

Recent progress on in-medium heavy flavor physics from lattice QCD

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Outline

- Introduction
- Spectral properties of quarkonia and open heavy flavor
- Heavy quark transport
- Other approaches
- Conclusion

Why heavy flavor?

- Heavy flavors = charm & bottom

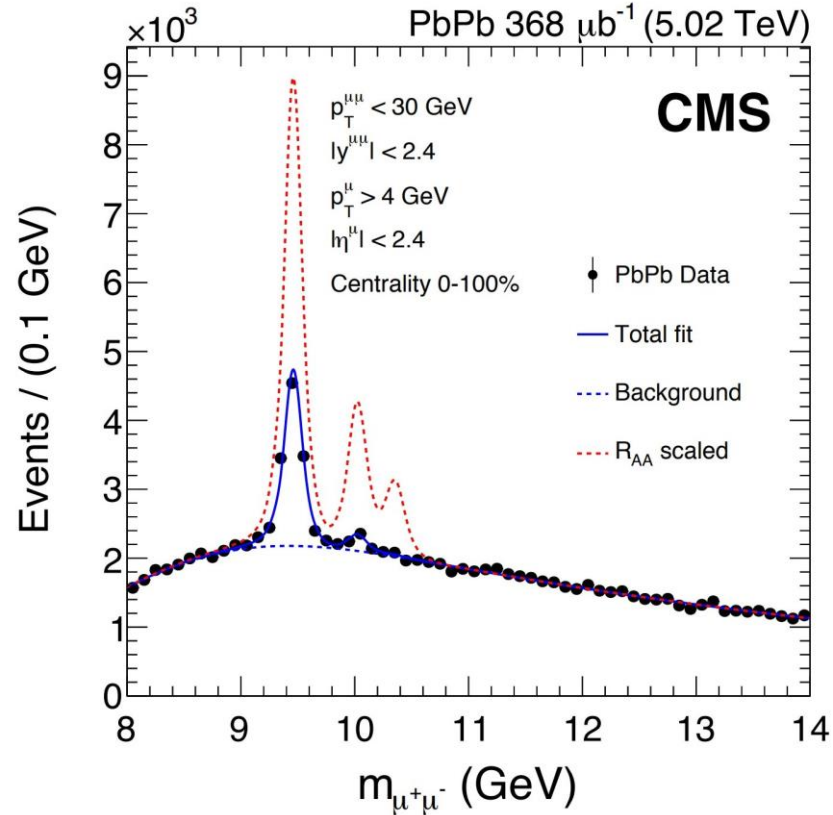
$\eta_c, J/\Psi, \chi_c, \eta_b, Y, \chi_b \dots$

D, B, \dots

$Q = c, b$
 $q = u, d, s$

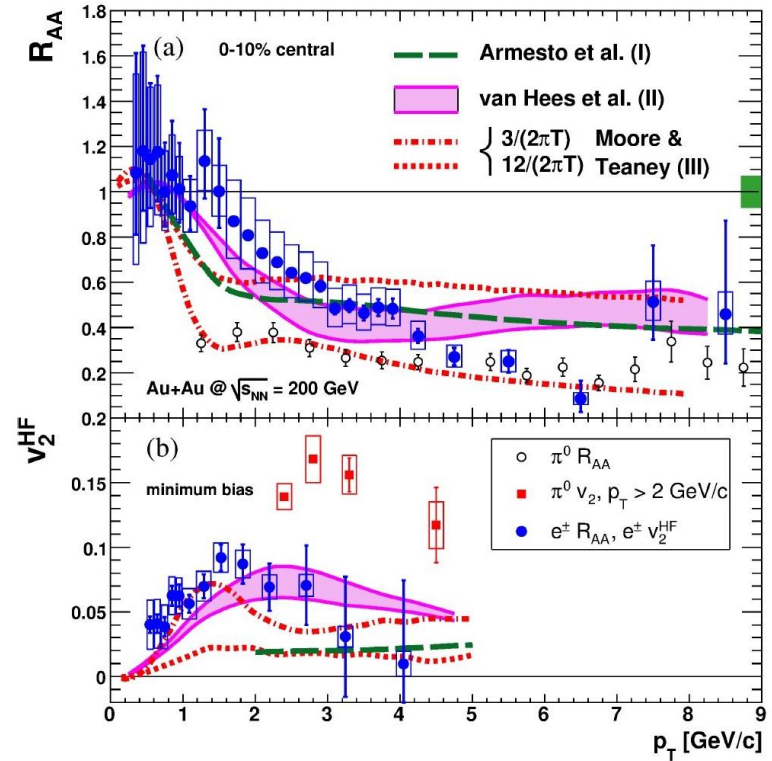
- **Quarkonia ($Q\bar{Q}$)** and **open heavy flavors ($Q\bar{q}, q\bar{Q}$)** are important probes to investigate **Quark-Gluon Plasma (QGP)** formed in heavy ion collisions.
 - Produced in the early stage of the collisions: experiencing entire evolution of QGP
 - A signal of QGP formation: color Debye screening in QGP \rightarrow suppression of quarkonium production [T. Matsui and H. Satz, PLB 178 \(1986\) 416](#)
 - Sequential suppression: different binding energy for different bound state \rightarrow different melting temperature

What should we understand?



CMS Collaboration@QM2018

Understanding suppression patterns
 → **dissociation temperatures**



PHENIX Collaboration, PRL 98 (2007) 172301

Inputs for transport models
 → **heavy quark diffusion coefficients**

In-medium properties of open/hidden heavy flavors ← All encoded in **spectral functions**

Spectral function

Euclidian (imaginary time) mesonic correlation function

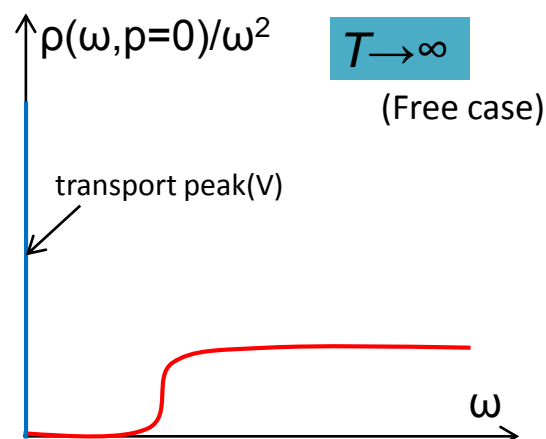
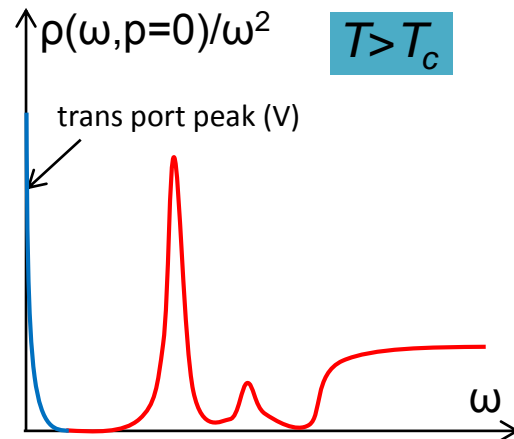
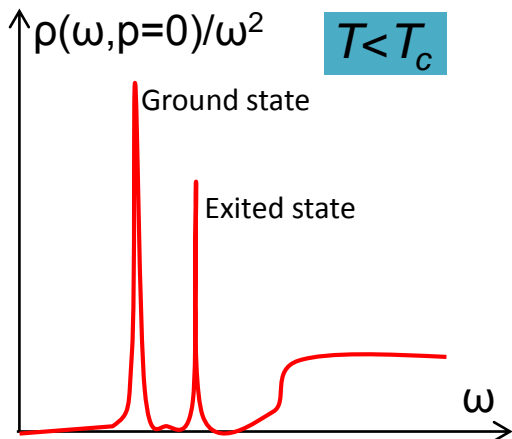
$$G_H(\tau, \vec{p}) \equiv \int d^3x e^{-i\vec{p}\cdot\vec{x}} \langle J_H(\tau, \vec{x}) J_H(0, \vec{0}) \rangle$$

$$= \int_0^\infty \frac{d\omega}{2\pi} \rho_H(\omega, \vec{p}) K(\omega, \tau)$$

Spectral function

$$J_H(\tau, \vec{x}) \equiv \bar{\psi}(\tau, \vec{x}) \Gamma_H \psi(\tau, \vec{x})$$

$$K(\omega, \tau) \equiv \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$



Heavy quark diffusion coefficient

$$D = \frac{1}{6\chi_{00}} \lim_{\omega \rightarrow 0} \sum_{i=1}^3 \frac{\rho_{ii}^V(\omega, \mathbf{0})}{\omega}$$

ρ_{ii}^V : Vector spectral function
 χ_{00} : Quark number susceptibility

Difficulties on lattice

- **Heavy quark mass m_Q is too heavy!**
 - Basically, fine and large lattices are needed to control lattice artifacts.
- **Obtaining spectral functions: an ill-posed inverse problem!**
 - Number of correlator data points \ll Number of frequency bins of spectral functions
 - A simple χ^2 fit gives infinite number of possible spectra within statistical uncertainties.

Overcoming difficulties

- **Heavy quark mass m_Q is too heavy!**
 - $m_Q \gg \Lambda_{\text{QCD}} \rightarrow$ separation of scales (especially for bottom)
 - Effective field theories: **Non-relativistic QCD (NRQCD), potential NRQCD (pNRQCD), ...**
Review: N. Brambilla et al., Rev. Mod. Phys. 77 (2005) 1423
- **Obtaining spectral functions: an ill-posed inverse problem!**
 - Adding prior information
 - Bayesian inference: **Maximum Entropy Method (MEM), Bayesian Reconstruction (BR) Method, ...**
 - **Phenomenologically motivated (perturbative) modeling of spectral functions**
 - Other approaches without spectral functions

Bayesian inference

- Bayes theorem

$$P[A|\bar{G}] = \frac{P[\bar{G}|A]P[A]}{P[\bar{G}]}$$

- Prior probability: prior information

$$P[A] \propto \exp(\alpha S)$$

- Likelihood function: usual χ^2 term

$$P[\bar{G}|A] \propto \exp(-\chi^2/2)$$

- Posterior probability

$$P[A|\bar{G}] \propto e^{-F}$$

$$\max P[A|\bar{G}] \leftrightarrow \min F \leftrightarrow \frac{\delta F}{\delta A} = 0$$

MEM: Shannon-Jaynes Entropy

→ Limitation of search space

M. Asakawa, T. Hatsuda and Y. Nakahara,
Prog.Part.Nucl.Phys. 46 (2001) 459-508

$$S = \int d\omega \left(A - D - A \log \left(\frac{A}{D} \right) \right)$$

BR: new regulator

→ No limitation of search space but
possibility of artificial peaks (ringing)

Y. Burnier and A. Rothkopf, PRL 111 (2013) 18, 182003

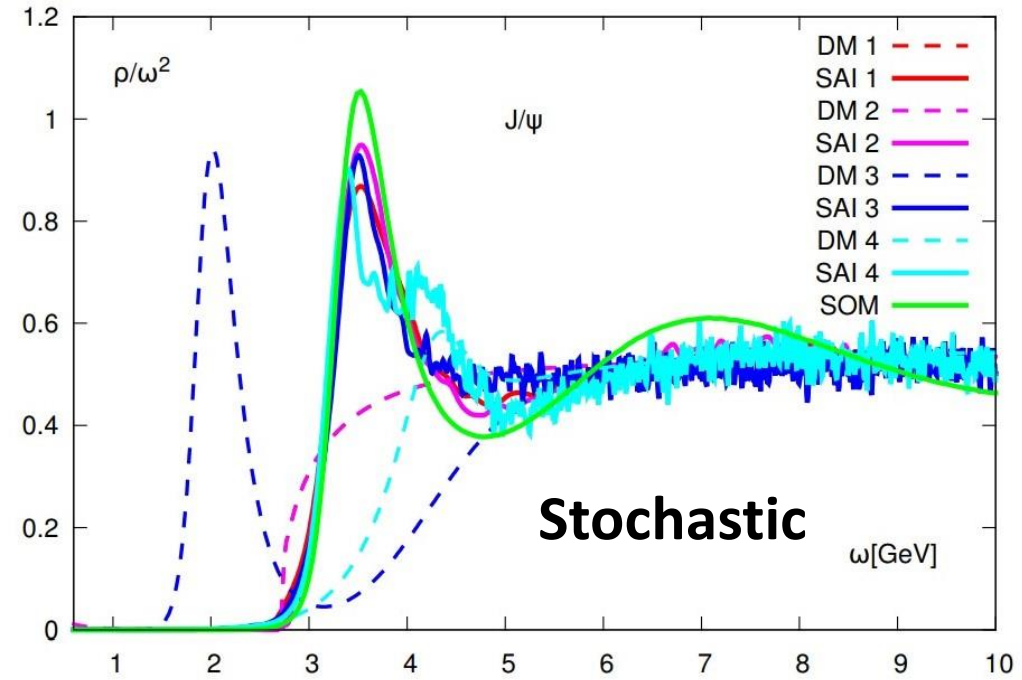
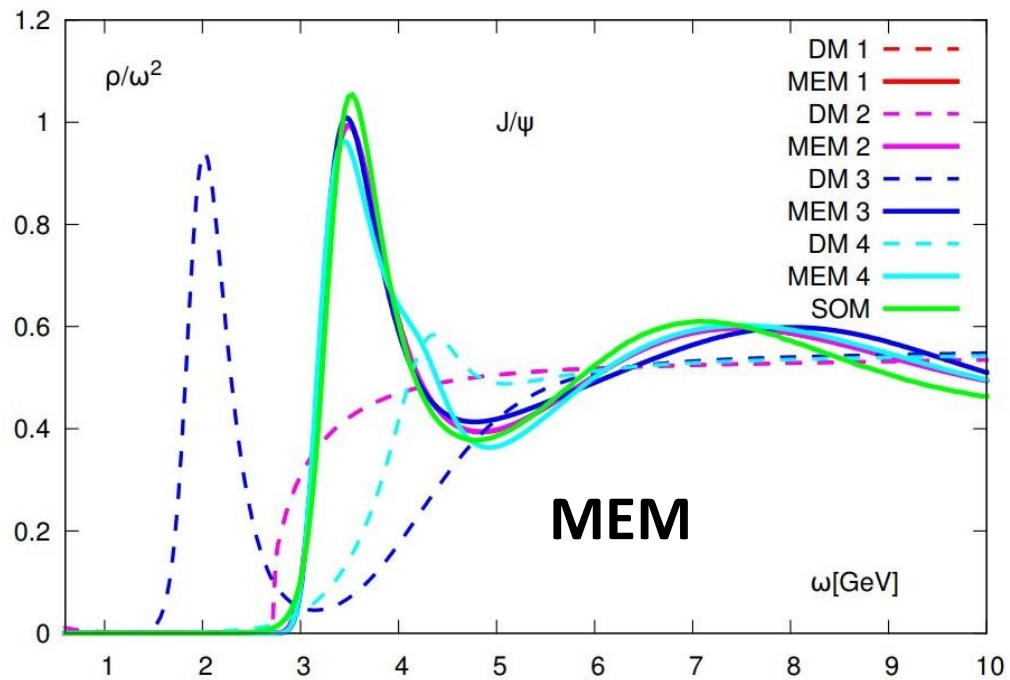
$$S = \int d\omega \left(1 - \frac{A}{D} + \log \left(\frac{A}{D} \right) \right)$$

D : Default model

Another option of Bayesian inference: Stochastic method

- Stochastically finding a spectral image
- Formally equivalent to MEM in a certain limit [K.S.D. Beach, arXiv:cond-mat/0403055](#)

J/ψ @ $0.75T_c$, $128^3 \times 96$, Quenched [H.-T. Ding, O. Kaczmarek, Swagato Mukherjee, HO and H.-T. Shu, PRD 97 \(2018\) 9, 094503](#)



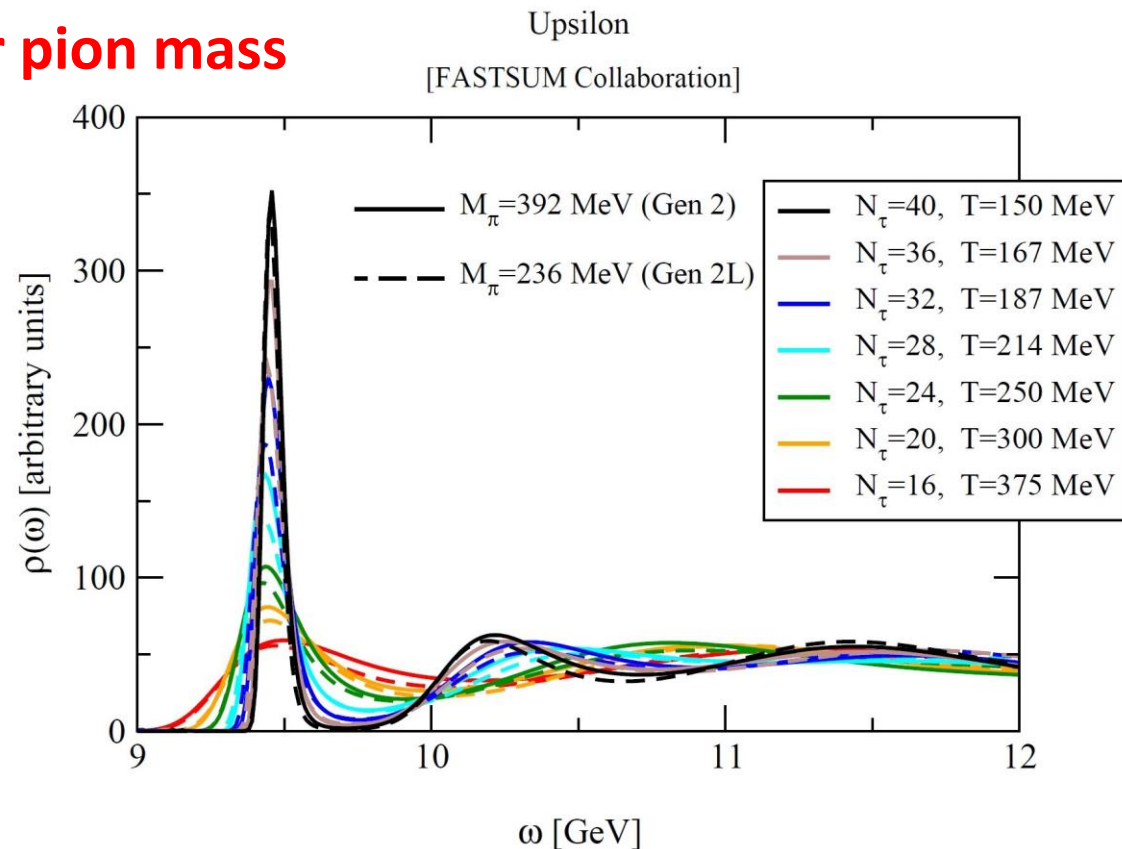
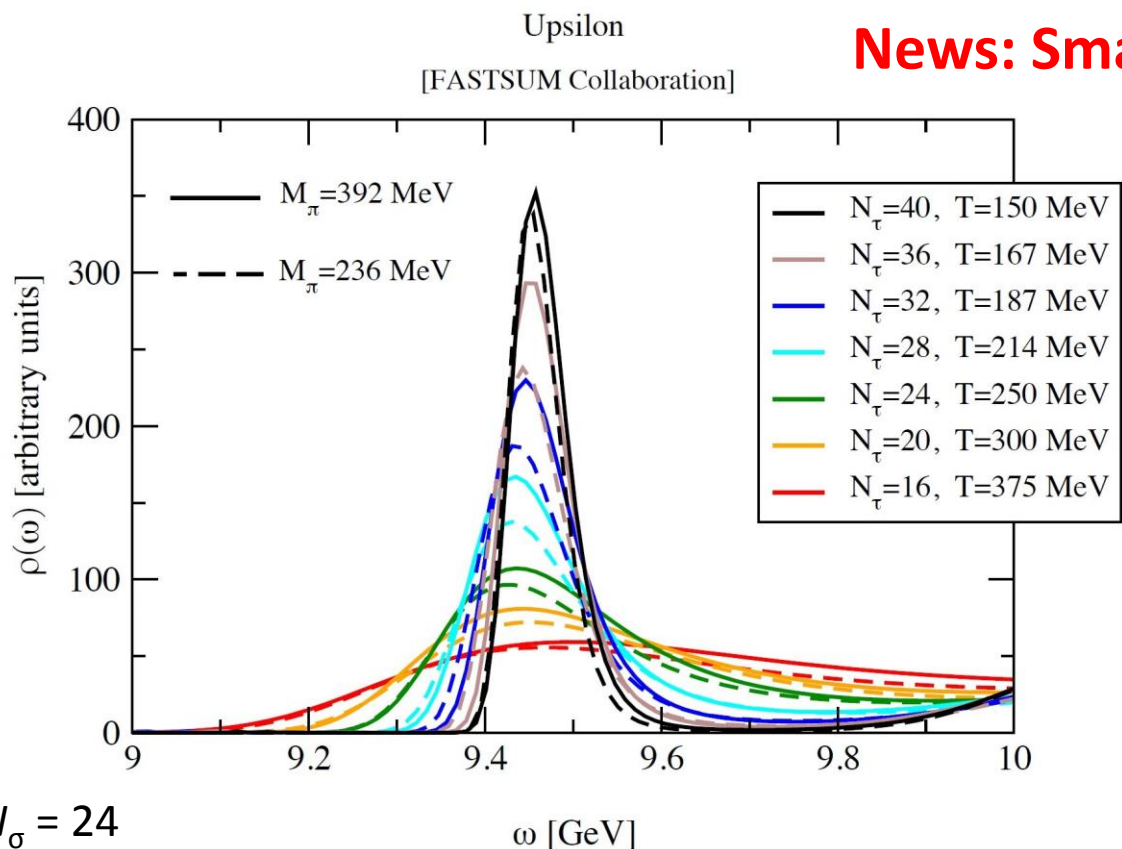
Qualitatively consistent results

Bottomonium spectral functions

- FASTSUM Collaboration, 2+1 flavor NRQCD, anisotropic $a_\sigma/a_\tau = 3.5$, MEM

Samuel Offler's talk on Wed. @10:20

News: Smaller pion mass

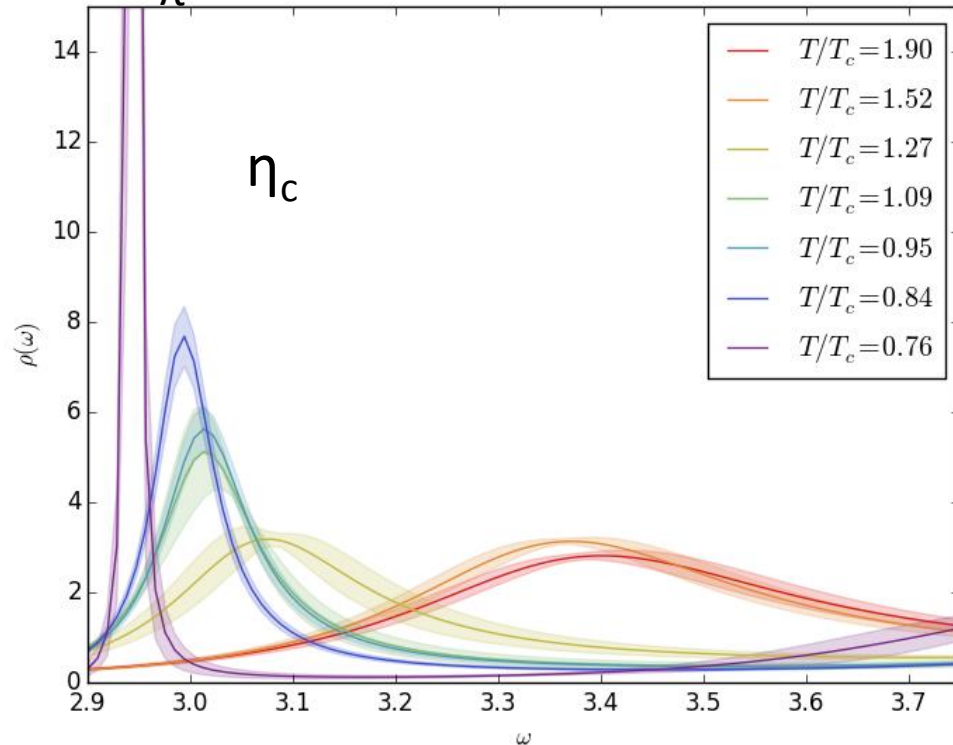


$N_\sigma = 24$
 $a_\tau^{-1} = 5.63(4)$ GeV
 $T_c \simeq 185$ MeV

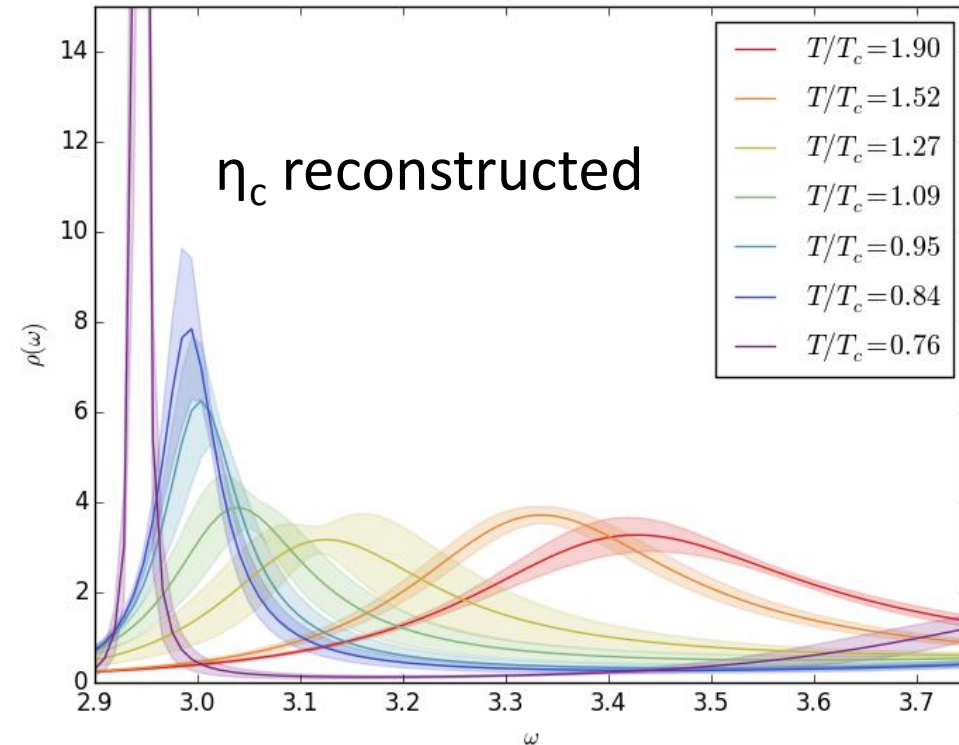
Virtually no effect of pion mass in the Y spectral function
Survival of Y(1S) up to the highest temperature ($1.9T_c$)

Open/hidden charm meson spectral functions

- FASTSUM Generation2 ensemble, 2+1 flavor Clover Wilson, anisotropic $a_\sigma/a_\tau = 3.5$, $m_\pi = 390$ MeV, BR method



R. Quinn *et al.*, arXiv:1903.11006 [hep-lat]

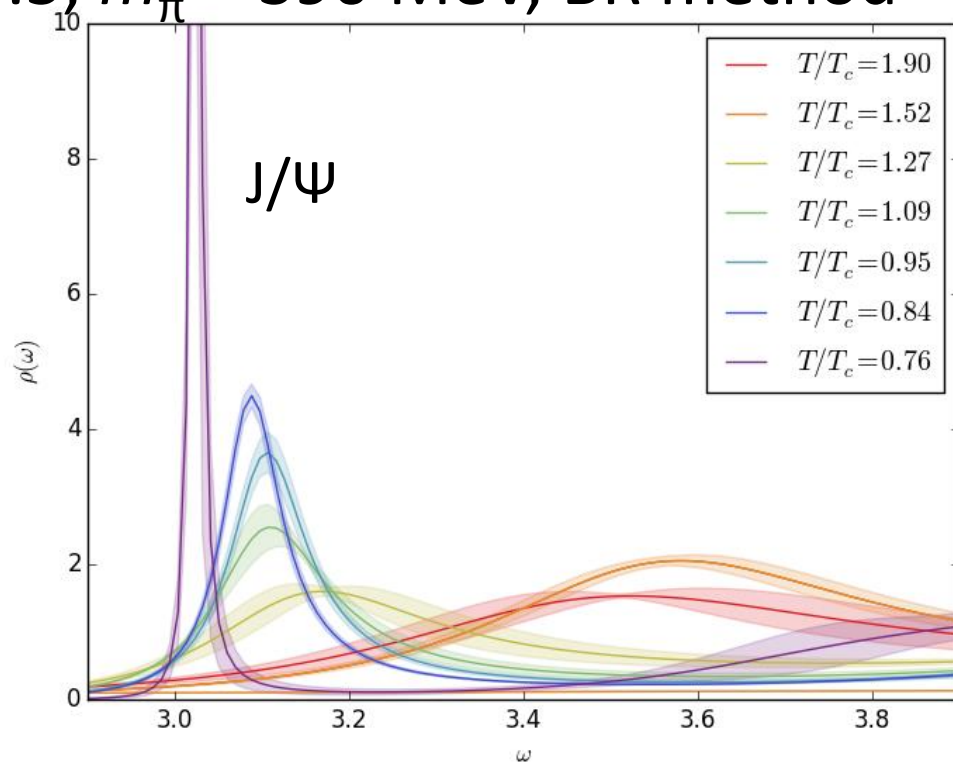


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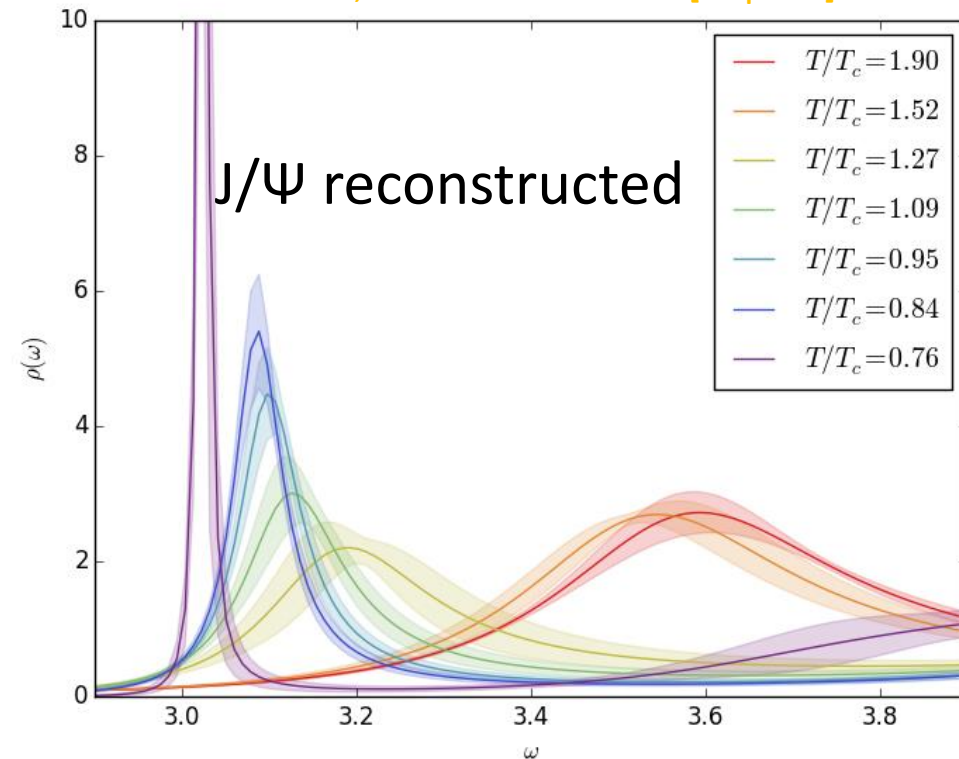
**Shifting and broadening of the peak in spectral functions from the thermal and reconstructed correlators are consistent with each other up to $1.9T_c$.
→ An effect of the limited number of temporal points available**

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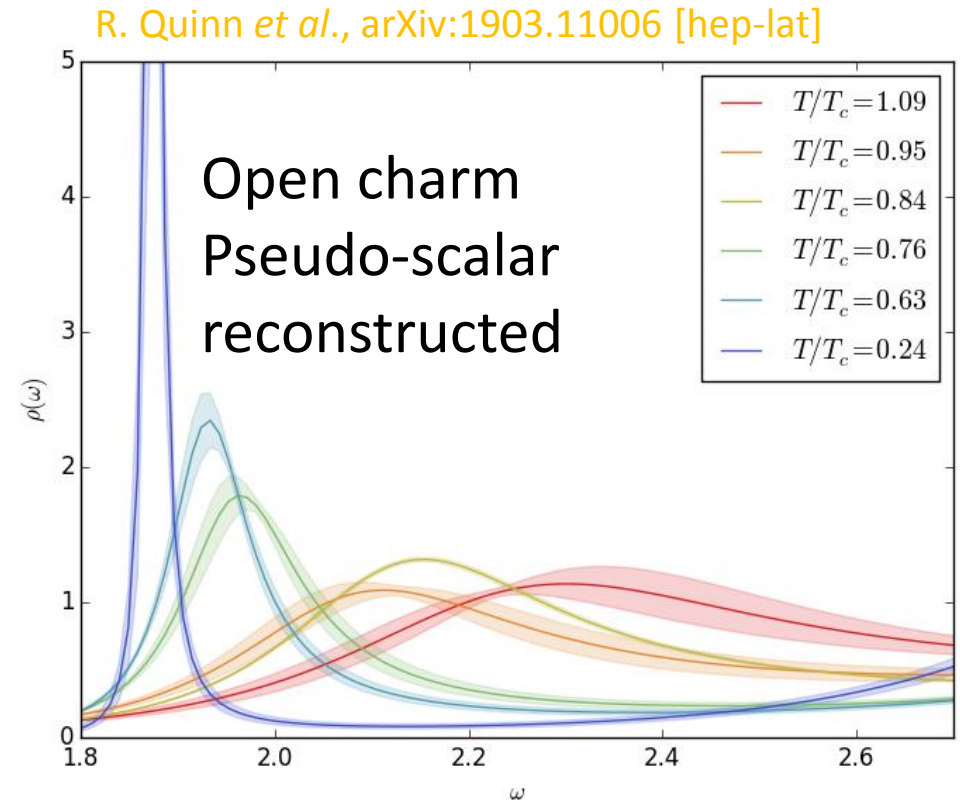
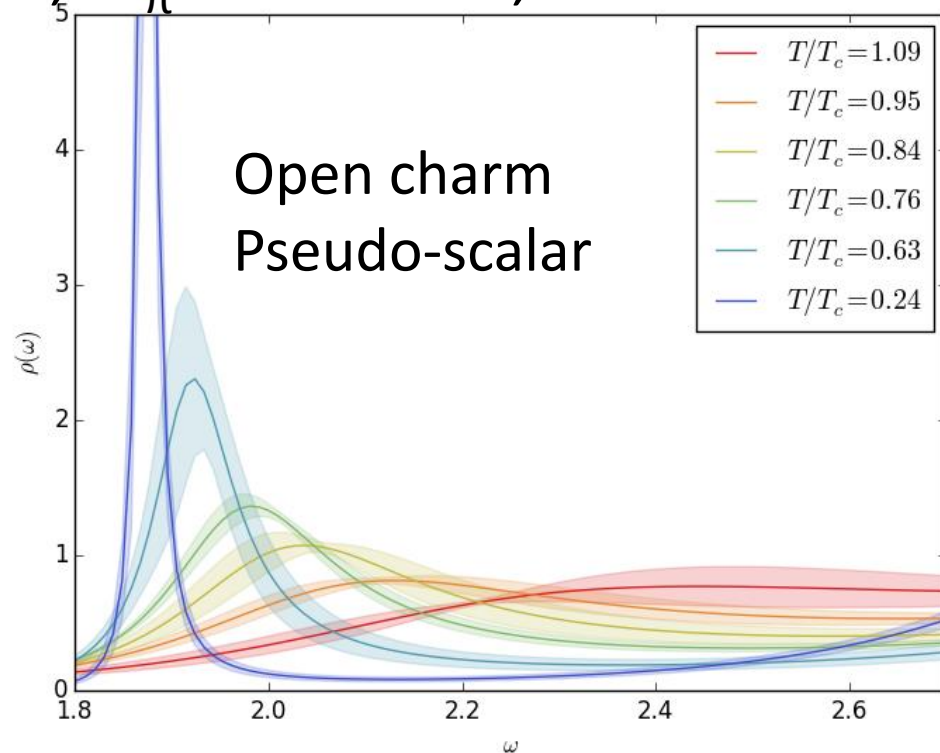


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**Shifting and broadening of the peak in spectral functions from the thermal and reconstructed correlators are consistent with each other up to $1.5T_c$.
→ An effect of the limited number of temporal points available**

Open/hidden charm meson spectral functions

- FASTSUM Generation2 ensemble, 2+1 flavor Clover Wilson, anisotropic $a_\sigma/a_\tau = 3.5$, $m_\pi = 390$ MeV, BR method



$N_\sigma = 24$
 $a_\tau^{-1} = 5.63(4)$ GeV
 $T_c \simeq 185$ MeV

Spectral functions from the thermal and reconstructed correlators are different already below T_c .
→ No signal of survival of open charm

Fit to a perturbative spectral function

- Quenched QCD, Clover Wilson, continuum extrapolated

Y. Burnier, H. -T. Ding, O. Kaczmarek, A. -L. Kruse, M. Laine, HO, H. Sandmeyer, JHEP11(2017)206

$$\rho^{pert}(\omega) = \underbrace{\rho^{vac}(\omega)}_{\text{Vacuum asymptotics}} \theta(\omega - \omega^{match}) + A^{match} \underbrace{\rho^{NRQCD}(\omega)}_{\text{pNRQCD}} \theta(\omega^{match} - \omega) \underbrace{\Phi(\omega)}_{\text{Suppression}}$$

- High energy ρ^{vac} : Vacuum asymptotics [Burnier, Laine, Eur.Phys.J.C 72 \(2012\) 1902](#)
- Threshold region ρ^{NRQCD} : pNRQCD [Laine, JHEP 0705:028,2007](#)
- Suppressed at low energy

Fit to a perturbative spectral function

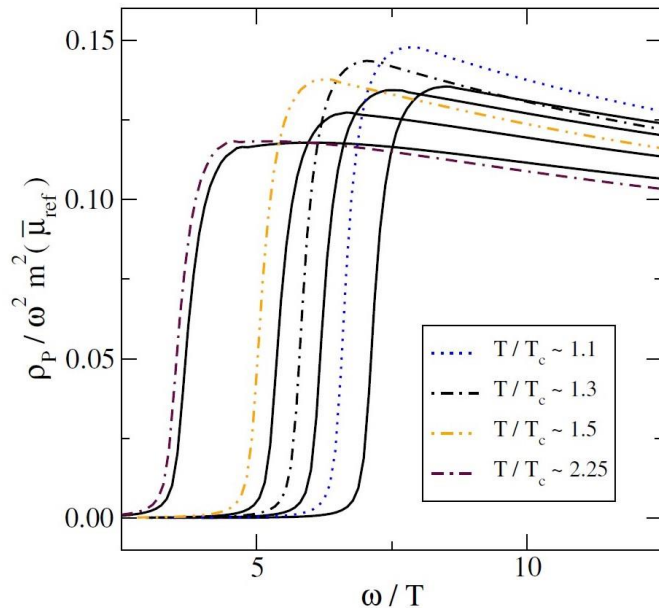
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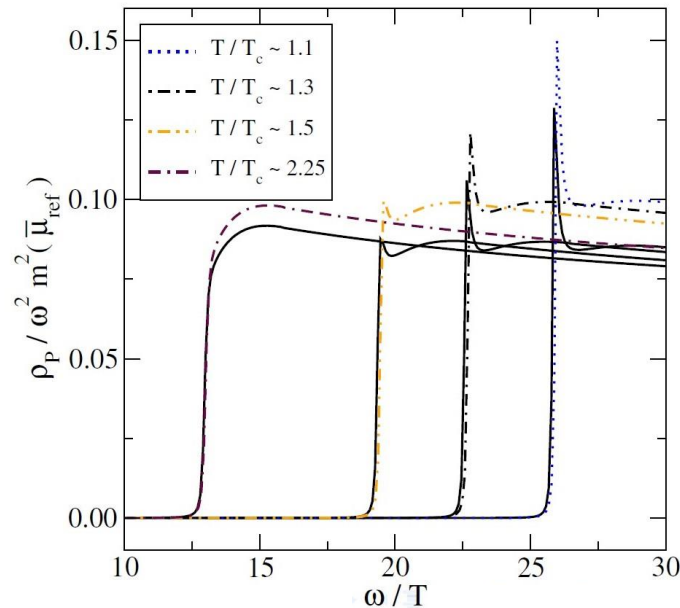
Charm, Ps

$m(\bar{\mu}_{ref}) = 1 \text{ GeV}$



Bottom, Ps

$m(\bar{\mu}_{ref}) = 5 \text{ GeV}$



$$\rho^{mod}(\omega) = \mathbf{A} \rho^{pert}(\omega - \mathbf{B})$$

- **Perturbative spectral function describes lattice data perfectly.**
- **No resonance peak needed for charm.**
- **A resonance peak may be well present for bottom at $T \lesssim 1.5 T_c$.**

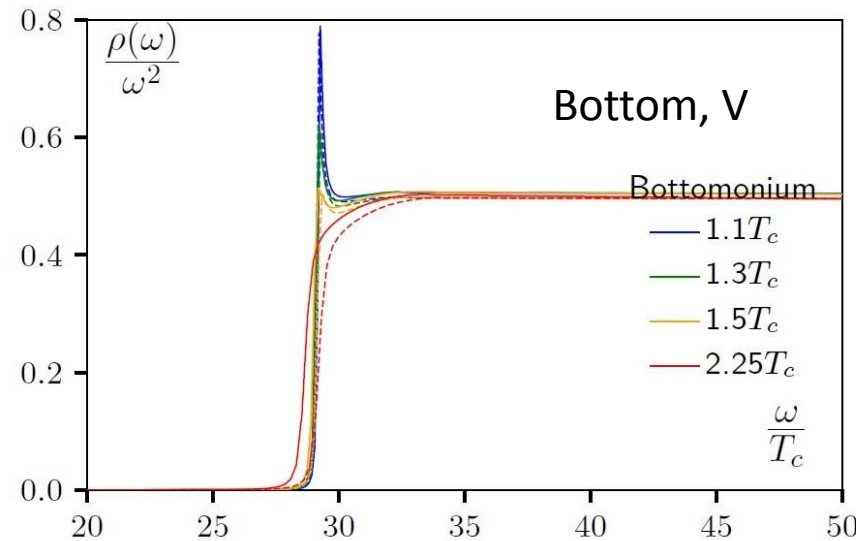
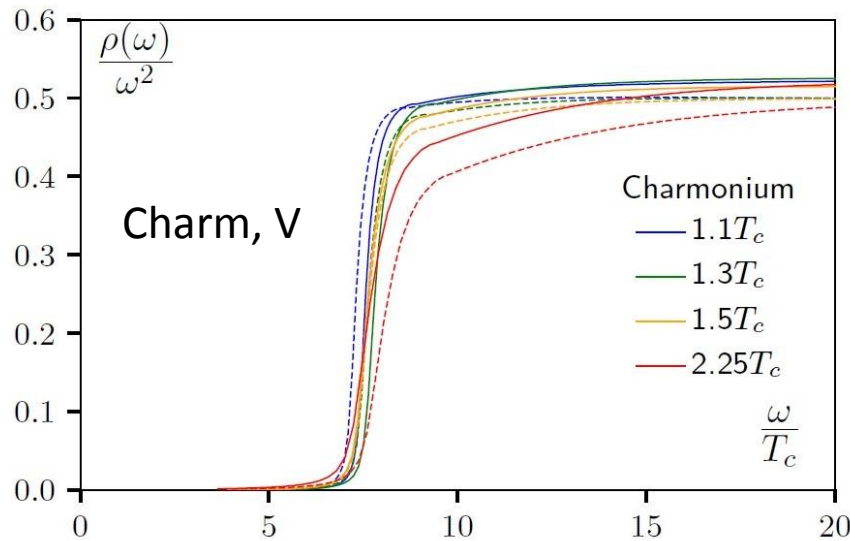
Fit to a perturbative spectral function

- Quenched QCD, Clover Wilson, continuum extrapolated

News: Vector channel & transport

Anna-Lena Kruse's talk on Wed. @9:20

$$\rho^{ii}(\omega) = A\rho^{pert}(\omega - B) + \rho^{trans}(\omega) \quad \rho^{trans}(\omega) = 3D\chi_q \frac{\omega\eta_D^2}{\omega^2 + \eta_D^2} \frac{1}{\cosh\left(\frac{\omega}{2\pi T}\right)}$$



- A, B fixed by fit for high ω
- $2\pi TD$ varied from 1 to 9
- η_D : solved

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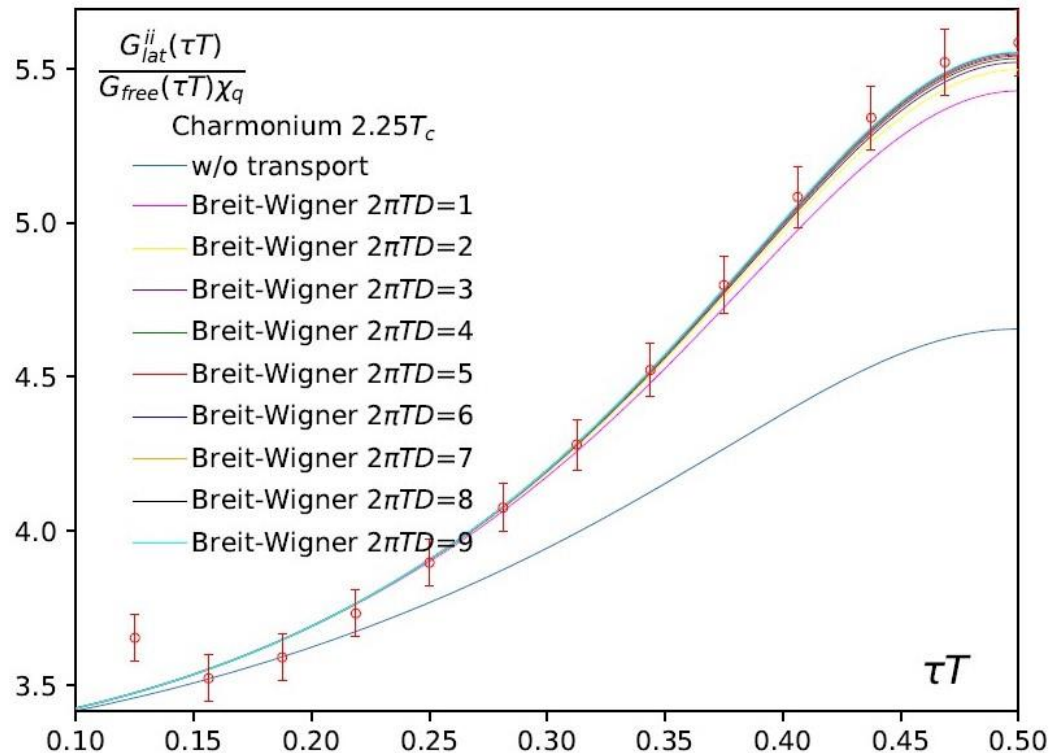
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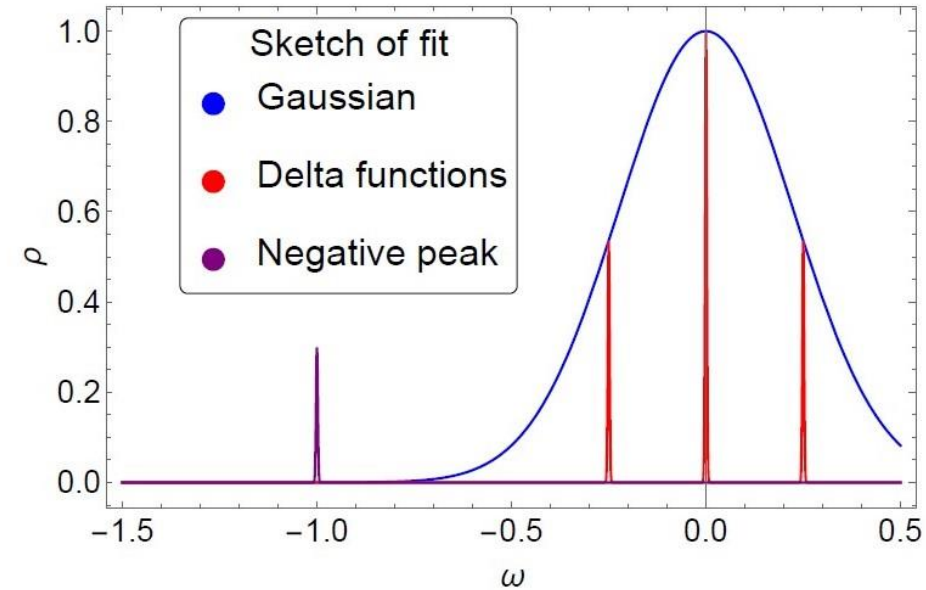
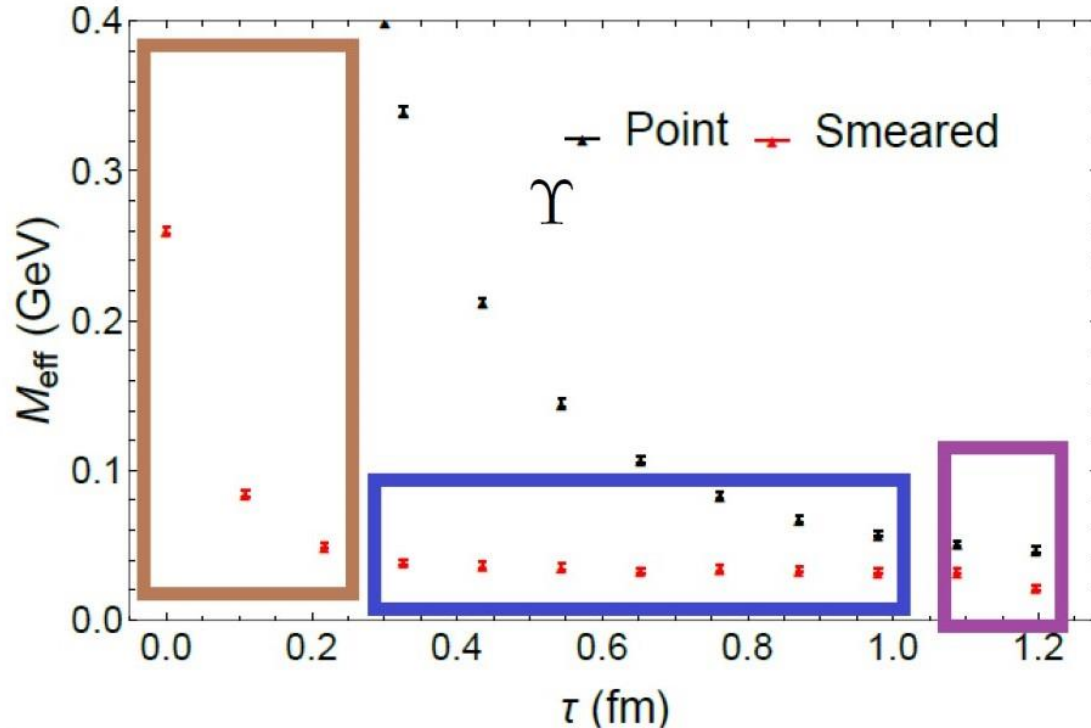
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- η_D : solved

Possible lower bound at $2\pi TD = 2$

Mass shift and width of SPF from smeared correlators

- HoTQCD ensembles, $N_\sigma = 48$, $N_\tau = 12$, $m_\pi = 160$ MeV, physical Kaon mass, NRQCD

Rasmus Larsen's poster



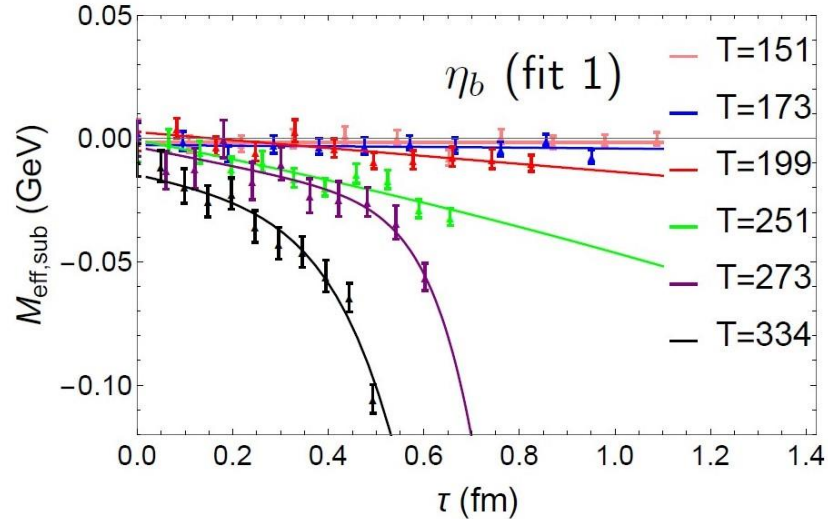
- Fit data with ansatz for spectral function $\rho(\omega)$ sketched in right plot
- Will subtract $T = 0$ continuum from finite temperature correlator

$$C_{sub, T>0}(\tau) = C_{T>0}(\tau) - C_{T=0}(\tau) + A \exp(-m_{T=0}\tau)$$

Mass shift and width of SPF from smeared correlators

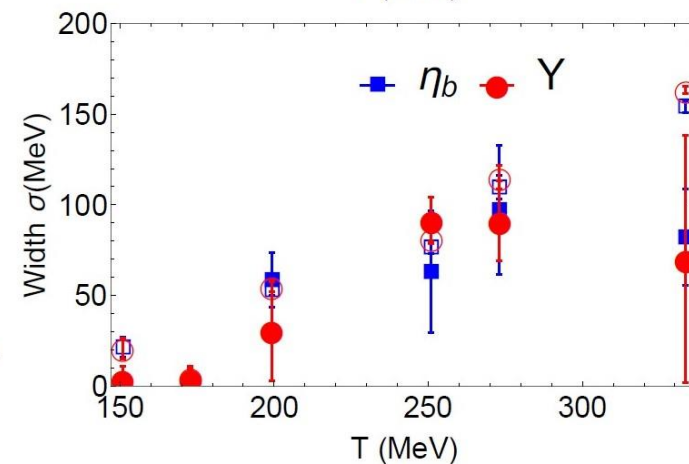
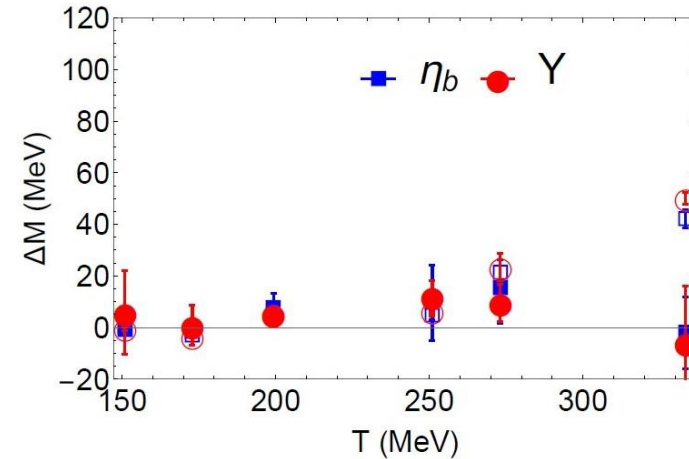
- HoTQCD ensembles, $N_\sigma = 48$, $N_\tau = 12$, $m_\pi = 160$ MeV, physical Kaon mass, NRQCD

- Zero temperature continuum subtracted
- (fit 1 filled) Main peak + small negative peak
- (fit 2 empty) Main peak and last 2 points removed



- Results for Υ and η_b very similar

Rasmus Larsen's poster

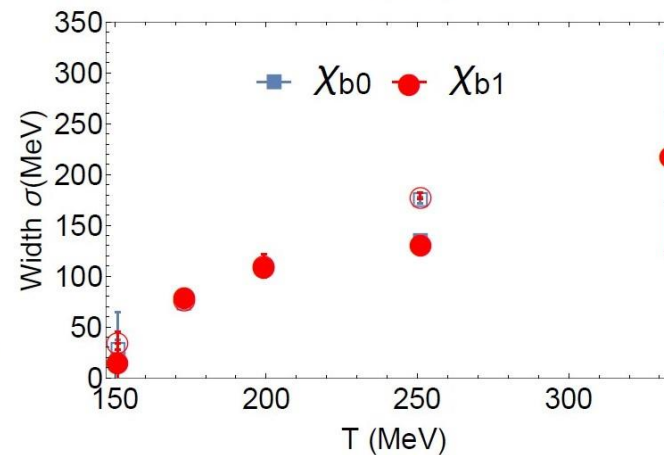
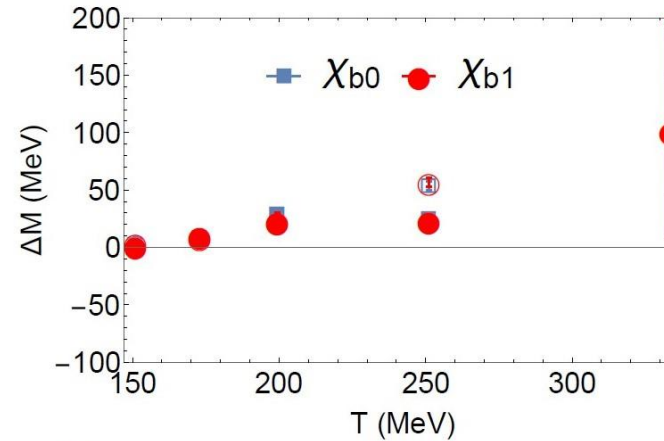
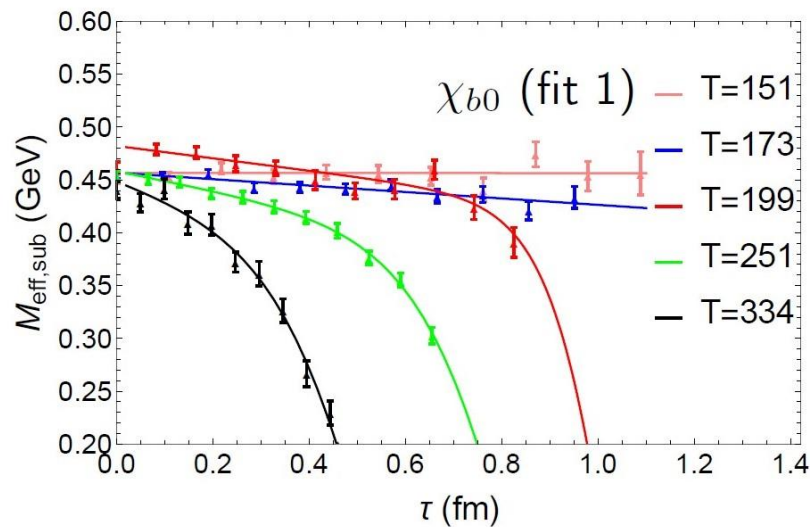


Mass shift and width of SPF from smeared correlators

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- (fit 1 filled) Main peak + small negative peak
- (fit 2 empty) Main peak and last 2 points removed

Rasmus Larsen's poster



- Results for χ_{b0} and χ_{b1} very similar

The other related talks/posters in this conference

- Real-Time-Evolution of Heavy-Quarkonium Bound States
Alexander Lehmann's talk on Wed. @11:10
- Euclidean correlation functions for transport coefficients under gradient flow
Hai-Tao Shu's talk on Wed. @11:30
- Heavy quark diffusion coefficient from lattice
Viljami Leino's talk on Wed. @11:50
- Non-perturbative study of heavy quark anti-quark potential at finite temperature
Dibyendu Bala's talk on Fri. @14:40
- Thermal Quarkonium Mass Shift from Euclidean Correlators
Alexander Maximilian Eller's poster
- Quarkonium suppression in non-equilibrium quark-gluon plasma
Zhandos Moldabekov's poster

Conclusions

- Quarkonia and open heavy flavors are important probes to investigate QGP.
- There are some difficulties for heavy flavor calculations on the lattice.
- Effective field theories are useful tool for bottom.
- There are a few approaches to reconstruct spectral functions.
- Full QCD studies for spectral functions of charmonia, bottomonia as well as open charm mesons have been already done.
- Continuum extrapolated quarkonia spectral functions have been studied with quenched QCD and perturbative models.
- A new kind of approach with smeared correlators to investigate quarkonium spectral function have been shown.
- There are variety of other approaches for heavy flavor physics.