Baryon bag simulation of QCD in the strong coupling limit

O. Orasch C. Gattringer P. Toerek [1] S. Chandrasekharan [2]

[1]University of Graz

[2]Duke University

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NAWI Graz Natural Sciences

UNI GRAZ

FWF Der Wissenschaftsfonds.

Introduction

- Worldline representations have been helpful in understanding and removing various kinds of sign problems
- Often worldline representations highlight certain features or symmetries of theories
- Good algorithms for worldline configurations, e.g., worm algorithms
- Here we revisit the old idea of a monomer-dimer-loop representation of strong coupling (SC) QCD
- How do the worldline DOF change when we increase the temperature to drive the system into the chirally symmetric phase?
- To analyze the relevant DOF we use the baryon bag picture ($\neq$ MIT bag)

Lattice model

QCD partition function for $\beta = 0$ (strong coupling)

$$Z = \int \mathrm{D}\left[\bar{\psi}\psi\right] \int_{\mathrm{SU}(3)} \mathrm{D}\left[U\right] \, \mathrm{e}^{S_F\left[\bar{\psi},\psi,U\right]}$$

with 1-flavor of staggered quarks on anisotropic lattices

$$S_F\left[\bar{\psi},\psi,U\right] = \sum_x \Big(2m\bar{\psi}_x\psi_x + \sum_\nu \xi_{x,\nu}\left[\mathrm{e}^{\mu\delta_{\nu,d}}\bar{\psi}_x U_{x,\nu}\psi_{x+\hat{\nu}} - \mathrm{e}^{-\mu\delta_{\nu,d}}\bar{\psi}_{x+\hat{\nu}} U^\dagger_{x,\nu}\psi_x\right]\Big)$$

$\xi_{x,1} = 1, \ldots, \xi_{x,d} = t(-)^{x_1+\cdots+x_{d-1}}$: staggered signs and anisotropic coupling t

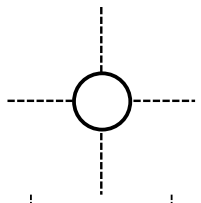
$\Rightarrow$ Temperature T is an increasing function of t [de Forcrand *et al.* 2017]

Monomers, dimers and baryon loops

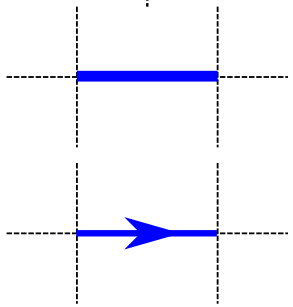
$\Rightarrow$ At SC gauge fields can be integrated out exactly [Rossi, Wolff 1984]

$\Rightarrow$ The remaining Grassmann integral can be mapped to a statistical system of the following objects [Rossi, Wolff '85, Karsch, Mütter '88]

Mass terms: $(\overline{\psi}_x \psi_x)^{n_x} \quad \rightarrow \quad$ Monomers n_x



Hops: $\begin{cases} (\overline{\psi}_x \psi_x \overline{\psi}_{x+\hat{\nu}} \psi_{x+\hat{\nu}})^{d_{x,\nu}} & \rightarrow \quad \text{Dimers } d_{x,\nu} \\ \cdots (\overline{\psi}_x \psi_{x+\hat{\nu}})^{3\ell_{x,\nu}} \cdots & \rightarrow \quad \text{Baryon loops } \ell_{x,\nu} \end{cases}$



Every non-vanishing contribution to the Grassmann integral must obey the constraint

$$n_x + \sum_\nu d_{x,\nu} + \sum_\nu \frac{3}{2} |\ell_{x,\nu}| = 3$$

Worldline representation of SC partition sum

$$Z = \sum_{\{n,d,\ell\}} w_n(m)\, w_d(t)\, w_\ell(\mu, t)$$

- Degrees of freedom:
  - Monomers (n) and dimers (d)
  - Directed self-avoiding Baryon loops (ℓ)
- Fermion sign problem: even $w_\ell(\mu = 0)$ is not strictly positive
- Karsch and Mütter realized: simulate U(3) and pair closed dimer chains with loops that occupy the same contour
- Is there a different way to get strictly positive weights? $\Rightarrow$ Baryon bags

Baryon bag representation of SC QCD

$$Z = \sum_{\{\mathcal{B}\}} \prod_{\mathcal{B}_i \in \mathcal{B}} \det D[\mathcal{B}_i] \ \times \ Z_{\overline{\mathcal{B}}}$$

[Gattringer 2018]

Each bag $\mathcal{B}_i$ has a weight of

$$\det D[\mathcal{B}_i] = \int_{\mathcal{B}_i} \prod_{x \in \mathcal{B}_i} \mathrm{d}B_x \mathrm{d}\bar{B}_x \exp\Big(\sum_{x,y \in \mathcal{B}_i} \bar{B}_x D^{(i)}_{xy} B_y \Big)$$

where $D^{(i)}$ is the FREE Dirac op. for the composite field $B_x = \psi^1_x \psi^2_x \psi^3_x$ ($\overline{B}_x = \overline{\psi}^3_x \overline{\psi}^2_x \overline{\psi}^1_x$) on some arbitrary domain $\mathcal{B}_i$ of space-time

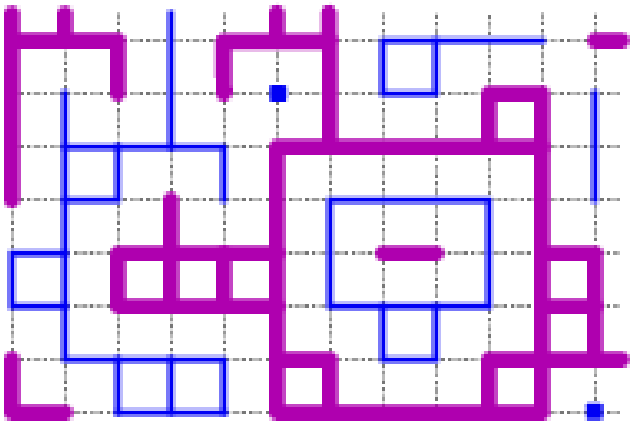
$$D^{(i)}_{x,y} = 2(4m^3)\delta_{x,y} + \sum_{\nu} \xi_{x,\nu} \left[\mathrm{e}^{3\mu\delta_{x,d}} \delta_{y,x+\hat{\nu}} - \mathrm{e}^{-3\mu\delta_{x,d}} \delta_{x,y+\hat{\nu}} \right] \ \forall x, y \ \in \mathcal{B}_i$$

$\Rightarrow$ In bag domain $\mathcal{B} = \cup_i \mathcal{B}_i$ resummation of 3-monomers, 3-dimers and baryon loops into a single determinant of a free composite field!

Baryon bag representation of SC QCD

- Baryon bag is a superposition of 3-monomers, 3-dimers and baryon loops
  $\Rightarrow$ Baryon terms fully saturate the Grassmann integrals
- Lattice decomposes into disjoint domains: $\mathcal{B} = \cup_i \mathcal{B}_i$ and $\overline{\mathcal{B}}$
- Complementary domain $\overline{\mathcal{B}}$ is filled with mesonic contributions

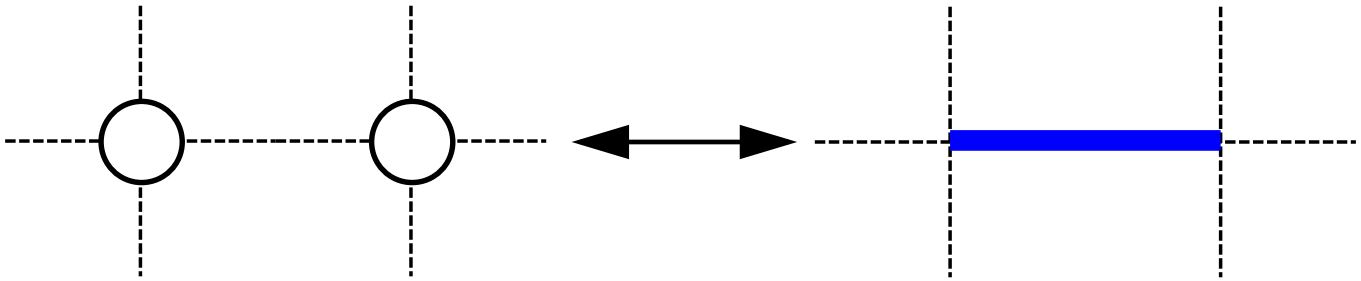
$$Z_{\overline{\mathcal{B}}} = \sum_{n,d} (2m)^{N(n)} \left(\frac{1}{3}\right)^{N(d_1)+N(d_2)}$$



- Bag weights are real and positive for $\mu = 0$
- Chiral symmetry for $m = 0$

Bag simulation

Monomer based updates for check of representation:



$\Rightarrow$ Local update good for a qualitative check of the representation

$\Rightarrow$ For numerically studying the chiral limit: better worm algorithm

Observables

- Chiral condensate

$$\langle \overline{\psi}\psi \rangle = \frac{1}{V}\frac{\partial}{\partial m}\ln Z$$

- Chiral susceptibility

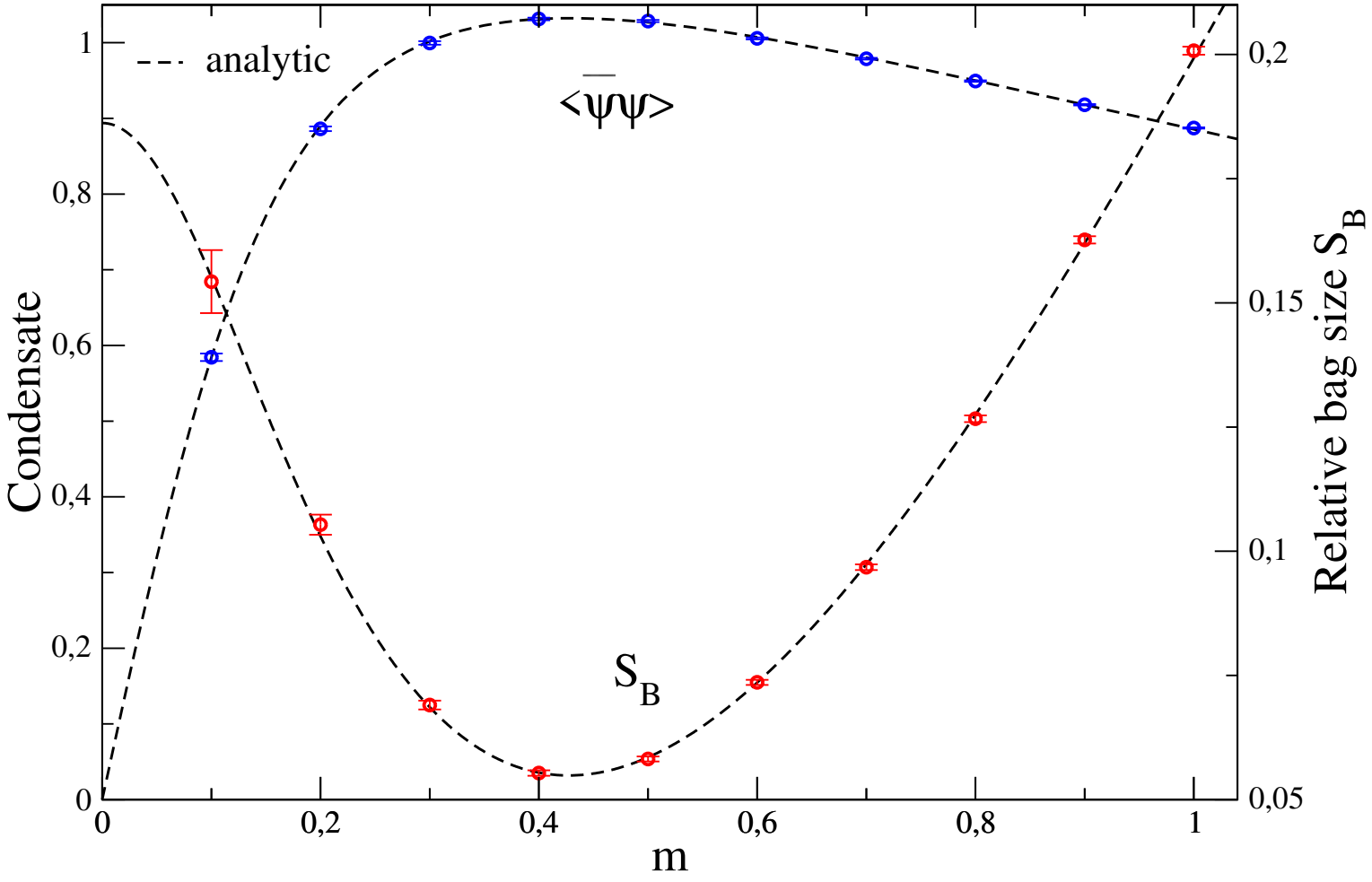
$$\chi_{\overline{\psi}\psi} \equiv \frac{\partial}{\partial m}\langle \overline{\psi}\psi \rangle + V\langle \overline{\psi}\psi \rangle^2$$

- Relative bag size

$$\langle S_B \rangle = \frac{1}{V}\Big\langle \sum_{\mathcal{B}_i \in \mathcal{B}} |\mathcal{B}_i| \Big\rangle$$

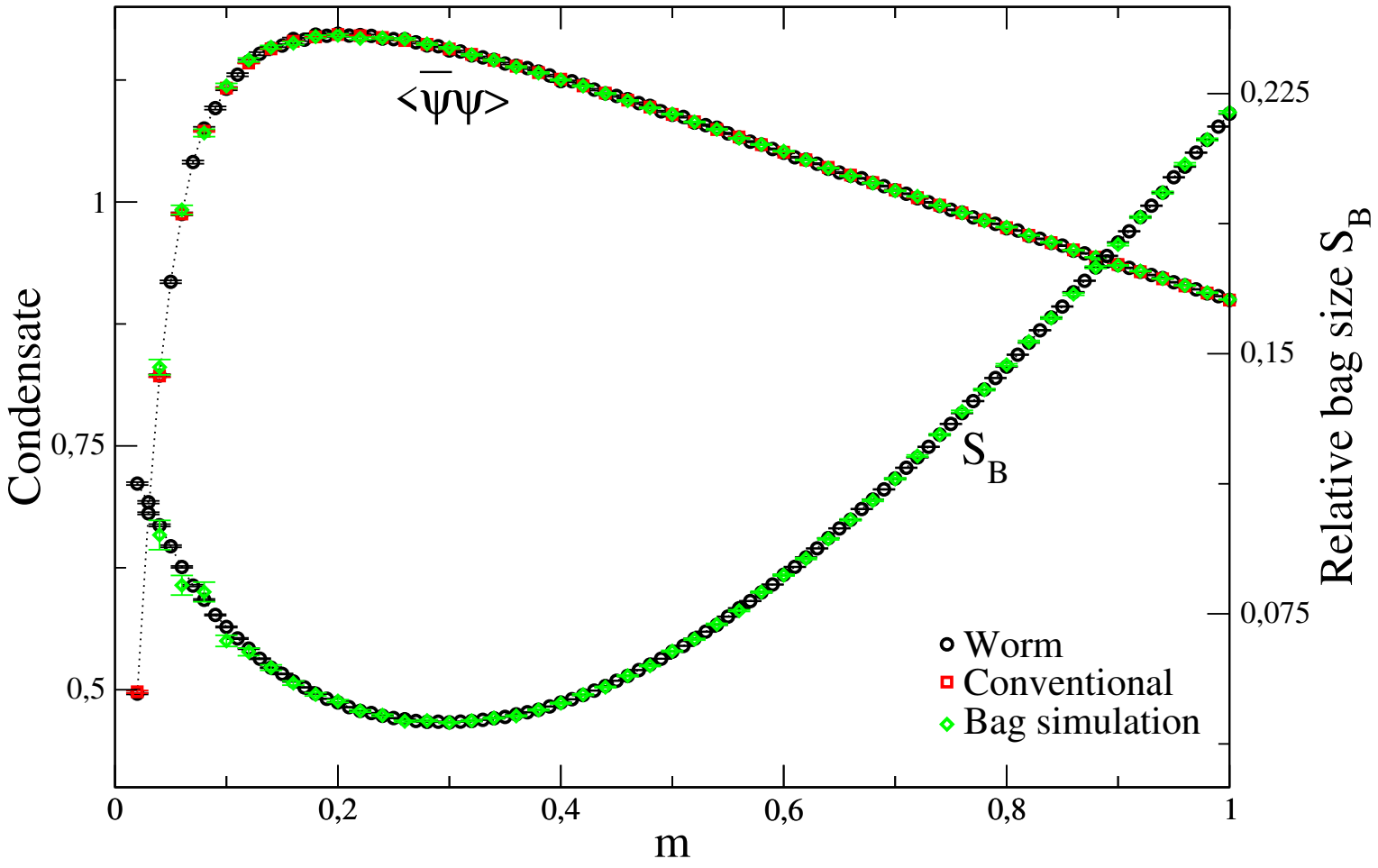
Cross-checks: 2x2

⇒ On a 2x2 lattice we can count all configs and provide analytic results!



Cross-checks: 4x4

⇒ On a 4x4 lattice we can use conventional simulations and a newly developed worm to validate our results!



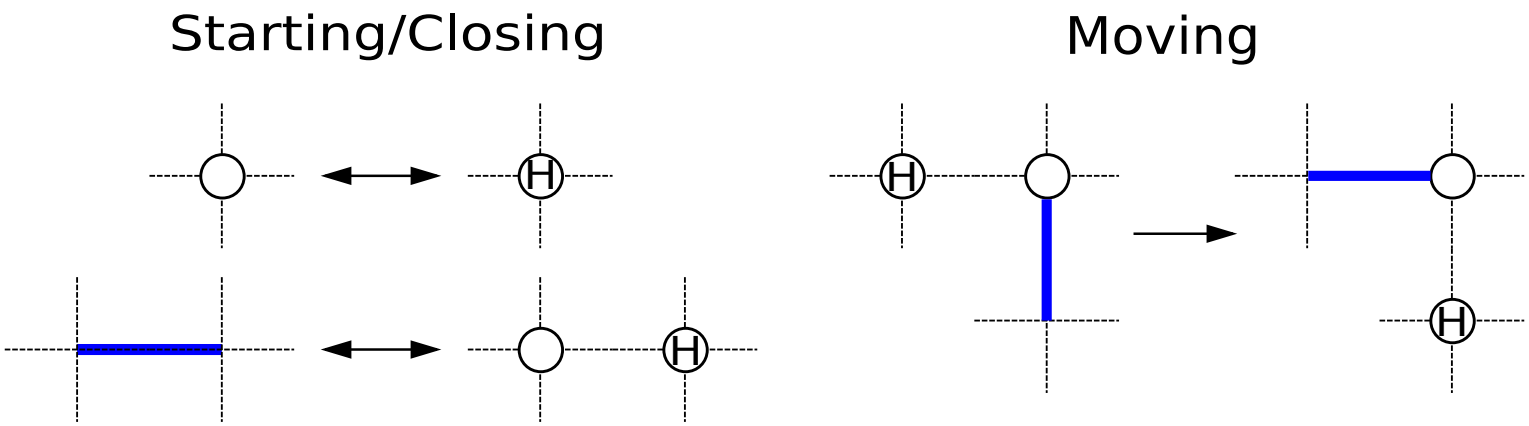
Issues and take-aways

- Representation is interesting as it organizes DOF in a new way
  - Still determinants to compute (Matrix size depends on parameters)
- Local update is good for a qualitative check
  - Bad in the (interesting) small m region
- Numerical study: We use a U(3) worm (no bags!)
  - S_B: sites occupied by 3-monomers, 3-dimers and baryon loops

Worm algorithm

The worm is a natural extension to $m \neq 0$ of the well known U(3) worm

[Adams, Chandrasekharan 2002]

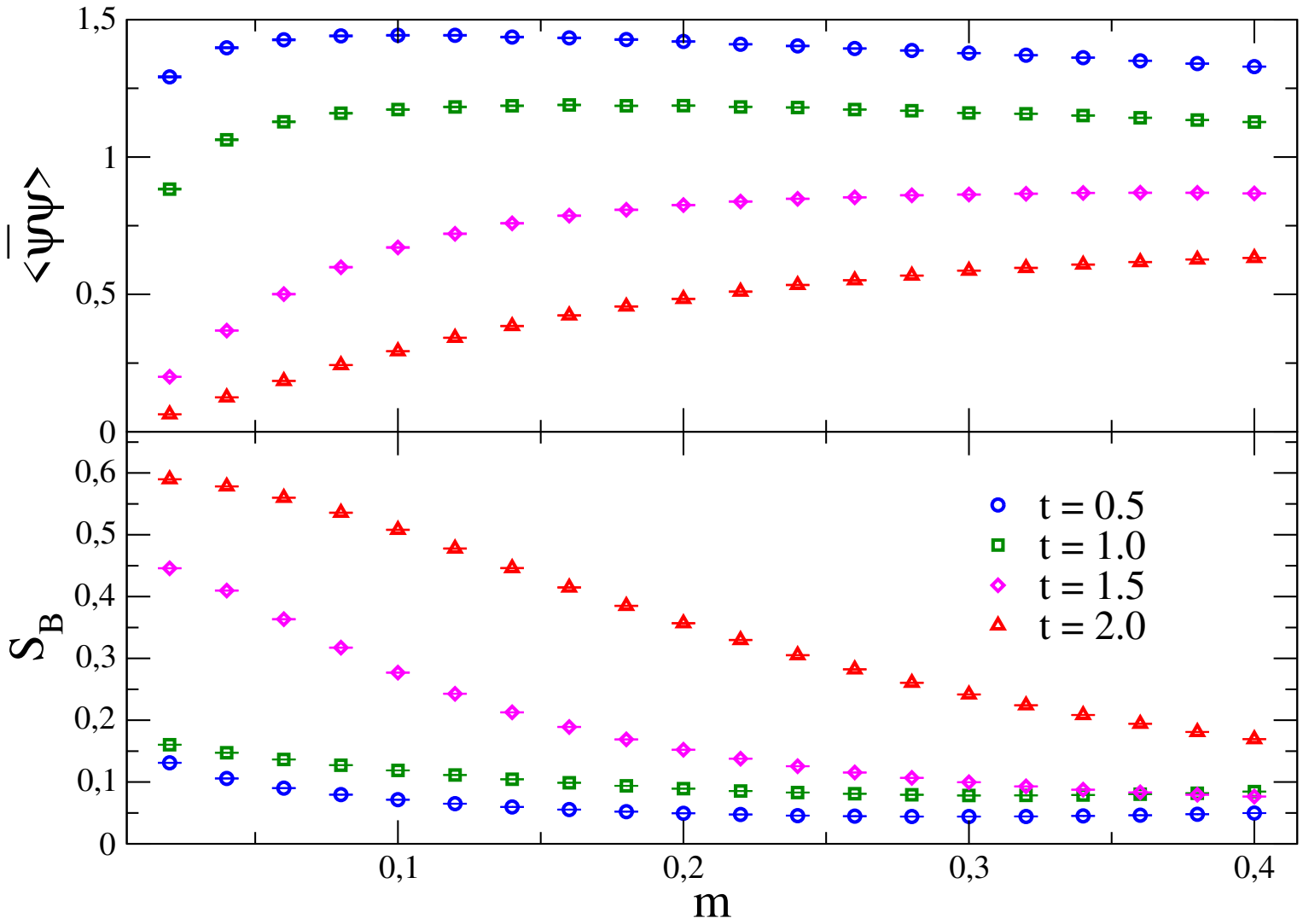


$\Rightarrow$ We measure observables on the U(3) ensemble and reweight to SU(3)!

[Karsch, Mütter 1988]

$$\langle \mathcal{O} \rangle_{\mathrm{SU}(3)} = \frac{\left\langle \mathcal{O} \prod_\ell \left[1 + f(t)\ \mathrm{sign}(\ell)\right] \right\rangle_{\mathrm{U}(3)}}{\left\langle \prod_\ell \left[1 + f(t)\ \mathrm{sign}(\ell)\right] \right\rangle_{\mathrm{U}(3)}}$$

2D Results, V = 16 x 4



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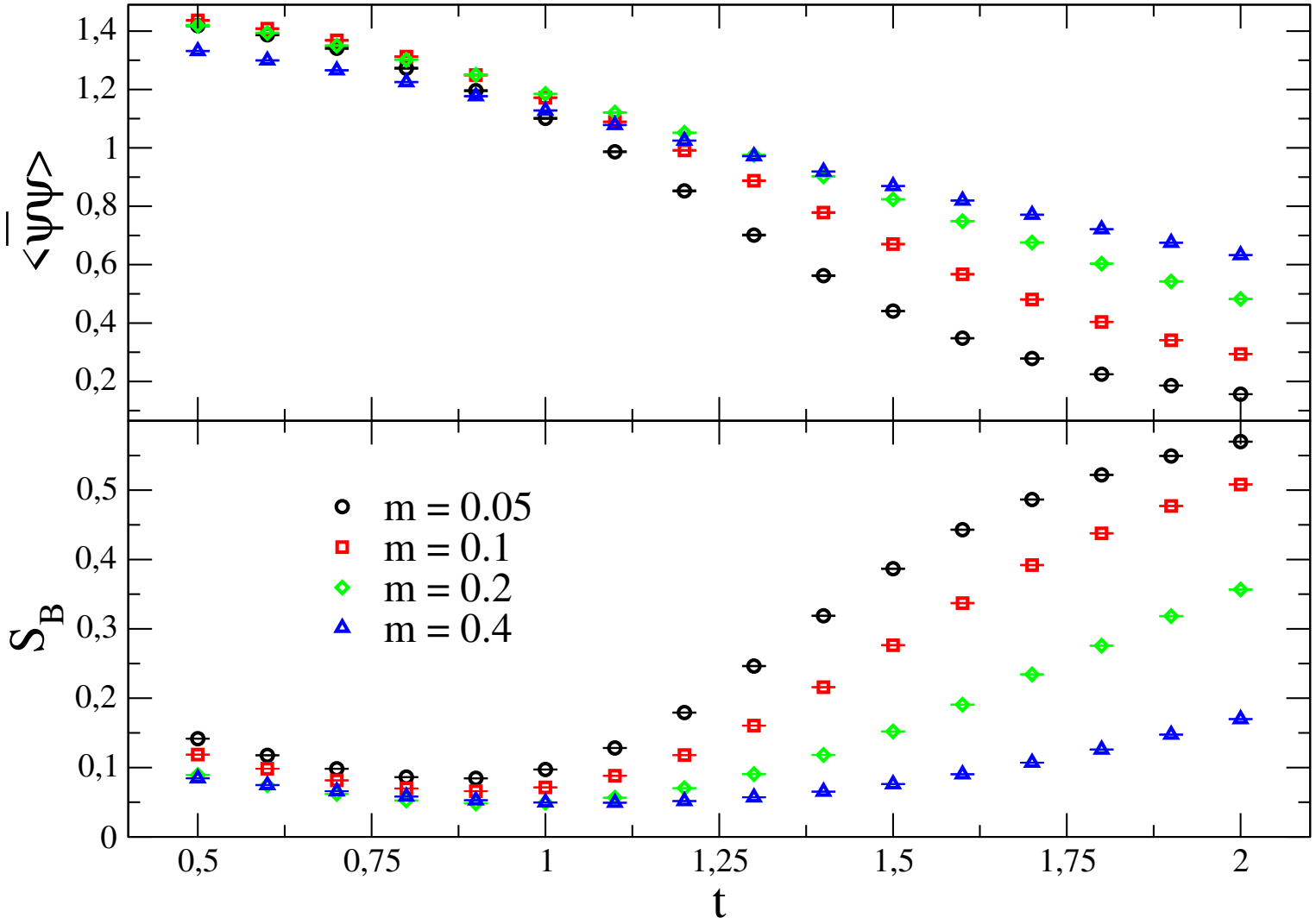
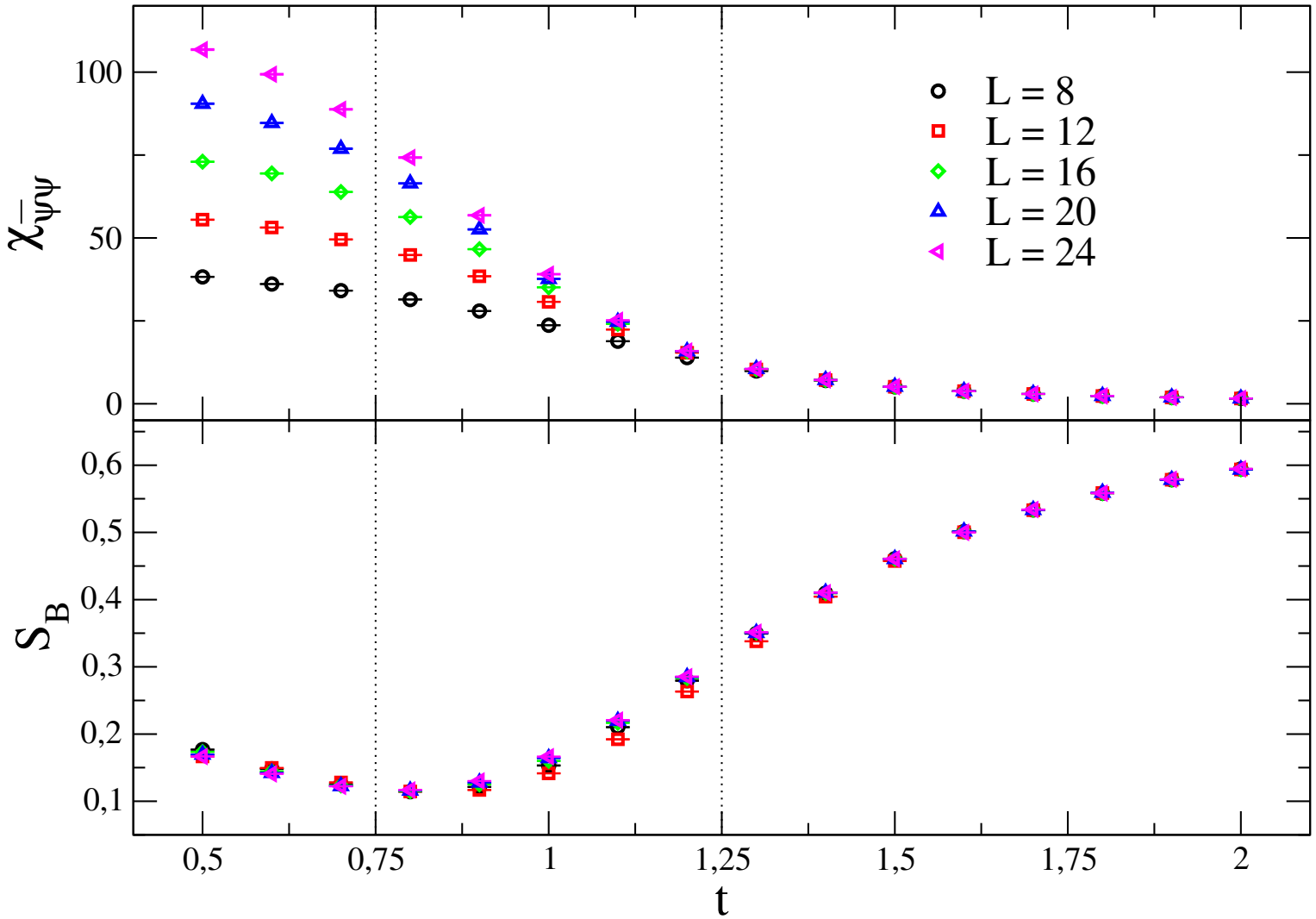


2D Results: Chiral limit, V = L x 4



Summary & Outlook

- Strong coupling limit: baryon bags $\mathcal{B}_i$ and complementary domain $\overline{\mathcal{B}}$
- Local updates in the baryon bag representation: inefficient for small m
- New worm algorithm for $m \neq 0$ and anisotropic coupling t
- Study of baryon bag shows interesting correlation with other observables

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- Worm with baryon bags?
- Explore the role of the baryon bags for observables
- Study quantum numbers of bag
- Add μ to study the strong coupling phase diagram in 4D

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Thanks for listening!