Baryon bag simulation of QCD in the strong coupling limit

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LATTICE XXXVII, Wuhan, June 19, 2019
Introduction

- Worldline representations have been helpful in understanding and removing various kinds of sign problems.

- Often worldline representations highlight certain features or symmetries of theories.

- Good algorithms for worldline configurations, e.g., worm algorithms.

- Here we revisit the old idea of a monomer-dimer-loop representation of strong coupling (SC) QCD.

- How do the worldline DOF change when we increase the temperature to drive the system into the chirally symmetric phase?

- To analyze the relevant DOF we use the baryon bag picture (≠ MIT bag).
Lattice model

QCD partition function for $\beta = 0$ (strong coupling)

$$Z = \int \mathcal{D}[\bar{\psi}\psi] \int \mathcal{D}[U] \ e^{S_F[\bar{\psi},\psi,U]}$$

with 1-flavor of staggered quarks on anisotropic lattices

$$S_F[\bar{\psi},\psi,U] = \sum_x \left( 2m\bar{\psi}_x\psi_x + \sum_\nu \xi_{x,\nu} \left( e^{\mu\delta_{\nu,d}} \bar{\psi}_x U_{x,\nu} \psi_{x+\hat{\nu}} - e^{-\mu\delta_{\nu,d}} \bar{\psi}_{x+\hat{\nu}} U_{x,\nu}^\dagger \psi_x \right) \right)$$

$\xi_{x,1} = 1, \ldots, \xi_{x,d} = t(-)^{x_1+\cdots+x_{d-1}}$: staggered signs and anisotropic coupling $t$

$\Rightarrow$ Temperature $T$ is an increasing function of $t$ [de Forcrand et al. 2017]
Monomers, dimers and baryon loops

⇒ At SC gauge fields can be integrated out exactly [Rossi, Wolff 1984]

⇒ The remaining Grassmann integral can be mapped to a statistical system of the following objects [Rossi, Wolff ’85, Karsch, Mütter ’88]

Mass terms: 
\[ (\bar{\psi}_x \psi_x)^{n_x} \rightarrow \text{Monomers } n_x \]

Hops:
\[
\begin{align*}
\left( \bar{\psi}_x \psi_x \bar{\psi}_{x+\hat{\nu}} \psi_{x+\hat{\nu}} \right)^{d_{x,\nu}} & \rightarrow \text{Dimers } d_{x,\nu} \\
\cdots \left( \bar{\psi}_x \psi_x \bar{\psi}_{x+\hat{\nu}} \right)^{3\ell_{x,\nu}} & \rightarrow \text{Baryon loops } \ell_{x,\nu}
\end{align*}
\]

Every non-vanishing contribution to the Grassmann integral must obey the constraint

\[ n_x + \sum_{\nu} d_{x,\nu} + \sum_{\nu} \frac{3}{2} |\ell_{x,\nu}| = 3 \]
Worldline representation of SC partition sum

$$Z = \sum_{\{n,d,\ell\}} w_n(m) \, w_d(t) \, w_\ell(\mu, t)$$

▶ Degrees of freedom:
  ▶ Monomers ($n$) and dimers ($d$)
  ▶ Directed self-avoiding Baryon loops ($\ell$)

▶ Fermion sign problem: even $w_\ell(\mu = 0)$ is not strictly positive

▶ Karsch and Mütter realized: simulate $U(3)$ and pair closed dimer chains with loops that occupy the same contour

▶ Is there a different way to get strictly positive weights? ⇒ Baryon bags
Baryon bag representation of SC QCD

\[ Z = \sum_{\{B\}} \prod_{B_i \in B} \det D[B_i] \times Z_B \]  

[Gattringer 2018]

Each bag \( B_i \) has a weight of

\[ \det D[B_i] = \int_{B_i} \prod_{x \in B_i} dB_x d\bar{B}_x \exp \left( \sum_{x,y \in B_i} \bar{B}_x D_x^{(i)} (B_y) \right) \]

where \( D^{(i)} \) is the FREE Dirac op. for the composite field

\( \bar{B}_x = \bar{\psi}_x \psi_x \psi_x \) on some arbitrary domain \( B_i \) of space-time

\[ D_x^{(i)} = 2(4m^3)\delta_{x,y} + \sum_{\nu} \xi_{x,\nu} \left[ e^{3\mu_d x} \delta_{y,x+\hat{\nu}} - e^{-3\mu_d x} \delta_{x,y+\hat{\nu}} \right] \quad \forall x, y \in B_i \]

\( \Rightarrow \) In bag domain \( B = \cup_i B_i \) resummation of 3-monomers, 3-dimers and baryon loops into a single determinant of a free composite field!
Baryon bag representation of SC QCD

- Baryon bag is a superposition of 3-monomers, 3-dimers and baryon loops
  ⇒ Baryon terms fully saturate the Grassmann integrals

- Lattice decomposes into disjoint domains: \( \mathcal{B} = \bigcup_i \mathcal{B}_i \) and \( \overline{\mathcal{B}} \)

- Complementary domain \( \overline{\mathcal{B}} \) is filled with mesonic contributions

\[
Z_{\overline{\mathcal{B}}} = \sum_{n,d} (2m)^{N(n)} \left( \frac{1}{3} \right)^{N(d_1) + N(d_2)}
\]

- Bag weights are real and positive for \( \mu = 0 \)

- Chiral symmetry for \( m = 0 \)
Bag simulation

Monomer based updates for check of representation:

⇒ Local update good for a qualitative check of the representation

⇒ For numerically studying the chiral limit: better worm algorithm
Observables

▶ Chiral condensate

\[ \langle \bar{\psi} \psi \rangle = \frac{1}{V} \frac{\partial}{\partial m} \ln Z \]

▶ Chiral susceptibility

\[ \chi_{\bar{\psi} \psi} \equiv \frac{\partial}{\partial m} \langle \bar{\psi} \psi \rangle + V \langle \bar{\psi} \psi \rangle^2 \]

▶ Relative bag size

\[ \langle S_B \rangle = \frac{1}{V} \left\langle \sum_{\mathcal{B}_i \in \mathcal{B}} |\mathcal{B}_i| \right\rangle \]
Cross-checks: 2x2

⇒ On a 2x2 lattice we can count all configs and provide analytic results!
Cross-checks: 4x4

⇒ On a 4x4 lattice we can use conventional simulations and a newly developed worm to validate our results!
Issues and take-aways

▶ Representation is interesting as it organizes DOF in a new way
  • Still determinants to compute (Matrix size depends on parameters)

▶ Local update is good for a qualitative check
  • Bad in the (interesting) small $m$ region

▶ Numerical study: We use a $\text{U}(3)$ worm (no bags!)
  • $S_B$: sites occupied by 3-monomers, 3-dimers and baryon loops
Worm algorithm

The worm is a natural extension to $m \neq 0$ of the well known U(3) worm

[Adams, Chandrasekharan 2002]

$\Rightarrow$ We measure observables on the U(3) ensemble and reweight to SU(3)!

[Karsch, Mütter 1988]

$$
\langle \mathcal{O} \rangle_{SU(3)} = \frac{\left\langle \mathcal{O} \prod_{\ell} \left[ 1 + f(t) \text{sign}(\ell) \right] \right\rangle_{U(3)}}{\left\langle \prod_{\ell} \left[ 1 + f(t) \text{sign}(\ell) \right] \right\rangle_{U(3)}}
$$
2D Results, $V = 16 \times 4$
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2D Results: Chiral limit, $V = L \times 4$
Summary & Outlook

- Strong coupling limit: baryon bags $B_i$ and complementary domain $\overline{B}$
- Local updates in the baryon bag representation: inefficient for small $m$
- New worm algorithm for $m \neq 0$ and anisotropic coupling $t$
- Study of baryon bag shows interesting correlation with other observables
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  - Worm with baryon bags?
  - Explore the role of the baryon bags for observables
  - Study quantum numbers of bag
  - Add $\mu$ to study the strong coupling phase diagram in 4D
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Thanks for listening!