Outline

• Introduction

• Spectral properties of quarkonia and open heavy flavor

• Heavy quark transport

• Other approaches

• Conclusion
Why heavy flavor?

- Heavy flavors = charm & bottom

  $\eta_c, J/\psi, \chi_c, \eta_b, Y, \chi_b \ldots$

  $Q = c, b$

  $q = u, d, s$

  $D, B, \ldots$

- Quarkonia ($Q\bar{Q}$) and open heavy flavors ($Qq$, $q\bar{Q}$) are important probes to investigate Quark-Gluon Plasma (QGP) formed in heavy ion collisions.
  - Produced in the early stage of the collisions: experiencing entire evolution of QGP
  - A signal of QGP formation: color Debye screening in QGP $\rightarrow$ suppression of quarkonium production
  - Sequential suppression: different binding energy for different bound state $\rightarrow$ different melting temperature
What should we understand?

Understanding suppression patterns
→ dissociation temperatures

In-medium properties of open/hidden heavy flavors ← All encoded in spectral functions

PHENIX Collaboration, PRL 98 (2007) 172301

CMS Collaboration@QM2018

Recent progress on in-medium heavy flavor physics from lattice QCD
Spectral function

**Euclidian (imaginary time) mesonic correlation function**

\[ G_H(\tau, \vec{p}) \equiv \int d^3x e^{-i\vec{p} \cdot \vec{x}} \langle J_H(\tau, \vec{x}) J_H(0, \vec{0}) \rangle \]

\[ = \int_0^\infty \frac{d\omega}{2\pi} \rho_H(\omega, \vec{p}) K(\omega, \tau) \]

\[ J_H(\tau, \vec{x}) \equiv \bar{\psi}(\tau, \vec{x}) \Gamma_H \psi(\tau, \vec{x}) \]

\[ K(\omega, \tau) \equiv \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)} \]

**Spectral function**

\[ \rho(\omega, p=0)/\omega^2 \]

- **T<T_c**
  - Ground state
  - Exited state

- **T>T_c**
  - Transport peak (V)

- **T→∞**
  - Transport peak (V)
  - (Free case)

**Heavy quark diffusion coefficient**

\[ D = \frac{1}{6\chi_{00}} \lim_{\omega→0} \sum_{i=1}^{3} \frac{\rho_{Vi}(\omega, 0)}{\omega} \]

- \( \rho_{Vi} \): Vector spectral function
- \( \chi_{00} \): Quark number susceptibility
Difficulties on lattice

- **Heavy quark mass** $m_Q$ **is too heavy!**
  - Basically, fine and large lattices are needed to control lattice artifacts.

- **Obtaining spectral functions: an ill-posted inverse problem!**
  - Number of correlator data points $<<$ Number of frequency bins of spectral functions
  - A simple $\chi^2$ fit gives infinite number of possible spectra within statistical uncertainties.
**Overcoming difficulties**

- **Heavy quark mass** \( m_Q \) **is too heavy!**
  - \( m_Q >> \Lambda_{\text{QCD}} \) → separation of scales (especially for bottom)
  - Effective field theories: *Non-relativistic QCD (NRQCD), potential NRQCD (pNRQCD)*, ...


- **Obtaining spectral functions: an ill-posted inverse problem!**
  - Adding prior information
  - Bayesian inference: *Maximum Entropy Method (MEM), Bayesian Reconstruction (BR) Method*, ...
  - Phenomenologically motivated (perturbative) modeling of spectral functions
  - Other approached without spectral functions
Bayesian inference

- **Bayes theorem**
  \[ P[A|\bar{G}] = \frac{P[\bar{G}|A]P[A]}{P[\bar{G}]} \]

- **Prior probability**: prior information
  \[ P[A] \propto \exp(\alpha S) \]

- **Likelihood function**: usual \( \chi^2 \) term
  \[ P[\bar{G}|A] \propto \exp \left( -\frac{\chi^2}{2} \right) \]

- **Posterior probability**
  \[ P[A|\bar{G}] \propto e^{-F} \]

\[ \max P[A|\bar{G}] \iff \min F \iff \frac{\delta F}{\delta A} = 0 \]

**MEM: Shannon-Jaynes Entropy**
- Limitation of search space
  \[ S = \int d\omega \left( A - D - A \log \left( \frac{A}{D} \right) \right) \]

**BR: new regulator**
- No limitation of search space but possibility of artificial peaks (ringing)
  \[ S = \int d\omega \left( 1 - \frac{A}{D} + \log \left( \frac{A}{D} \right) \right) \]

\[ D: \text{Default model} \]
Another option of Bayesian inference: Stochastic method

- Stochastically finding a spectral image
- Formally equivalent to MEM in a certain limit

$J/\psi @ 0.75T_c$, $128^3 \times 96$, Quenched


Qualitatively consistent results
Bottomonium spectral functions

- FASTSUM Collaboration, 2+1 flavor NRQCD, anisotropic $a_\sigma/a_\tau = 3.5$, MEM

**News: Smaller pion mass**

Virtually no effect of pion mass in the $\Upsilon$ spectral function

Survival of $\Upsilon(1S)$ up to the highest temperature (1.9$T_c$)

$N_\sigma = 24$

$a^{-1}_\tau = 5.63(4)$ GeV

$T_c \approx 185$ MeV

Samuel Offler’s talk on Wed. @10:20
Open/hidden charm meson spectral functions

- FASTSUM Generation2 ensemble, 2+1 flavor Clover Wilson, anisotropic $a_\sigma/a_\tau = 3.5$, $m_\pi = 390$ MeV, BR method

\[ T_c = 185 \text{ MeV} \]

Shifting and broadening of the peak in spectral functions from the thermal and reconstructed correlators are consistent with each other up to $1.9T_c$.

$N_\sigma = 24$

$\sigma^{-1} = 5.63(4) \text{ GeV}$

$T_c \approx 185$ MeV

An effect of the limited number of temporal points available

R. Quinn et al., arXiv:1903.11006 [hep-lat]
Open/hidden charm meson spectral functions

- FASTSUM Generation2 ensemble, 2+1 flavor Clover Wilson, anisotropic $a_σ/a_τ = 3.5$, $m_π = 390$ MeV, BR method

Shifting and broadening of the peak in spectral functions from the thermal and reconstructed correlators are consistent with each other up to $1.5T_c$.

→ An effect of the limited number of temporal points available

$N_σ = 24$
$a^{-1}_τ = 5.63(4)$ GeV
$T_c ≃ 185$ MeV
Open/hidden charm meson spectral functions

- FASTSUM Generation2 ensemble, 2+1 flavor Clover Wilson, anisotropic $a_\sigma/a_\tau = 3.5$, $m_\pi = 390$ MeV, BR method

$$N_\sigma = 24$$
$$a^{-1}_\tau = 5.63(4)\text{ GeV}$$
$$T_c \approx 185\text{ MeV}$$

Spectral functions from the thermal and reconstructed correlators are different already below $T_c$.

$\rightarrow$ No signal of survival of open charm

R. Quinn et al., arXiv:1903.11006 [hep-lat]
Fit to a perturbative spectral function

- Quenched QCD, Clover Wilson, continuum extrapolated

\[
\rho^{\text{pert}}(\omega) = \rho^{\text{vac}}(\omega) \theta(\omega - \omega^{\text{match}}) + A^{\text{match}} \rho^{\text{NRQCD}}(\omega) \theta(\omega^{\text{match}} - \omega) \Phi(\omega)
\]

Vacuum asymptotics

Suppression

- High energy $\rho^{\text{vac}}$: Vacuum asymptotics


- Threshold region $\rho^{\text{NRQCD}}$: pNRQCD

  Laine, JHEP 0705:028, 2007

- Suppressed at low energy
Fit to a perturbative spectral function

- Quenched QCD, Clover Wilson, continuum extrapolated

\[
\rho_{\text{pert}}(\omega) = \rho_{\text{vac}}(\omega) \theta(\omega - \omega_{\text{match}}) + A_{\text{match}} \rho_{\text{NRQCD}}^{\text{NRQCD}}(\omega) \theta(\omega_{\text{match}} - \omega) \Phi(\omega)
\]

Vacuum asymptotics

Recent progress on in-medium heavy flavor physics from lattice QCD

- Perturbative spectral function describes lattice data perfectly.
- No resonance peak needed for charm.
- A resonance peak may be well present for bottom at \( T \lesssim 1.5 T_c \).

Charm, Ps

Bottom, Ps

\[
\rho_{\text{mod}}(\omega) = A \rho_{\text{pert}}(\omega - B)
\]
Fit to a perturbative spectral function

- Quenched QCD, Clover Wilson, continuum extrapolated

News: Vector channel & transport

Anna-Lena Kruse’s talk on Wed. @9:20

\[ \rho^{\text{II}}(\omega) = A \rho^{\text{pert}}(\omega - B) + \rho^{\text{trans}}(\omega) \quad \rho^{\text{trans}}(\omega) = 3D \chi_q \frac{\omega \eta D^2}{\omega^2 + \eta D^2} \frac{1}{\cosh\left(\frac{\omega}{2\pi T}\right)} \]

- Perturbative spectral function describes lattice data perfectly.
- No resonance peak needed for charm.
- A resonance peak may be well present for bottom at \( T \lesssim 1.5 T_c \).
Fit to a perturbative spectral function

- Quenched QCD, Clover Wilson, continuum extrapolated

**News: Vector channel & transport**

Anna-Lena Kruse’s talk on Wed. @9:20

\[
\rho^\text{tot}(\omega) = A \rho^{\text{pert}}(\omega - B) + \rho^{\text{trans}}(\omega)
\]

\[
\rho^{\text{trans}}(\omega) = 3D \chi_q \frac{\omega \eta_D^2}{\omega^2 + \eta_D^2} \frac{1}{\cosh(\frac{\omega}{2\pi T})}
\]

- A, B fixed by fit for high \( \omega \)
- \( 2\pi TD \) varied from 1 to 9
- \( \eta_D \): solved

Possible lower bound at \( 2\pi TD = 2 \)
Mass shift and width of SPF from smeared correlators

- HoTQCD ensembles, $N_\sigma = 48, N_\tau = 12, m_\pi = 160$ MeV, physical Kaon mass, NRQCD

- Fit data with ansatz for spectral function $\rho(\omega)$ sketched in right plot
- Will subtract $T = 0$ continuum from finite temperature correlator

$$C_{sub,T>0}(\tau) = C_{T>0}(\tau) - C_{T=0}(\tau) + A \exp(-m_{T=0} \tau)$$
Mass shift and width of SPF from smeared correlators

- HoTQCD ensembles, $N_{\sigma} = 48$, $N_{\tau} = 12$, $m_{\pi} = 160$ MeV, physical Kaon mass, NRQCD
  - Zero temperature continuum subtracted
  - (fit 1 filled) Main peak + small negative peak
  - (fit 2 empty) Main peak and last 2 points removed

- Results for $\Upsilon$ and $\eta_b$ very similar

Rasmus Larsen’s poster
Mass shift and width of SPF from smeared correlators

- HoTQCD ensembles, $N_\sigma = 48$, $N_\tau = 12$, $m_\pi = 160$ MeV, physical Kaon mass, NRQCD

- Zero temperature continuum subtracted
- (fit 1 filled) Main peak + small negative peak
- (fit 2 empty) Main peak and last 2 points removed

- Results for $\chi_{b0}$ and $\chi_{b1}$ very similar

Rasmus Larsen’s poster
The other related talks/posters in this conference

• Real-Time-Evolution of Heavy-Quarkonium Bound States  
  Alexander Lehmann’s talk on Wed. @11:10

• Euclidean correlation functions for transport coefficients under gradient flow  
  Hai-Tao Shu’s talk on Wed. @11:30

• Heavy quark diffusion coefficient from lattice  
  Viljami Leino’s talk on Wed. @11:50

• Non-perturbative study of heavy quark anti-quark potential at finite temperature  
  Dibyendu Bala’s talk on Fri. @14:40

• Thermal Quarkonium Mass Shift from Euclidean Correlators  
  Alexander Maximilian Eller’s poster

• Quarkonium suppression in non-equilibrium quark-gluon plasma  
  Zhandos Moldabekov’s poster
Conclusions

• Quarkonia and open heavy flavors are important probes to investigate QGP.
• There are some difficulties for heavy flavor calculations on the lattice.
• Effective field theories are useful tool for bottom.
• There are a few approaches to reconstruct spectral functions.
• Full QCD studies for spectral functions of charmonia, bottomonia as well as open charm mesons have been already done.
• Continuum extrapolated quarkonia spectral functions have been studied with quenched QCD and perturbative models.
• A new kind of approach with smeared correlators to investigate quarkonium spectral function have been shown.
• There are variety of other approaches for heavy flavor physics.