Non-perturbative study of heavy $Q\bar{Q}$ potential at finite temperature

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outline

- Quarkonia provides an important tool to probe quark-gluon plasma.
- One way to study quarkonia is to define an in medium potential.
- The potential is useful for the study of quarkonia states at finite temperature e.g. modification of quarkonia peak in dilepton production process.

We consider the quarkonium operator $M(r, t_m)$ in singlet channel.
$D(r, t_m) =< M^\dagger(r, t_m)M(r, 0) >$ in the heavy quark limit satisfy Schrodinger equation

$$[-\frac{\nabla^2}{m_Q} + V(r)]D(r, t_m) = i\frac{\partial D(r, t_m)}{\partial t_m}$$

$$V(r) = i \lim_{t_m \to \infty} \frac{\partial log(w_m(r, t_m))}{\partial t_m}$$

M. Laine et al JHEP03(2007)054

$w(r, t) = \int e^{-\omega t}\rho(r, \omega)$ and $w_m(r, t_m) = \int e^{-i\omega t_m}\rho(r, \omega)$

A. Rothkopf,T. Hatsuda & S. Sasaki, Phys. Rev. Lett, 108, 162001 (2012)
Y. Burnier,O. Kaczmarek & A. Rothkopf, Phys. Rev. Lett, 114, 082001 (2015) BR
P. Petreczky & J. Weber, Nucl Phys A 00 (2018) 1-4. MODEL

Here we will give another method to extract the potential.

- We have used anisotropic lattice pure gauge action

β	ξ_b	a_t	ξ
6.64	2.55	$\frac{1}{45T_c}$	3
6.35	2.51	$\frac{1}{30T_c}$	3
6.04	2.44	$\frac{1}{19T_c}$	3

- We change the temperature from 0.75Tc to 2.0Tc by varying the the temporal extent of the lattice.
- $U_{x,x+i} = \alpha U_{x,x+i} + staple$
  where $\alpha = 2.5$

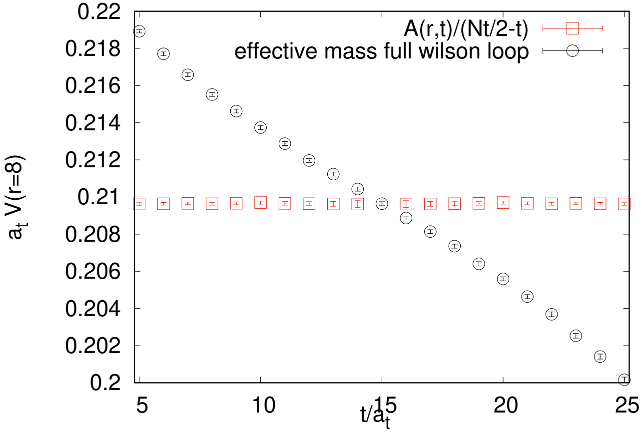
- We can write the smeared Wilson loop as

$$log(w(r,t)) = \frac{1}{2}log(\frac{w(r,t)}{w(r,\beta-t)}) + \frac{1}{2}log(w(r,t)w(r,\beta-t))$$

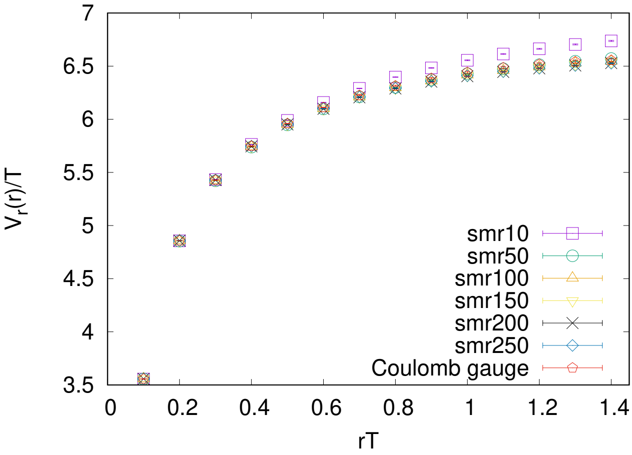
- From perturbation theory we can see

$$A(r,t) = \frac{1}{2}log(\frac{w(r,t)}{w(r,\beta-t)}) = (\frac{\beta}{2}-t)V_r(r)$$

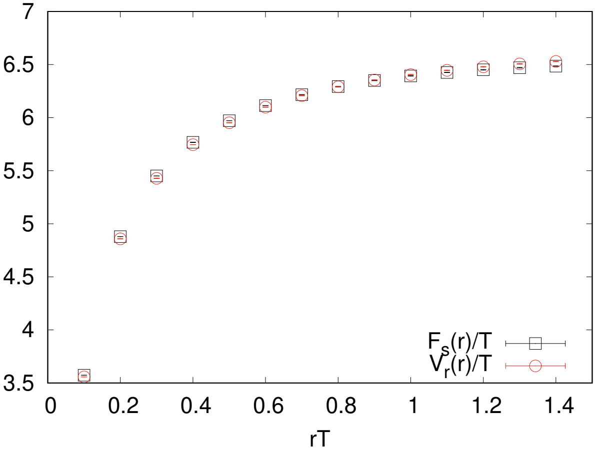
$Y.Burnier\&A.Rothkopf, Phys.Rev.D87, 114019(2013)$


A(r,t)/(Nt/2-t)
effective mass full wilson loop
a_t V(r=8)
t/a_t
0.22
0.218
0.216
0.214
0.212
0.21
0.208
0.206
0.204
0.202
0.2
5
10
15
20
25

- Variation of real part with smearing



Comparison with free energy. $F_s(r,\beta) = -\frac{log(w(r,\beta))}{\beta}$



$$P(r,t) = \frac{1}{2} log(w(r,t)w(r,\beta - t)) = \int_0^\infty (e^{-\omega t} + e^{-\omega(\beta - t)})\rho_p(r,\omega)$$

if

$$\lim_{t_m \to \infty} \frac{\partial P(r,t_m)}{\partial t_m} = constant$$

then

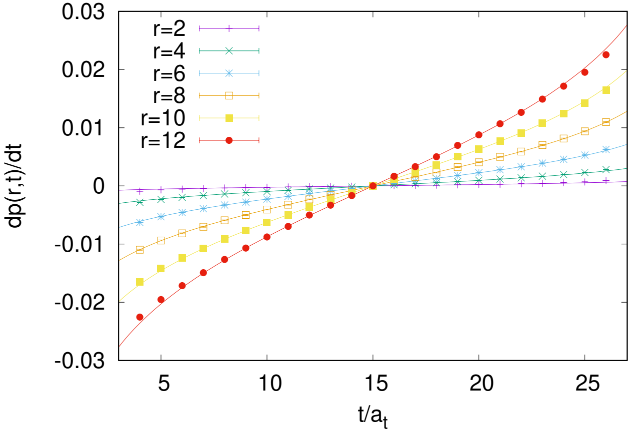
$$\rho_p(r,\omega) = \frac{V_i(r)}{\pi\omega^2} + \dots$$

which implies

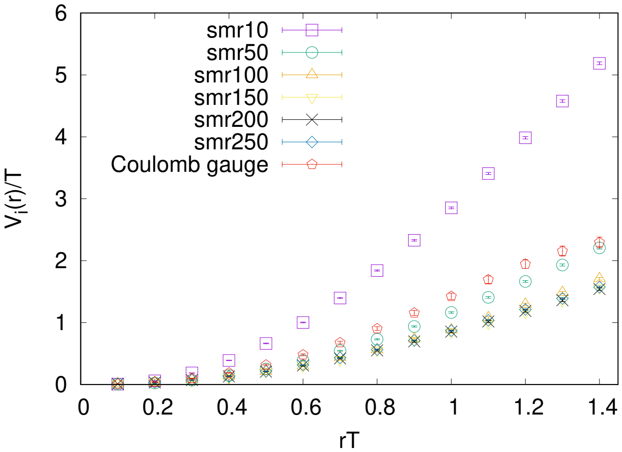
$$\frac{\partial P(r,t)}{\partial t} = \frac{V_i(r)}{\pi} log(\frac{t}{\beta - t}) + \dots$$

This equation is supported by perturbation theory

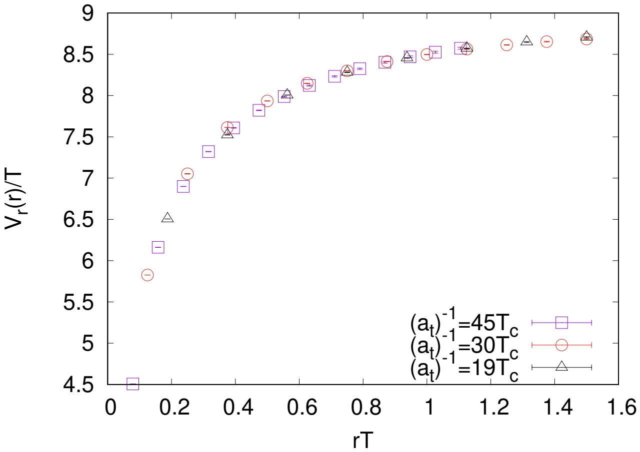
Fitting with the function



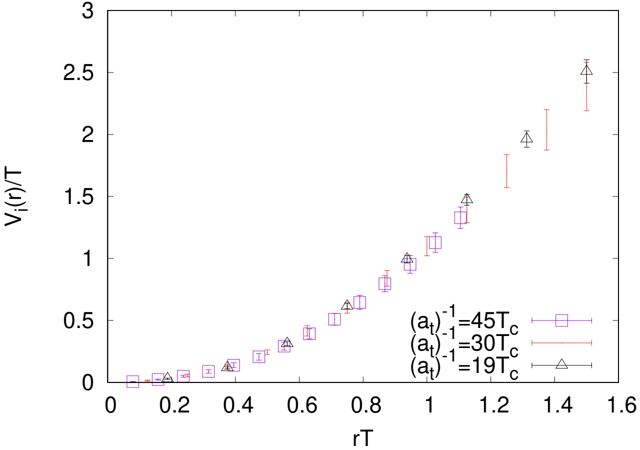
- Variation of imaginary part with smearing



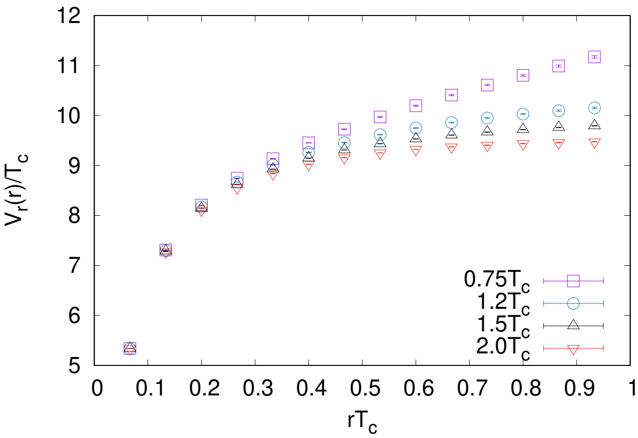
- Variation of real part with lattice spacing at 1.2Tc.



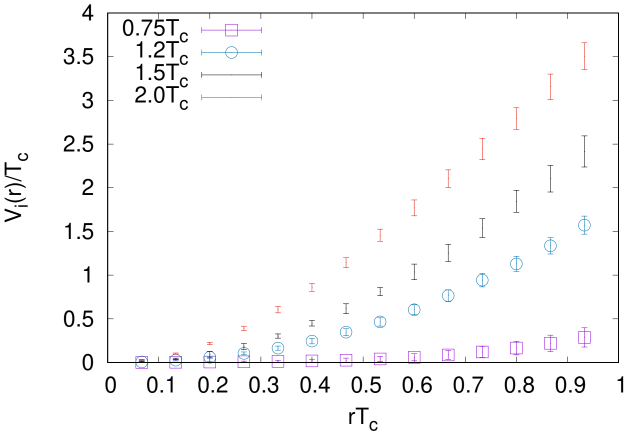
- Varition of imaginary part with lattice spacing at 1.2Tc.



- Potential at different temperature



- Potential at different temperature

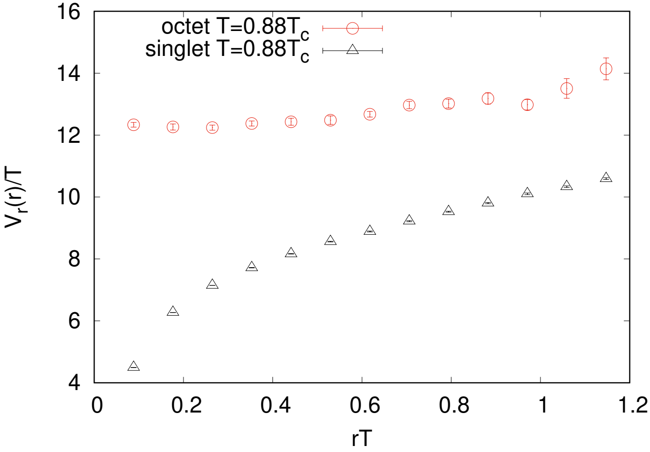


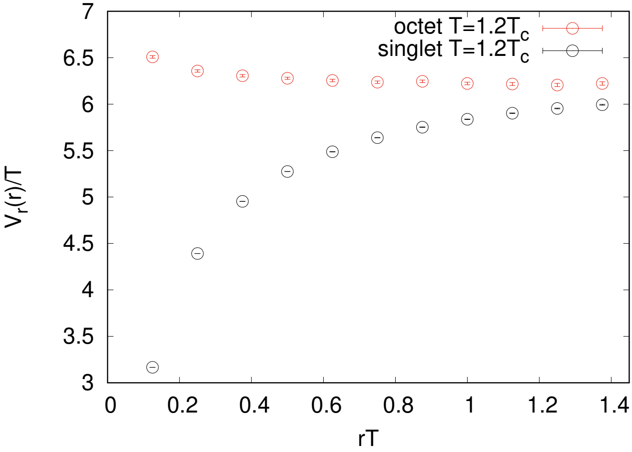
In short

$$\frac{\partial log(w(r,t))}{\partial t} = -V_r(r) + \frac{V_i(r)}{\pi} log(\frac{t}{\beta - t}) + ...$$

$$V(r) = \lim_{t_m \to \infty} i \frac{\partial log(w(r, it_m))}{\partial t_m} = V_r(r) - iV_i(r)$$

- In the QGP the quark antiquark pair can also be in an octet state.
- For octet state
  $M(r = |\vec{x} - \vec{y}|, t_m) = \bar{\psi}(\vec{x}, t_m)U(\vec{x}, \vec{z}; t_m)T^a U(\vec{z}, \vec{y}; t_m)\psi(\vec{y}, t_m)$.
- We can not implement this operator directly in lattice.
- To make it gauge invariant we use the following operator
  $M(r = |\vec{x} - \vec{y}|, t_m) = \bar{\psi}(\vec{x}, t_m)U(\vec{x}, \vec{z}; t_m)T^a B_a(z)U(\vec{z}, \vec{y}; t_m)\psi(\vec{y}, t_m)$.
- However the potential should not depend on any particular gluonic operator.


octet T=0.88T$_c$
singlet T=0.88T$_c$
$V_r(r)/T$
rT
16
14
12
10
8
6
4
0
0.2
0.4
0.6
0.8
1
1.2


octet T=1.2T$_c$
singlet T=1.2T$_c$
$V_r(r)/T$
rT
7
6.5
6
5.5
5
4.5
4
3.5
3
0
0.2
0.4
0.6
0.8
1
1.2
1.4

Conclusion

- We proposed a method to obtain the potential directly from smeared thermal wilson loop.
- Real part of this potential is screended and close to the singlet free energy.
- Continiuum limit has been shown both for real and imaginary part.
- Real part of octet potential has been obtained and it does not show any increasing behaviour.

- $M(r = |\vec{x} - \vec{y}|, t_m) = \bar{\psi}(\vec{x}, t_m) U(\vec{x}, \vec{y}; t_m) \psi(\vec{y}, t_m)$
- $$S_L = \frac{\beta_s}{3} \sum_x \sum_{\substack{i>j \\ i\neq 4}} \mathrm{Re}\ \mathrm{Tr}(1 - P_{ij}(x)) + \frac{\beta_t}{3} \sum_x \sum_{i\neq 4} \mathrm{Re}\ \mathrm{Tr}(1 - P_{4i}(x))$$