Heavy Quark Diffusion Coefficient from the Lattice

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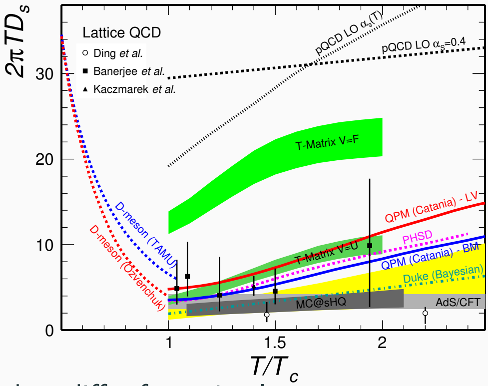
- Charm and bottom quarks much heavier than RHIC/LHC temperatures
  - $\rightarrow$ Probe for early time physics
- Nuclear suppression factor R_{AA} and elliptic flow ν_2 follow from Heavy quark diffusion coefficients D



- Experimental results for R_{AA} and ν_2 differ from simple perturbative estimates
- Both R_{AA} and ν_2 can be calculated from diffusion constant D of heavy quark in medium
- D can be tuned to match experimental results

Figure from: X. Dong CIPANP (2018)

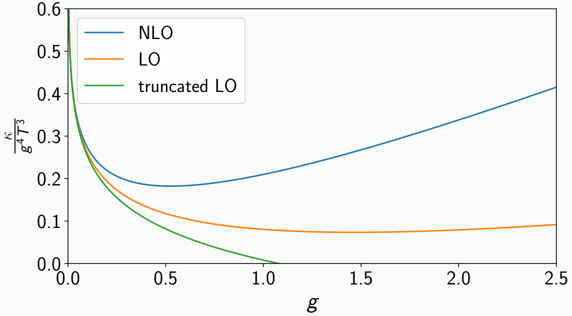
Heavy Quark in medium

- Heavy quark energy doesn't change much in collision with a thermal quark

$$E_k \sim T\,, \quad p \sim \sqrt{MT} \gg T$$

- HQ momentum is changed by random kicks from the medium
  $\rightarrow$ Brownian motion; Follows Langevin dynamics

$$\frac{dp_i}{dt} = \frac{\kappa}{2MT} p_i + \xi_i(t)$$
$$\langle \xi(t)\xi(t') \rangle = \kappa\delta(t - t')$$

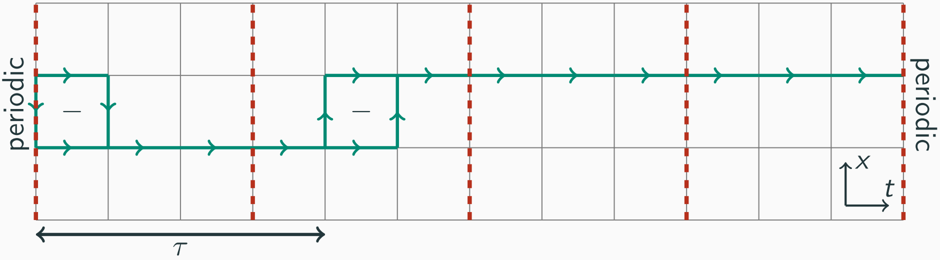


- In position space
  $\langle x^2(t) \rangle = 6Dt$
  with $D = 2T^2/\kappa$

- κ needs nonperturbative measurement

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)

Euclidean $\langle EE \rangle$ correlator



$$G_{\mathrm{E}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\langle \mathrm{Re\,Tr}\,[U(\beta,\tau) gE_i(\tau,0) U(\tau,0) gE_i(0,0)] \rangle}{\langle \mathrm{Re\,Tr}\,[U(\beta,0)] \rangle}$$

- Related to the diffusion coefficient κ by:

$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho(\omega) \frac{\cosh\left(\frac{\beta}{2} - \tau\right)\omega}{\sinh\frac{\beta\omega}{2}}, \quad \kappa = \lim_{\omega\to 0} \frac{2T}{\omega}\rho(\omega)$$

- No transport peak $\omega \to 0$

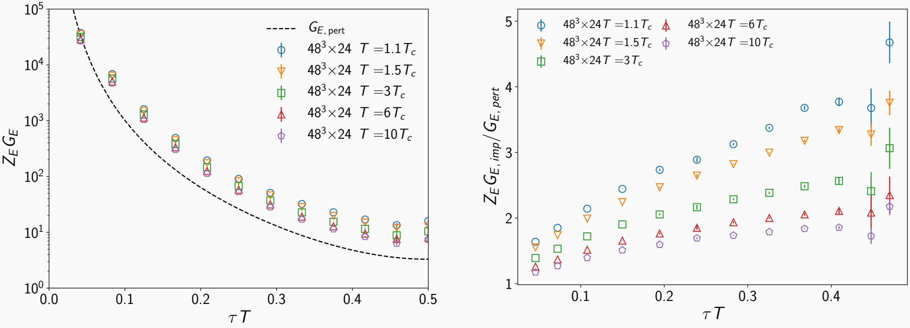
Caron-Huot *et.al.*JHEP04 (2009)

Lattice parameters

- Quenched multilevel simulations
- 4 sublattices with 2000 updates
- Spatial lattice size 48^3
- Vary the temporal size from 12 to 24 to perform continuum limit
- Temperatures between $1.1\,T_c - 10\,T_c$
- Set β for each N_t , T with (Francis *et.al.*PRD91 (2015))
- Previous similar studies (Meyer NJP13 (2011), Ding *et.al.*JPG38 (2011), Banarjee *et.al.*PRD85 (2012), Francis *et.al.*PRD92 (2015))

T/T_c	$N_t \times N_s^3$	β	$N_{\rm conf}$
1.1	12×48^3	6.407	1350
	16×48^3	6.621	1549
	20×48^3	6.795	926
	20×48^3	6.940	1082
1.5	12×48^3	6.639	1126
	16×48^3	6.872	2778
	20×48^3	7.044	1162
	24×48^3	7.192	2316
3	12×48^3	7.193	673
	16×48^3	7.432	310
	24×48^3	7.774	576
6	12×48^3	7.774	450
	16×48^3	8.019	313
	24×48^3	8.367	525
10	12×48^3	8.211	1356
	16×48^3	8.458	2769
	20×48^3	8.651	1153
	24×48^3	8.808	2150

Lattice correlator

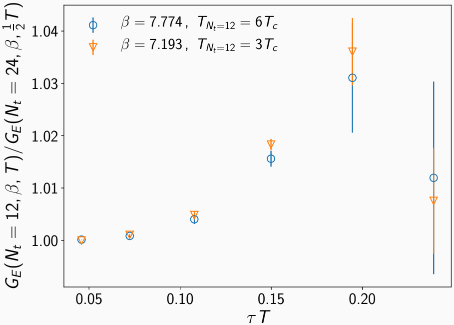


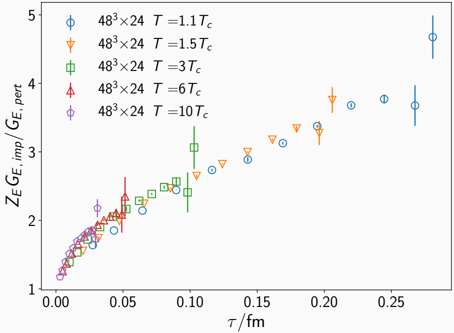
$$G_{E,\,\mathrm{pert}} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)}\right]$$

Tree-level improvement:

$$G_{E,\,\mathrm{pert}}(\bar\tau \bar T) = G_{E,\,\mathrm{LOlat}}(\tau T)$$

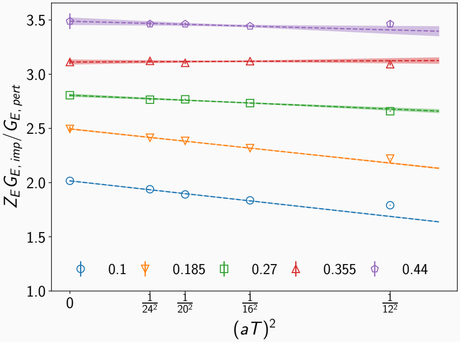
When do thermal effects start

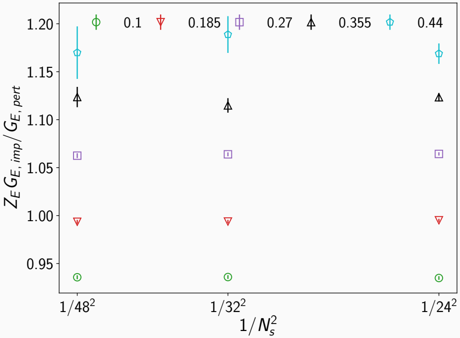




- On small physical separation every T shares a scaling (apart from finite size effects)
- Thermal effect nonexistent for $\tau < 0.10$
- Fairly small even on bigger separations

Continuum limit and finite size effects





- Use 3 largest lattices for continuum limit
- Check systematics by including the $N_t = 12$ point
- $\chi^2/\mathrm{d.o.f.} < 5$ for $\tau T > 0.20$ when using 3 largest lattices (< 10 with $N_t = 12$)
- Preliminary: finite size effects seem to be reasonably small

Spectral function

$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho(\omega) \frac{\cosh\left(\frac{\omega}{T}[\tau T - \frac{1}{2}]\right)}{\sinh \frac{\omega}{2T}}$$

$$\kappa = \lim_{\omega \to 0} \frac{2T}{\omega} \rho(\omega)\,, \qquad \gamma = -\frac{1}{3N_c} \int_0^\infty \frac{d\omega}{2\pi} \frac{\rho(\omega)}{\omega}$$

- Assume simple behavior on IR ($\omega \ll T$):

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

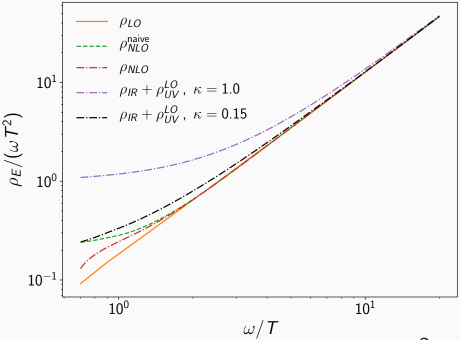
- Perturbative behavior in UV ($\omega \gg T$):

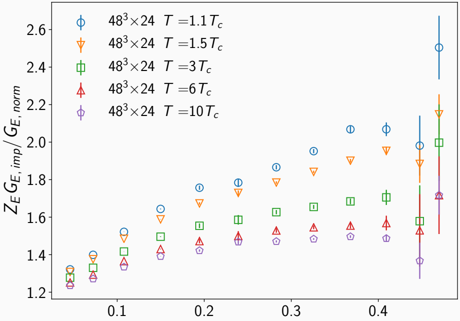
$$\rho_{\mathrm{UV}}^{\mathrm{LO}}(\omega) = \frac{g^2(\mu_\omega) C_F \omega^3}{6\pi}\,, \qquad \rho_{\mathrm{UV}}^{\mathrm{NLO}} \text{ from (Burnier } \textit{et.al.}\text{JHEP08 (2010))}$$

$$\mu_\omega = \max(\omega, \pi T) \qquad \text{or from EQCD}$$

- Use 5-loop running for the coupling

g^2 effects

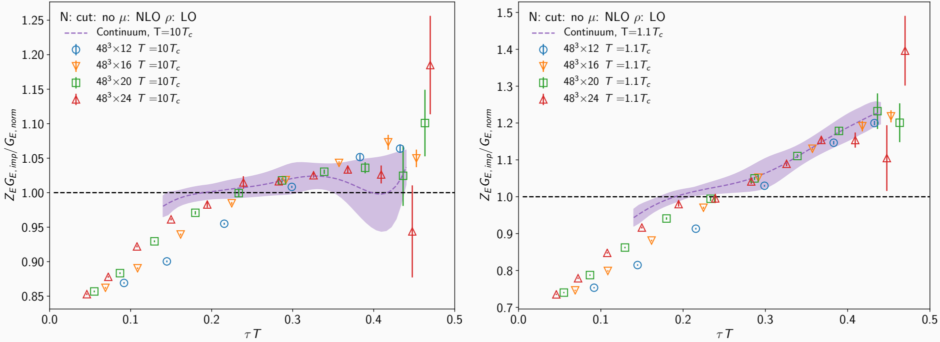




$$G_{E,\mathrm{norm}} = \int_{\omega_{\mathrm{cut}}}^{\infty} \frac{\mathrm{d}\omega}{\pi} \frac{g^2(\mu_\omega) C_F \omega^3}{6\pi} \frac{\cosh\left(\frac{\omega}{T}[\tau T - \frac{1}{2}]\right)}{\sinh\frac{\omega}{2T}}$$

- Include running of the coupling to the normalization
- How to set the scale $\mu_\omega \sim c\omega$, c affects absolute value of g^2
- Helps to discern running coupling effects from thermal effects
- How to set ω_{cut}?

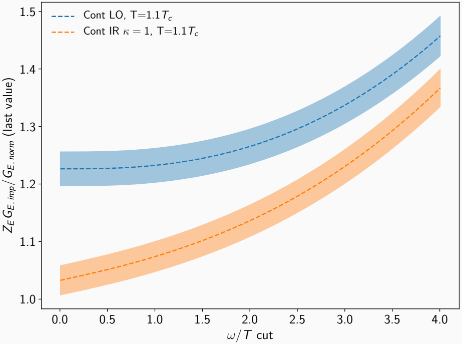
g^2 divided continuum limit



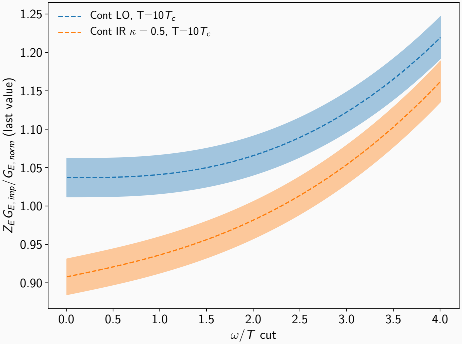
$T = 10\,T_c$ $\qquad$ $T = 1.1\,T_c$

- Normalize data to 1 at small $\tau T \sim 0.19$
- In figures $\omega_{\text{cut}} = 0$
- $\tau T < 0.16$ continuum limit not in control
- $\tau T > 0.45$, need bigger lattices/statistics
- Errors includes a preliminary estimate of systematic errors

Effect of ω cut



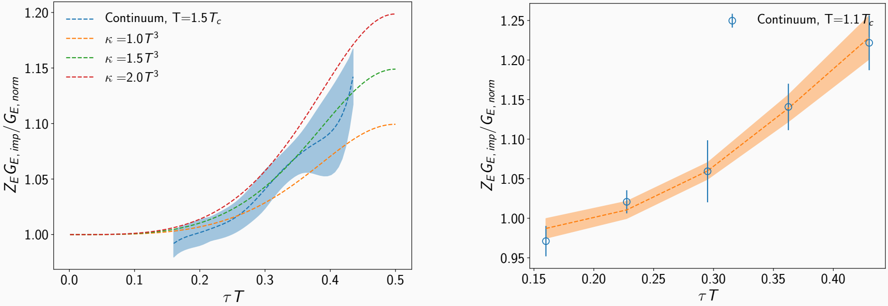
$T = 10\,T_c$



$T = 1.1\,T_c$

- What is the maximum value reached by continuum limit for each ω_{cut}?
- Thermal effects are small but existent

Size of κ



How to find or limit the value of κ?

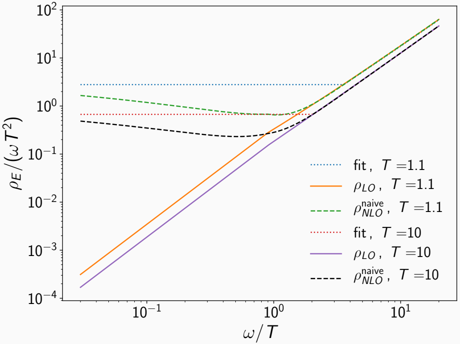
- Choose $\omega_{\rm cut}$ (here πT) and find limiting curve
- Fit to ansatz. Here: $\max(\rho_{\rm IR}, c\rho_{\rm UV}^{\rm LO})$ (and no cut)

(Francis *et.al.*PRDD92 (2015), Banerjee *et.al.*PRD85 (2011))

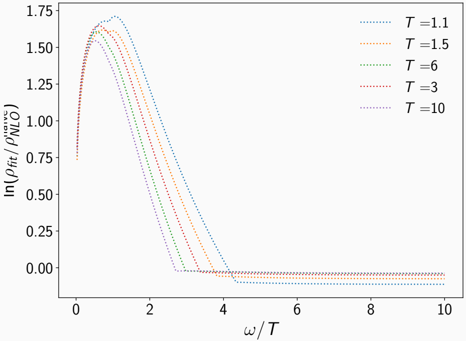
Preliminary

T/T_c	1.1	1.5	3	6	10
κ	2.79(95)	2.02(21)	1.55(18)	1.03(19)	0.67(26)

How close are we to NLO spectral function?



$T = 10\,T_c$



$T = 1.5\,T_c$

- The naive NLO spectral function seems to be close to our fits

Conclusions and Future prospects

- We observe that κ decreases as Temperature increases
- results very close to the naive NLO behavior
- Analysis still need other fit ansatzes and refinement to properly limit the maximum value of κ
- The preliminary measured numbers are in agreement with existing results
- Future: try measuring γ

Thank You