Non-perturbative study of heavy $Q\bar{Q}$ potential at finite temperature

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Quarkonia provides an important tool to probe quark-gluon plasma.
One way to study quarkonia is to define an in medium potential.
The potential is useful for the study of quarkonia states at finite temperature e.g. modification of quarkonia peak in dilepton production process.
We consider the quarkonium operator $M(r, t_m)$ in singlet channel. $D(r, t_m) = \langle M^\dagger(r, t_m)M(r, 0) \rangle$ in the heavy quark limit satisfy Schrodinger equation

$$\left[-\nabla^2 + V(r)\right]D(r, t_m) = i \frac{\partial D(r, t_m)}{\partial t_m}$$

$$V(r) = i \lim_{t_m \to \infty} \frac{\partial \log(w_m(r, t_m))}{\partial t_m}$$

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$w(r, t) = \int e^{-\omega t} \rho(r, \omega)$ and $w_m(r, t_m) = \int e^{-i\omega t_m} \rho(r, \omega)$


Here we will give another method to extract the potential.
We have used anisotropic lattice pure gauge action

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\xi_b$</th>
<th>$a_t$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.64</td>
<td>2.55</td>
<td>$\frac{1}{45T_c}$</td>
<td>3</td>
</tr>
<tr>
<td>6.35</td>
<td>2.51</td>
<td>$\frac{1}{30T_c}$</td>
<td>3</td>
</tr>
<tr>
<td>6.04</td>
<td>2.44</td>
<td>$\frac{1}{19T_c}$</td>
<td>3</td>
</tr>
</tbody>
</table>

We change the temperature from $0.75T_c$ to $2.0T_c$ by varying the the temporal extent of the lattice.

$U_{x,x+i} = \alpha U_{x,x+i} + staple$

where $\alpha = 2.5$
We can write the smeared Wilson loop as

$$\log(w(r, t)) = \frac{1}{2} \log\left( \frac{w(r, t)}{w(r, \beta - t)} \right) + \frac{1}{2} \log(w(r, t)w(r, \beta - t))$$

From perturbation theory we can see

$$A(r, t) = \frac{1}{2} \log\left( \frac{w(r, t)}{w(r, \beta - t)} \right) = \left( \frac{\beta}{2} - t \right) V_r(r)$$

Method and Results

Non-perturbative study of heavy $Q\bar{Q}$ potential at finite temperature
Variation of real part with smearing
Comparison with free energy. $F_s(r, \beta) = -\frac{\log(w(r, \beta))}{\beta}$
\[ P(r, t) = \frac{1}{2} \log(w(r, t)w(r, \beta - t)) = \int_0^\infty (e^{-\omega t} + e^{-\omega(\beta - t)})\rho_p(r, \omega) \]

if

\[ \lim_{t_m \to \infty} \frac{\partial P(r, t_m)}{\partial t_m} = \text{constant} \]

then

\[ \rho_p(r, \omega) = \frac{V_i(r)}{\pi \omega^2} + \ldots \]

which implies

\[ \frac{\partial P(r, t)}{\partial t} = \frac{V_i(r)}{\pi} \log\left(\frac{t}{\beta - t}\right) + \ldots \]

This equation is supported by perturbation theory.
Fitting with the function
Variation of imaginary part with smearing
Variation of real part with lattice spacing at $1.2T_c$. 

\begin{align*}
(a_t)^{-1} &= 45T_c \\
(a_t)^{-1} &= 30T_c \\
(a_t)^{-1} &= 19T_c
\end{align*}
- Variation of imaginary part with lattice spacing at 1.2Tc.
- Potential at different temperature
Method and Results

Potential at different temperature

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In short

\[
\frac{\partial \log(w(r, t))}{\partial t} = -V_r(r) + \frac{V_i(r)}{\pi} \log\left(\frac{t}{\beta - t}\right) + \ldots
\]

\[
V(r) = \lim_{t_m \to \infty} i \frac{\partial \log(w(r, it_m))}{\partial t_m} = V_r(r) - iV_i(r)
\]
In the QGP the quark antiquark pair can also be in an octet state.

For octet state
\[ M(r = |\vec{x} - \vec{y}|, t_m) = \bar{\psi}(\vec{x}, t_m)U(\vec{x}, \vec{z}; t_m)T^a U(\vec{z}, \vec{y}; t_m)\psi(\vec{y}, t_m). \]

We can not implement this operator directly in lattice.

To make it gauge invariant we use the following operator
\[ M(r = |\vec{x} - \vec{y}|, t_m) = \bar{\psi}(\vec{x}, t_m)U(\vec{x}, \vec{z}; t_m)T^a B_a(z)U(\vec{z}, \vec{y}; t_m)\psi(\vec{y}, t_m). \]

However the potential should not depend on any particular gluonic operator.
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Conclusion

- We proposed a method to obtain the potential directly from smeared thermal wilson loop.
- Real part of this potential is screened and close to the singlet free energy.
- Continuum limit has been shown both for real and imaginary part.
- Real part of octet potential has been obtained and it does not show any increasing behaviour.
\[ M(r = |\vec{x} - \vec{y}|, t_m) = \bar{\psi}(\vec{x}, t_m) U(\vec{x}, \vec{y}; t_m) \psi(\vec{y}, t_m) \]

\[ S_L = \frac{\beta_s}{3} \sum_x \sum_{i > j \atop i \neq 4} \text{Re } \text{Tr}(1 - P_{ij}(x)) + \frac{\beta_t}{3} \sum_x \sum_{i \neq 4} \text{Re } \text{Tr}(1 - P_{4i}(x)) \]