Full $\mathcal{O}(a)$-improvement of EQCD

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EQCD as an effective theory for hot QCD

- 3D **effective theory** for hot QCD ($T > 2T_c$) [1]

$$S_{\rm QCD} = \int d\tau \int d^3x \left(\overline{\psi}(\not{D}+m)\psi + \frac{1}{2g^2}\mathrm{Tr}\, F^{\mu\nu}F_{\mu\nu}\right)$$

- $A^0 \to \Phi$
- Integrate out Matsubara $\neq 0$-modes
- EFT matching known to $\mathcal{O}(g^4)$

$$S_{\rm EQCD,c} = \int d^3x \left(\frac{1}{2g_{3d}^2}\mathrm{Tr}\, F^{ij}F^{ij} + \mathrm{Tr}\, D^i\Phi D^i\Phi + m_D^2 \mathrm{Tr}\,\Phi^2 + \lambda\left(\mathrm{Tr}\,\Phi^2\right)^2\right)$$

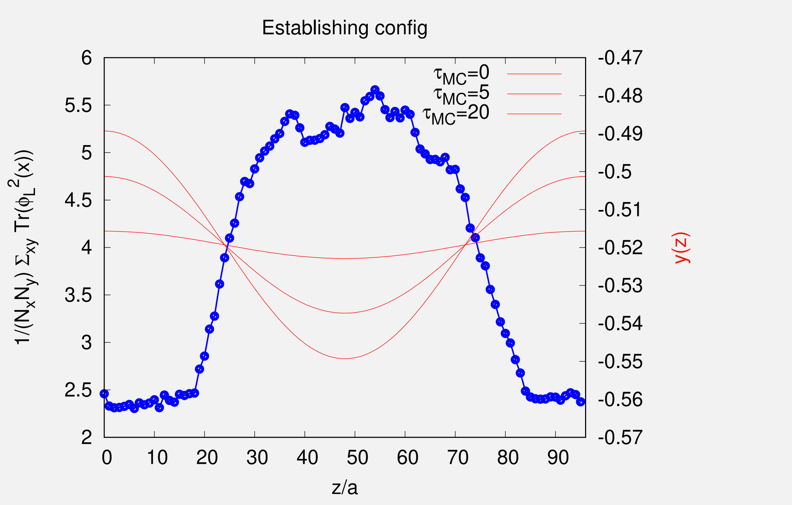
- Lattice parameters $y \equiv \frac{m_D^2}{g_{3d}^4}\Big|_{\mu=g_{3d}^2}$ and $x \equiv \frac{\lambda}{g_{3d}^2}$
- Lattice-continuum matching known to $\mathcal{O}(a)$
- Except δy (3-loop effect)

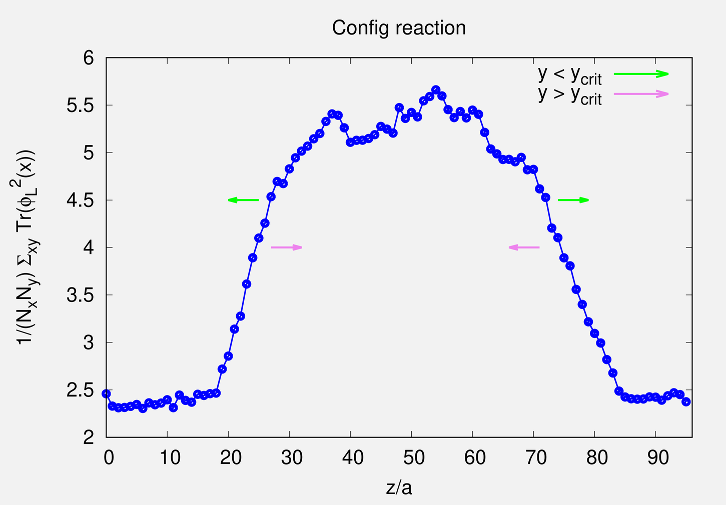
$$S_{\rm EQCD,L} = \beta\sum_{x,i>j}\left(1-\tfrac{1}{3}\Box_{ij}\right) + 2\sum_{x,i}\mathrm{Tr}\left(\Phi_L^2(x) - \Phi_L(x)U_i(x)\Phi_L(x+a\hat{i})U_i^\dagger(x)\right) + \sum_x\left(Z_2(y+\delta y)\mathrm{Tr}\,\Phi_L^2(x) + Z_4(x+\delta x)\left(\mathrm{Tr}\,\Phi_L^2(x)\right)^2\right)$$

- Information about thermodynamics from lattice [2]
- Also correct EFT for $\mathcal{O}(g)$-corrections to the **jet broadening** coefficient $\hat{q}$ and $C(b_\perp)$ [3]
- First lattice study of $\hat{q}$ promising, but lacks continuum limit [4]

Our method

- Measure $\mathcal{O}(a)$-dependence of δy numerically by fitting to **line of constant physics**
- Line of constant physics provided by **EQCD phase transition**
- Phase transition is first order in interesting parameter range [5]
- Multicanonical reweighting ineffective
- New method:
  - prepare configuration in which both **phases coexist** and are **permanently compared** to each other
  - tune mass to its **critical value** $y_{\rm crit}$





- Prepare configuration with coexisting phases by temporarily introducing a z-dependent mass:

$$y(z) = y_{\rm crit,est} + \Delta y\cos(2\pi z/L_z),$$

  gradually shrink $\Delta y \to 0$
- Tune y to critical value according to:

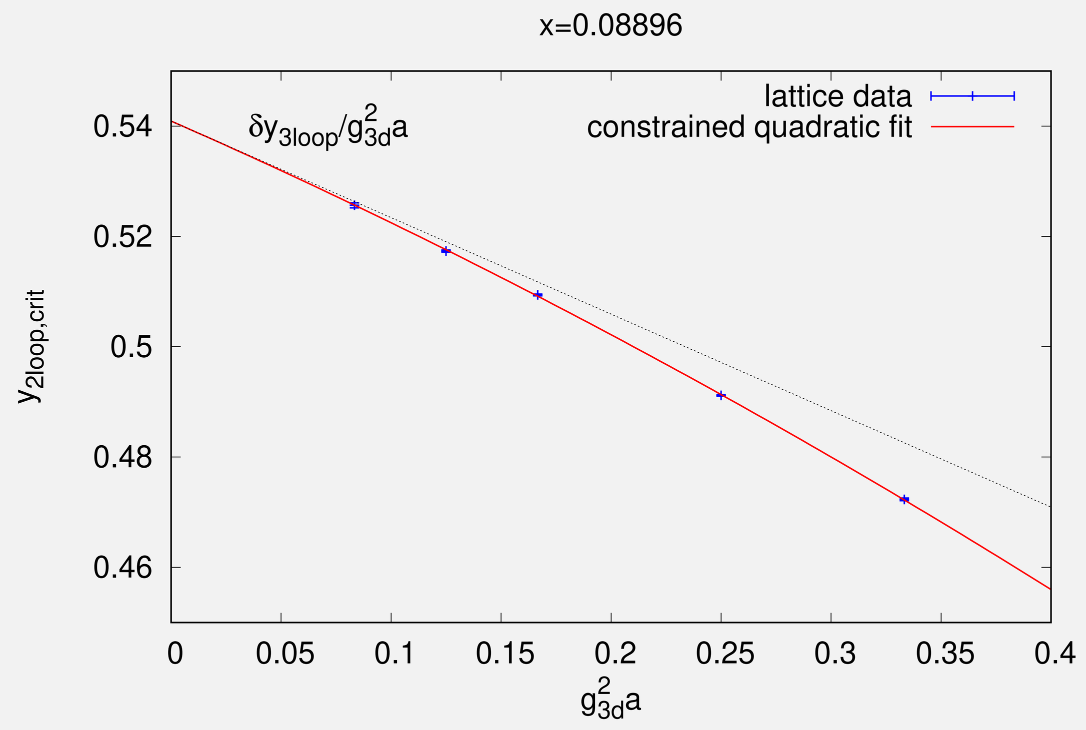
$$y_{\rm L,new} = y_{\rm L,old} + c_B\cdot\left(\frac{\frac{1}{V}\sum_x \mathrm{Tr}\,\Phi^2 - \mathrm{Tr}\,\Phi^2_{\rm symm}}{\mathrm{Tr}\,\Phi^2_{\rm brok} - \mathrm{Tr}\,\Phi^2_{\rm symm}} - 0.5\right),$$

  measure $\mathrm{Tr}\,\Phi^2_{\rm brok}$ and $\mathrm{Tr}\,\Phi^2_{\rm symm}$ in separate simulations
- Phase discriminator consists of difference of $\mathrm{Tr}\,\Phi^2$ operators, i.e. is free of $\mathcal{O}(a)$ errors

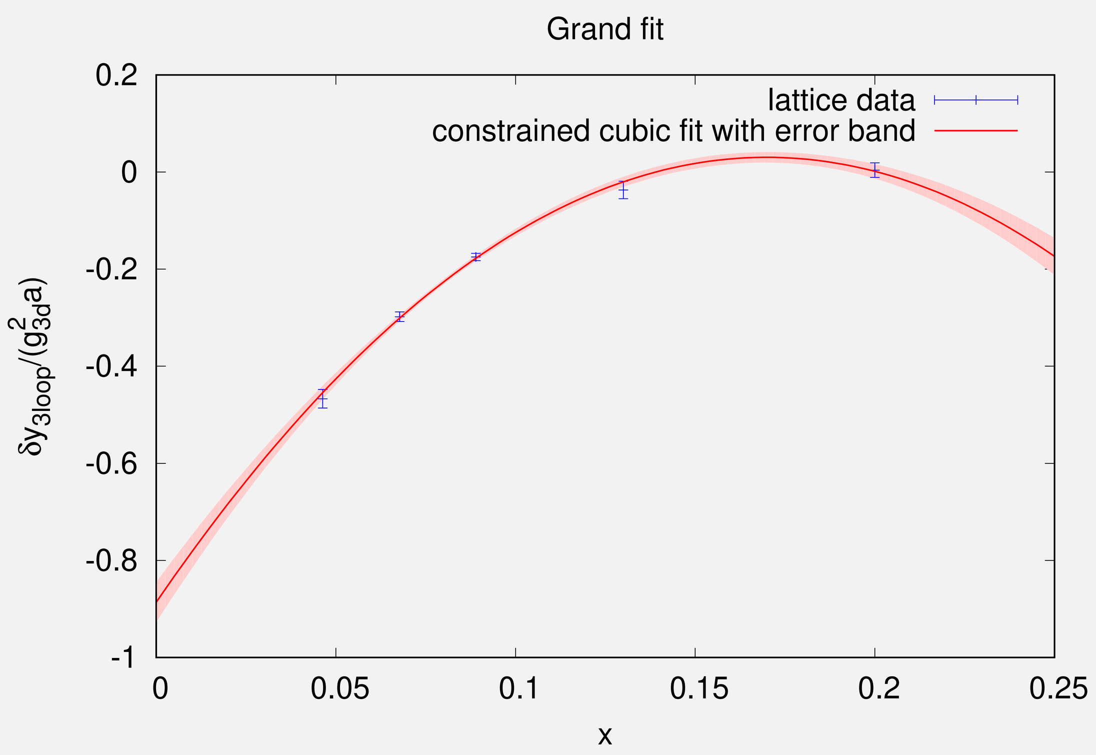
CRC-TR 211 Strong-interaction matter under extreme conditions · DFG Deutsche Forschungsgemeinschaft German Research Foundation · Lattice 19 Wuhan·China June 16-22

Results

- Strategy:
  1. measure $y_{\rm crit}$ at 5 different lattice spacings
  2. subtract known contributions [6]
  3. **extract slope** at $g_{3d}^2 a = 0$ using constrained curve fitting [7]



- Repeat for **5 different x** and limiting, pure scalar case



- Obtain **grand fit**

$$\frac{\delta y_{\rm 3loop}}{g_{3d}^2 a}(x) = 0.0151(55)\,x^3 - 31.8(28)\,x^2 + 10.80(74)\,x - 0.886(41)$$

  for the mass $\mathcal{O}(g_{3d}^2 a)$-improvement as main result
- Find updated (continuum-extrapolated) version of EQCD phase diagram as a corollary
- Further interesting information on the strength of the transition and the $\mathcal{O}(g_{3d}^2 a)$-improvement of $\mathrm{Tr}\,\Phi^2$ is provided [6]

Conclusion and outlook

- Numerically determined the missing three-loop renormalization coefficient of y
- Methodology allows efficient determination of spot of a first-order phase transition on the lattice
- Many byproducts: continuum-extrapolated EQCD phase diagram, study of the transition strength and $\mathrm{Tr}\,\Phi^2$ operator renormalization
- Full improvement allows studying EQCD free from $\mathcal{O}(a)$-errors, in particular error of $\hat{q}$ and $C(b_\perp)$ is down to $\mathcal{O}\left(a^2/b_\perp^2\right)$

References

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