Continuous Time Simulations of Strong Coupling LQCD at Finite Baryon Density

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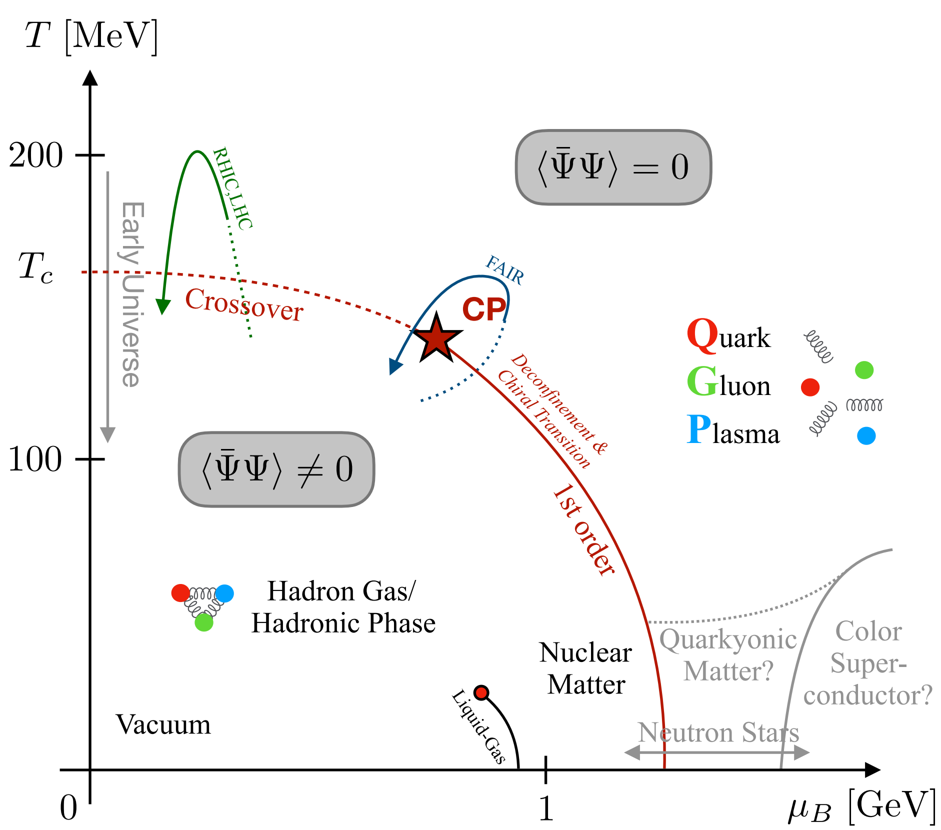
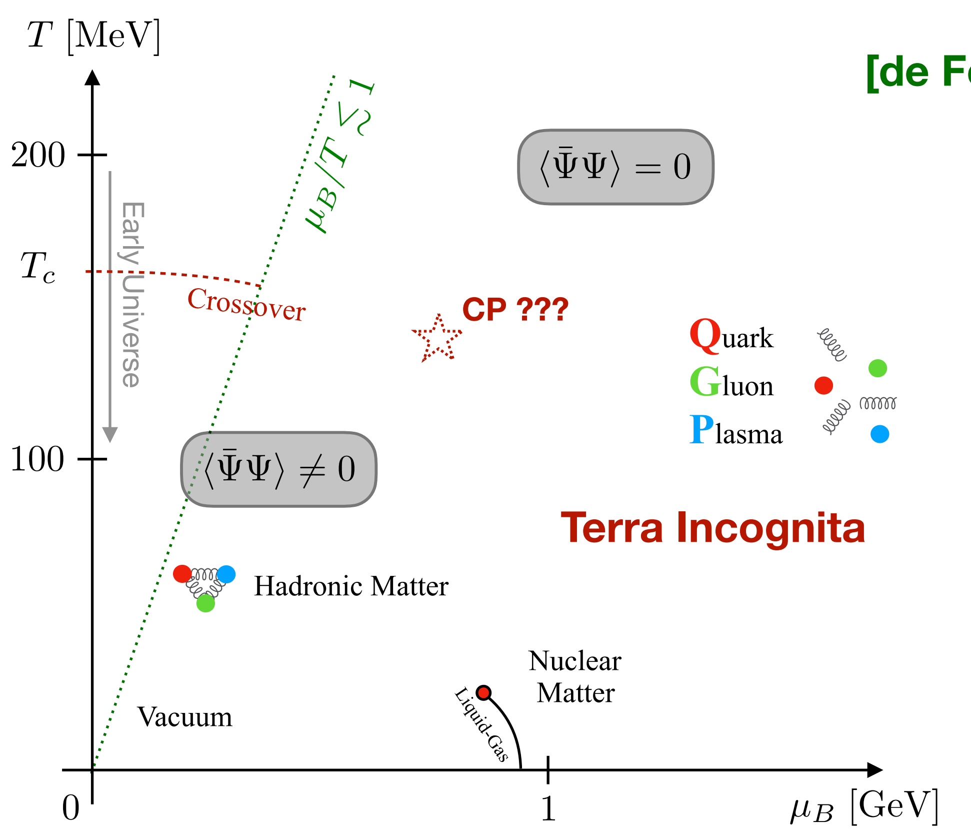
Motivation: QCD Phase Diagram and Sign Problem



Educated guess



Measurement

[de Forcrand et. al., 2015]

- **Sign/complex phase problem**
- **Access to T/μ_B-plane via: Reweighting, *Taylor series expansion*, Imaginary μ_B ...**

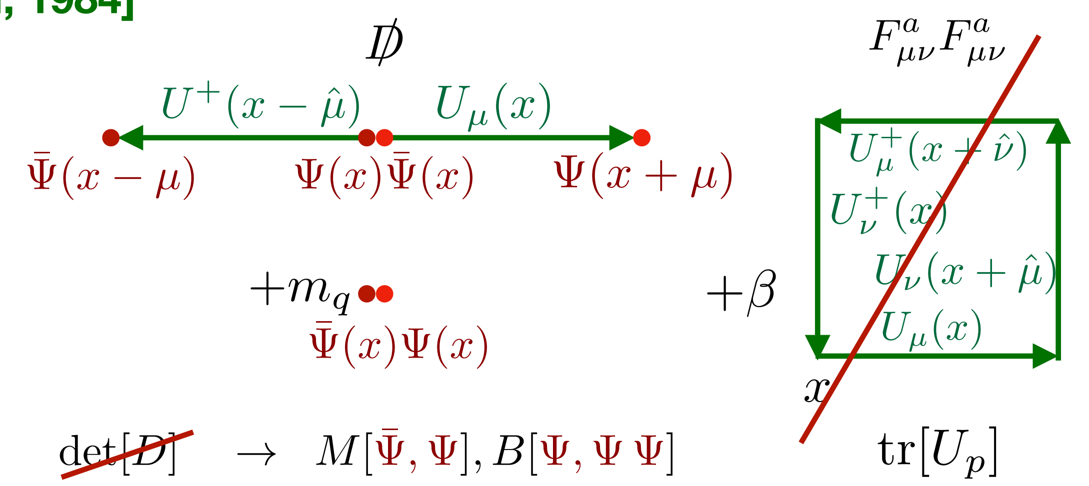
Tackle sign problem: Lefshetz Thimbles, Complex Langevin, Dual representations ...

Sign problem is representation dependent!

For lattice QCD, a dual representation is well known in the strong coupling limit

QCD in the Strong Coupling Limit

Study regime with a mild *sign problem*: [Wolff & Rossi, 1984]



Strong Coupling Limit: $g \rightarrow \infty, \beta = \dfrac{2N_c}{g^2} \rightarrow 0$

- **Change of integration order** → **gauge fields $U_\mu(x)$ first!**

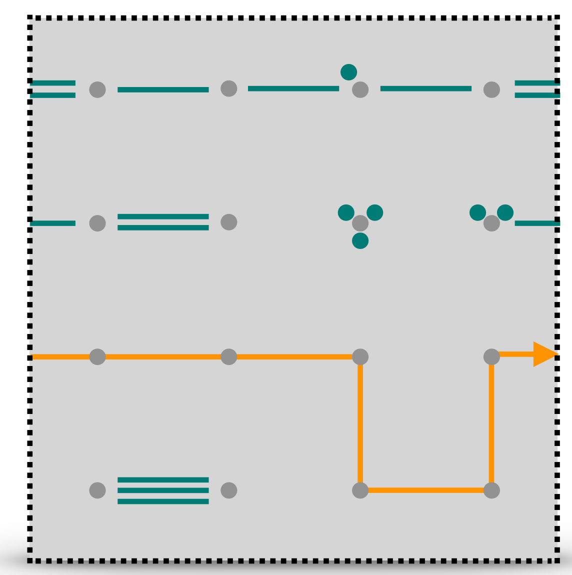
new dual degrees of freedom:

- **bosonic ("mesonic"(M)) and fermionic ("baryonic"(B)) color-singlet states**

SC-partition function for *staggered fermions*:

$$\mathcal{Z}_{SC} = \sum_{\{n,k,l\}} \underbrace{\prod_x \frac{N_c!}{n_x!}(2am_q)^{n_x}}_{\text{monomers}} \underbrace{\prod_{b=(x,\mu)} \frac{(N_c-k_b)!}{N_c!k_b!}\gamma^{2k_b\delta_{\mu 0}}}_{\text{mesonic hoppings/dimers}} \underbrace{\prod_l w(l,\mu)}_{\text{baryonic hoppings}}$$

Chiral Limit **Mesonic** **Baryonic**



QCD in the Strong Coupling Limit

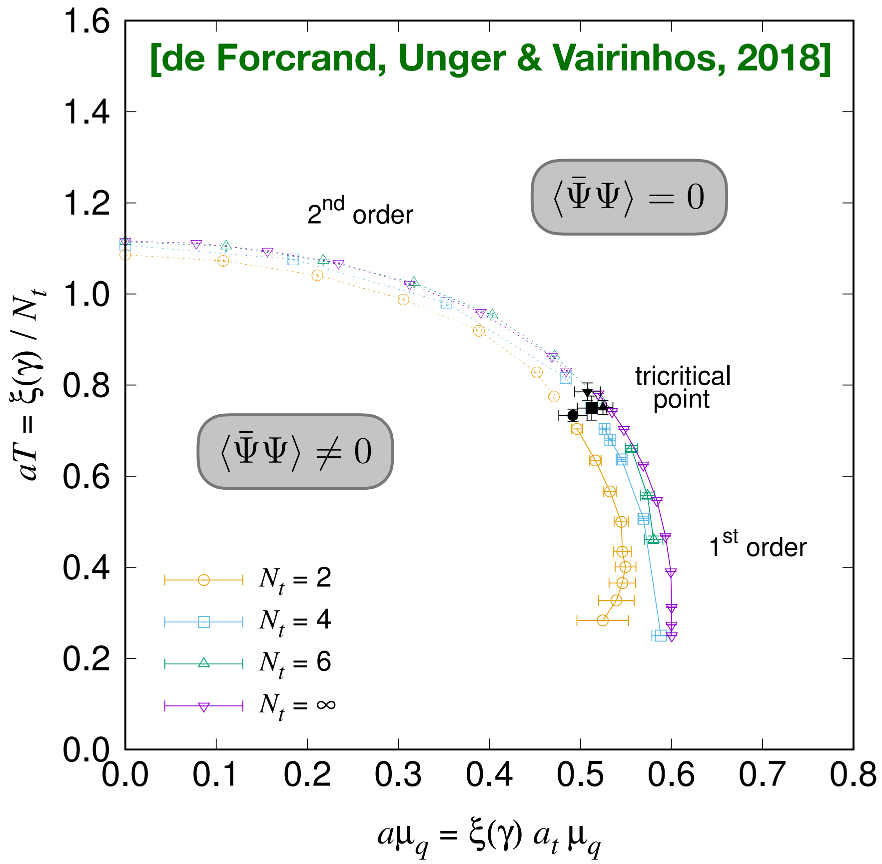
Advantage:

- (almost) no sign problem
- Integer based algorithm (fast)

Restriction by *Grassmann constraint*: $n_x + \sum_{\pm\mu} k_{x\mu} = N_c, \quad \sum_{\pm\mu} l_{x\mu} = 0, \quad \forall x$

Exactly Nc — Closed loops

"complete" phase diagram can be calculated



Features:

- Chiral symmetry breaking
- (Tri)-critical endpoint

Downside:

- maximally coarse lattice (opposite of continuum limit)
- difficult to include gauge corrections $\beta > 0$ to make lattice finer

Continuous Time Limit within Strong Coupling QCD

No discretization errors due to finite N_τ and work sign problem free

1st: Introduction of *anisotropy* for continuous temperature variation:

$$aT = \frac{1}{N_\tau} \Rightarrow aT = \frac{\xi(\gamma)}{N_\tau}, \qquad \underbrace{\xi(\gamma) = a/a_\tau}_{\text{anisotropy parameter}}$$

2nd: Gamma dependence of $\xi(\gamma)$ non trivial: [de Forcrand, Unger & Vairinhos, 2018]

$$\xi(\gamma) \approx \kappa\gamma^2 + \frac{\gamma^2}{1+\lambda\gamma^4} \qquad , \kappa = 0.781 \text{ for SU}(3)$$

Definition of the *Continuous Time Limit* as:

$$N_\tau \to \infty,\ \gamma \to \infty,\ \text{with } \frac{\xi(\gamma)}{N_\tau} = \frac{\kappa\gamma^2}{N_\tau} = aT \text{ fixed}$$

Continuous Time Limit within Strong Coupling QCD

Simplified partition function:

$N_c = 3:$

$$Z_{CT}(T) = \sum_{k\in 2\mathbb{N}} \left(\frac{1}{2aT}\right)^k \sum_{\mathcal{G}'\in\Gamma_k} e^{\mu_B B/T} \hat{\nu}_{\mathcal{T}}^{N_{\mathcal{T}}}$$

$$\text{with } k = \sum_{b=(x,\hat{I})} k_b, \; N_{\mathcal{T}} = \sum_x n_{\mathcal{T}}(x)$$

$$\hat{\nu}_{\mathcal{T}} = \frac{2}{\sqrt{3}} :$$

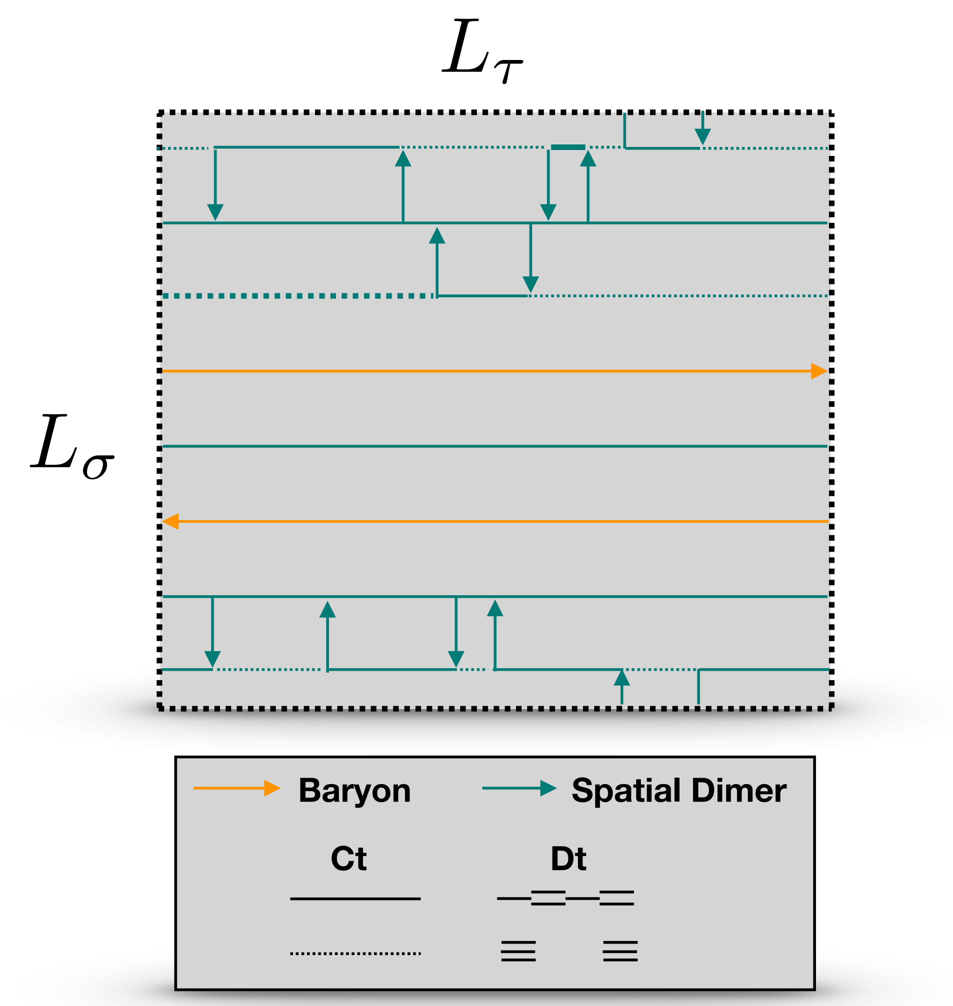
$$\hat{\nu}_L = 1 :$$

Comments:

- Only one parameter left (temperature T)
- Baryons become static (non-relativistic, but finite mass)

Sign problem is absent

- No multiple spatial dimers
- Simpler dual observables



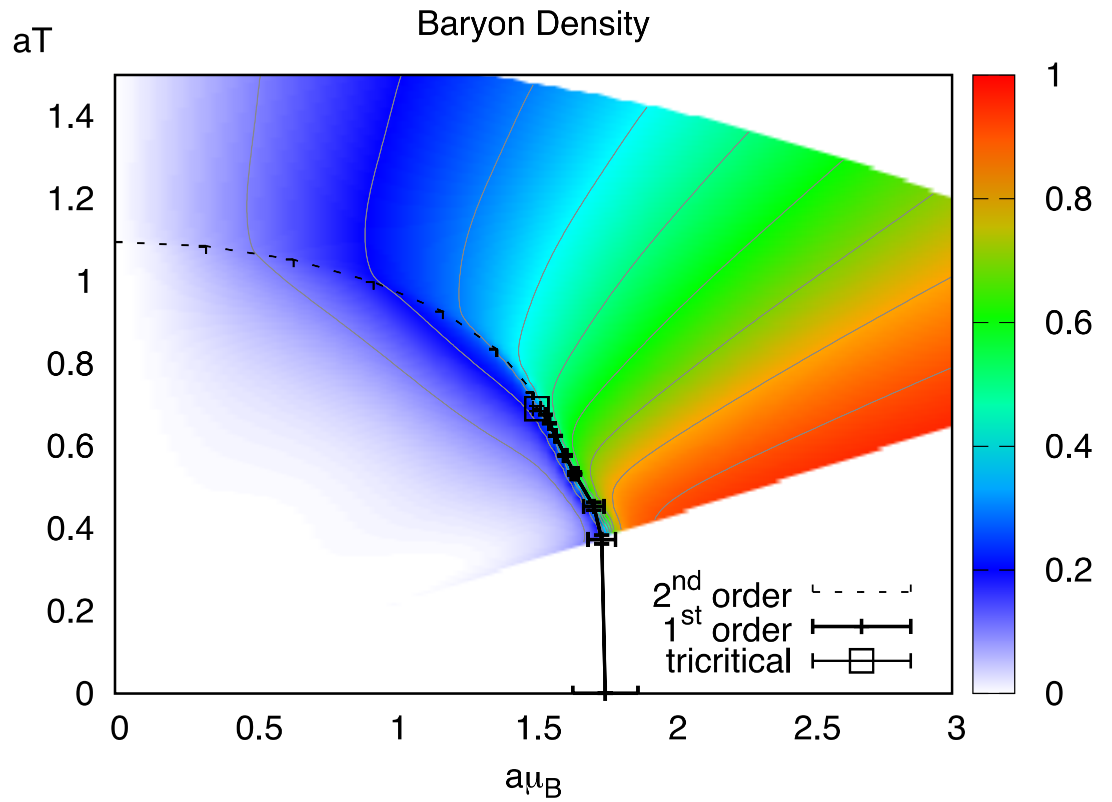
Observables from CT partition function

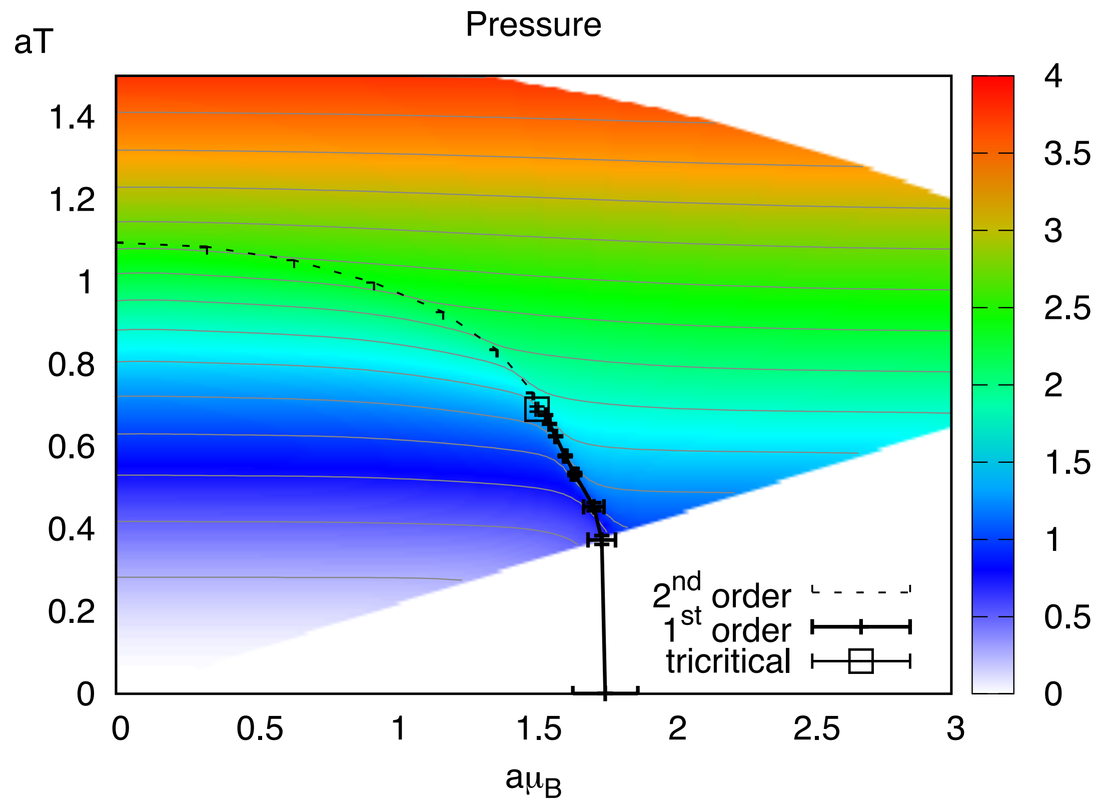
- **Sample *two-point correlation functions*** $C(\tau, \vec{x})$
  - **Obtain chiral susceptibility**
  - **Construct temporal correlators**

  ⟶ **extract *pole masses* (possible: transport coefficients)** **[arXiv:1811.01614]**

- **Baryon density:** $n_B = \frac{\langle r_l \rangle}{V_\sigma}$
  - **Simple for static baryons in CT limit:** $n_B = \frac{\langle B - A \rangle}{V_\sigma}$

- **Pressure/energy density(** $\epsilon - 3p = 0$ **):** $p \sim -\langle n_{D_\sigma} \rangle$





Taylor expansion of the pressure

- Apply **Taylor expansion method** to reconstruct finite density pressure results measured in **dual variable.** → **Validity of method in dual variables**

- **Taylor expansion of the pressure**

$$\Delta p = p(T,\mu_B) - p(T,0) = \frac{T}{V}\sum_{n=1}^{\infty}\frac{1}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n} \left.\frac{\partial^{2n}\log\mathcal{Z}}{\partial(\mu_B/T)^{2n}}\right|_{\mu_B=0}$$

$$= \frac{T}{V}\sum_{n=1}^{\infty}\frac{1}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n} \kappa_{2n}(\omega_l)\cdot V_\sigma^{2n}$$

cumulants of winding number

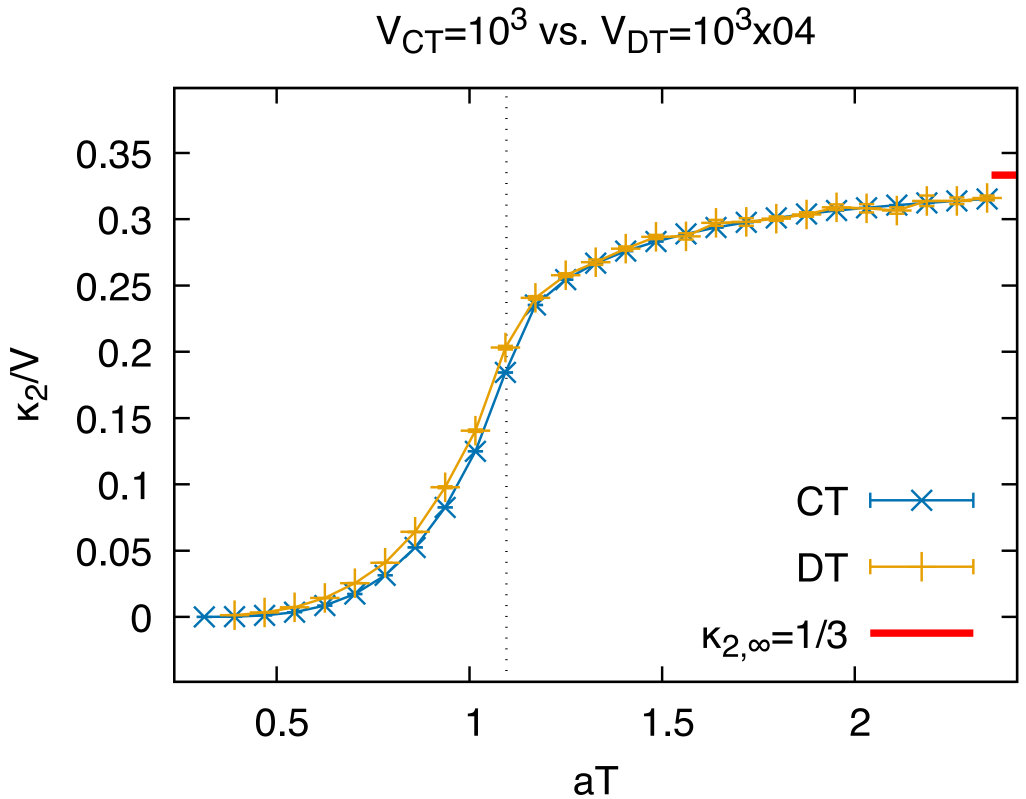
- **Measure baryon density cumulants:**

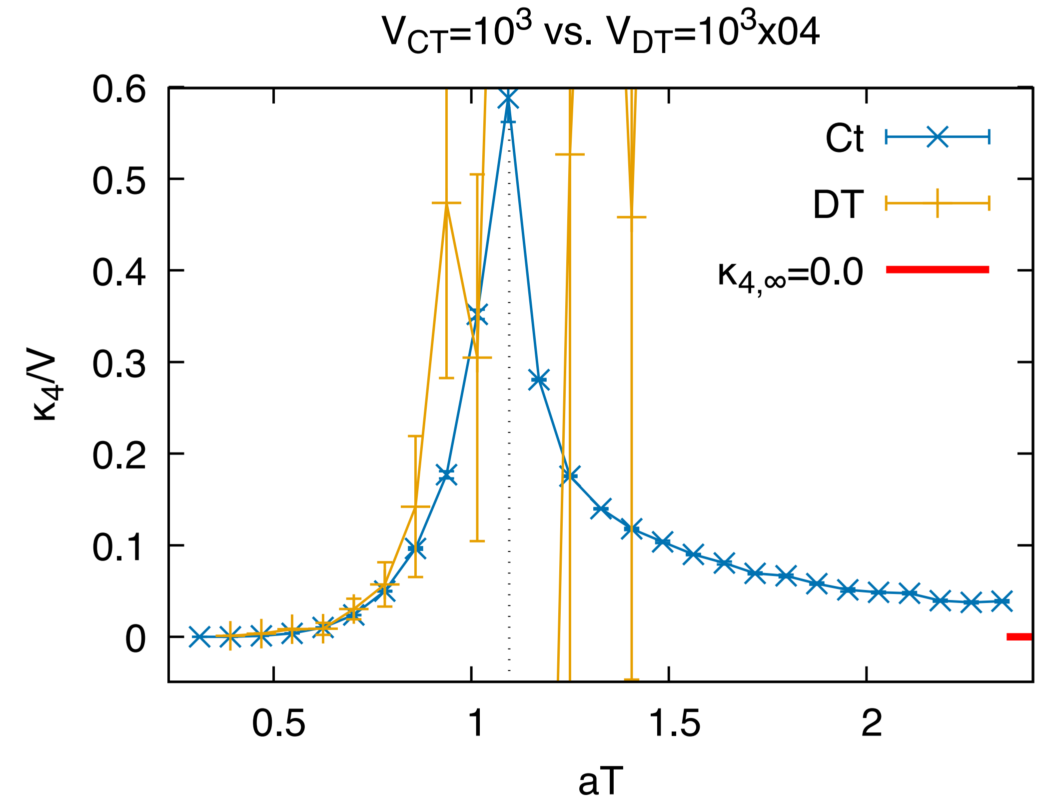
$$n_B^i = \frac{\langle (B-A)^i\rangle}{N_\sigma^3} = \langle \omega_l^i\rangle = \mu_i(\omega_l)$$

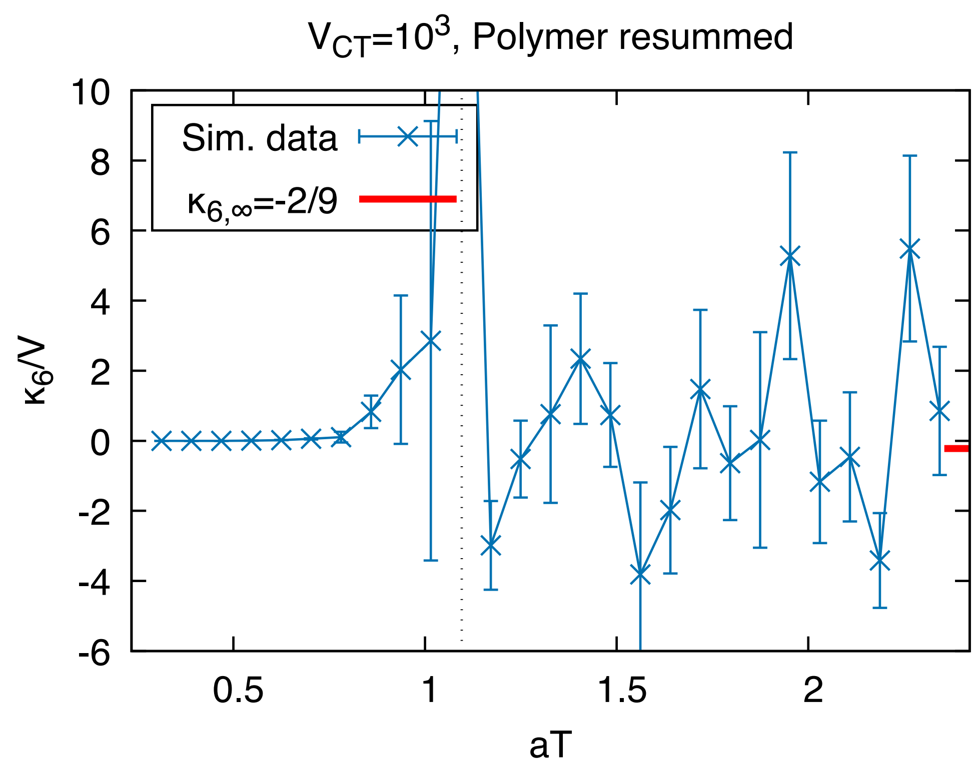
- **Finally we obtain:**

$$\kappa_2(\omega_l) = \mu_2 - \mu_1^2$$
$$\kappa_4(\omega_l) = -6\mu_1^4 + 12\mu_1^2\mu_2 - 3\mu_2^2 - 4\mu_1\mu_3 + \mu_4$$
$$\ldots$$

Results: Taylor coefficients

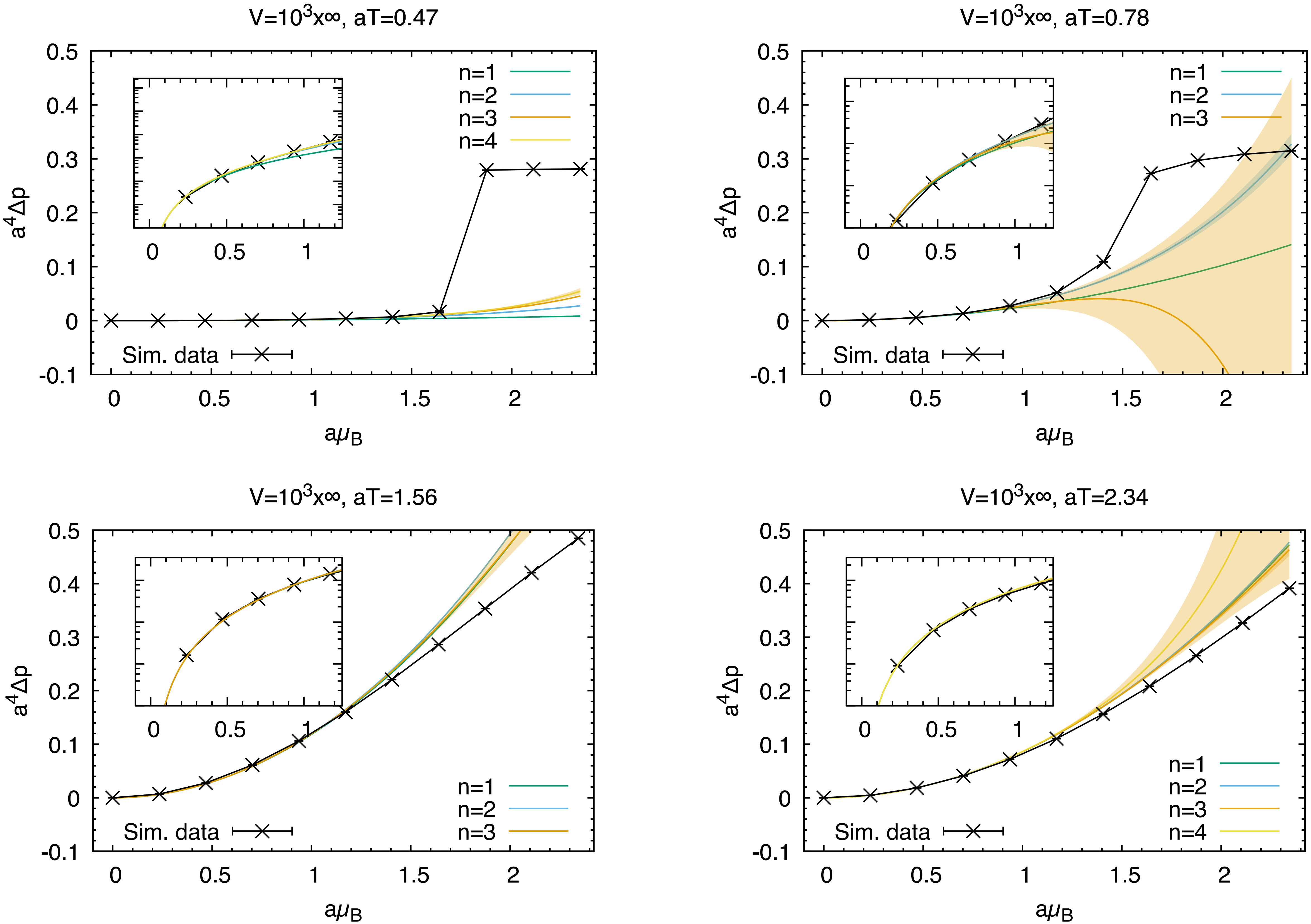






κ_6 noisy, but possible to get

Results: Taylor expansion vs. CT Finite muB



Polymer Site Histograms

- **Improve on accuracy by measurement of histograms in a Polymer resummation scheme**
- **The polymer weight encodes static dimers and static baryons as:**

$$\omega_p = 2\cosh(\mu_B B/T) + N_c + 1$$

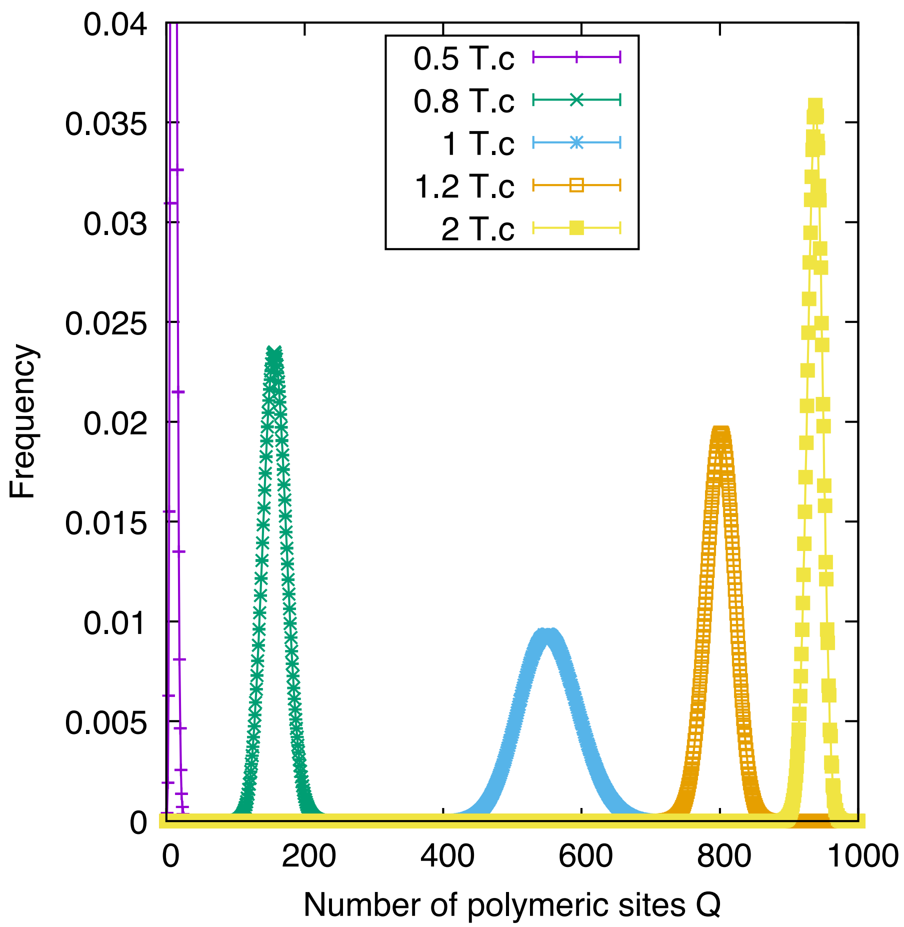
$P : B, A$

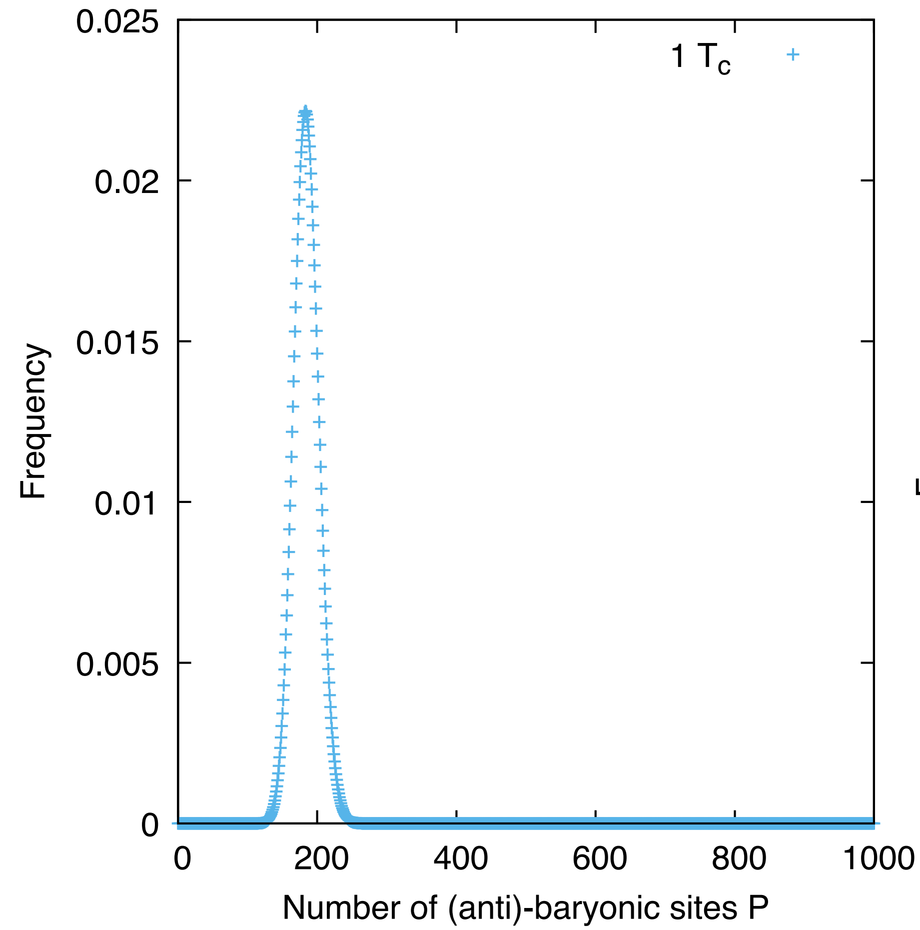
$Q : B, A, D$

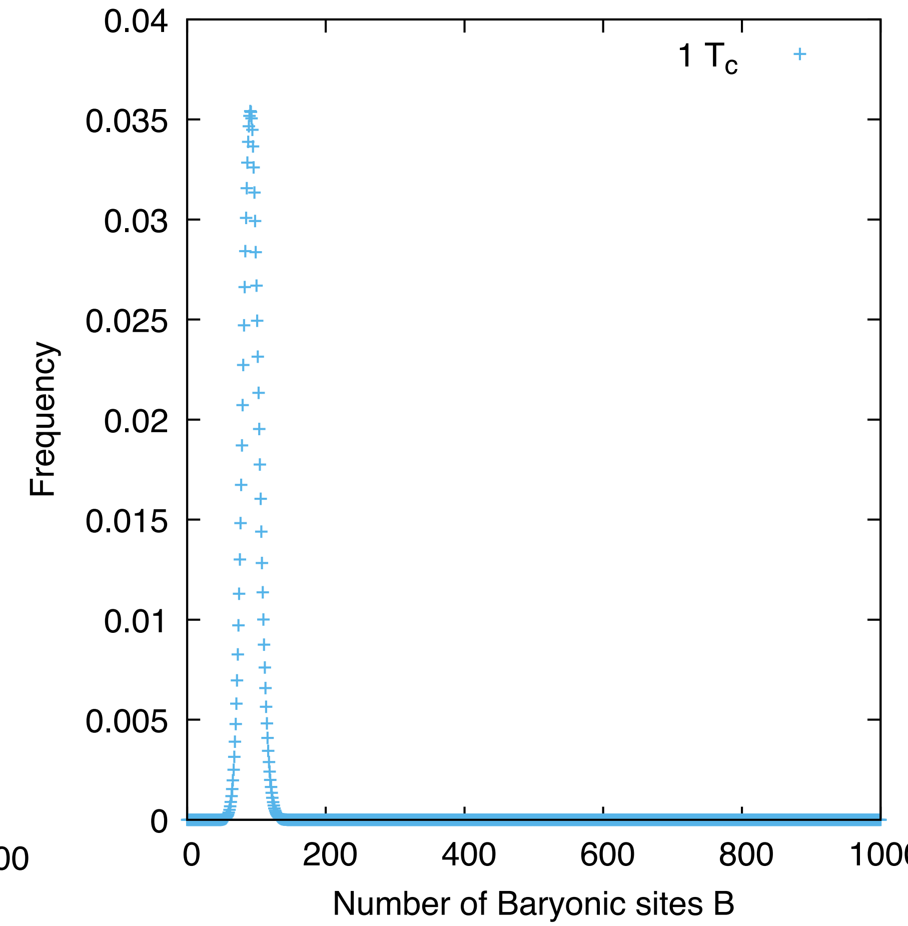
- **Measure histograms $H_{V,T,\mu_B}(Q)$ in polymer number**
- **Extract baryon density moments fully analytically! (Only histogram is measured!)**

$$h_{V,T,\mu_B}(Q,P) = H_{V,T,\mu_B}(Q) \binom{Q}{P} (N_c+1)^{Q-P} \left(\frac{2\cosh \mu_B/T}{2\cosh(\mu_B/T + N_c + 1)} \right)^P$$

Polymer Site Histograms



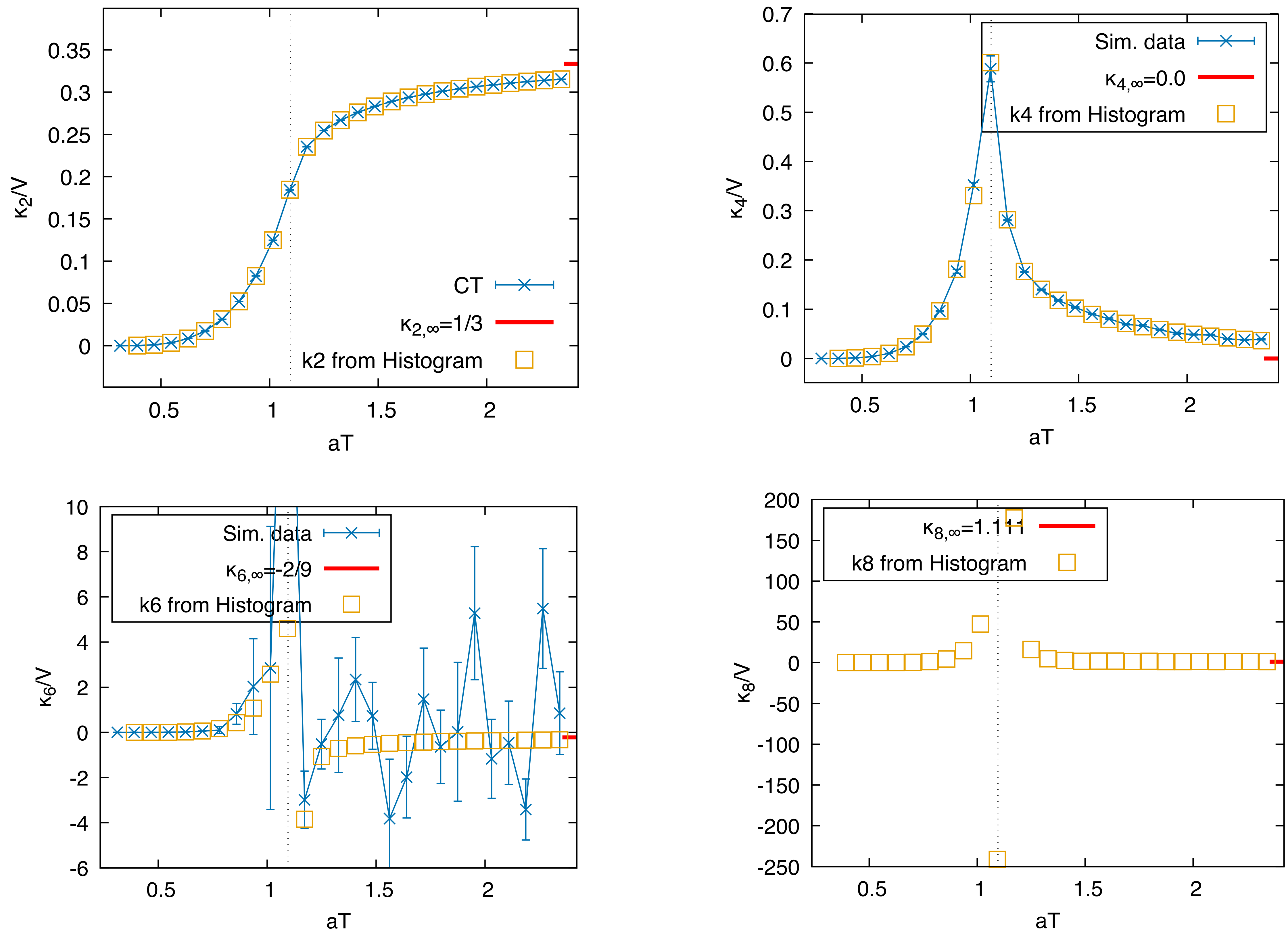




- **Reconstruct functions $\langle f(B, P-B)\rangle$ that make up the baryon density moments $\mu_n(\omega_l)$**

$$\langle f(B, P-B)\rangle_{V,T,\mu_B} = \underbrace{\left(\sum_Q h_{V,T,\mu_B}(Q,P)\right)}_{H_{V,T,\mu_B}(P)} q_{\mu_B/T}(B,P) f(B, P-B)$$

Results: Taylor Coefficients from Histograms



Summary and Outlook

Summary

- Continuous Time formalism naturally introduces static baryons

→ Static baryons improve accuracy of coefficients κ_{2n}, but computational effort too great to go beyond κ_6 on $V = 10^3 \times \infty$ lattice

- Histograms with full polymer resummation (Q) greatly improve Taylor coefficients

Taylor

- Error propagation for Histograms …
- Data available for $\kappa_{10} \ldots \kappa_{16}$
- Radius of convergence

General

- Ct finite beta corrections
- Finite quark mass
- …

Thank you

Backup

Concept: Polymer resummation

Usual algorithm:

1. Mesonic update + **2. Baryonic update**

Static lines either mesonic or (anti)-baryonic

$N_c + 1$ mesonic line types with weight: $\omega_m = 1.0$

(anti)-baryonic line types with weight: $\omega_\pm = e^{\pm\mu_B B/T}$

First resummation scheme:

Resummation in (anti)-baryon sector: $\omega_b = 2\cosh(\mu_B B/T)$ $\rightarrow$ No distinction

Second resummation scheme::

Resummation in both sectors: $\omega_p = 2\cosh(\mu_B B/T) + N_c + 1$

Benefits: More accurate algorithm?

Concept: Polymer resummation