Exploring the QCD phase diagram at finite density by the complex Langevim method on a 16^3 x 32 lattice

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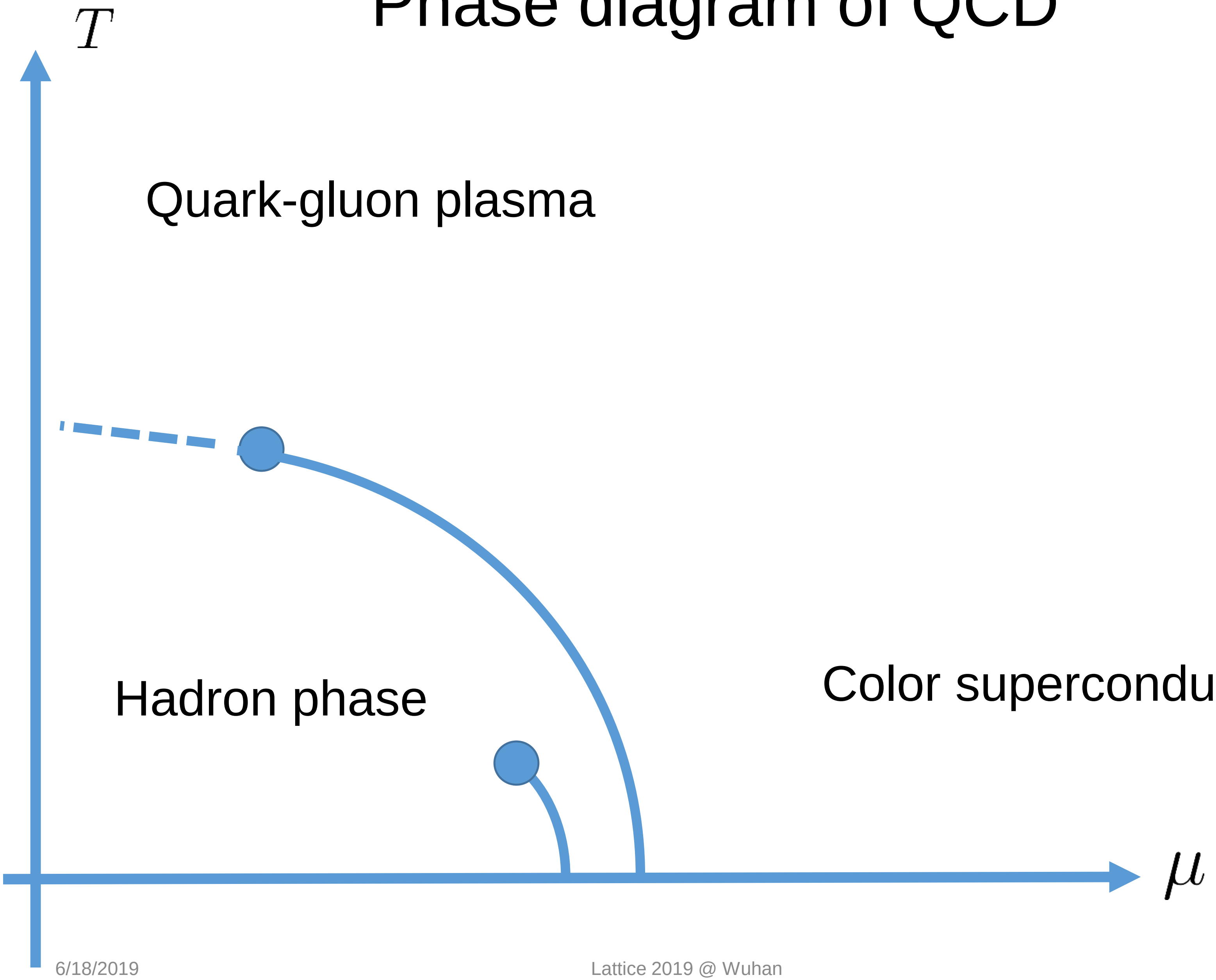
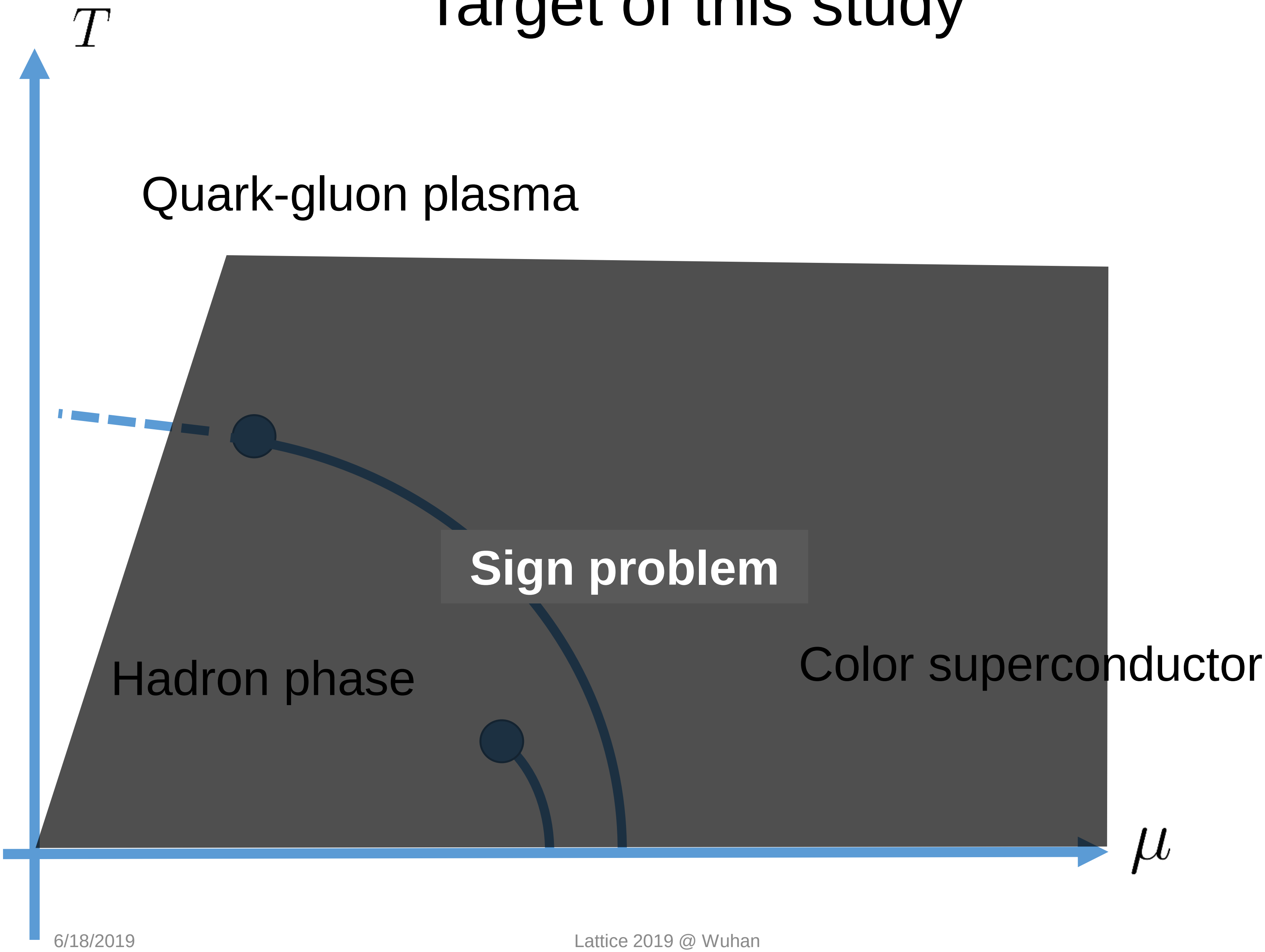
Phase diagram of QCD



Target of this study



QCD at finite density

Partition function：

$$Z = \int dU \det M[U,\mu] \mathrm{e}^{-S_g[U]}$$

$$S_\mathrm{g} = -\frac{\beta}{6} \sum_x \sum_{\mu<v} \mathrm{tr}\left(U_{x,\mu v} + U_{x,\mu v}^{-1}\right)$$

$$M = \gamma_\mu D_\mu + m + \mu\gamma_4$$

The origin of the sign problem：

$$\det\left(M(\mu)^\dagger\right) = \det M(-\mu)$$ … complex number !

Complex Langevin Method (CLM)

[Parisi ’83], [Klauder ’84]
[Aarts, Seiler, Stamatescu ’09]
[Aarts, James, Seiler, Stamatescu ’11]
[Sexty ’14] [Fodor, Katz, Sexty, Torok ’15]
[Sinclair, Kogut ’16]
[Nishimura, Shimasaki ’15]
[Nagata, Nishimura, Shimasaki ’15]

Complexify the link variable:

$$U_{x\mu} \in SU(3) \to \mathcal{U}_{x\mu} \in SL(3, \mathbb{C})$$

Solve the Langevin equation:

$$\mathcal{U}_{x\mu}(t+\epsilon) = \exp\left[i\left(-\epsilon \mathcal{D}_{x\mu} S[\mathcal{U}] + \sqrt{\epsilon}\eta_{x\mu}(t)\right)\right]\mathcal{U}_{x\mu}(t)$$

Drift term — Gaussian noise

Calculate observables:

$$\langle O(\mathcal{U})\rangle_{\text{CLM}} = \lim_{T\to\infty}\frac{1}{T}\int_{t_0}^{t_0+T} dt O(\mathcal{U}) P_{\text{eq}}(\mathcal{U})$$

$\langle\cdots\rangle_{\text{CLM}}$ Noise average

Validity of the CLM 1/2

(1) Excursion problem

Complexified link variables deviate from SU(3) subspace

(2) Singular drift problem

The drift term becomes large due to the zero eigenvalues of the Dirac operator

$$v_{x,\nu} = D_{x,\nu} S[\mathcal{U}] \propto \mathrm{tr}\left(\boxed{M[\mathcal{U}]^{-1}} \mathcal{D}_{x,\nu} M[\mathcal{U}] \right)$$

Drift term

$$M = \gamma_\mu D_\mu + m + \mu \gamma_4$$

If one of these happens, CLM ≠ path integral quantization.

Validity of the CLM 2/2

[Nagata, Nishimura, Shimasaki '15]

We can judge the validity by histograms of the drift term.

Norm of the drift term: $v = \sqrt{\frac{1}{3} \max_{x,\nu} \mathrm{tr}\left(v_{x,\nu}^{\dagger} v_{x,\nu}\right)}$

The histogram shows exponential fall-off: CLM = path integral

The histogram shows power-law fall-off: CLM ≠ path integral

Lattice setup

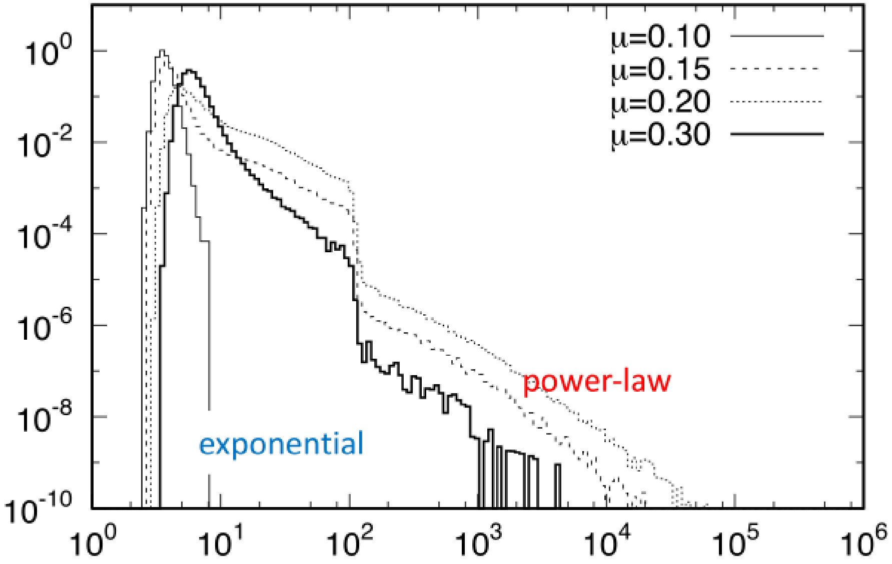
- N_f = 4 (staggered fermions),
- Lattice size: $8^3 \times 16$, $16^3 \times 32$
- β = 5.7 (a ~ 0.045 fm),
- ma = 0.01, μa = 0.1 – 0.5.

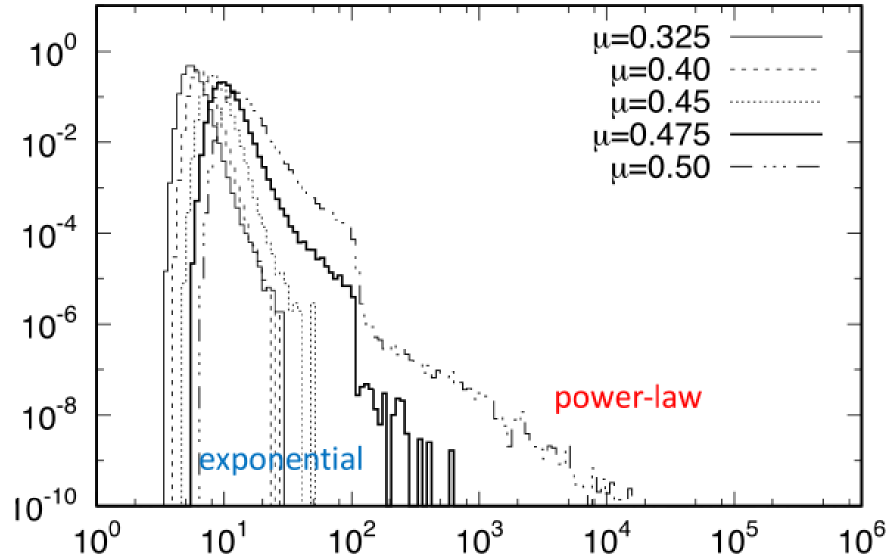
c.f) N_f=2 is studied in [Kogut, Sinclair ’19] (next talk)

Histograms of the drift term

$8^3 \times 16$ lattice

[ST, Ito, Matsufuru, Nishimura, Shimasaki, Tsuchiya '18]



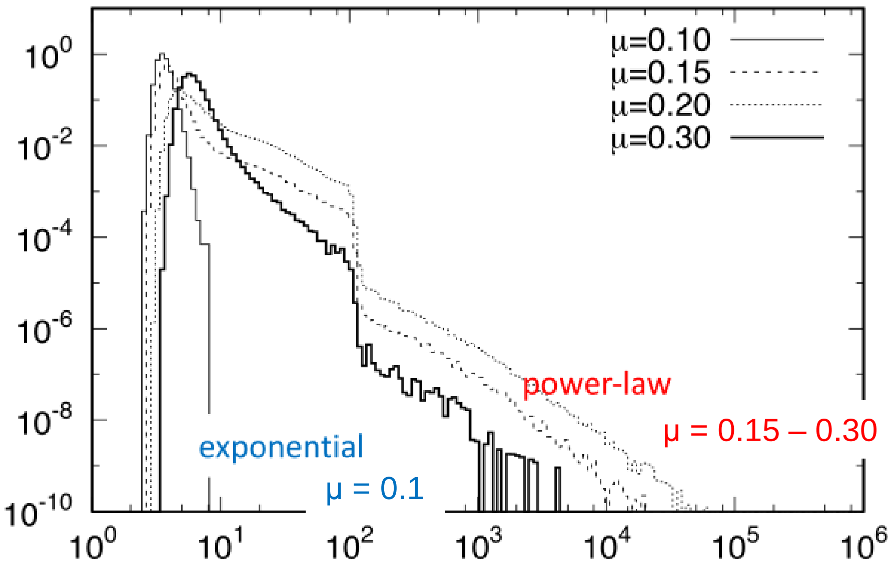


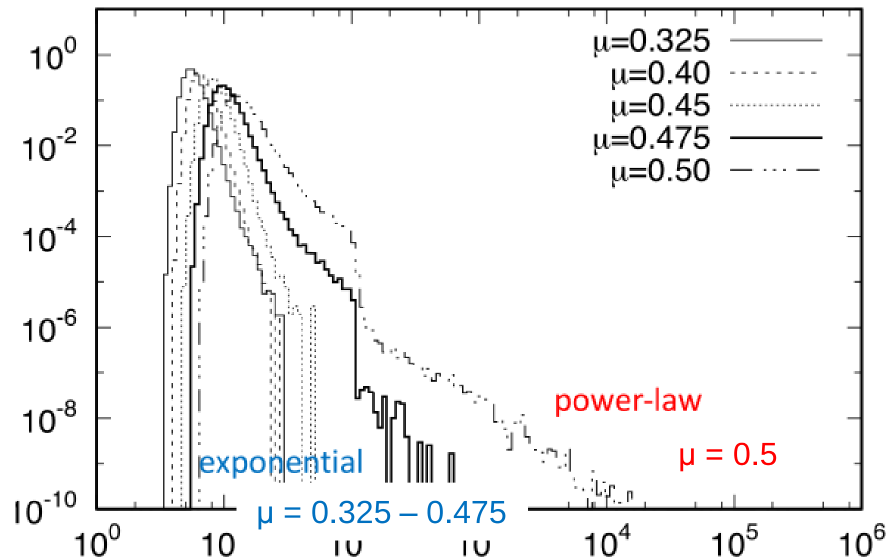
$\mu a = 0.1 – 0.5$

Histograms of the drift term

$8^3 \times 16$ lattice

[ST, Ito, Matsufuru, Nishimura, Shimasaki, Tsuchiya ’18]



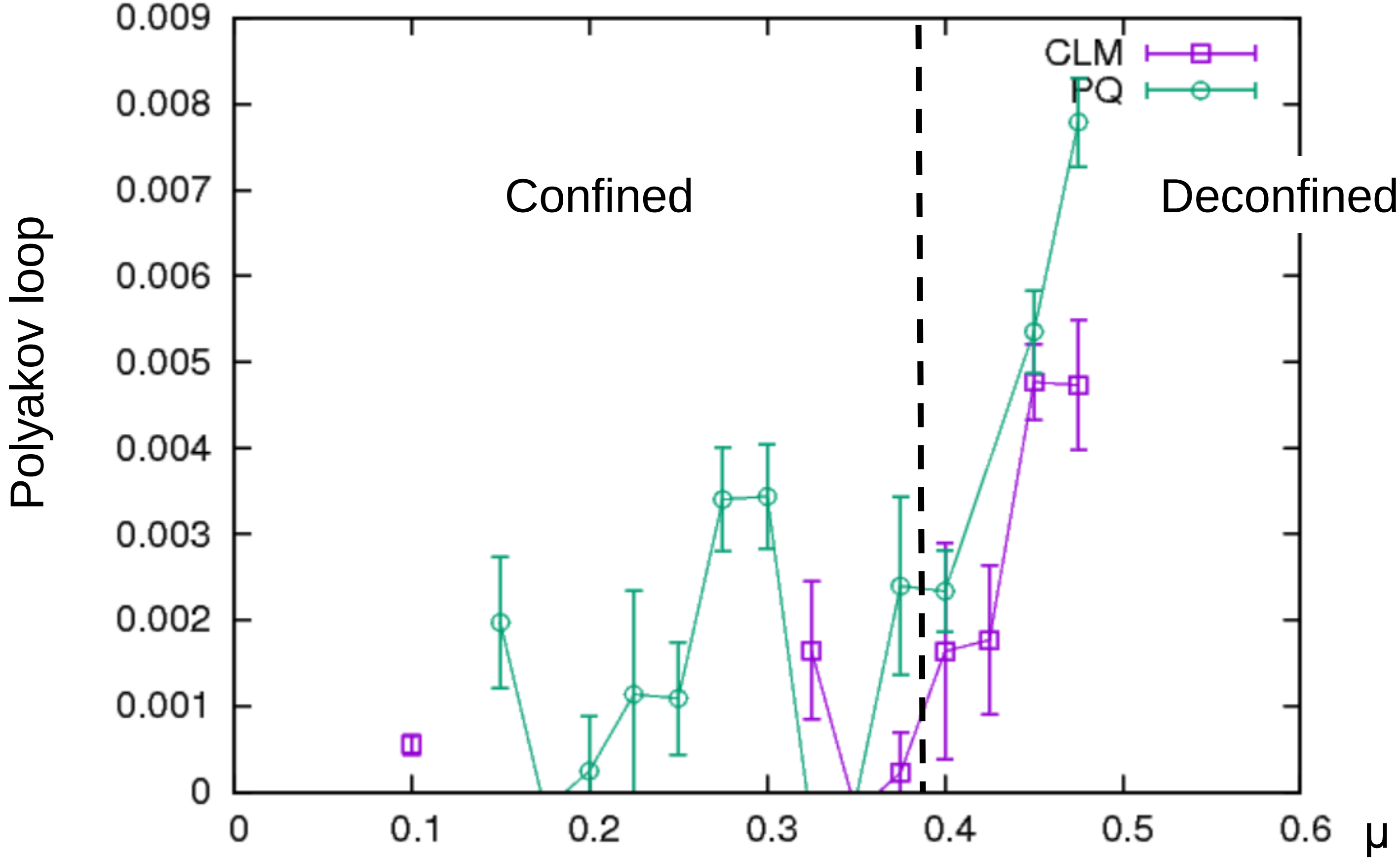


μa = 0.1, 0.15-0.30, 0.325-0.475 – 0.5

CLM is justified

Results on $8^3 \times 16$ lattice 1/2

[ST, Ito, Matsufuru, Nishimura, Shimasaki, Tsuchiya' 18]

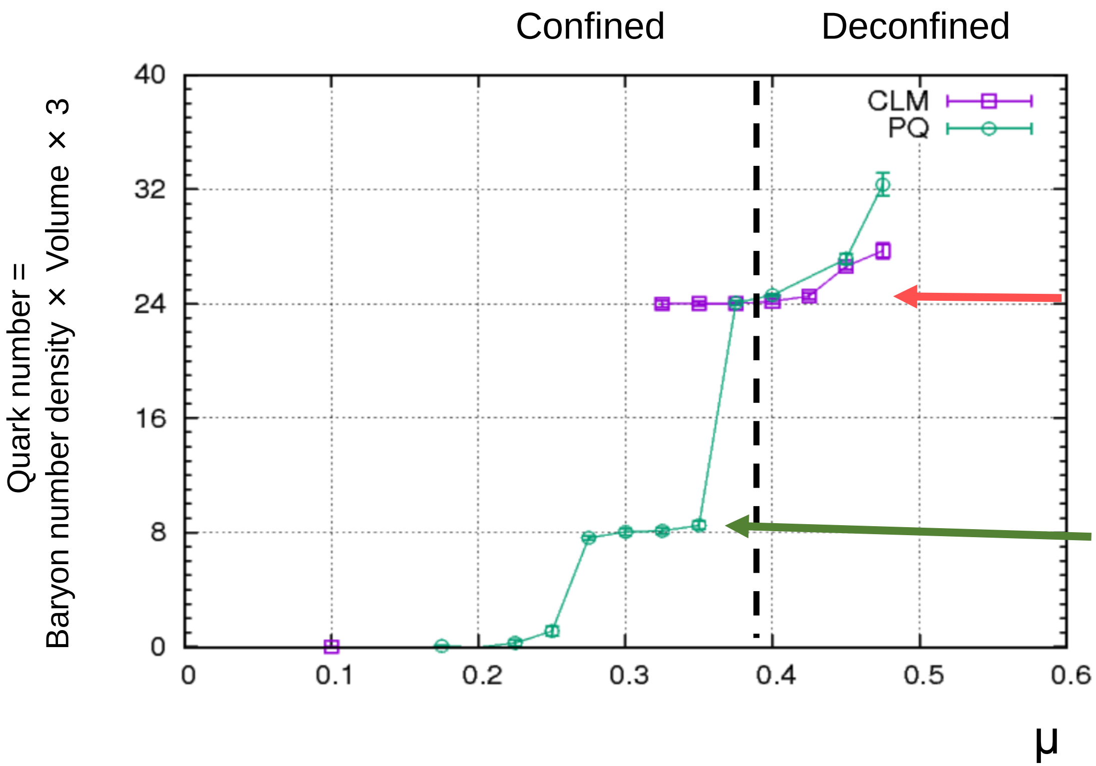


PQ = Phase Quenched
CLM: only reliable data are shown

Reliable data can be obtained
in the high density region

Results on $8^3 \times 16$ lattice 2/2

[ST, Ito, Matsufuru, Nishimura, Shimasaki, Tsuchiya' 18]



CLM predicts a plateau structure in the confined phase, where the quark number is 24.

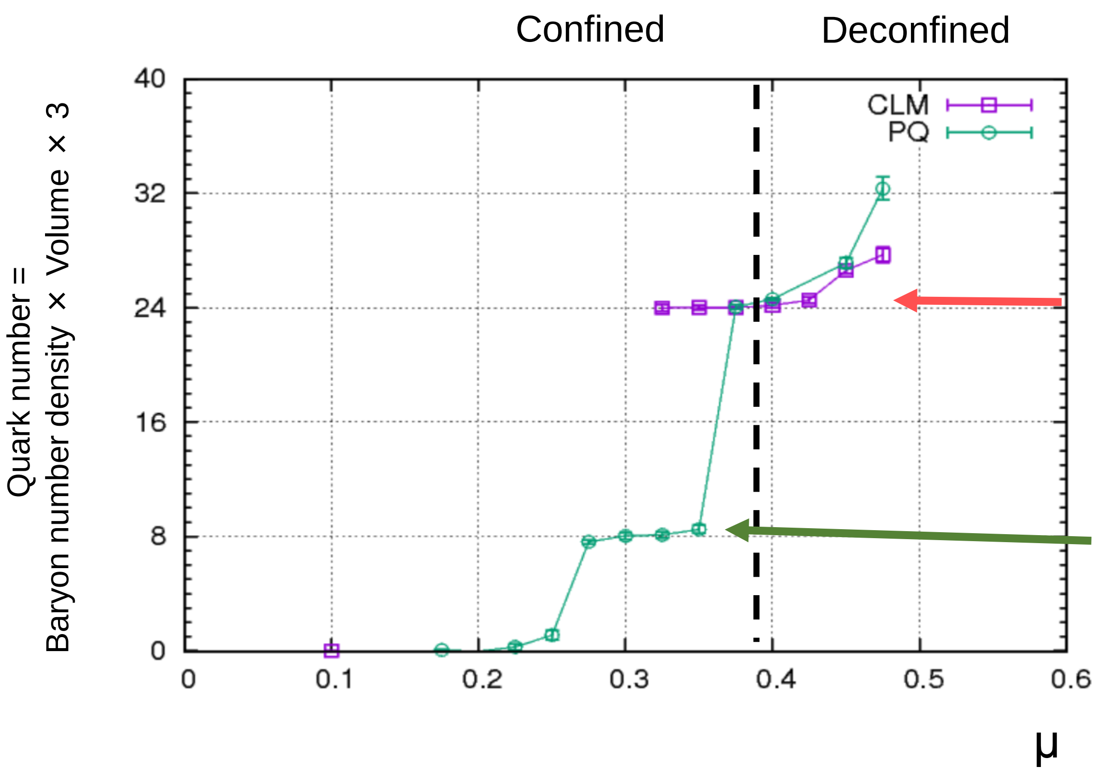
↓

8 baryons are packed in a box.

On the other hand, another plateau found in the PQ simulation has different height. It corresponds to meson creation.

Results on $8^3 \times 16$ lattice 2/2

[ST, Ito, Matsufuru, Nishimura, Shimasaki, Tsuchiya' 18]



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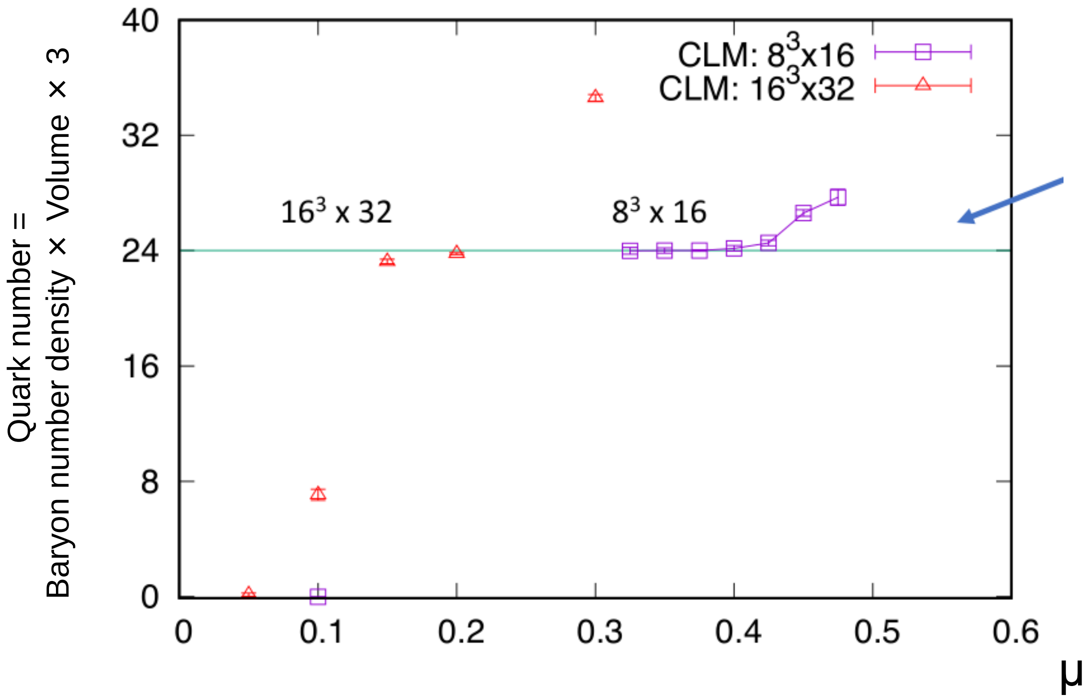
Caveat: finite volume effect is large.

$a \sim 0.045$ fm, $L = 0.36$ fm,

$\mu a = 0.3$ corresponds to $\mu \sim 1300$ MeV

Results on $16^3 \times 32$ lattice



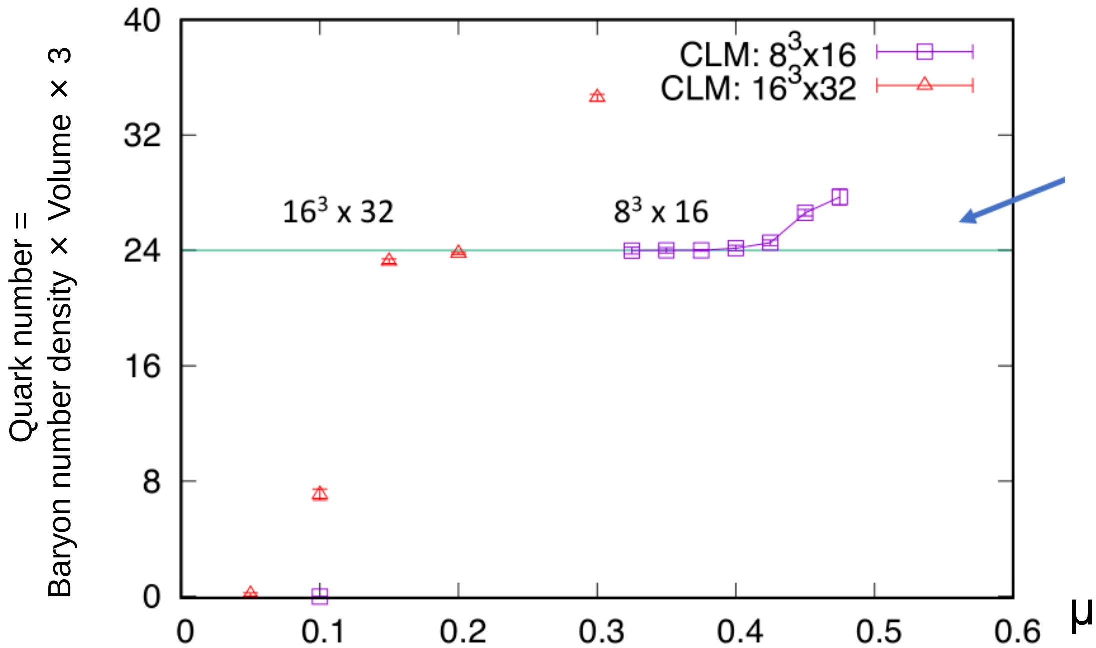
Use same β: β=5.7

CLM simulation on $16^3 \times 32$ lattice also show the plateau structure at N_{quark} = 24.

- 8 baryons are created again.
- The critical μ is lower than the previous case.

Reduction of the finite volume effect → creation energy per baryon decreases

Comparison with ρ_0



Baryon number densities are

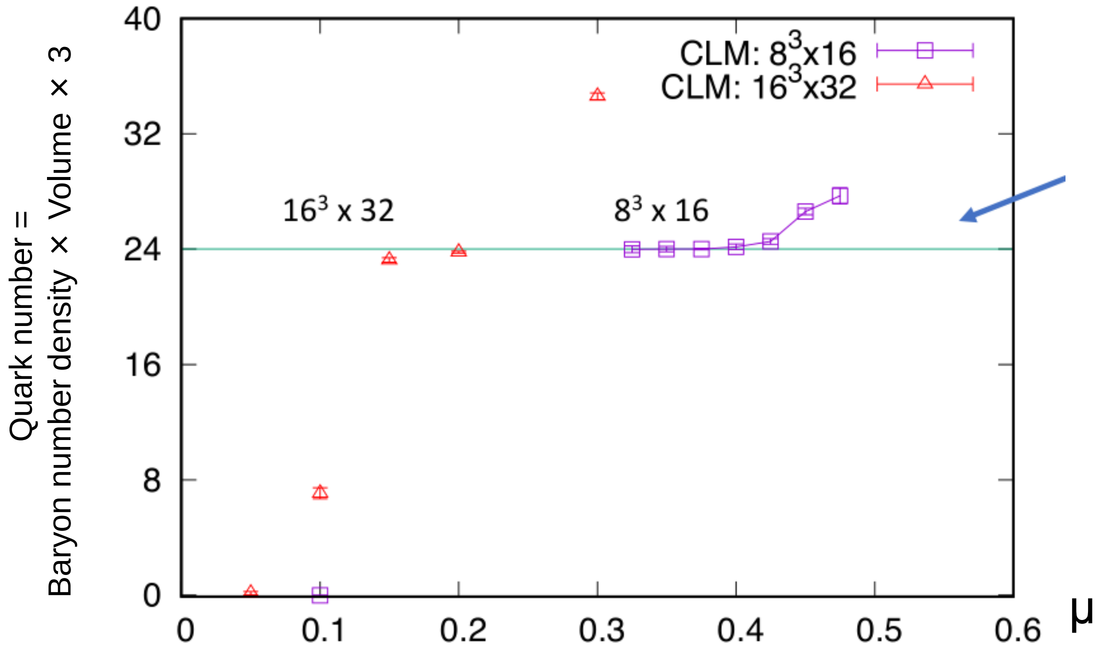
$\langle n \rangle = 0.016\ (8^3 \times 16)$,
$\langle n \rangle = 0.002\ (16^3 \times 32)$

In physical unit,
$\langle n \rangle = 171.5\ \mathrm{fm}^{-3}\ (8^3 \times 16)$,
$\langle n \rangle = 21.4\ \mathrm{fm}^{-3}\ (16^3 \times 32)$

($\beta = 5.7$, $a \sim 0.045$ fm)

$\langle n \rangle$ is much higher than the normal nuclear density $\rho_0 = 0.17\ \mathrm{fm}^{-3}$

Increase the lattice size further



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(β = 5.7, a ~ 0.045 fm)

Speculation: quark number remains to be 24 until <n> decreases down to ρ_0 (around Ns = 80)

- If the volume is sufficiently large, the quark number should be (quark num.) = 3 × (Volume) × ρ_0
  $\sim 4.6 \times 10^{-5} \times Ns^3$ … ← $\rho_0 = 0.17$ fm^{-3}, (Volume) = $(Ns\ a)^3$
- (quark num.) = 24 ⇔ $Ns^3 = 80^3$

Summary

- We have extended our our previous work on a $8^3 \times 16$ lattice to a $16^3 \times 32$ lattice with the same beta to reduce finite size effects.
- The plateau of the baryon number starts at much lower chemical potential. The energy cost for creating a baryon seems to be reduced considerably due to the increase of lattice size.
- The height of the plateau corresponds to 24 quarks (8 baryons), which turns out to coincide with our previous result on a $8^3 \times 16$ lattice.
- Eventually, the height should grow linearly with the lattice volume since the nuclear density should be reproduced from the height.
- What happens if we increase the lattice size further? Speculation: the height of the plateau remains to be the same baryon number until the density decreases down to the nuclear density (around Ns=80, due to the chosen large beta=5.7).

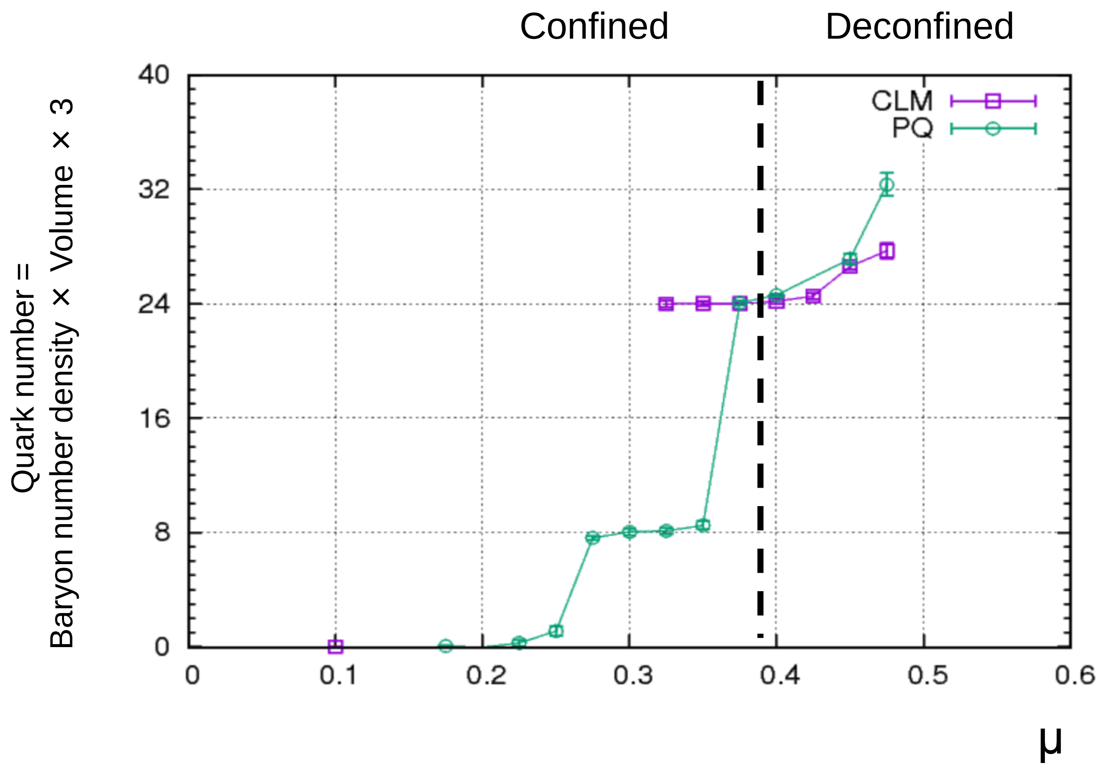
Future plans:

Increase lattice size, use more finer lattice (larger β) and study quark matter phase. For instance, can we approach to color-superconductor phase?

Appendix

Phase quenched simulation

[ST, Ito, Matsufuru, Nishimura, Shimasaki, Tsuchiya' 18]



In the phase quenched simulation, μ is interpreted as the isospin chemical potential due to

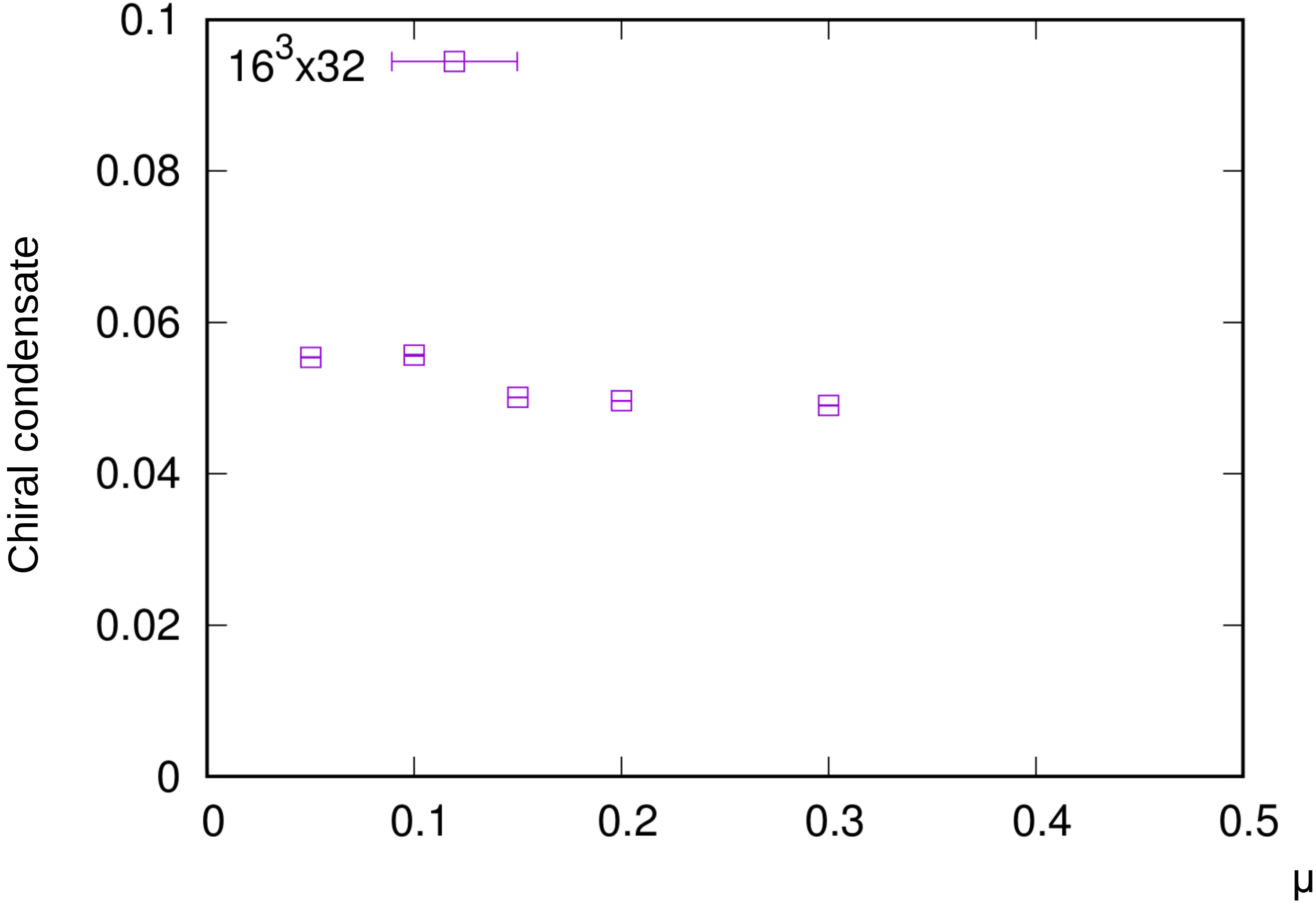
$$\det D(-\mu) = (\det D(\mu))^*$$

Therefore, the number density represents

$$\langle n \rangle = \langle (n_u - n_d)/3 \rangle$$

Number density of up and down quarks

Results on $16^3 \times 32$ lattice



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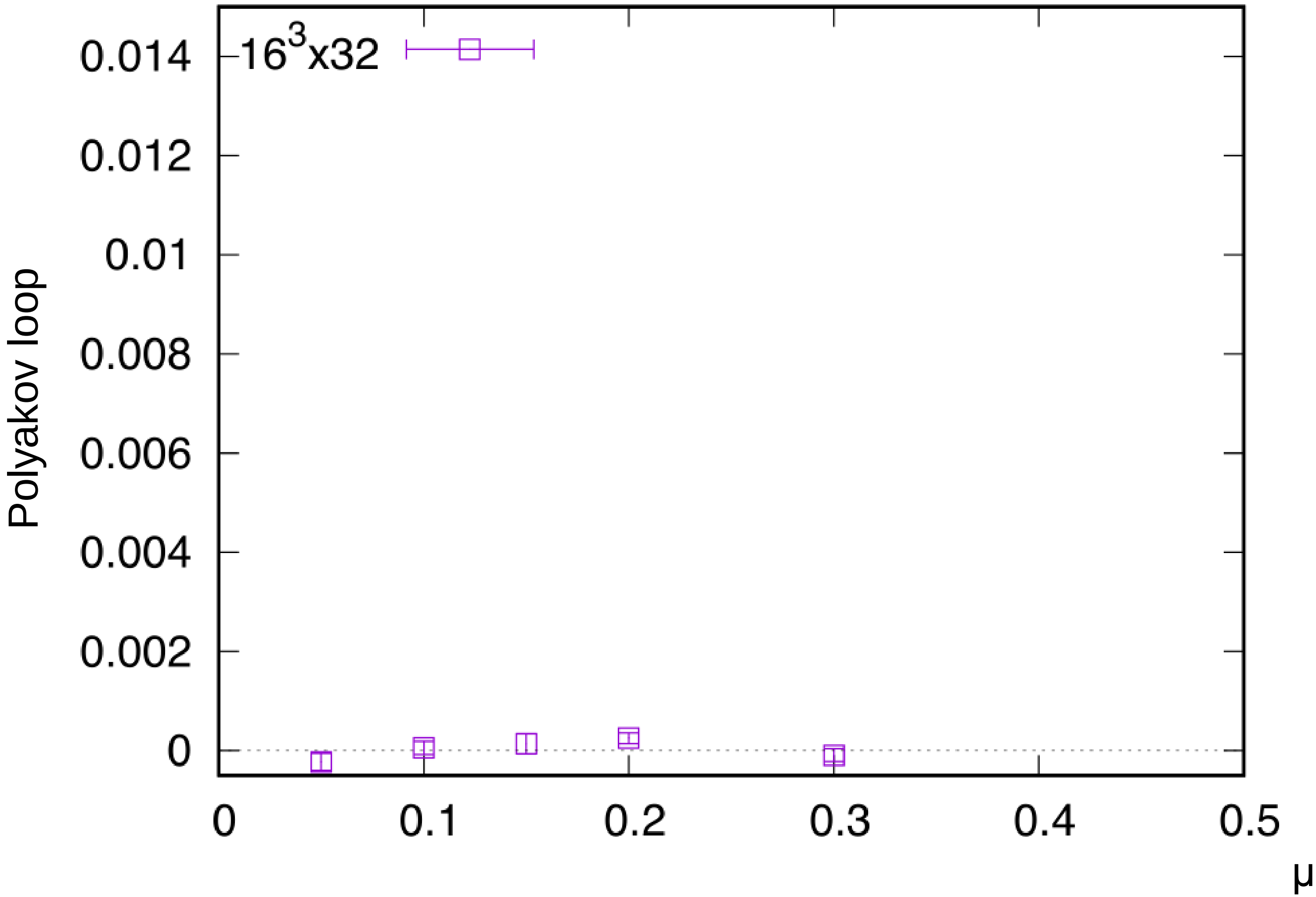
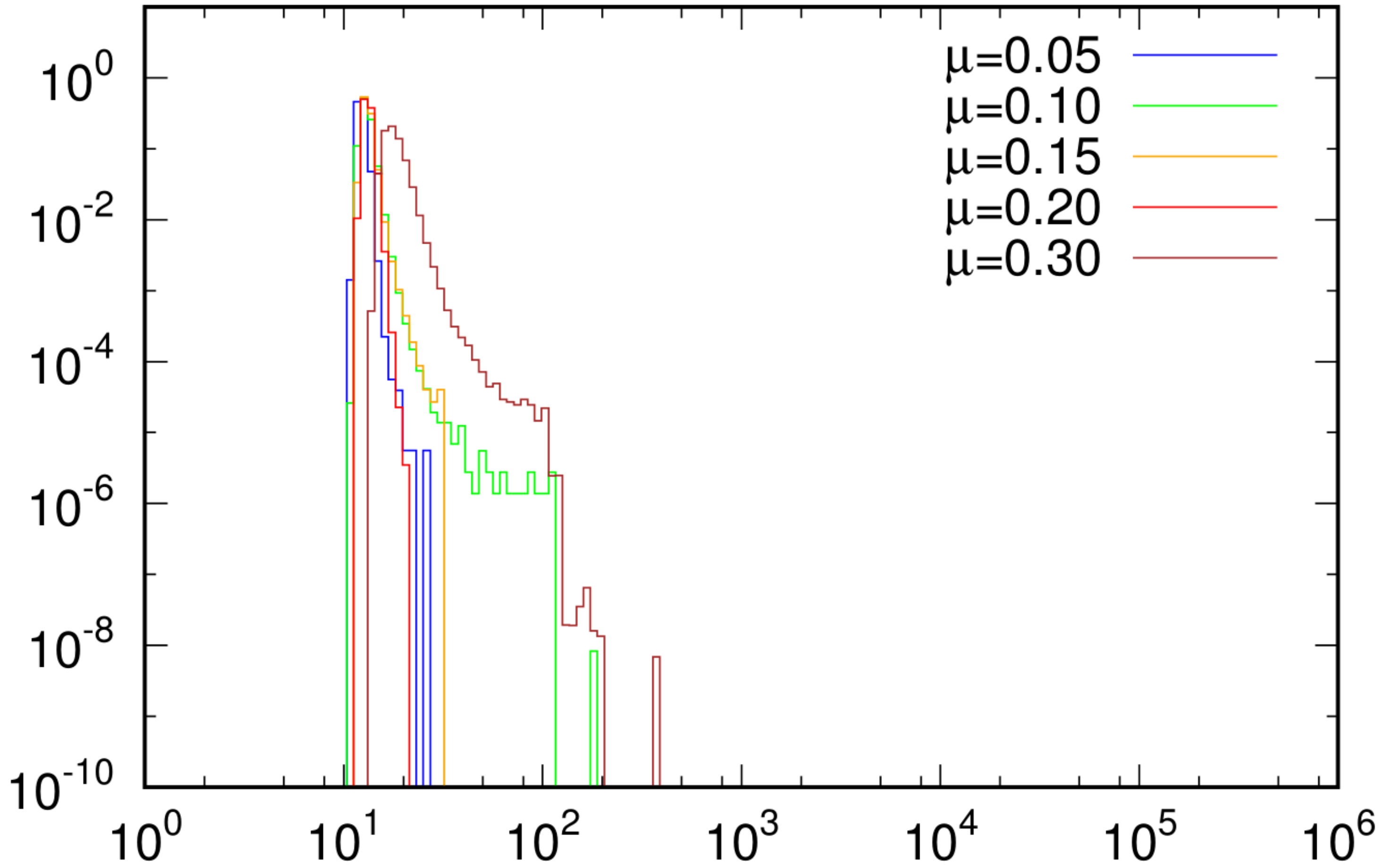
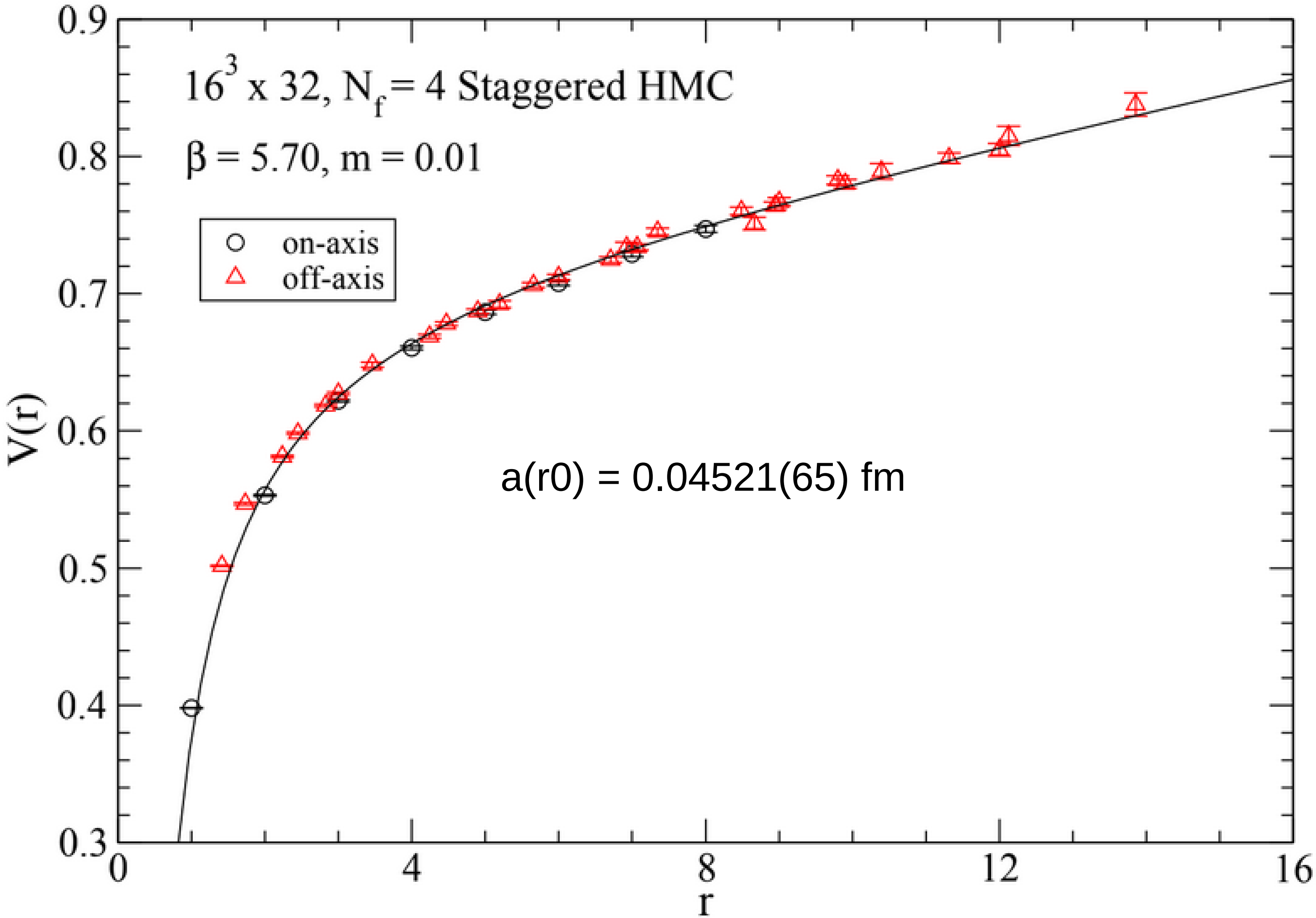


Results on $16^3 \times 32$ lattice



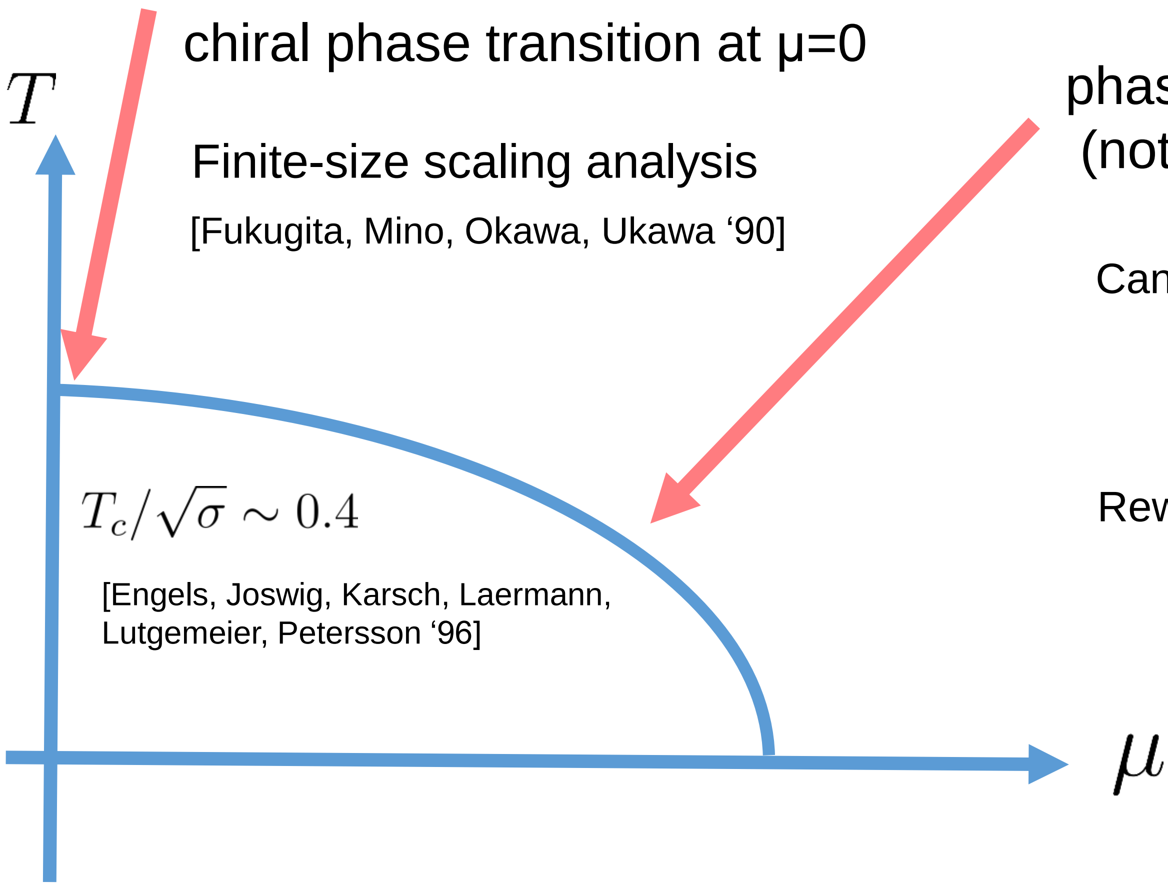

16^3 x 32, N_f = 4 Staggered HMC
β = 5.70, m = 0.01
on-axis
off-axis
V(r)
r
a(r0) = 0.04521(65) fm

Phase diagram of QCD with 4-flavor staggered fermion

1st order
chiral phase transition at μ=0

Finite-size scaling analysis

[Fukugita, Mino, Okawa, Ukawa '90]



$T_c/\sqrt{\sigma} \sim 0.4$

[Engels, Joswig, Karsch, Laermann, Lutgemeier, Petersson '96]

phase transition at finite μ
(not completely established)

Canonical method

[de Forcrand, Kratochvila '06]
[Li, Alexandru, Liu, Meng '10]

Reweighting and complex Langevin

[Fodor, Katz, Sexty, Torok '15]