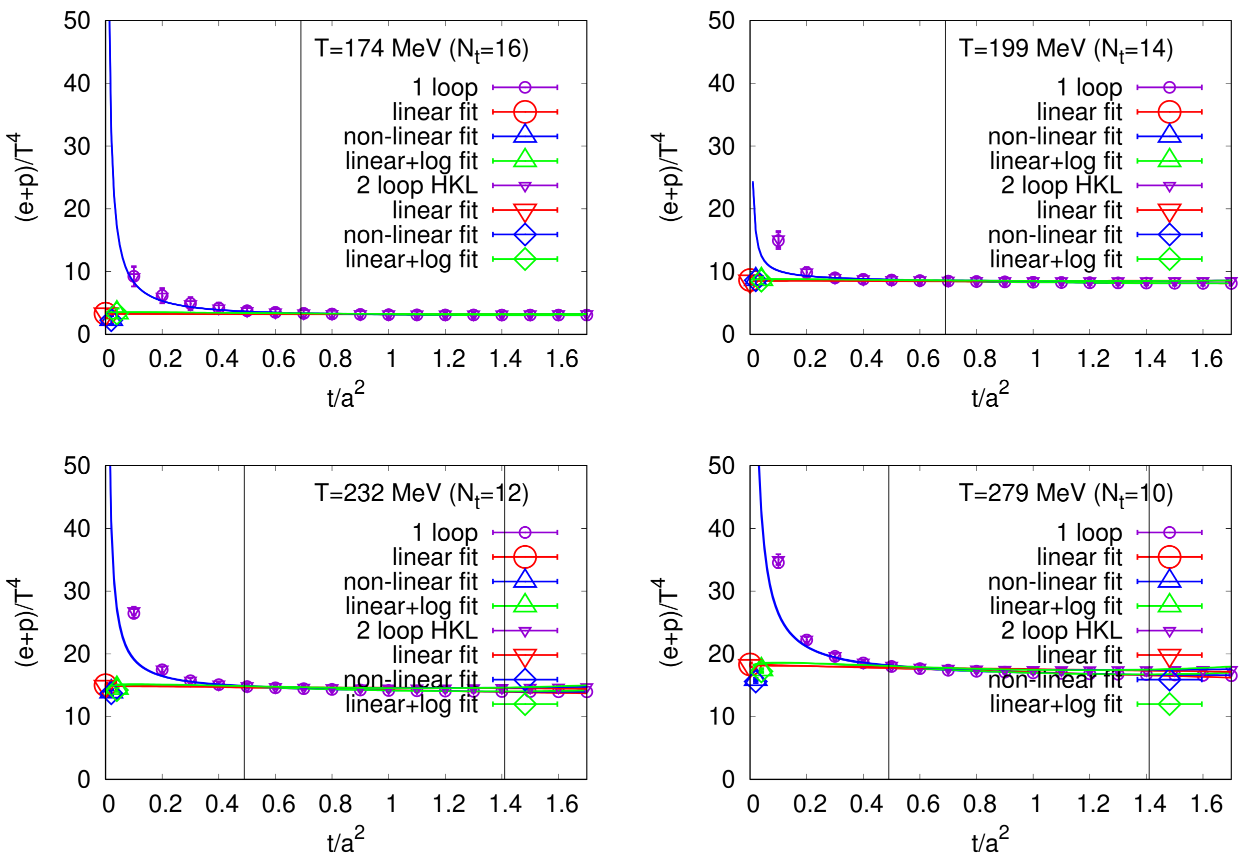
To test the effects of 2-loop coefficients in full QCD,
we have revisited the case of QCD with heavy u,d quarks.

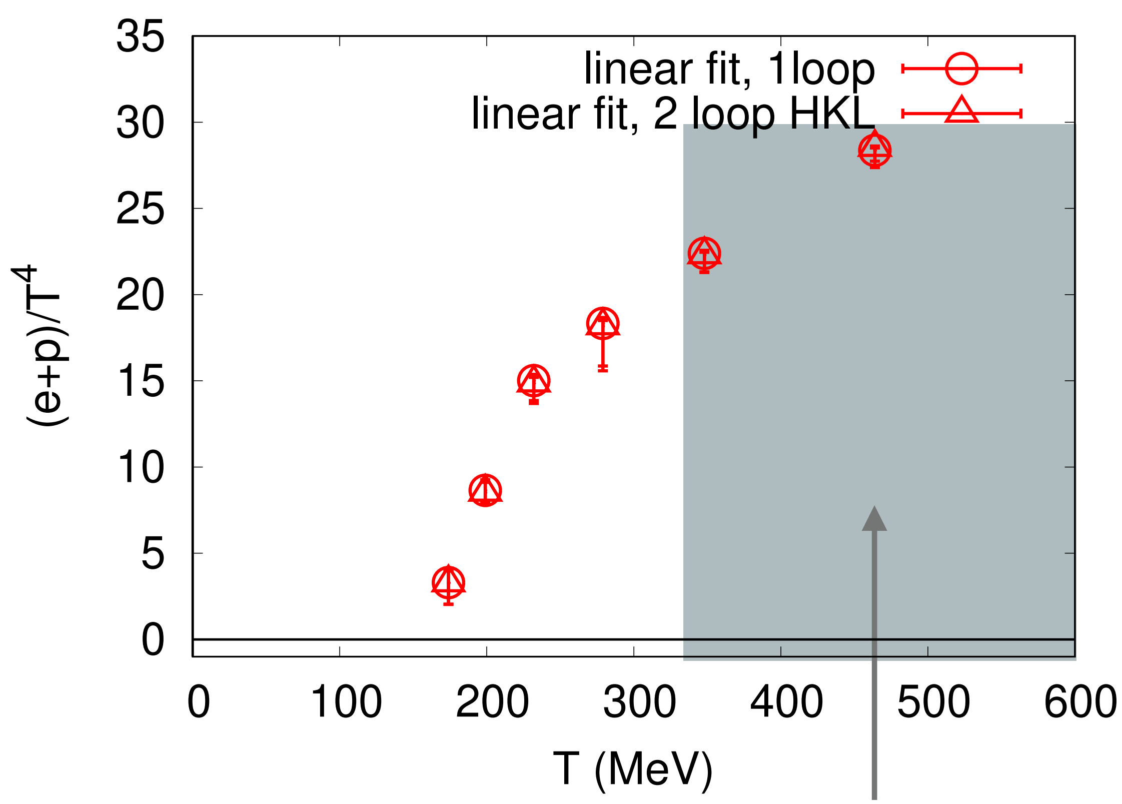
- $m_{PS}/m_V \approx 0.63$
- $a \approx 0.07$fm

Preliminary

We adopt μ_0 as the ren. scale. (The difference from $1/\sqrt{(8t)}$ was small in EoS on this fine lattice.)

➤ $(e+p)/T^4$ **in which EoM not used. <= trace-less combination of EMT**





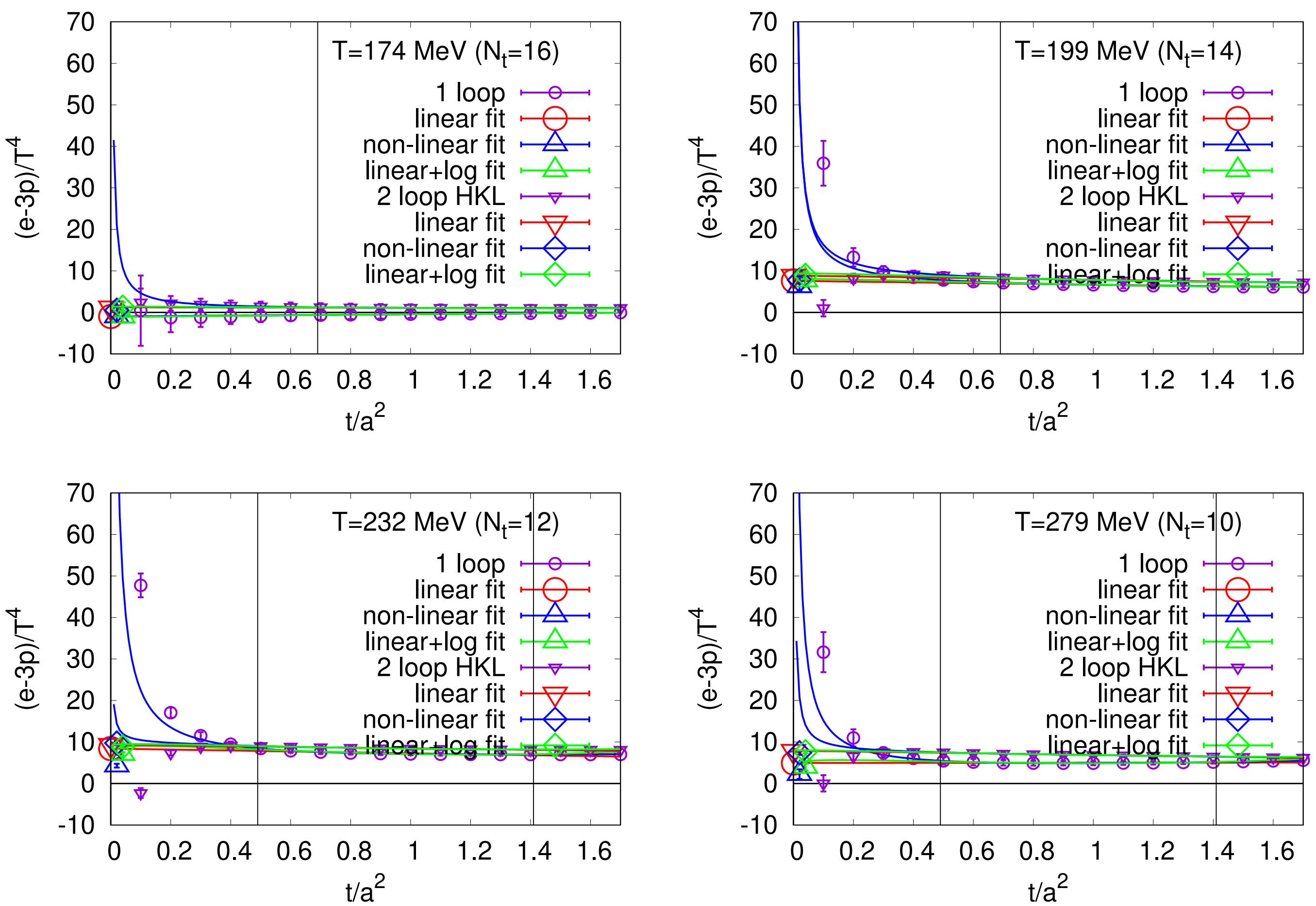
O($(aT)^2$) lattice artifacts at $Nt \leq 8$

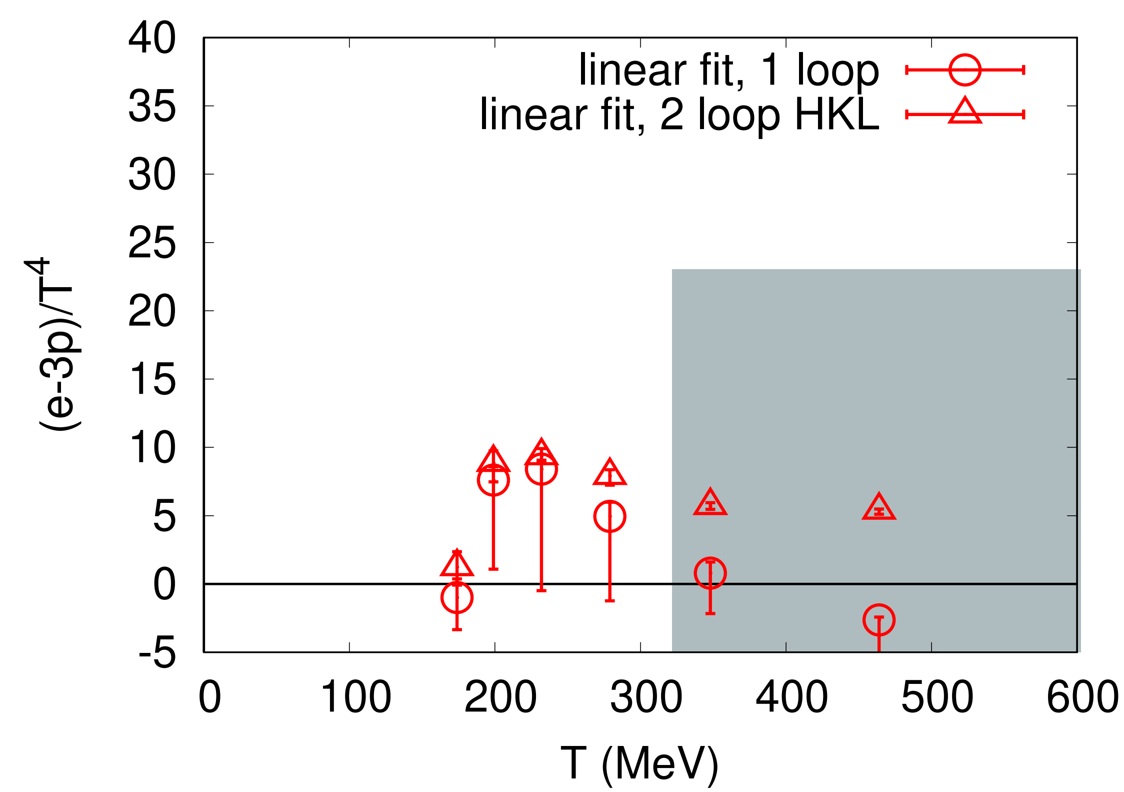
- ☑ **1-loop and 2-loop results are completely consistent with each other.**
- ☑ 1-loop results sufficiently flat in t to extract the $t \to 0$ limit on this fine lattice for this operator..
  => no apparent improvements with 2-loop coeff's. in this case.

(2+1)-flavor **heavy** QCD with **2-loop** coefficients

Preliminary

➤ $(e\text{-}3p)/T^4$ in which EoM is used in the 2-loop HKL coefficients.





☑ **1-loop (w/o EoM) and 2-loop (w/ EoM) results are well consistent with each other.**

☑ 1-loop results sufficiently flat in *t*

=> no apparent improvements with 2-loop coeff's. in this case.

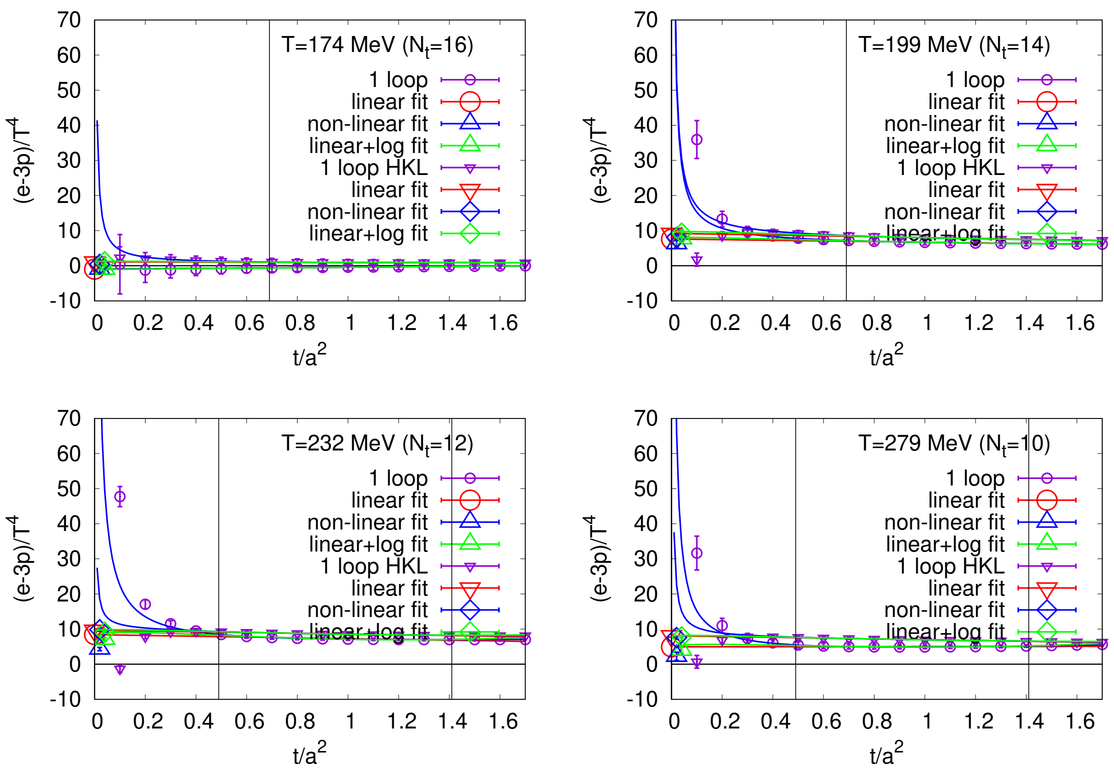
(2+1)-flavor **heavy** QCD with ~~2-loop~~ 1-loop coefficients

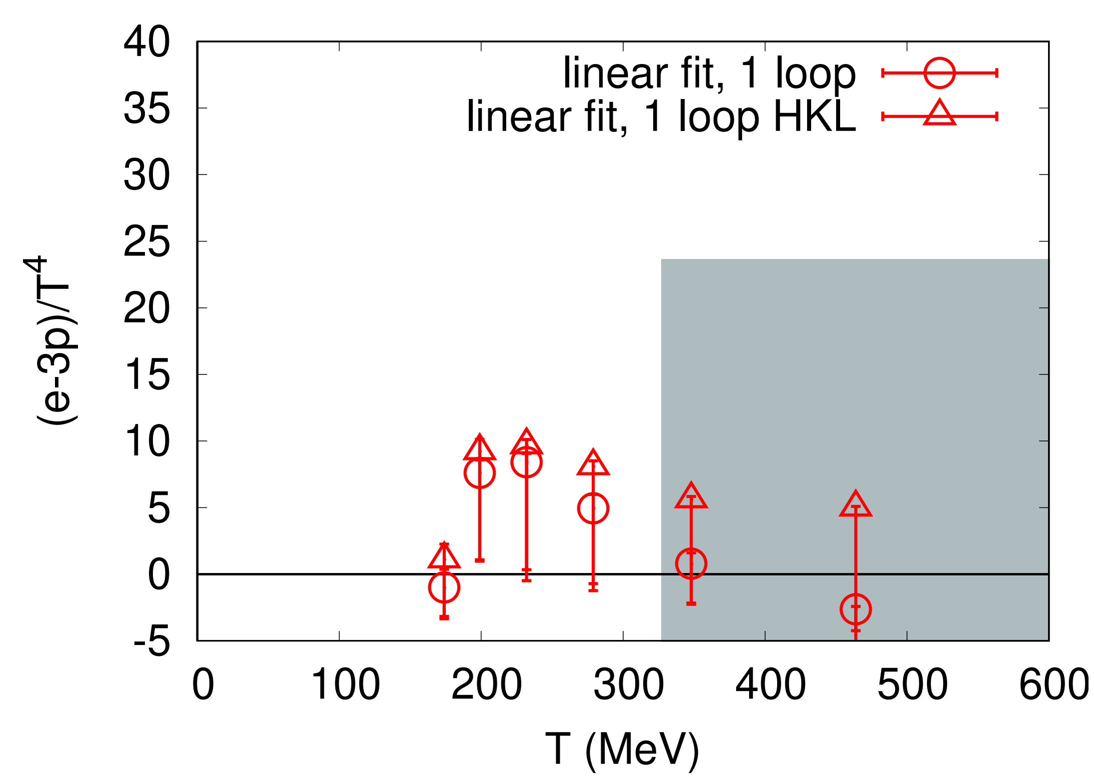
Preliminary

➤ $(e\text{-}3p)/T^4$ in which EoM is used in the HKL coefficients.

To identify the effects of EoM, we compare

- 1-loop Makino-Suzuki coefficients w/o EoM — Makino-Suzuki, PTEP 2014, 063B02 [E: 2015. 079202]
- 1-loop HKL coefficients w/ EoM — Harlander-Kluth-Lange, EPJC 78:944 (2018)





☑ They are consistent with each other.

=> Effect of EoM looks small in EoS.

Summary

2+1 flavor phys. pt. QCD thermodynamics with GF

- slightly coarser lattice ($a \approx 0.09$fm), $32^3 \times Nt$ (Nt=4,5,...,16, 18): $T \approx$ 122, 237-549MeV

☑ **Similar to the heavy QCD case. The method seems to work well here too.**

☑ **Choosing μ_0 as the renormalization scale helps on coarse lattices.**

☑ **T_{pc}^{phys} < 157 MeV** <= Need more statistics/data at low-T 's.

Simulations at Nt=15, 17, 20 are coming.

2+1 flv. heavy QCD revisited with 2-loop coefficients by HKL

☑ **Works well** for EoS.

But, unlike the qQCD case, no particular benefits so far for EoS.

<= t-dep. well flat in our case.

They may help on coarse lattices / other observables.

☑ **Effect of EoM looks small in EoS.**

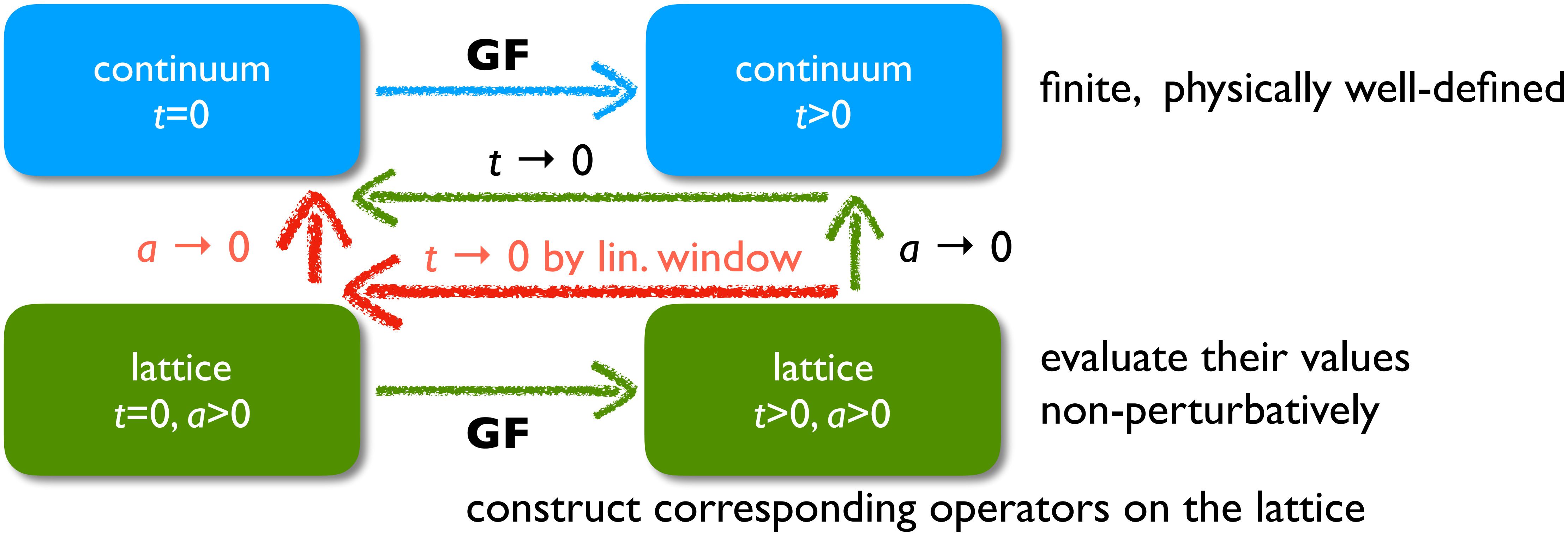
We are applying 2-loop coefficients to chiral cond./suscept. and also to the phys. pt.

BACKUP

Suzuki's method

Hiroshi Suzuki, PTEP 2013, 083B03 (2013) [E: 2015, 079201]
Makino-Suzuki, PTEP 2014, 063B02 (2014) [E: 2015. 079202]
Hieda-Suzuki, Mod.Phys.Lett.A31, 1650214 (2016)

General method to correctly calculate any renormalized observables making use of the finiteness of GF



Finiteness of GF
➡ Can evaluate flowed operators non-perturbatively on the lattice.

Define renormalized operators in the continuum, and evaluate their flowed ones on the lattice in the cont. lim.. Effects of the flow can be removed by taking $t \to 0$.

Applicable to any observables related to symmetries violated on the lattice (translational inv., rotational inv., chiral sym. etc.).

The order of $a \to 0$ and $t \to 0$ may be interchanged when linear window is identified.

Taniguchi et al.(WHOT-QCD), Phys.Rev. D 96, 014509 (2017)

Full QCD EMT by GF

Taniguchi et al.(WHOT-QCD), Phys.Rev. D 96, 014509 (2017)

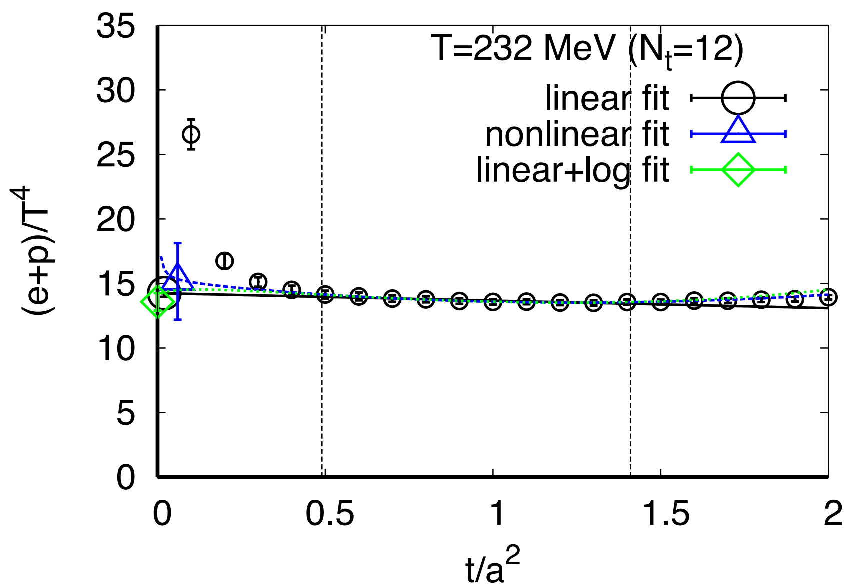
At $a \neq 0$, additional mixing with unwanted operators:

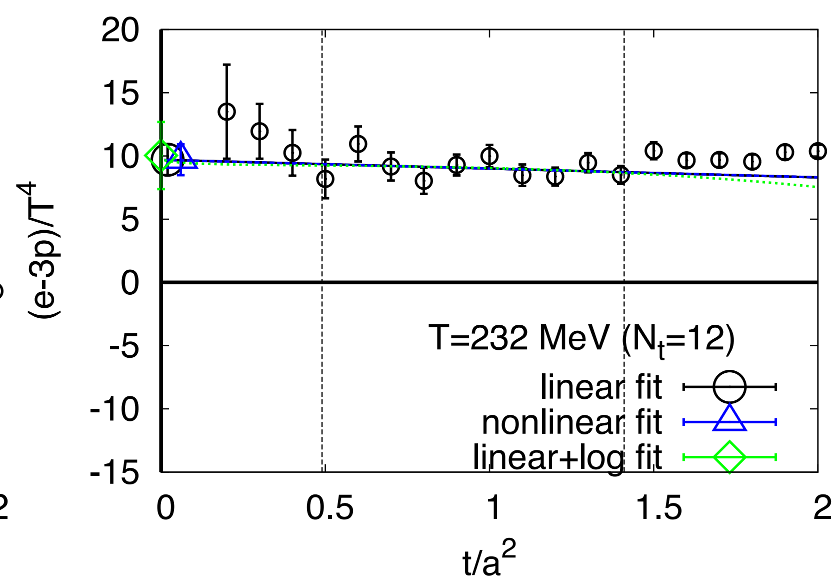
$$T_{\mu\nu}(t,x,a) = T_{\mu\nu}(t,x) + A_{\mu\nu}\frac{a^2}{t} + \sum_f B_{f\mu\nu}(am_f)^2 + C_{\mu\nu}(aT)^2 + D_{\mu\nu}(a\Lambda_{\rm QCD})^2 + a^2 S'_{\mu\nu}(x) + \mathcal{O}(a^4),$$

Note: lattice artifacts of NP-clover is $O(a^2)$.

Singular term at $t \approx 0$ due to mixing with D=4 ops.
=> should be handled properly in the $t \to 0$ extrapolation.

We **remove the contamination of singular terms by identifying "linear windows"** in which the linear term looks dominating.





Taniguchi et al., PRD96, 014509 ('17)

$$\langle T_{\mu\nu}(t,a)\rangle = \langle T_{\mu\nu}\rangle + t\,S_{\mu\nu}$$ linear fit

$$\langle T_{\mu\nu}(t,a)\rangle = \langle T_{\mu\nu}\rangle + A_{\mu\nu}\frac{a^2}{t} + t\,S_{\mu\nu} + t^2 R_{\mu\nu}$$ nonlinear fit inspired from a^2/t as well as next-leading t corrections.

$$\langle T_{\mu\nu}(t,a)\rangle = \langle T_{\mu\nu}\rangle + t\,S_{\mu\nu} + \frac{Q_{\mu\nu}}{\log^2(\sqrt{8t}/a)}$$ linear+log fit for higher-order corrections to the one-loop coeff's. c_i.

Note: also in the case $a \to 0$ is taken first, one have to remove singular data at small t by hand to get a reliable $a \to 0$ value.

The method seems to work when we are close to the continuum limit.