The 37th International Symposium on Lattice Field Theory

Axial U(1) symmetry and mesonic correlators at high temperature in N_f=2 lattice QCD

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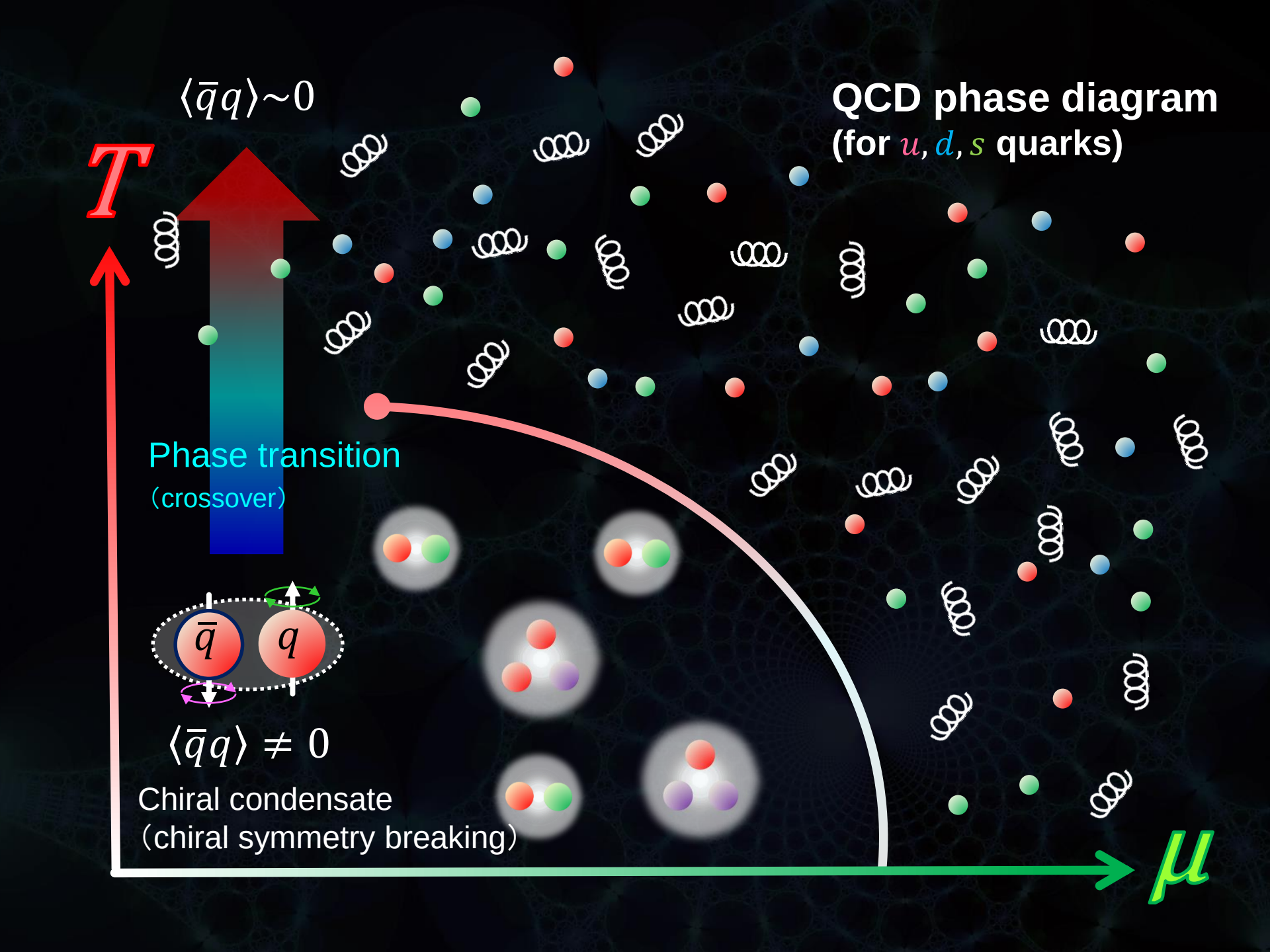
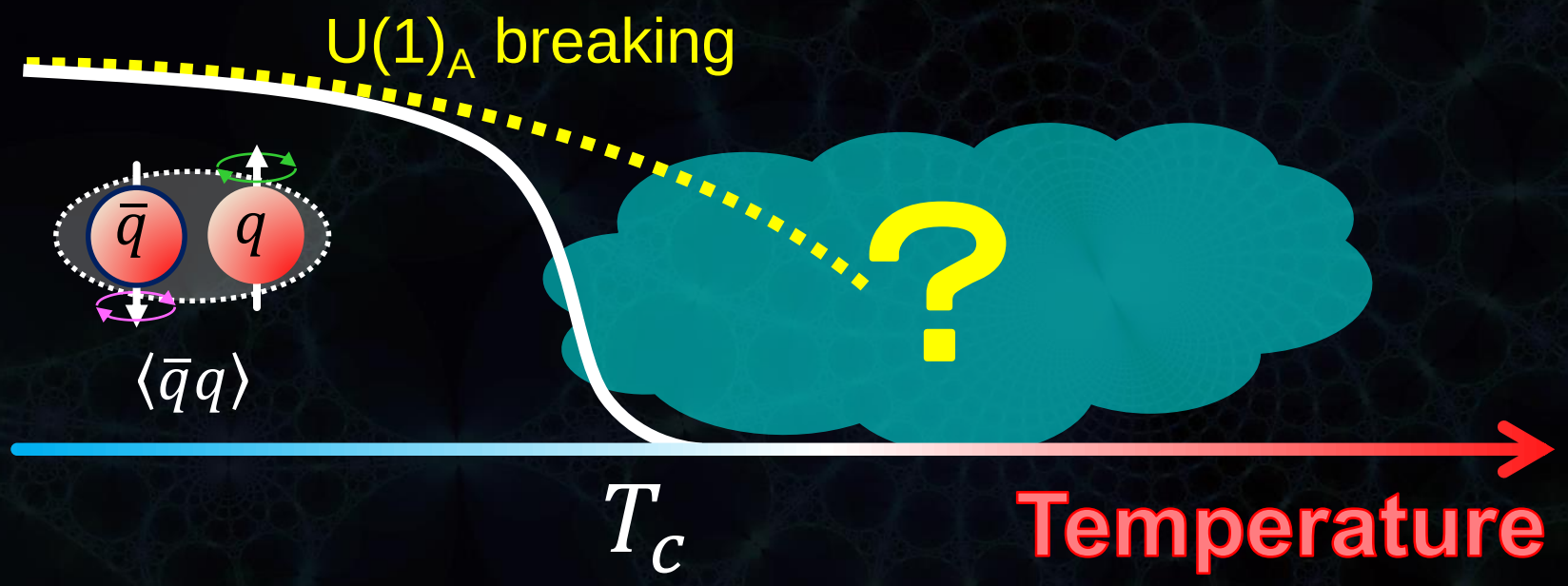

QCD phase diagram
(for u, d, s quarks)
$\langle \bar{q}q \rangle \sim 0$
T
Phase transition
(crossover)
$\bar{q}$
q
$\langle \bar{q}q \rangle \neq 0$
Chiral condensate
(chiral symmetry breaking)
μ

$U(1)_A$ symmetry（in vacuum, broken by anomaly） restored above Tc？

- Above Tc, chiral symmetry breaking by $\langle \bar{q}q \rangle$ is restored

  ⇒How about <u>$U(1)_A$ symmetry?</u>

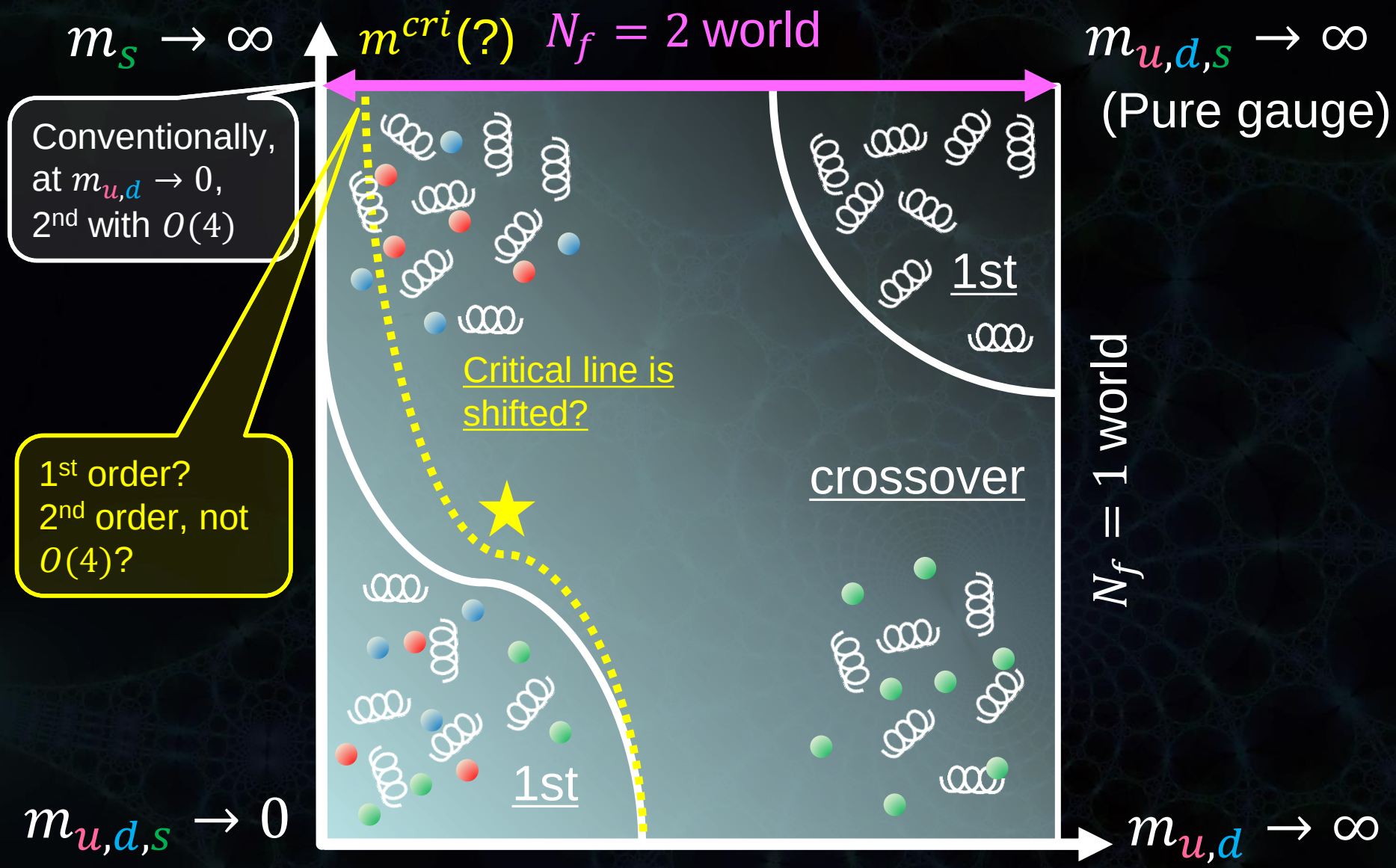
$$\Delta_{\pi-\delta} = \int_0^\infty d^4x \, [\pi^a(x)\pi^a(x) - \delta^a(x)\delta^a(x)]$$



Cf.) S. Aoki, H. Fukaya, and Y. Taniguchi, PRD86

If $U(1)_A$ is restored…

Colombia plot is modified?



$U(1)_A$ symmetry above Tc ⇒Long-standing problem in QCD

- Gross-Pisarski-Yaffe (Dilute instanton gas model, 1981) restored at enough high T
- Cohen (1996) w/o zero mode (or instanton)⇒restored
- Aoki-Fukaya-Taniguchi (theory, 2012) zero mode suppressed, restored in chiral limit at $N_f = 2$
- HotQCD (DW, 2012) broken
- JLQCD (topology fixed overlap, 2013) restored
- TWQCD (optimal DW, 2013) restored
- LLNL/RBC (DW, 2014) broken (restored at higher T?)
- Dick et al. (overlap on HISQ, 2015) broken
- Sharma et al. (overlap on DW, 2015,2016,2018) broken
- Brandt et al. (Wilson, 2016,2019) restored (broken at larger V?)
- Ishikawa et al. (Wilson, 2017) restored
- JLQCD (reweighted overlap on DW, 2016) restored
- Gomez Nicola-Ruiz de Elvira (theory, 2017) restored
- Rohrhofer et al. (DW, 2017) restored

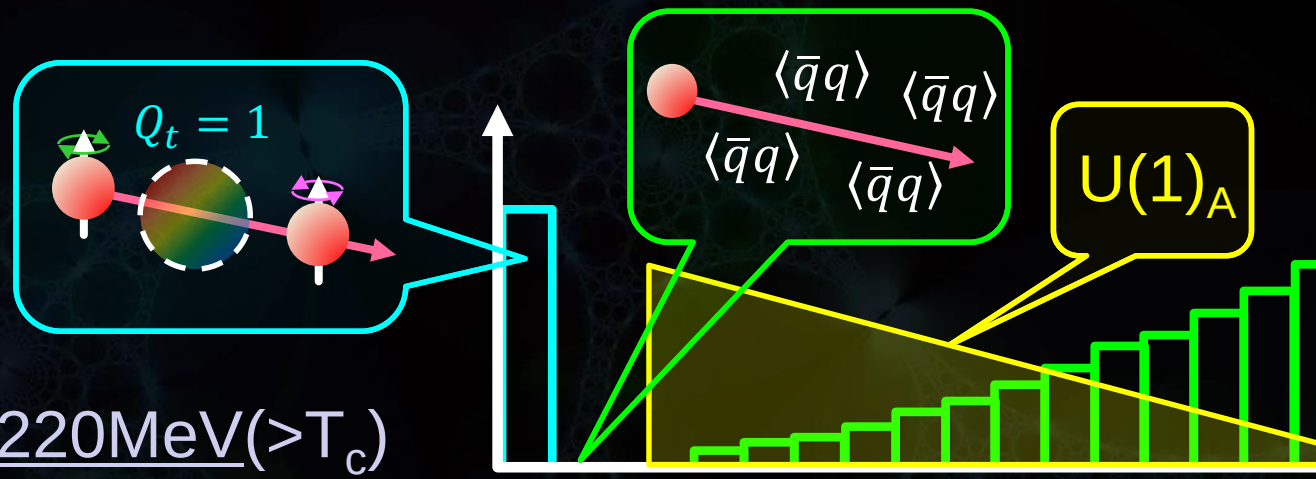
⇒ Many suggestions from lattice QCD (and models)…

Lattice study with chiral fermion

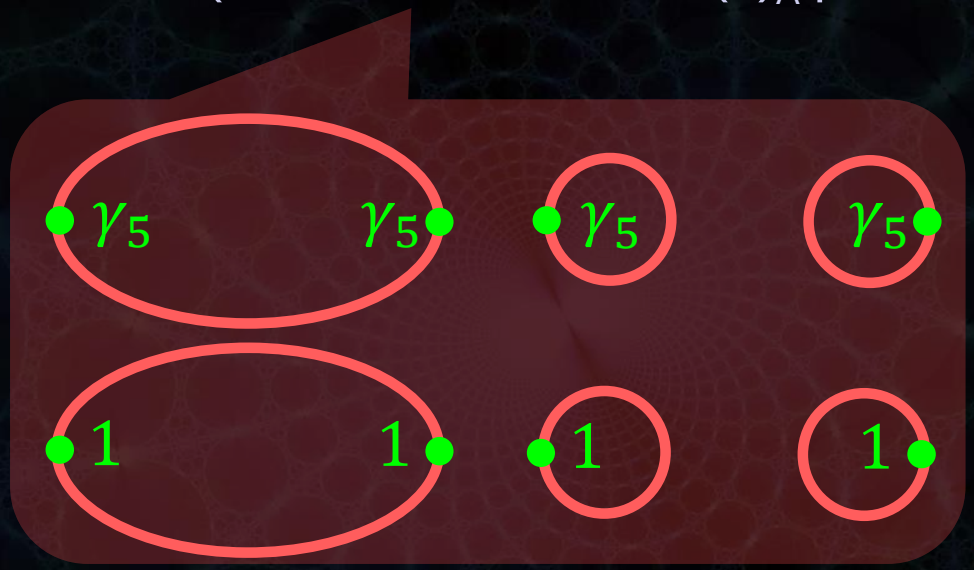
by JLQCD Collaboration (2013-2019) ⇒ $U(1)_A$ symmetry restored

	valence/sea quark	Setup
G. Cossu et al. PRD87 (2013)	OV on OV (Topology fixed sector)	
A. Tomiya et al. PRD96 (2017)	DW on DW OV on DW OV on (reweighted) OV	1/a=1.7GeV (a=0.11fm)
This work (JLQCD, 2019)	OV on DW OV on (reweighted) OV	1/a=2.6GeV (a=0.076fm) (Finer lattice)

Outline

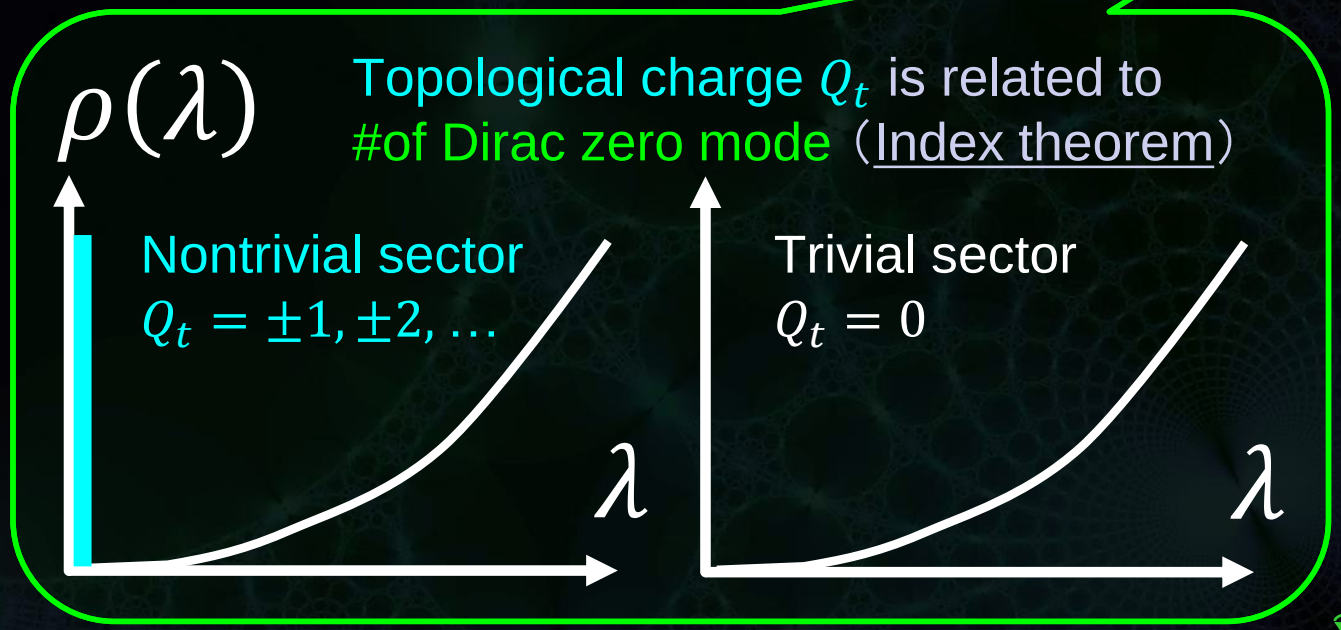


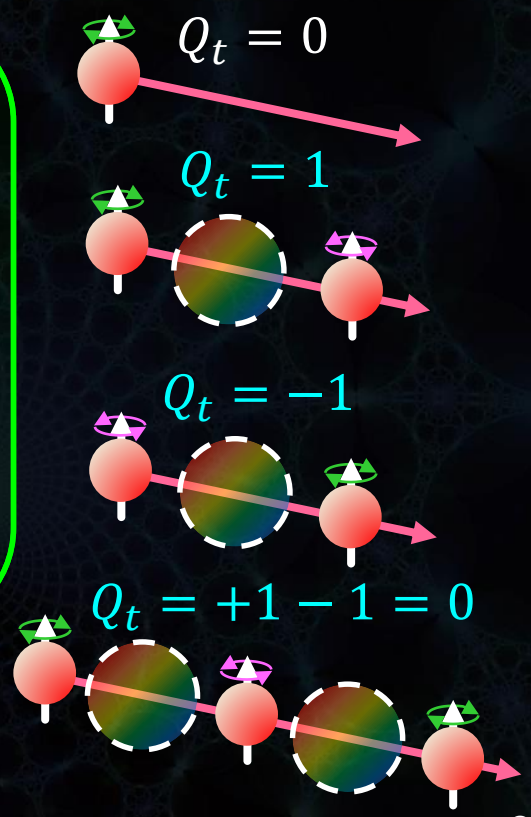
1. Introduction
2. Results at T=220MeV(>T_c)
   - 3-1: Topological susceptibility
   - 3-2: U(1)$_A$ susceptibility (and finite-V effect)
   - 3-3: Mesonic correlators (conn. and disc. for U(1)$_A$ partners)
3. Summary



Topological susceptibility and zero mode of Dirac spectra

$$\chi_t \equiv \frac{\langle Q_t^2 \rangle}{V}, \qquad Q_t = n_+ - n_-$$

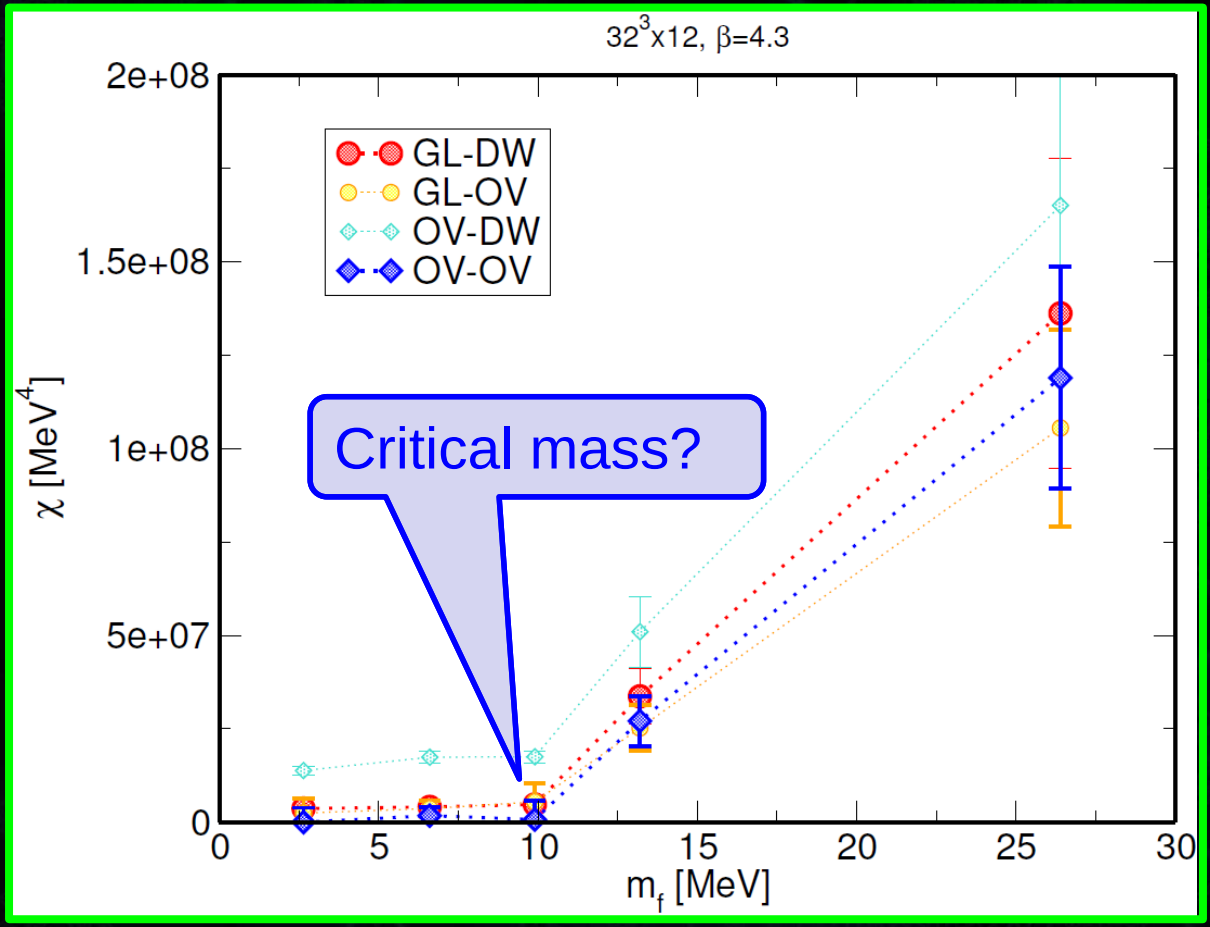




Cf.) Gluonic definition: $Q_t \equiv \frac{g^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}$

JLQCD, preliminary (2019)

Top. susceptibility at T = 220MeV



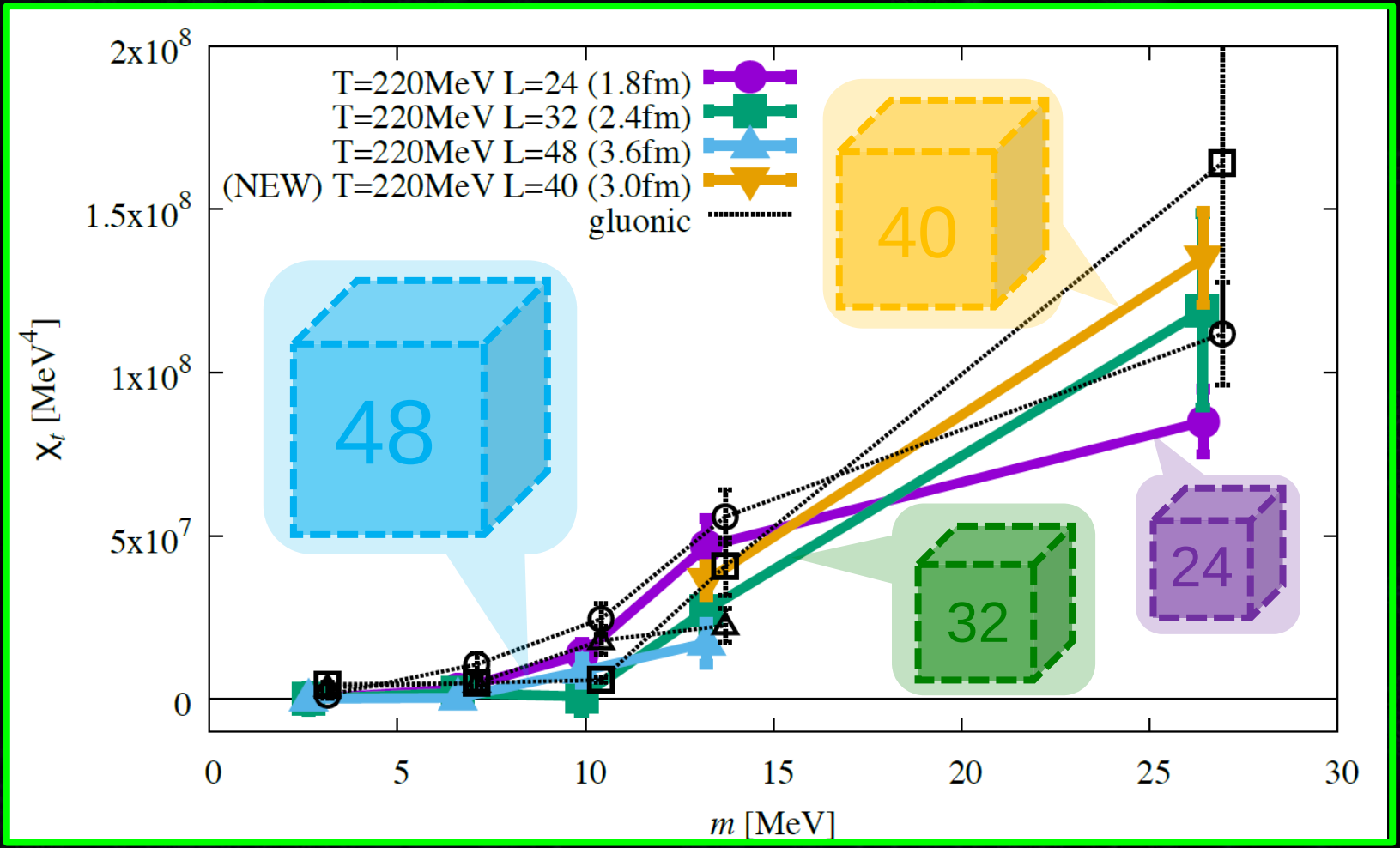
⇒Small m_q: $\chi_t = 0$

⇒Around m_q~10MeV, we found a jump (critical mass?)

Cf.) S. Aoki, H. Fukaya, and Y. Taniguchi PRD86 (2012), 114512

JLQCD, preliminary (2019)

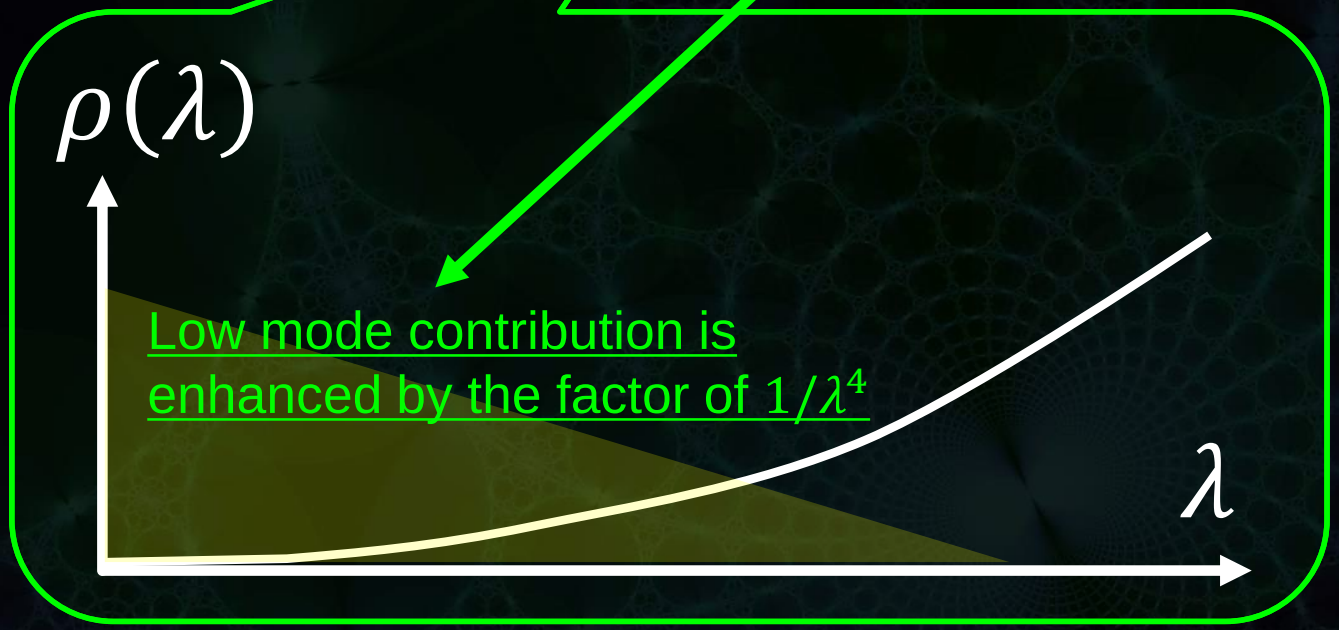
Top. susceptibility (Volume dependence)



⇒Small m_q: <u>no volume dependence</u>

$U(1)_A$ susceptibility and low modes of Dirac spectra

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{2m^2}{(\lambda^2 + m^2)^2}$$



Cf.) Banks-Casher relation: $\langle \bar{q}q \rangle = \lim_{m\to 0} \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{2m}{\lambda^2 + m^2}$

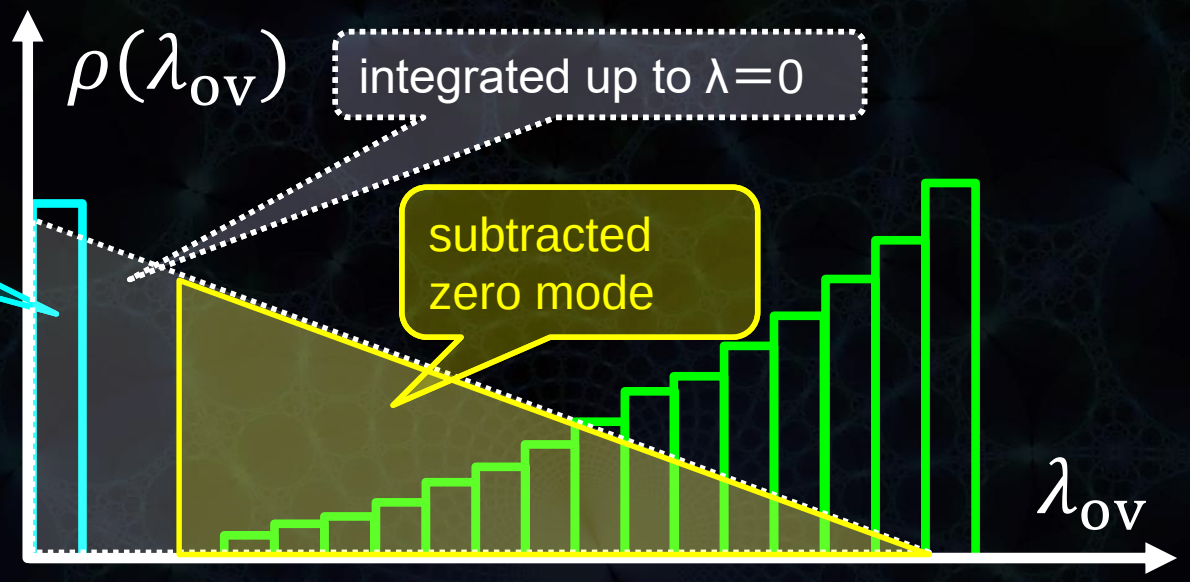
S. Aoki, H. Fukaya, and Y. Taniguchi PRD86 (2012), 114512
A. Tomiya et al. (JLQCD) PRD96 (2017), 034509

Note:

$U(1)_A$ susc. = Low modes + ~~Zero~~ mode ?

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda\, \rho(\lambda) \frac{2m^2}{(\lambda^2+m^2)^2} \Rightarrow \Delta^{\rm ov}_{\pi-\delta} \equiv \frac{1}{V(1-m^2)^2} \sum_i \frac{2m^2(1-\lambda_{\rm ov}^{(i)2})^2}{\lambda_{\rm ov}^{(i)4}}$$

The factor of $1/\lambda^4$ enhances zero-mode contribution?



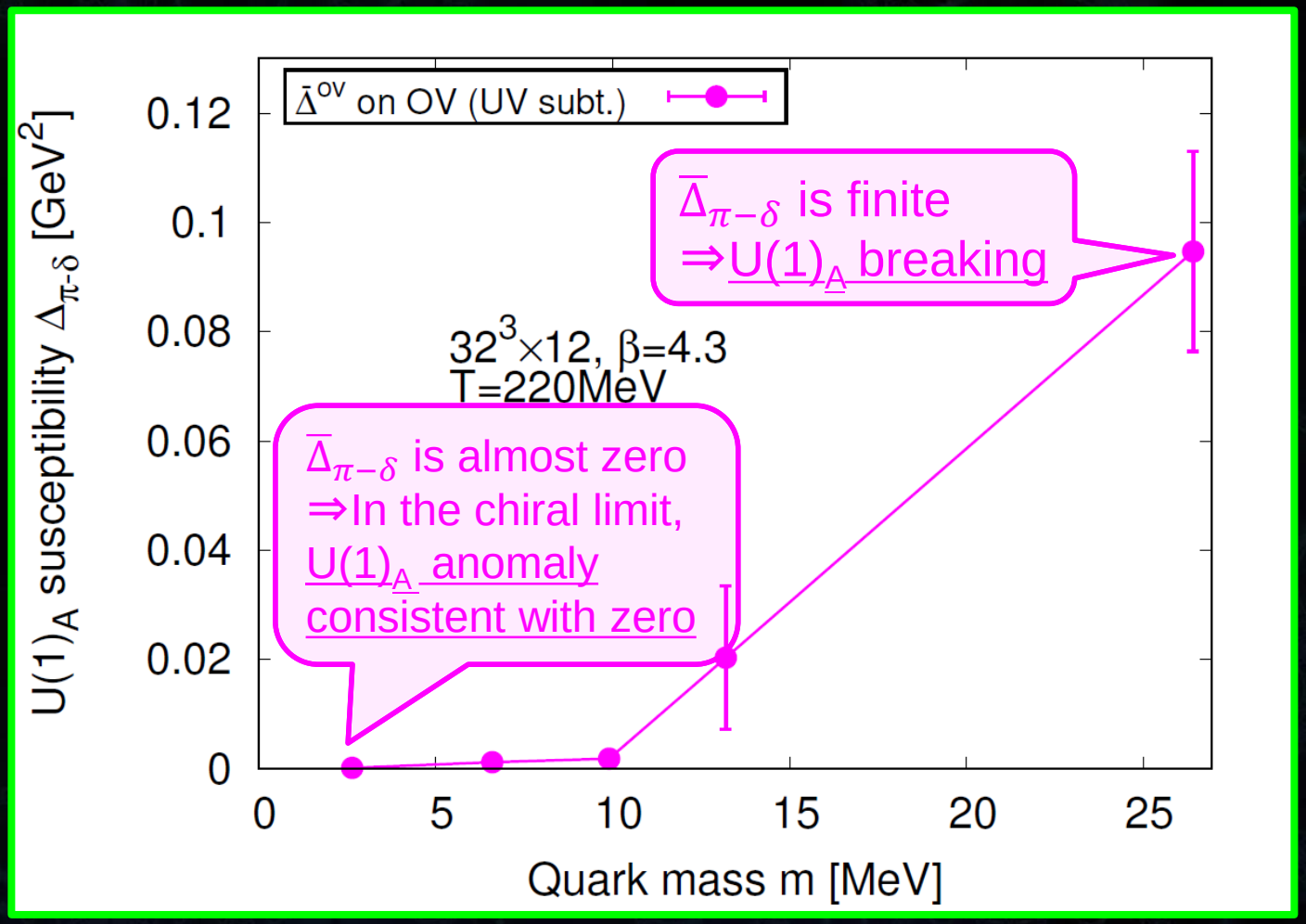
In $V \to \infty$ limit, we know zero-mode contribution is suppressed:

$$\Delta^{\rm ov}_{0-mode} = \frac{2N_0}{Vm^2} (\propto 1/\sqrt{V})$$

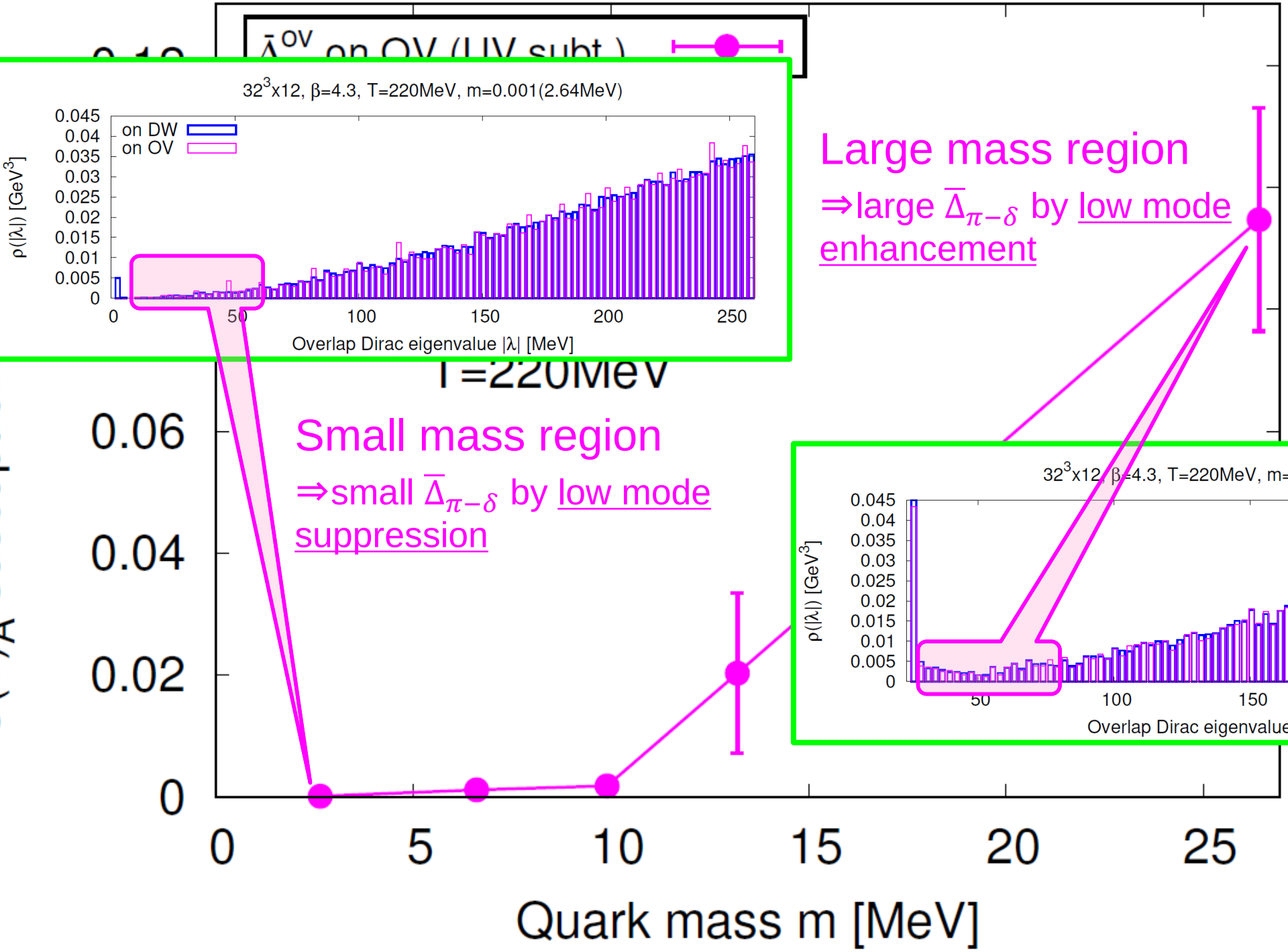
New order parameter: we subtract zero mode

$$\bar{\Delta}^{\rm ov}_{\pi-\delta} \equiv \Delta^{\rm ov}_{\pi-\delta} - \frac{2N_0}{Vm^2}$$

$U(1)_A$ susceptibility at T = 220MeV

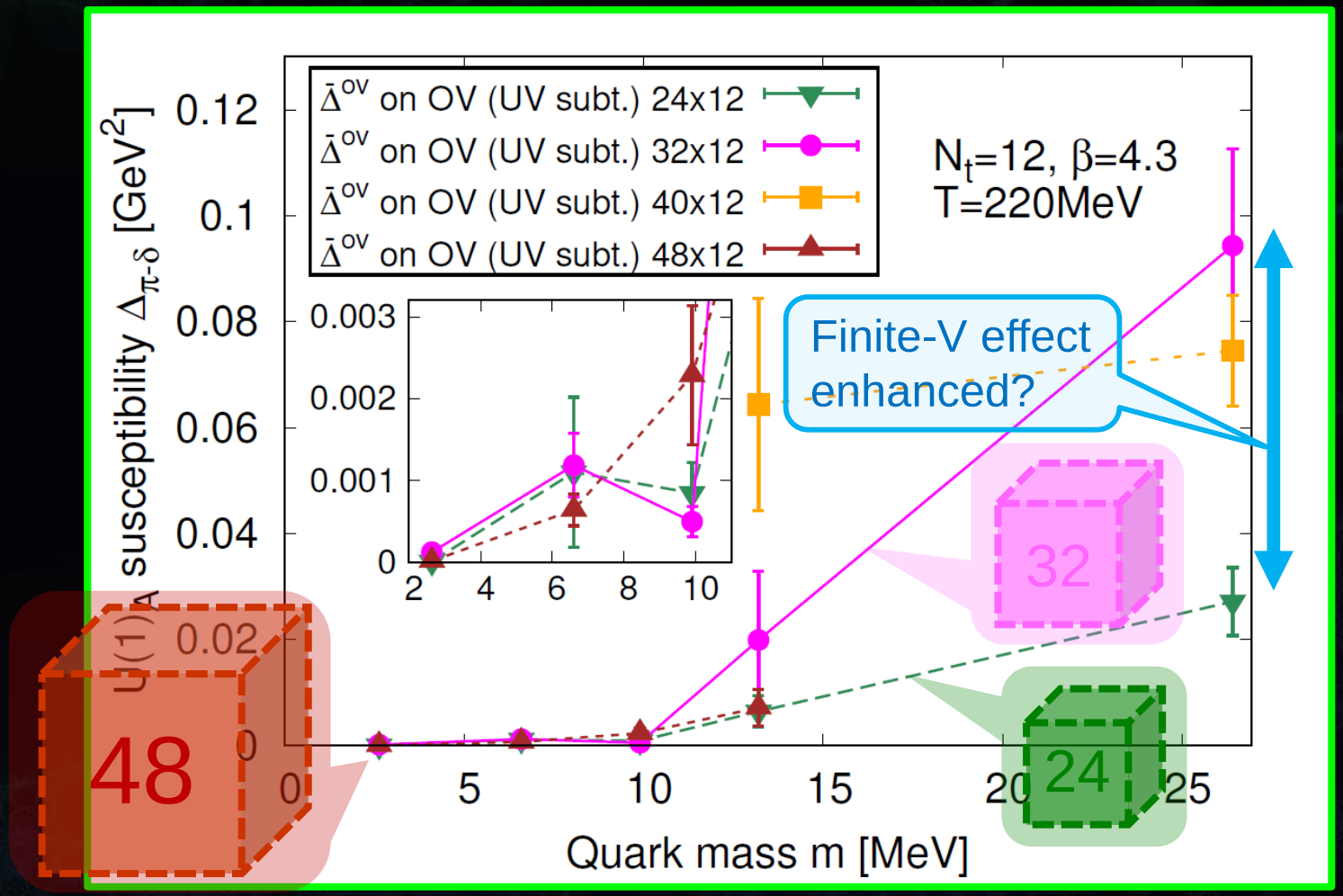


⇒At m_q =2.6MeV, we found suppression of 10^{-4}GeV^2


32^3x12, β=4.3, T=220MeV, m=0.001(2.64MeV)
on DW
on OV
ρ(|λ|) [GeV^3]
Overlap Dirac eigenvalue |λ| [MeV]
Large mass region
⇒large $\bar{\Delta}_{\pi-\delta}$ by low mode enhancement
Small mass region
⇒small $\bar{\Delta}_{\pi-\delta}$ by low mode suppression
T=220MeV
0.06
0.04
0.02
0
0
5
10
15
20
25
Quark mass m [MeV]

JLQCD, preliminary (2019)

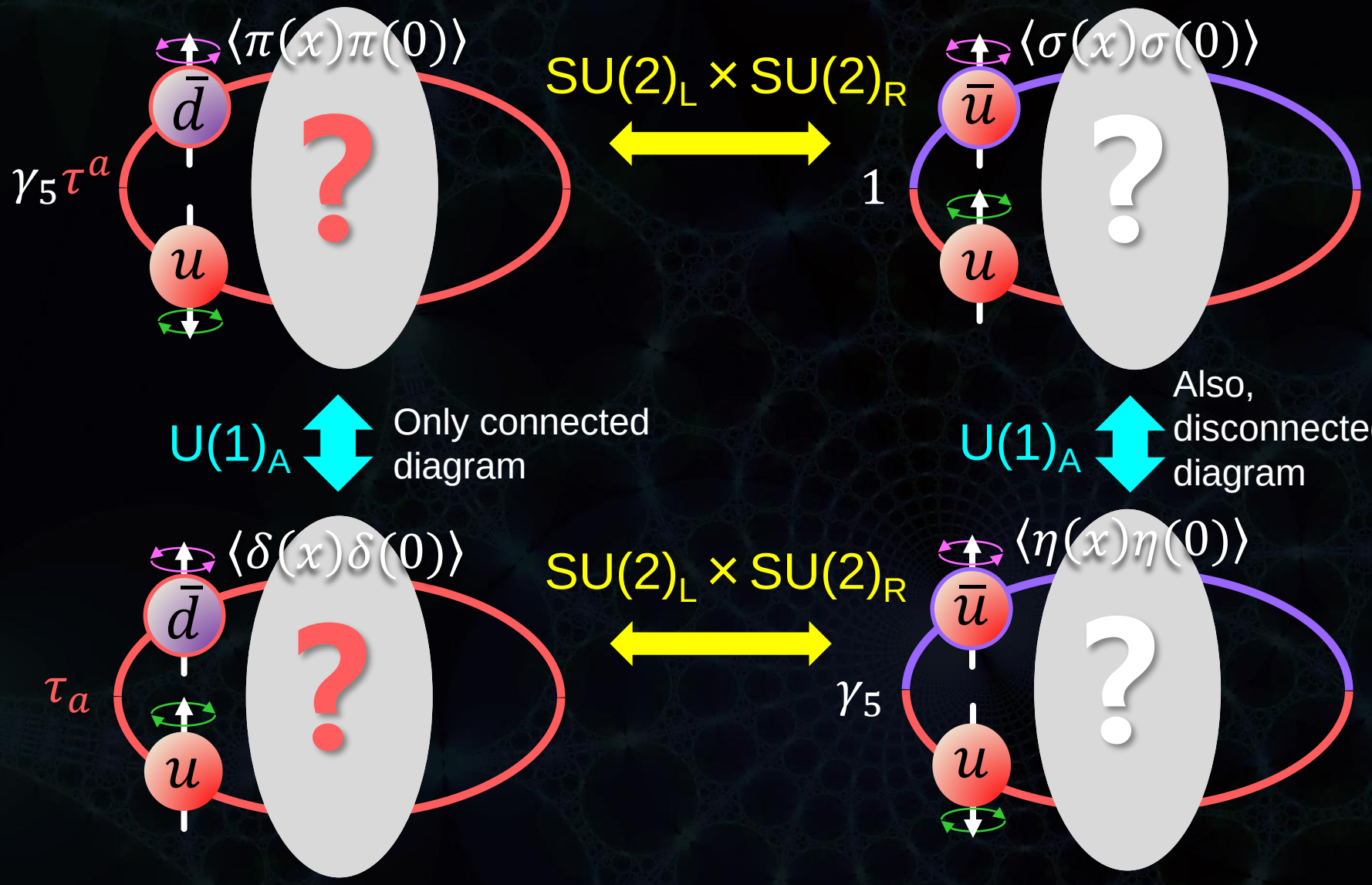
$U(1)_A$ susceptibility (Volume dependence)



⇒Small m_q: V-dependence seems to be small

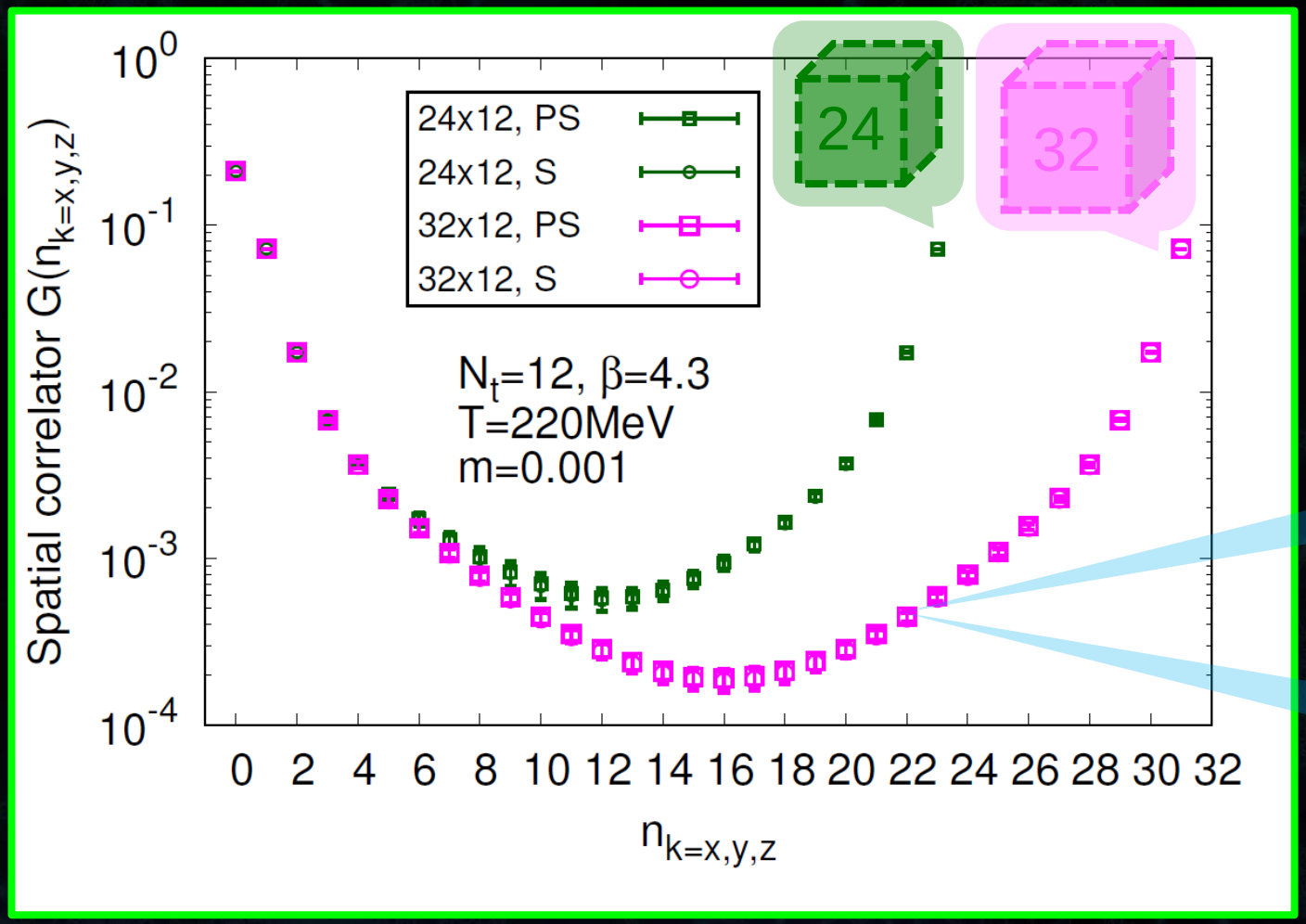
⇒Large m_q: we found a finite-V effect btw 24 and 32 lattice

Mesonic correlators (PS/S for $N_f = 2$)



JLQCD, preliminary (2019)

PS-S(Connected) Correlators: $U(1)_A$ partners

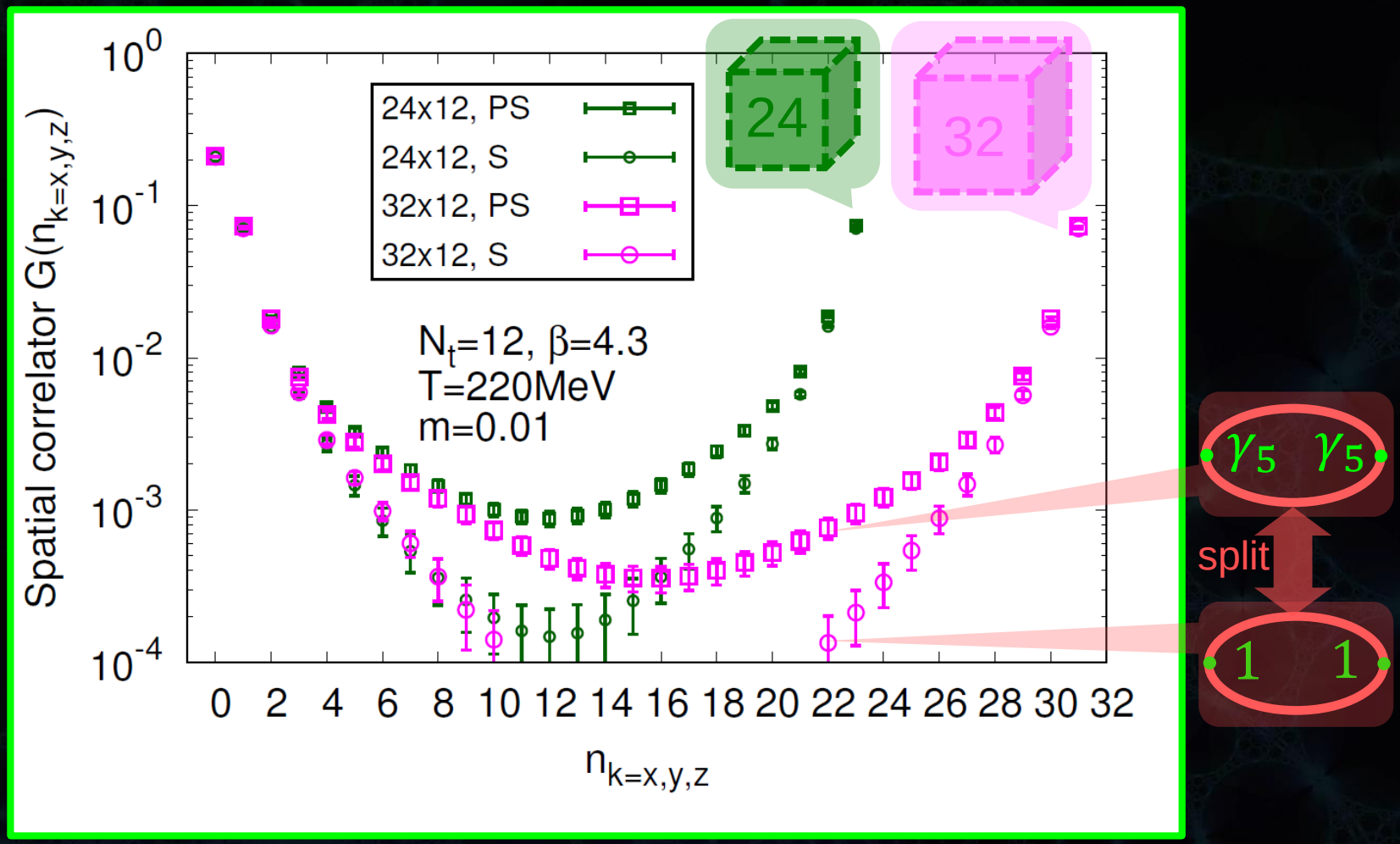


⇒Small m_q : $U(1)_A$ restoration, Large m_q : $U(1)_A$ breaking

(Details of conn. correlators ⇒ Next talk by C. Rohrhofer)

JLQCD, preliminary (2019)

PS-S(Connected) Correlators: $U(1)_A$ partners



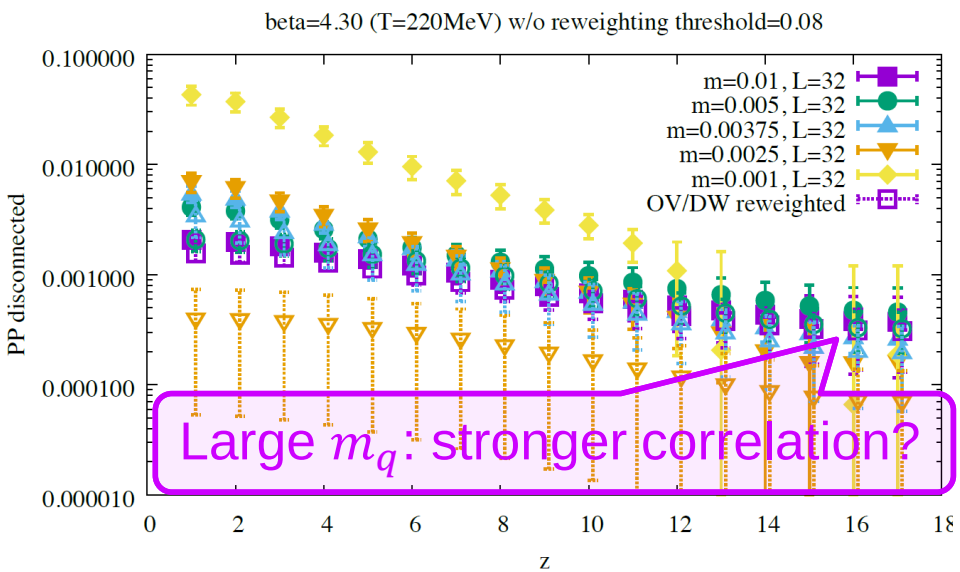
⇒Small m_q : $U(1)_A$ restoration, Large m_q : $U(1)_A$ breaking

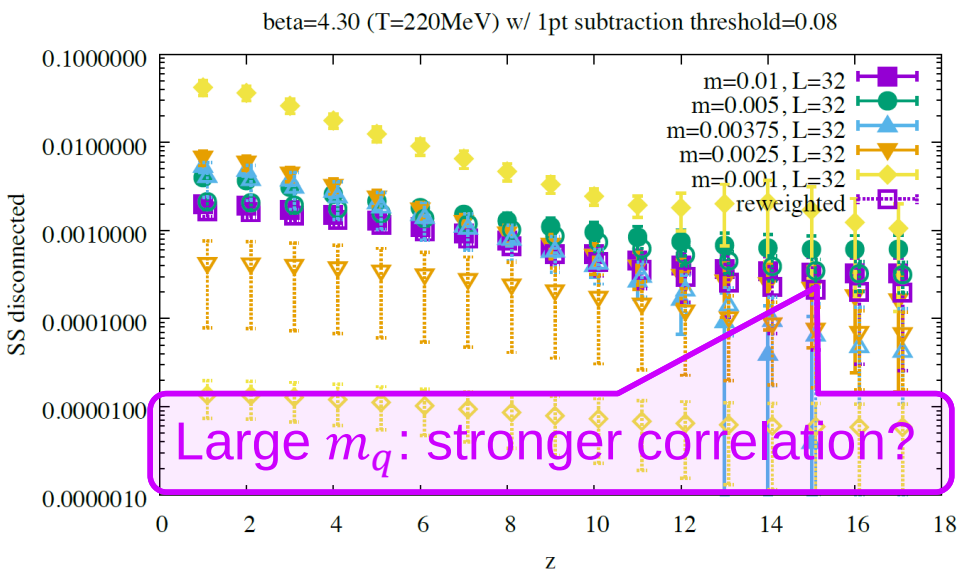
(Details of conn. correlators ⇒ Next talk by C. Rohrhofer)

JLQCD, preliminary (2019)

PS and Scalar(Disconnected) correlators

from OV Dirac modes



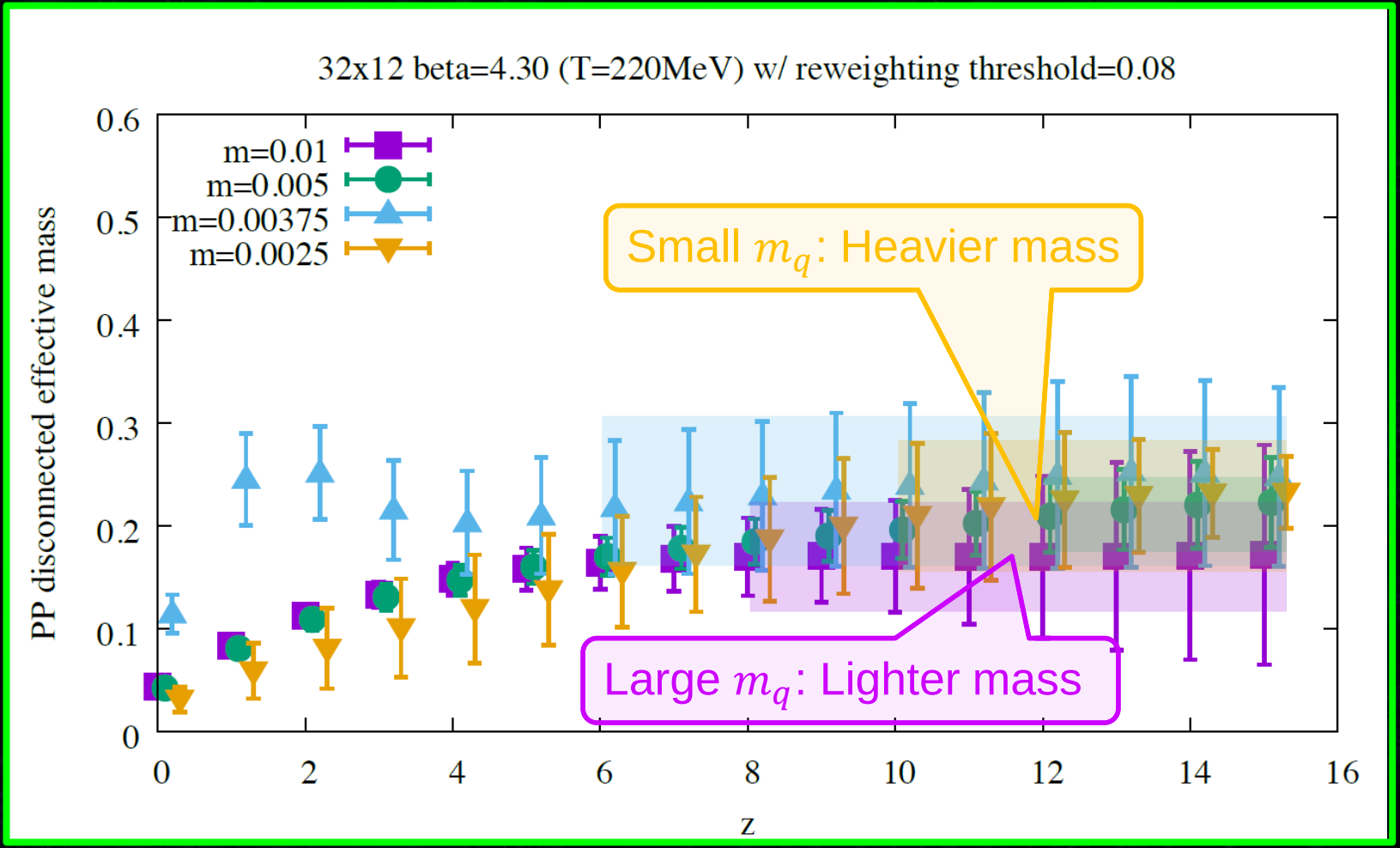


⇒ Large m_q: Spatial correlation becomes stronger?

⇒ Let's check screening masses!

JLQCD, preliminary (2019)

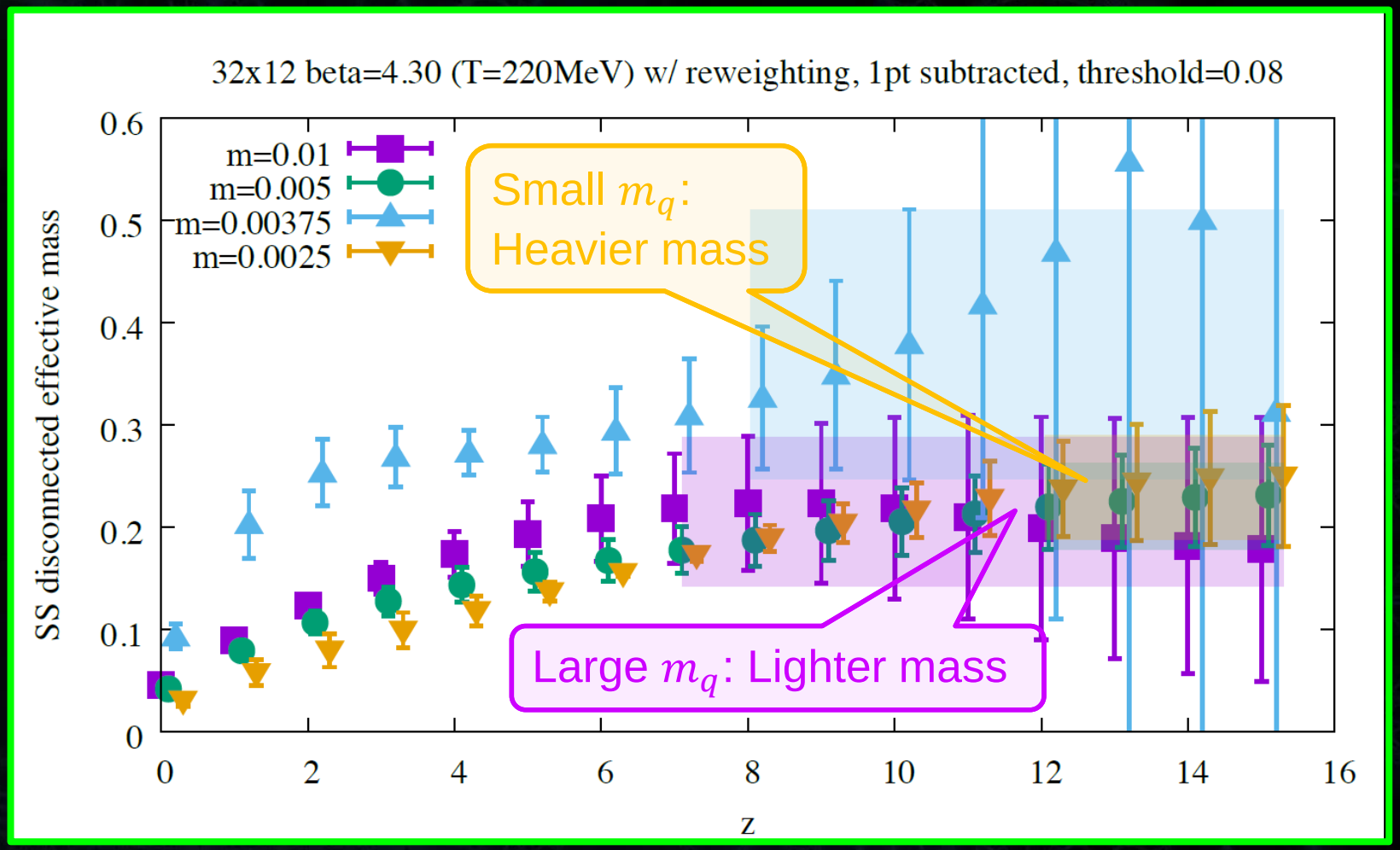
PS(Disconnected) screening mass



⇒ Large m_q: m_{PS}^{dis}[~450MeV] $<$ m_{PS}^{con}[~850MeV]

JLQCD, preliminary (2019)

Scalar(Disconnected) screening mass

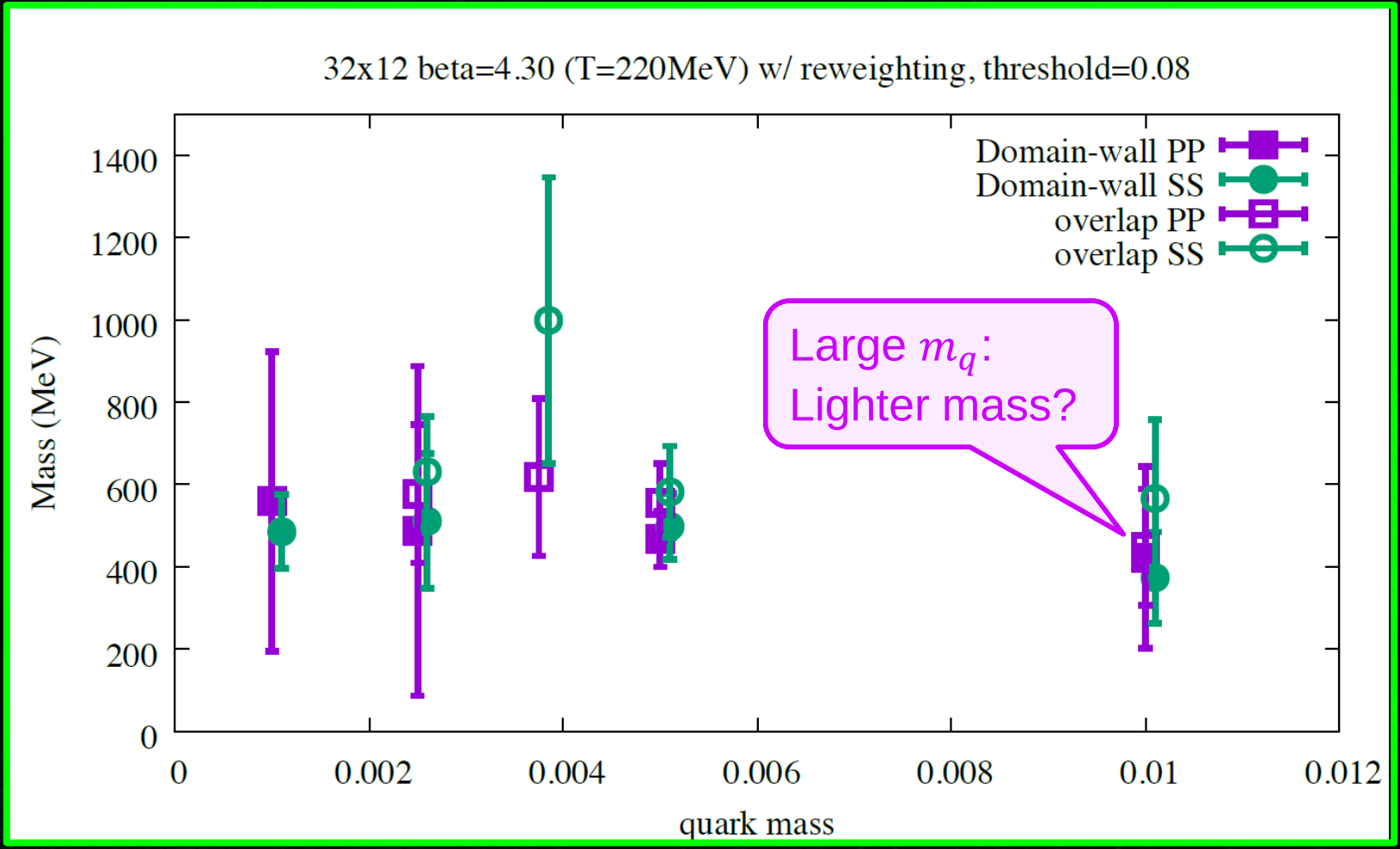


⇒ Large m_q: m_S^{dis}[~500MeV] $<$ m_S^{con}[~1050MeV]

JLQCD, preliminary (2019)

(Disconnected) Screening mass

from OV and DW Dirac modes



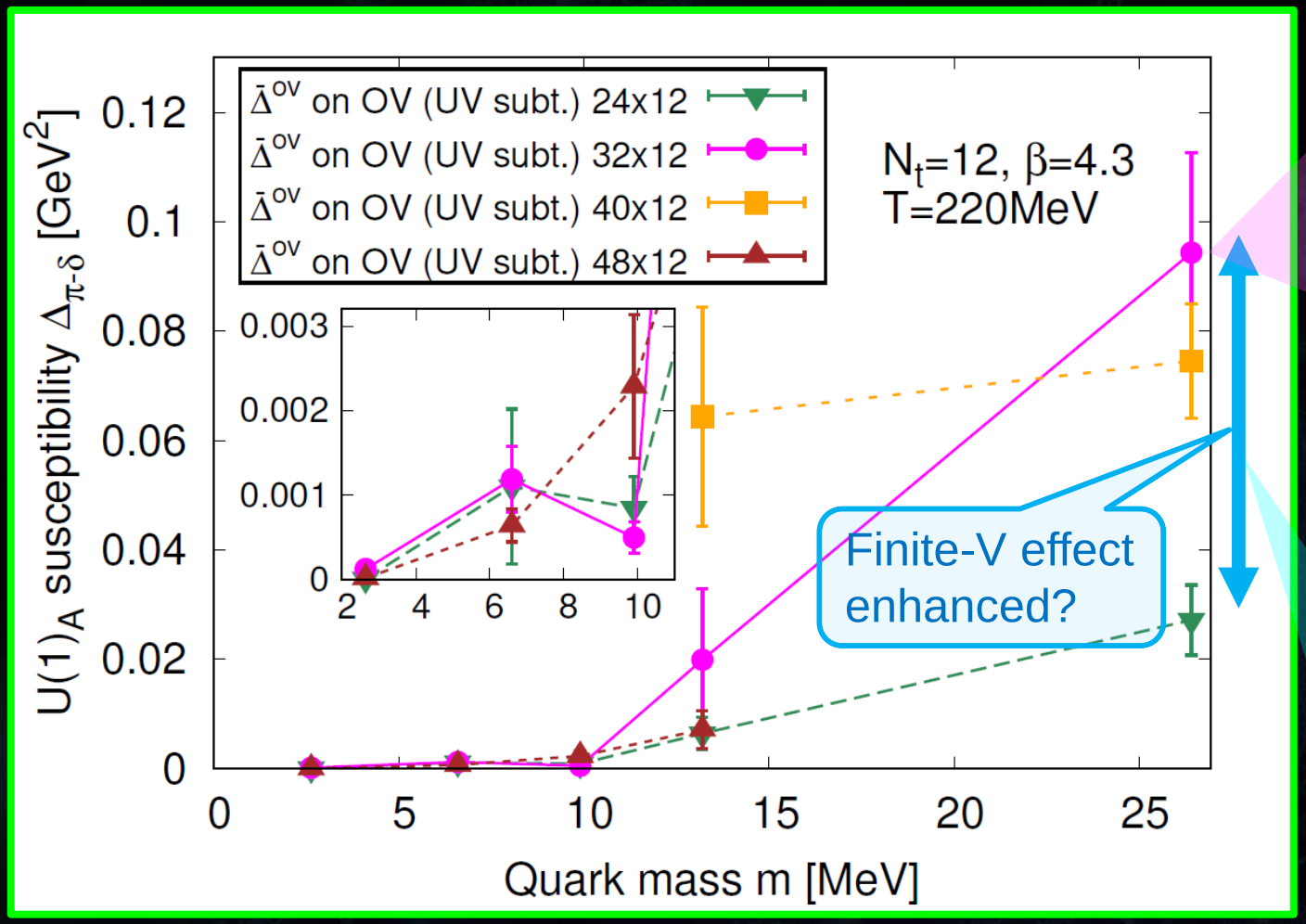
⇒ Large m_q: Lighter screening masses? ⇒ Long-distance correlations

（Finite-V effect between L=24 [~1.8fm] and L=32 [~2.4fm]?）

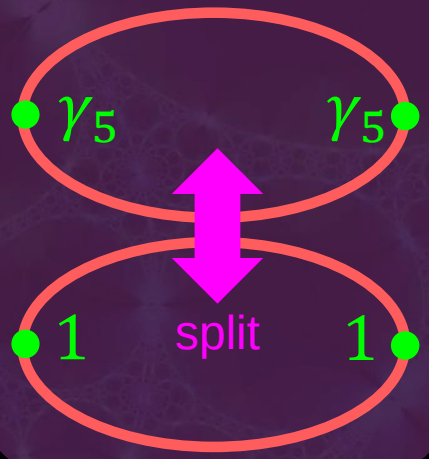
JLQCD, preliminary (2019)

Short summary:

$U(1)_A$ and Correlators



$U(1)_A$ violation by conn. diag.



Finite-V effect by disc. diag.?

Summary and Outlook

- We study high-temperature phase ($T > T_c$) at $N_f = 2$
- Top. susceptibility drops to be consistent with zero at m_q= a few MeV
- $U(1)_A$ susceptibility is also strongly suppressed in the chiral limit
- At $L \geq$2.4fm, V-dep. is under control, but at $L =$1.8fm, we find a sizable effect at large m_q (caused by disc. correlation?)

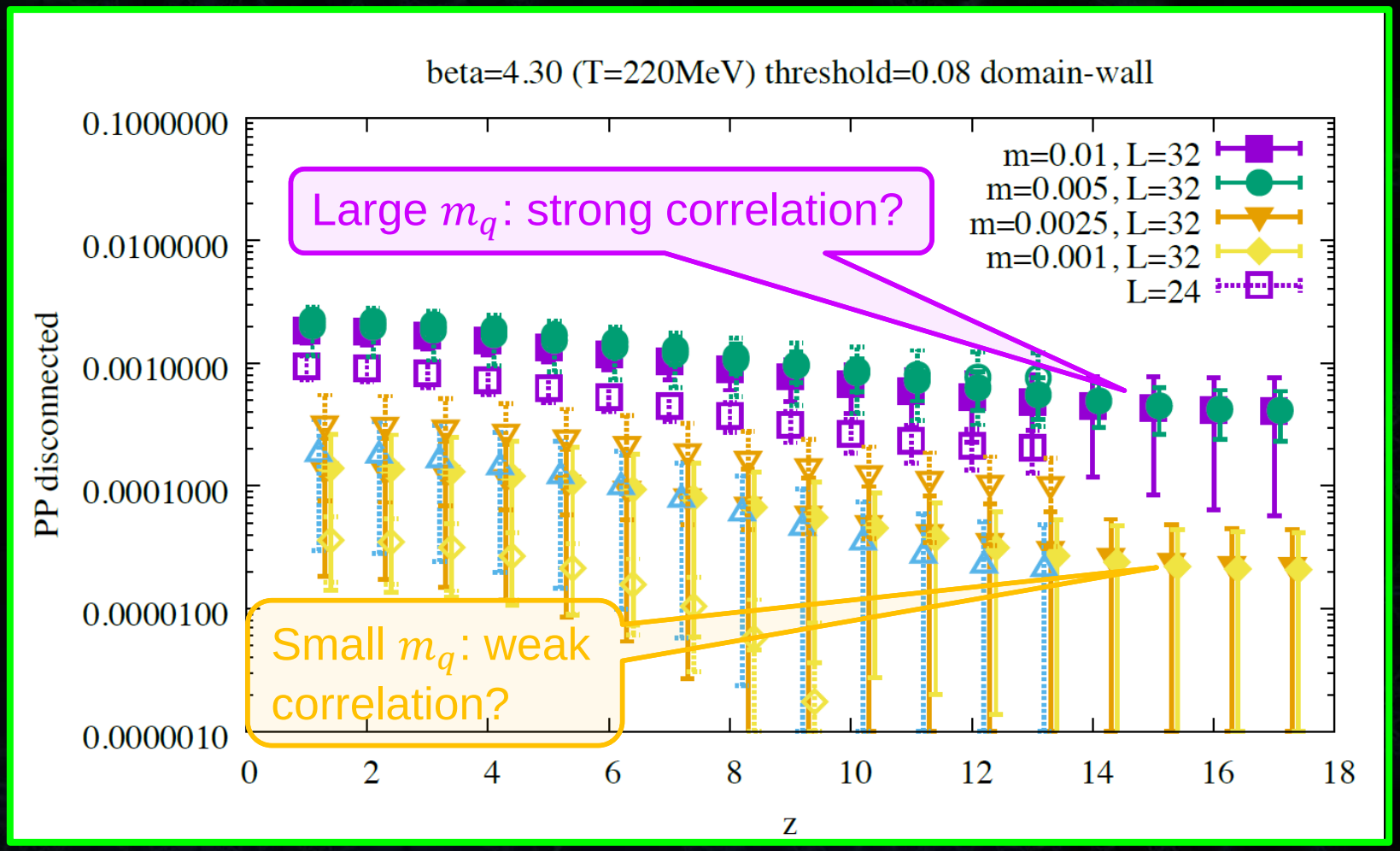
- Symmetries for mesonic/baryonic correlators
  (⇒ Next talk by C. Rohrhofer)
- Near T_c ($N_t = 14$ [T~190MeV]?, chiral transition?)
- $N_f = 2 + 1$ sector

Backup

JLQCD, preliminary (2019)

PS(Disconnected) correlator

from DW Dirac modes

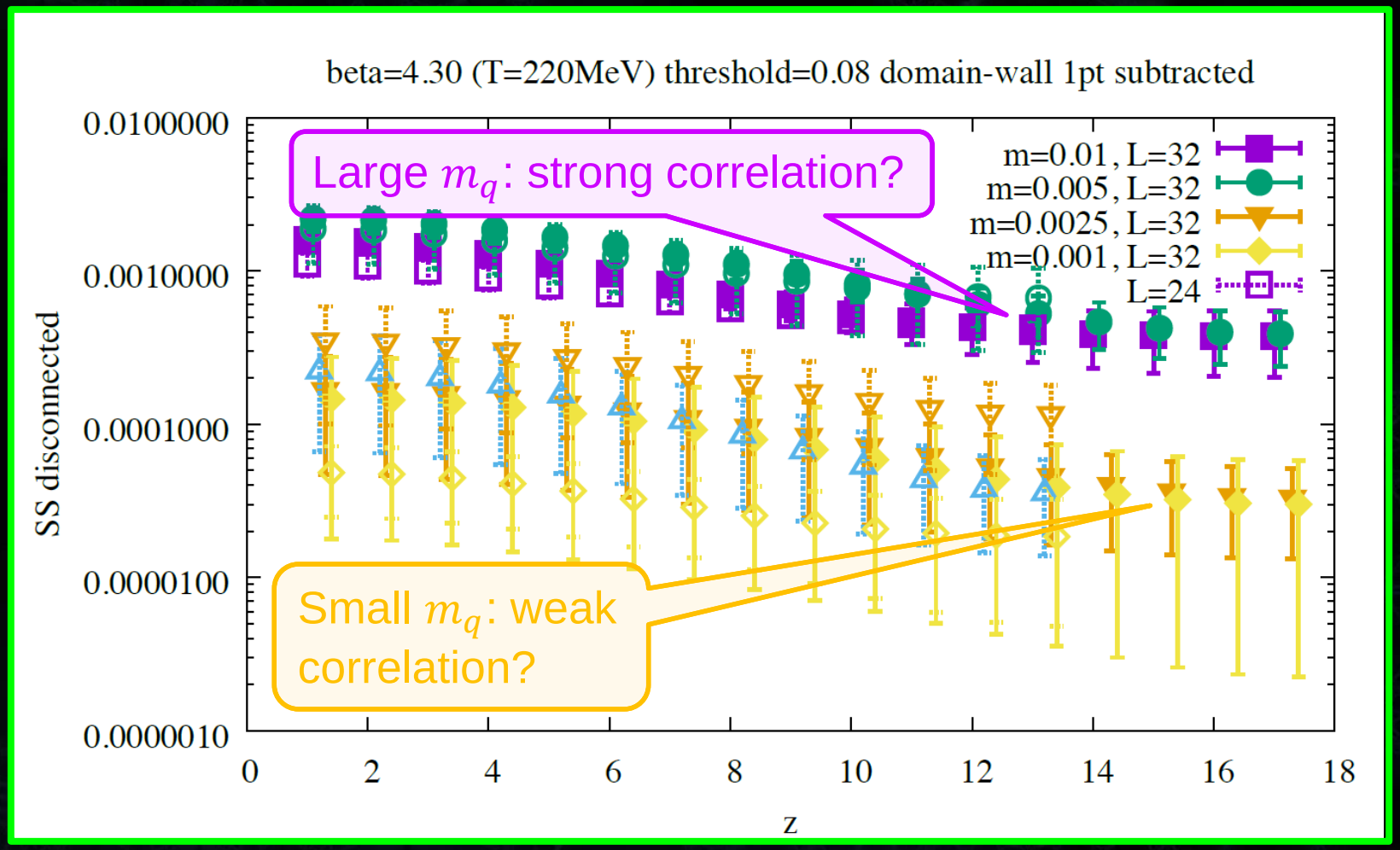


⇒ Large m_q: Correlation becomes strong ⇒ screening masses?

JLQCD, preliminary (2019)

Scalar(Disconnected) correlator

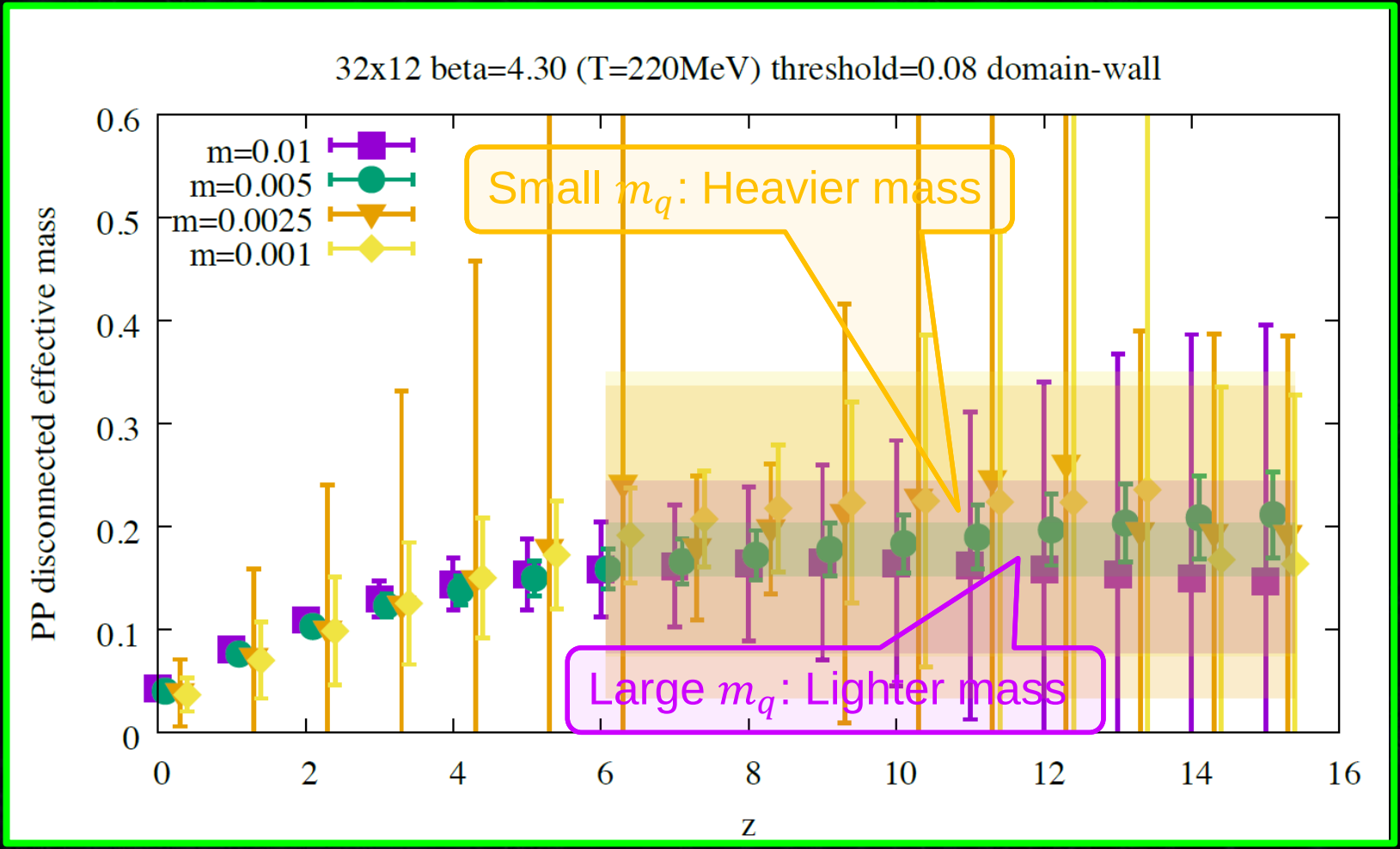
from DW Dirac modes



⇒ Large m_q: Correlation becomes strong ⇒ screening masses?

JLQCD, preliminary (2019)

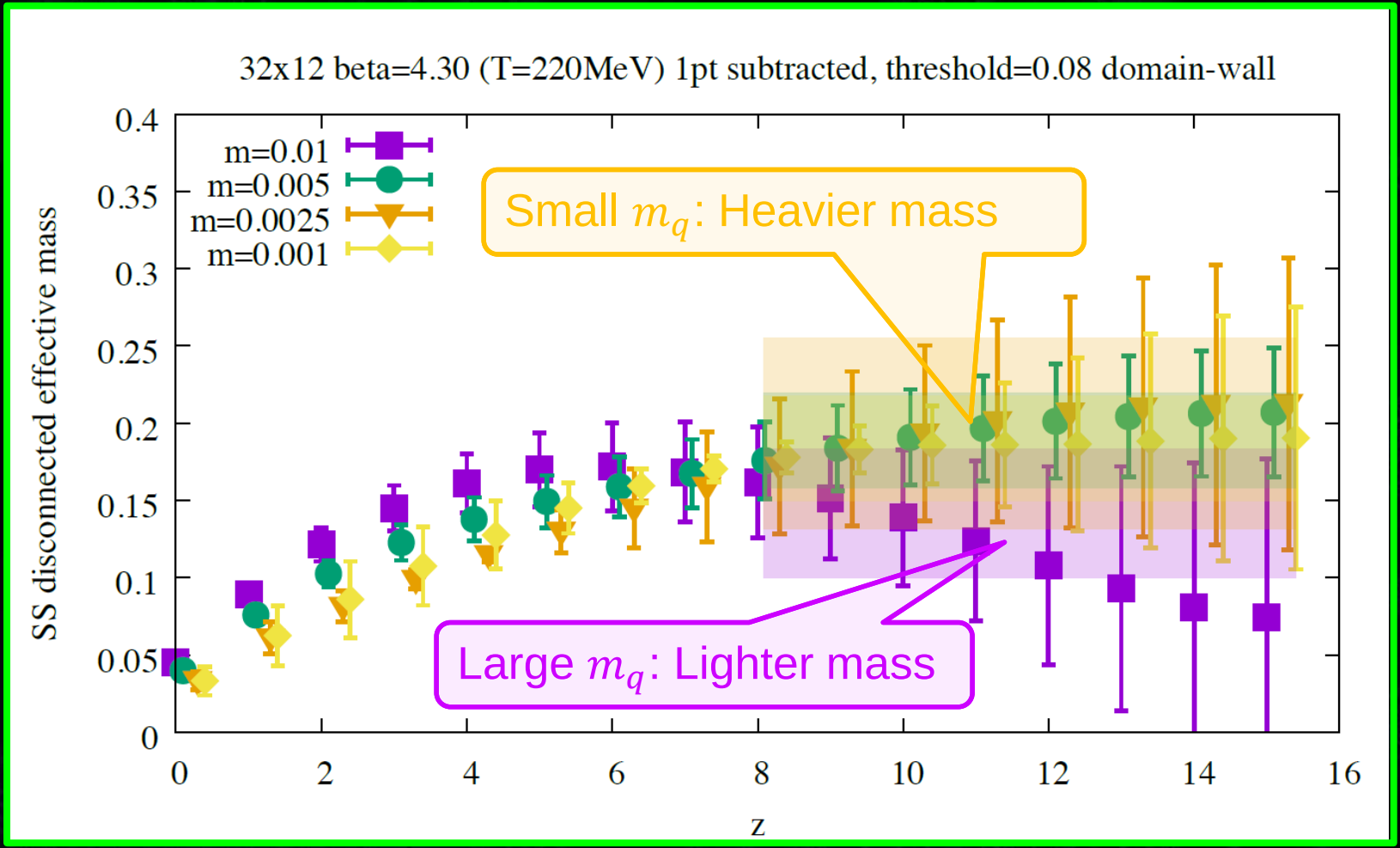
PS(Disconnected) screening mass



⇒ Large m_q: m_{PS}^{dis}[~350MeV] < m_{PS}^{con}[~850MeV]

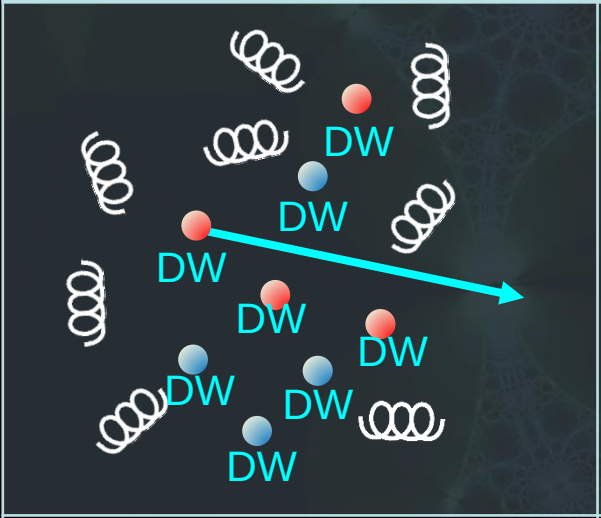
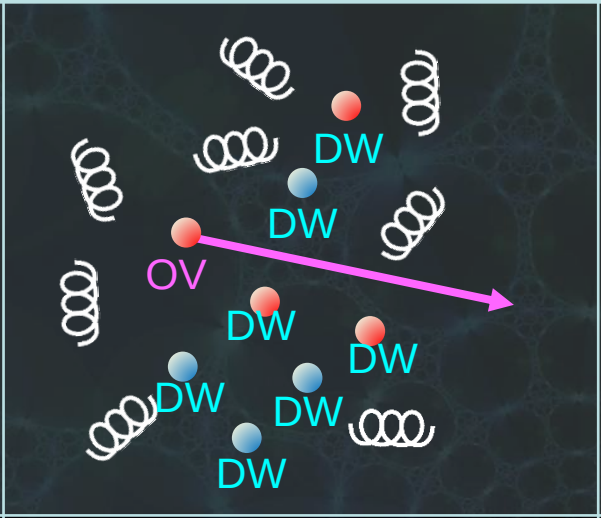
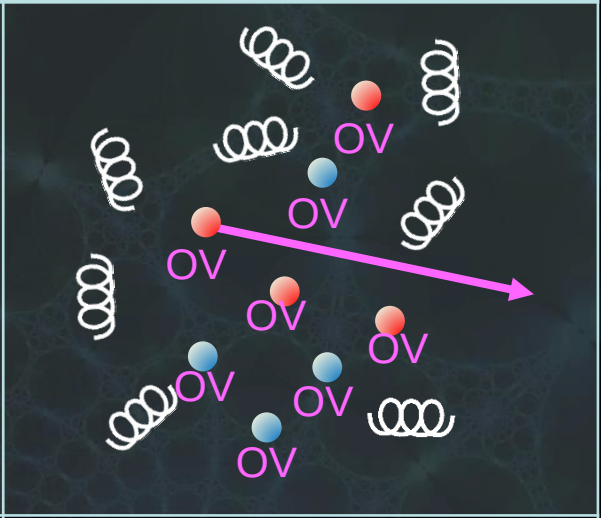
JLQCD, preliminary (2019)

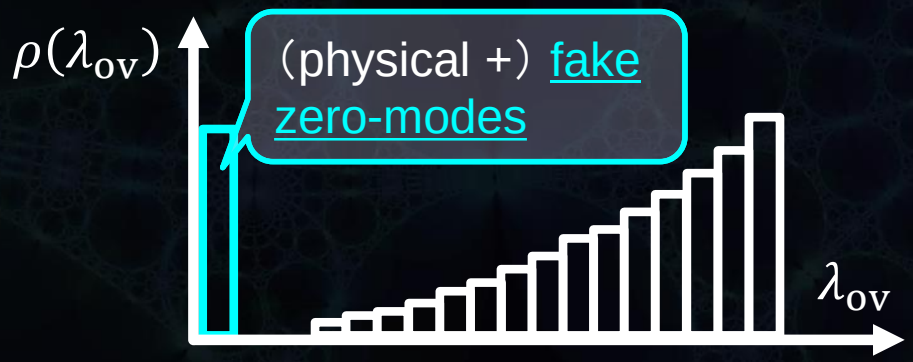
Scalar(Disconnected) screening mass



⇒ Large m_q: m_{PS}^{dis}[~350MeV] $<$ m_{PS}^{con}[~1050MeV]

Valence quark and Sea quark

DW on DW	OV on DW	OV on OV
		
Almost good chiral symmetry	Fake zero-mode appears as an artifact	Exact chiral symmetry, but, very high cost



DW / OV reweighting

⇒can remove fake zero mode

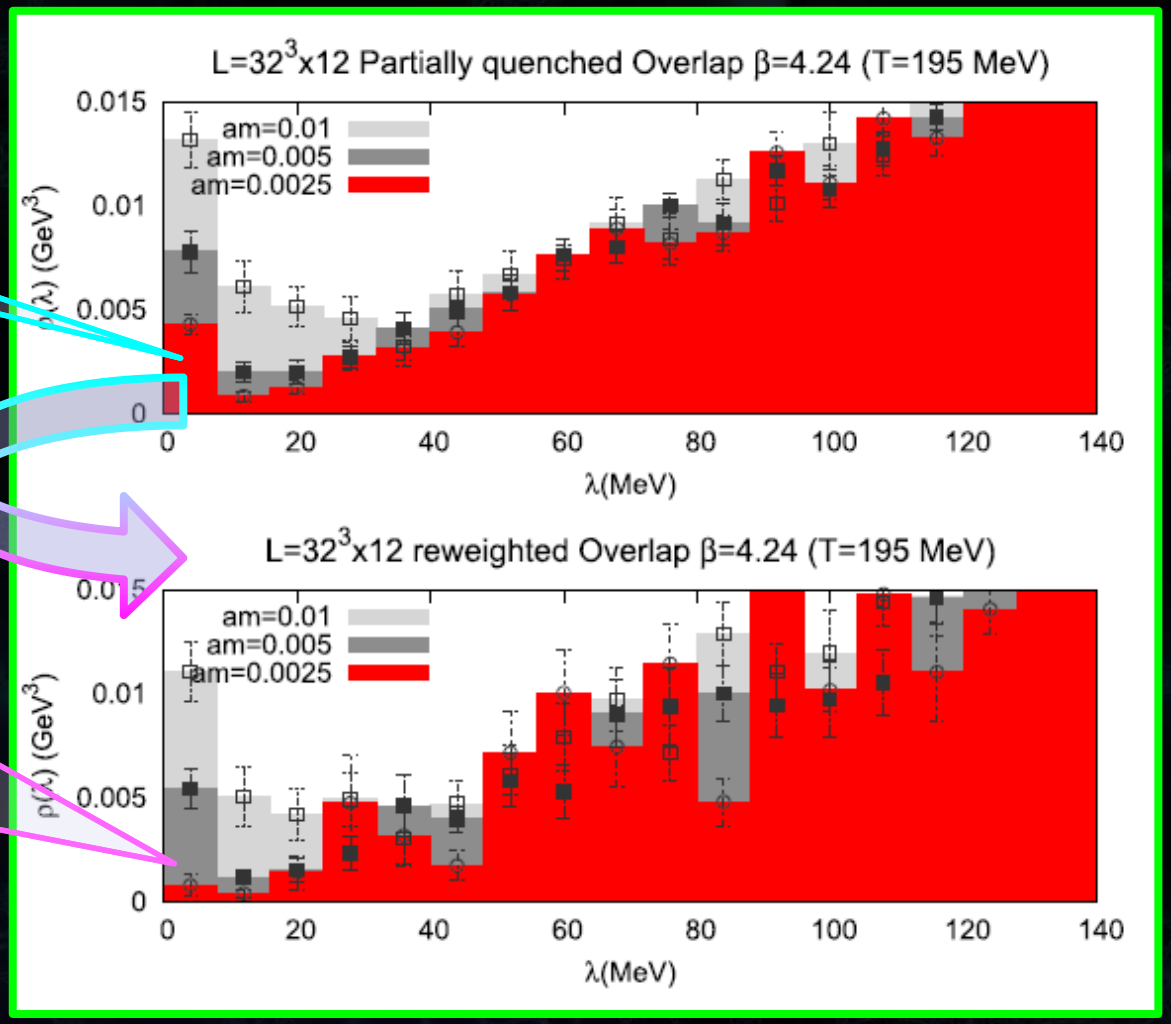
A. Tomiya et al. (JLQCD) PRD96 (2017) 034509

A. Tomiya et al. (JLQCD) PRD96 (2017) 034509

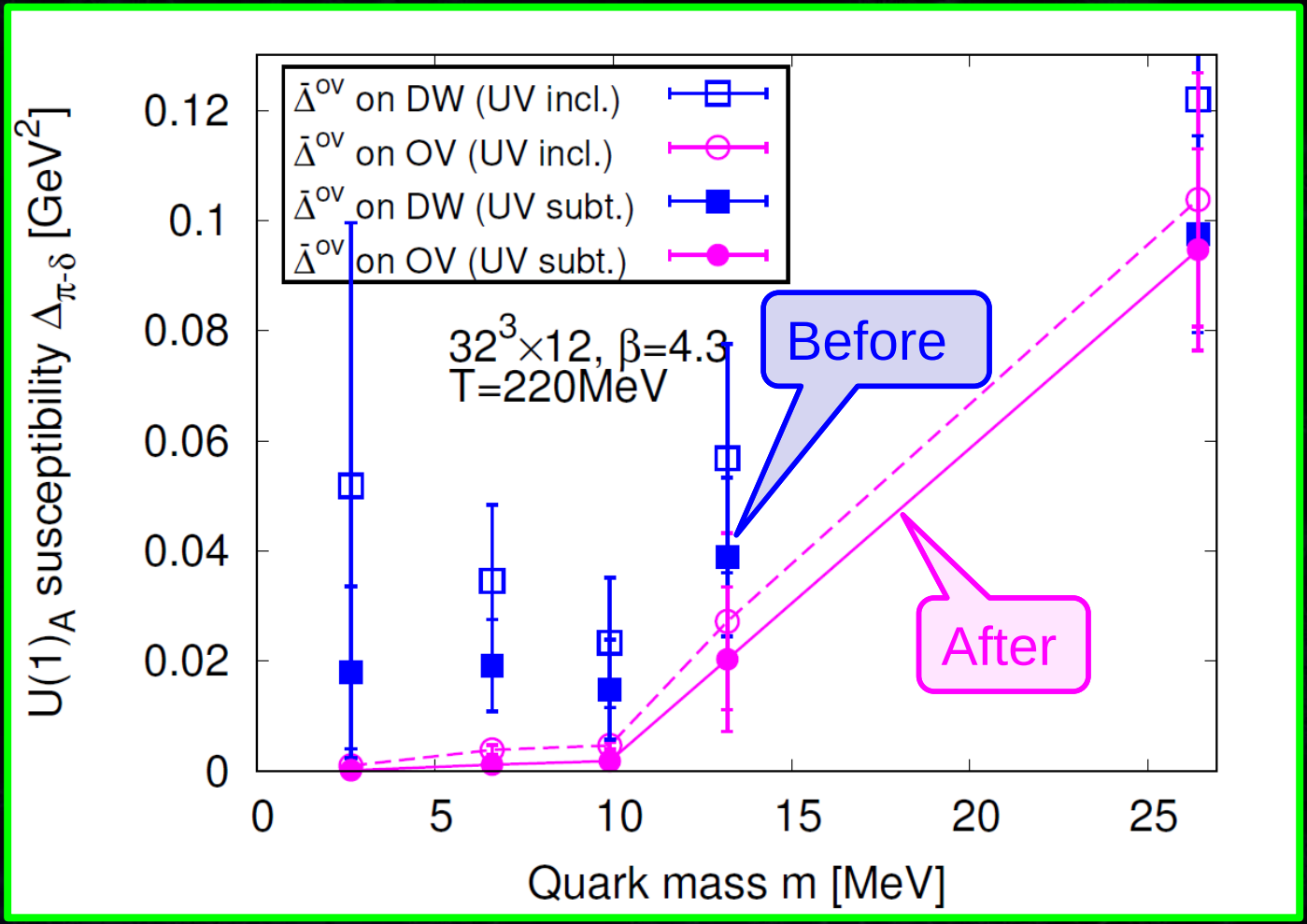
DW/OV reweighting removes fake zero-modes

OV on DW:
Fake zero-modes by partially quenched

OV on OV:
removed fake zero-modes
⇒ Only physical zero-modes survive!



$U(1)_A$ susceptibility (DW/OV reweighting)



⇒DW/OV reweighting is crucial in small m region

S. Aoki, H. Fukaya, and Y. Taniguchi PRD86 (2012), 114512
A. Tomiya et al. (JLQCD) PRD96 (2017), 034509

Note 1 :

$U(1)_A$ susc. = Low modes + ~~Zero~~ mode ?

$$\Delta_{\pi-\delta} \equiv \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{2m^2}{(\lambda^2+m^2)^2}$$

$$\rho_{0-mode}(\lambda) = \frac{1}{V}\sum_{0-mode} \delta(\lambda)$$

$$\Delta_{\text{zero}} = \int_0^\infty d\lambda\, \frac{1}{V}\sum_{0-mode} \delta(\lambda)\frac{2m^2}{(\lambda^2+m^2)^2}$$

$$= \frac{1}{V}\sum_{0-mode}\frac{2m^2}{m^4}$$

$$= \frac{1}{V}\sum_{0-mode}\frac{2}{m^2} = \frac{2N_0}{Vm^2}$$

$$\langle N_{L+R}^2\rangle = \mathcal{O}(V)$$
$$\langle N_{L+R}\rangle = \mathcal{O}(\sqrt{V})$$

$$\lim_{V\to\infty}\Delta_{\text{zero}} = 0$$

Zero mode contributions in $\Delta_{\pi-\delta}$ will be suppressed in $V \to \infty$ limit

Note 2:

$U(1)_A$ susc. = Physics + ~~Ultraviolet divergence?~~

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{2m^2}{(\lambda^2+m^2)^2}$$

$\rho(\lambda) \sim \lambda^3$ $\sim 1/\lambda^4$

$$\Delta_{\pi-\delta}^{\mathrm{ov}} \propto m^2 \ln\Lambda + \cdots$$

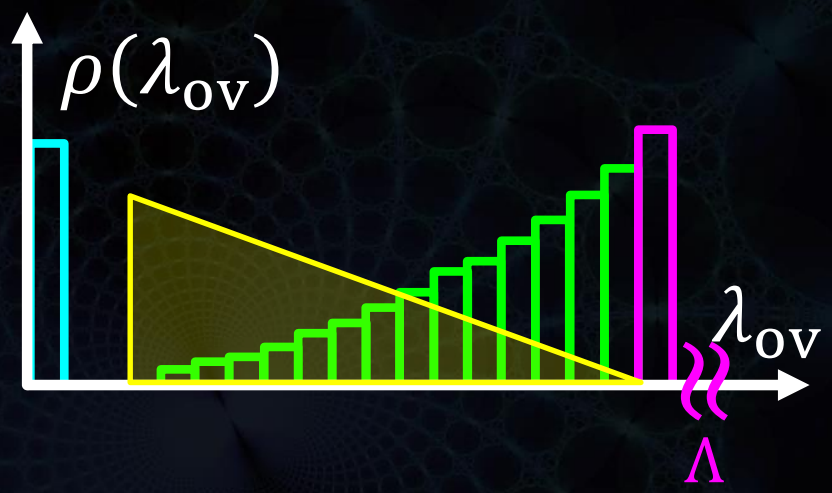
The term depends on cutoff Λ and valence quark mass m

We assume valence quark mass dependence of $\Delta_{\pi-\delta}$ (for small m):

$$\Delta_{\pi-\delta}(m) = \frac{a}{m^2} + b + cm^2 + O(m^4)$$

Zero-mode (disappears in $V \to \infty$)

$m^2 \ln\Lambda$ (disappears in $m \to 0$)



⇒ From 3 eqs. for $\Delta_{\pi-\delta}(m_1)$, $\Delta_{\pi-\delta}(m_2)$, $\Delta_{\pi-\delta}(m_3)$, a and c are eliminated

⇒ $\Delta_{\pi-\delta} \sim b + O(m^4)$ (, that depends on sea quark mass)