Euclidean correlation functions for transport coefficients under gradient flow

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in collaboration with

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Outline

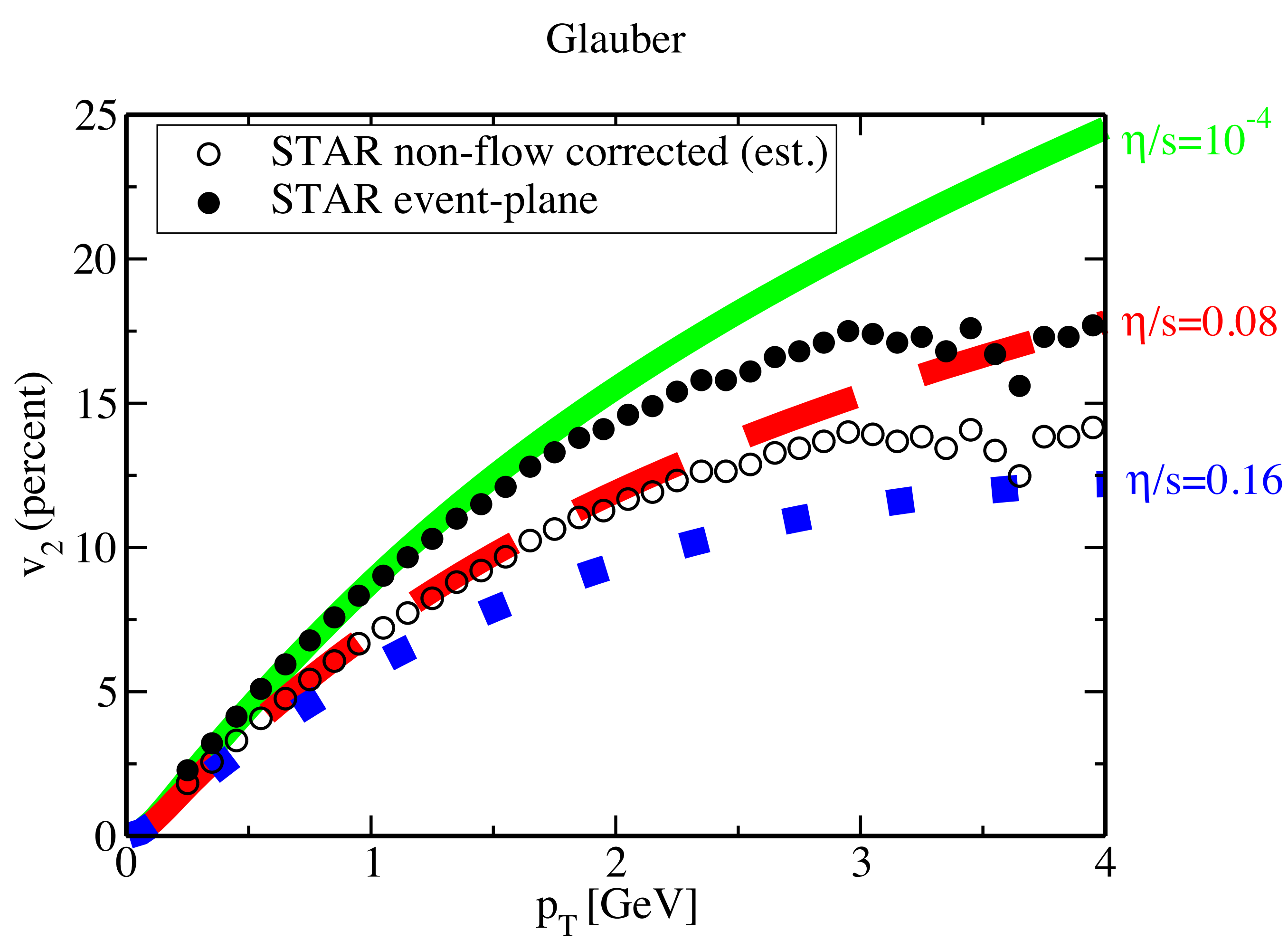
- Motivation
- Transport coefficients from euclidean correlation functions
- Application of gradient flow on the lattice as noise reduction technique
- Correlation functions on the lattice
- Summary & Outlook

Motivation

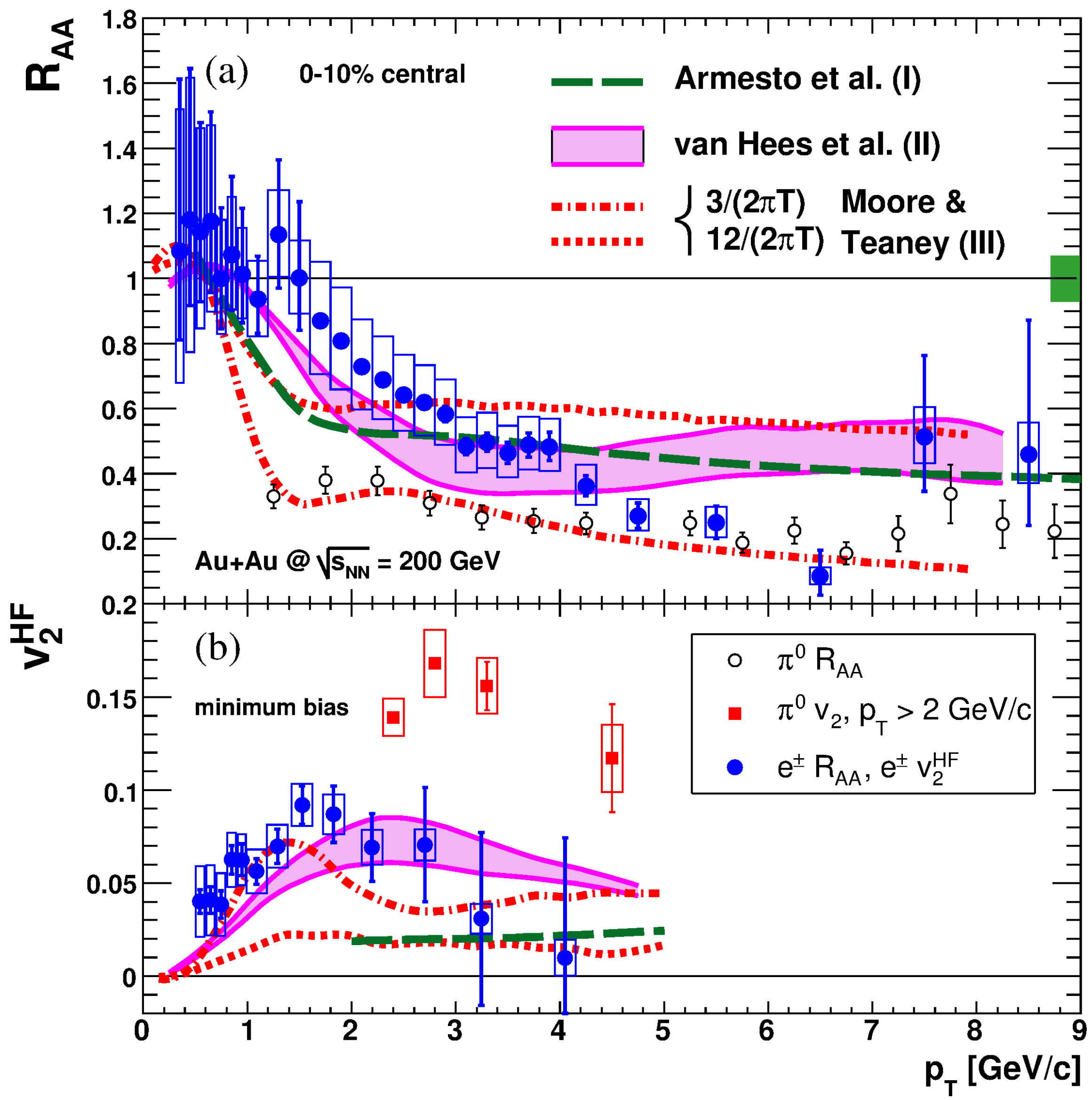
- Transport coefficients as crucial inputs for hydro/transport models to describe the experimental data

  ✴ Heavy quark (momentum) diffusion coefficient

  ✴ Shear viscosity & bulk viscosity



[M. Luzum and P. Romatschke, Phys. Rev. C 78, 034915 (2008)]



[PHENIX, PRL98(2007)172301]

Analytic continuation

$$G_X(\tau,\vec{p}) = \int d^3\vec{x}\; e^{-i\vec{p}\cdot\vec{x}} \langle \hat{\mathcal{O}}_X(\tau,\vec{x}) \hat{\mathcal{O}}_X(0,\vec{0}) \rangle = \int \frac{d\omega}{2\pi} \frac{\cosh(\omega(\tau T - 1/2)/T)}{\sinh(\omega/2T)} \rho_X(\omega,\vec{p},T)$$

spectral reconstruction: see Anna-Lena Kruse & Hiroshi Ohno's talk

HQ momentum diffusion coefficient:

$$\kappa = \lim_{\omega\to 0} \frac{2T\rho_{ii}(\omega)}{\omega} \quad \Longleftrightarrow \quad \hat{\mathcal{O}}_X(x) = E_i(x)$$

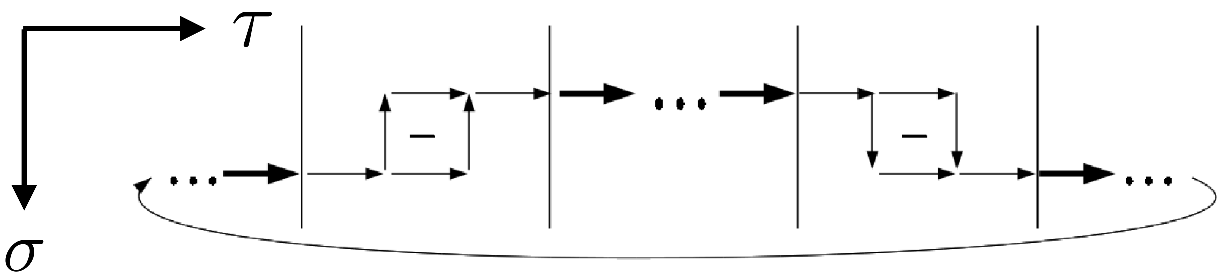
Shear viscosity: $$\eta(T) = \pi \lim_{\omega\to 0} \lim_{\mathbf{p}\to 0} \frac{\rho^{12,12}(\omega,\mathbf{p},T)}{2\omega} \quad \Longleftrightarrow \quad \hat{\mathcal{O}}_X(x) = T_{12}(x)$$

Bulk viscosity: $$\zeta(T) = \frac{\pi}{18} \sum_{i,j=1}^{3} \lim_{\omega\to 0} \lim_{\mathbf{p}\to 0} \frac{\rho^{ii,jj}(\omega,\mathbf{p},T)}{\omega} \quad \Longleftrightarrow \quad \hat{\mathcal{O}}_X(x) = T_{ii}(x)$$

For correlators of topological charge, see Lukas Mazur's talk

Color-electric correlators

Large-mass limit drives current-current correlator to be a “color-electric” correlator (in pure gauge) along a Polyakov loop [S. Caron-Huot et al., JHEP 0904 (2009) 053]



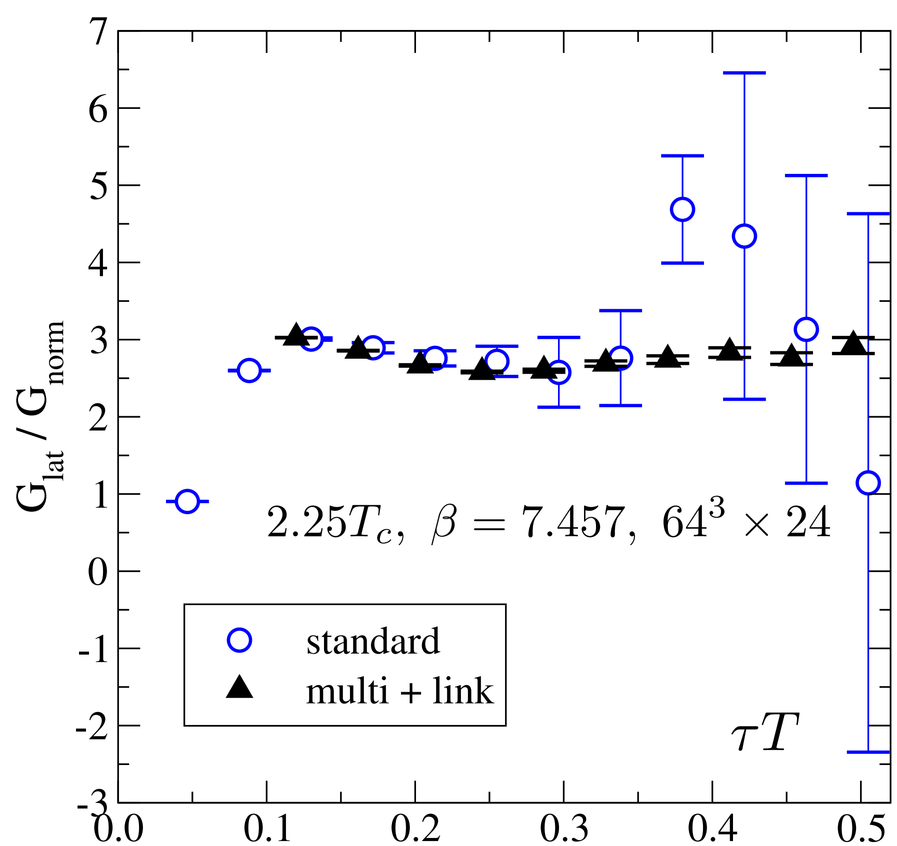
- available on the lattice
- gives access to “mass shift” of quarkonia (vacuum subtraction required)

$$G_{EE}(\tau) = -\frac{1}{3}\sum_{ii=1}^{3}\frac{\langle \mathrm{Re\ Tr}[U(\beta,\tau)gE_i(\tau,\vec{0})U(\tau,0)gE_i(0,\vec{0})]\rangle}{\langle \mathrm{Re\ Tr}[U(\beta,0)]\rangle}$$

$$\delta m = \frac{3}{2}a_0^2\gamma,\ \ \gamma = -\int_0^\beta d\tau\ G_{EE}(\tau)$$

[A M. Eller, J. Ghiglieri, G. Moore, PRD.99.094042]

- Lattice calculations of color-electric correlators: see A M. Eller’s poster



[A.Francis, et al., Lattice 2011, PRD92(2015)116003]

✤Multi-level [Luscher & Weisz, JHEP09 (2001)010]
✤Link-integration [Forcrand &Roiesnel, PLB151(1985)77]

However, only works for gluonic fields
Need Gradient Flow for dynamic quarks,
but start from gauge field first …

[Luscher & Weisz, JHEP1102(2011)051]
[Narayanan & Neuberger, JHEP0603(2006)064]

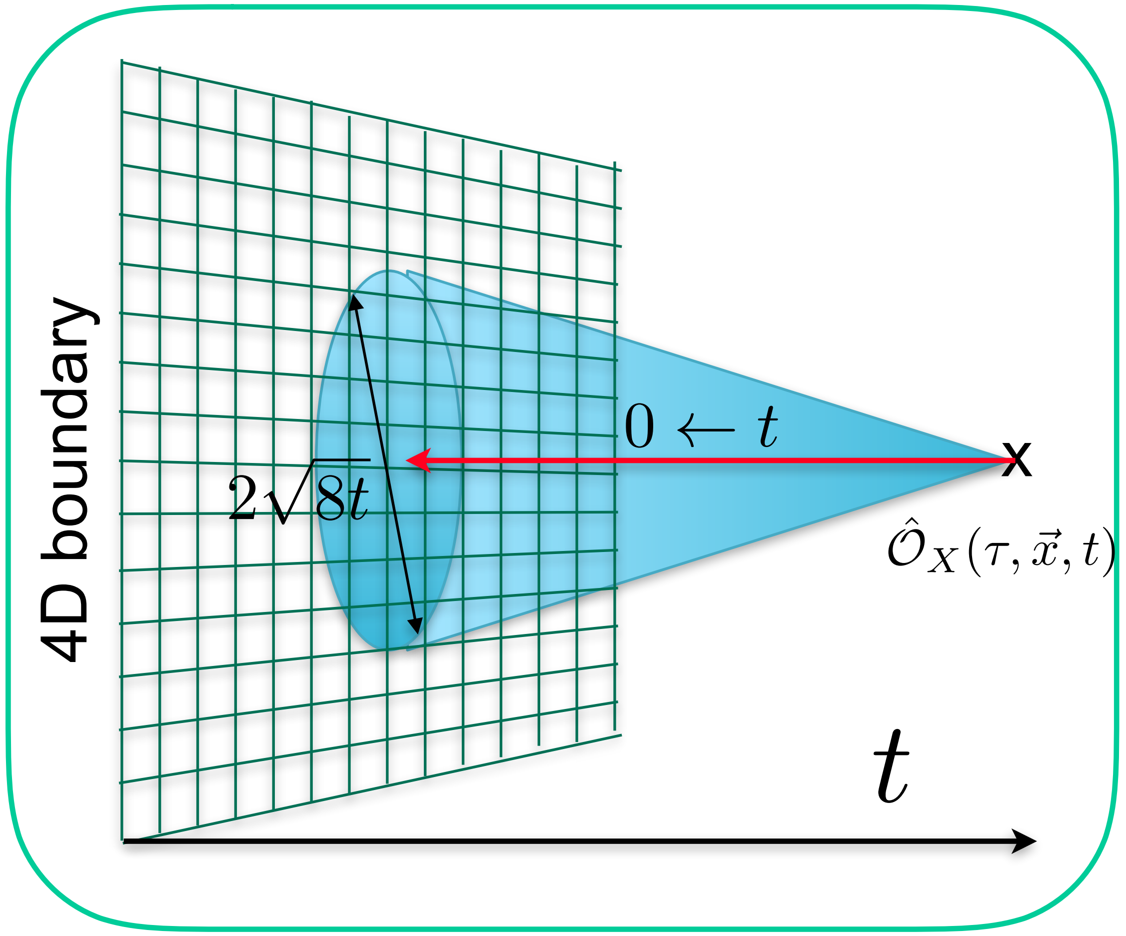
Gradient flow

Gradient flow as a "diffusion" equation along extra dimension "*t*"

$\partial_t B(x,t) = D_\nu G_{\nu\mu}(x,t)$ with initial condition: $B_\nu(x,t)|_{t=0} = A_\nu(x)$

Small flow time expansion

$$\mathcal{O}(x,t) \xrightarrow{t\to 0} \sum_k c_k(t)\mathcal{O}_k^R(x)$$



Applications:

running coupling / topo. charge / scale setting

defining operators / noise reduction / ...

Example: construction of $T_{\mu\nu}$

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left(\frac{U_{\mu\nu}(t,x)}{\alpha_U(t)} + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}E(t,x)_{subt}\right)$$

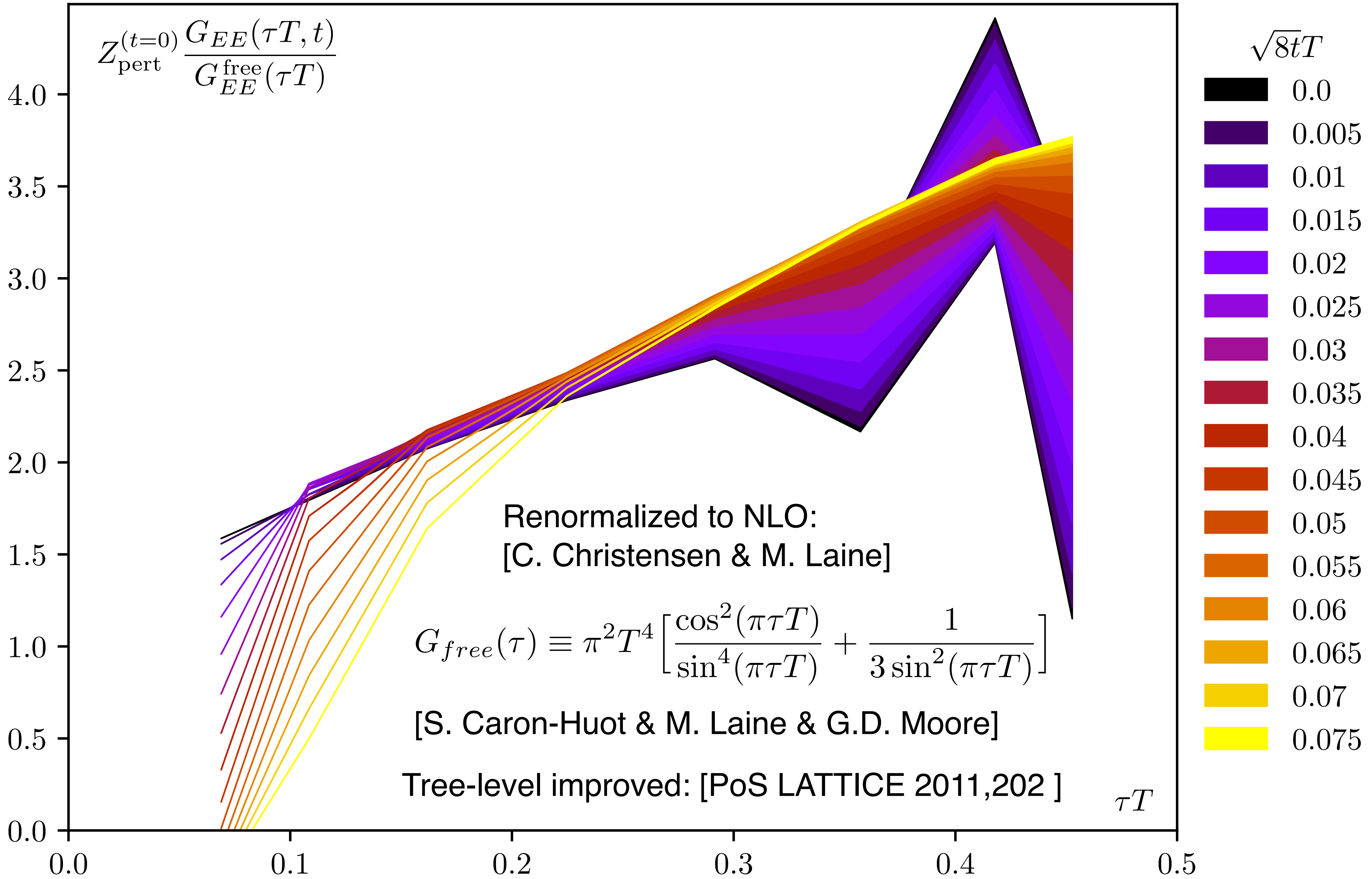
$$\begin{cases} E(t,x) = \frac{1}{4}G^a_{\rho\sigma}(x,t)G^a_{\rho\sigma}(x,t) \\ U_{\mu\nu}(x,t) = G^a_{\mu\rho}(x,t)G^a_{\nu\rho}(x,t) - \delta_{\mu\nu}E(t,x) \end{cases}$$

Lattice set-up

β	a[fm](a^{-1}[GeV])	N_σ	N_τ	T/T_c	#confs.	#meas.
6.8736	0.026 (7.496)	64	16	1.50	10000	10000
			64	0.00	10000	-
7.0350	0.022 (9.119)	80	20	1.50	10000	10000
7.1920	0.018 (11.19)	96	16	2.25	10000	-
			24	1.50	10000	3000
			28	1.29	10000	-
			32	1.13	10000	-
			48	0.75	10000	-
7.5440	0.012 (17.01)	144	36	1.50	-	-
7.7930	0.009 (22.78)	192	48	1.50	-	-

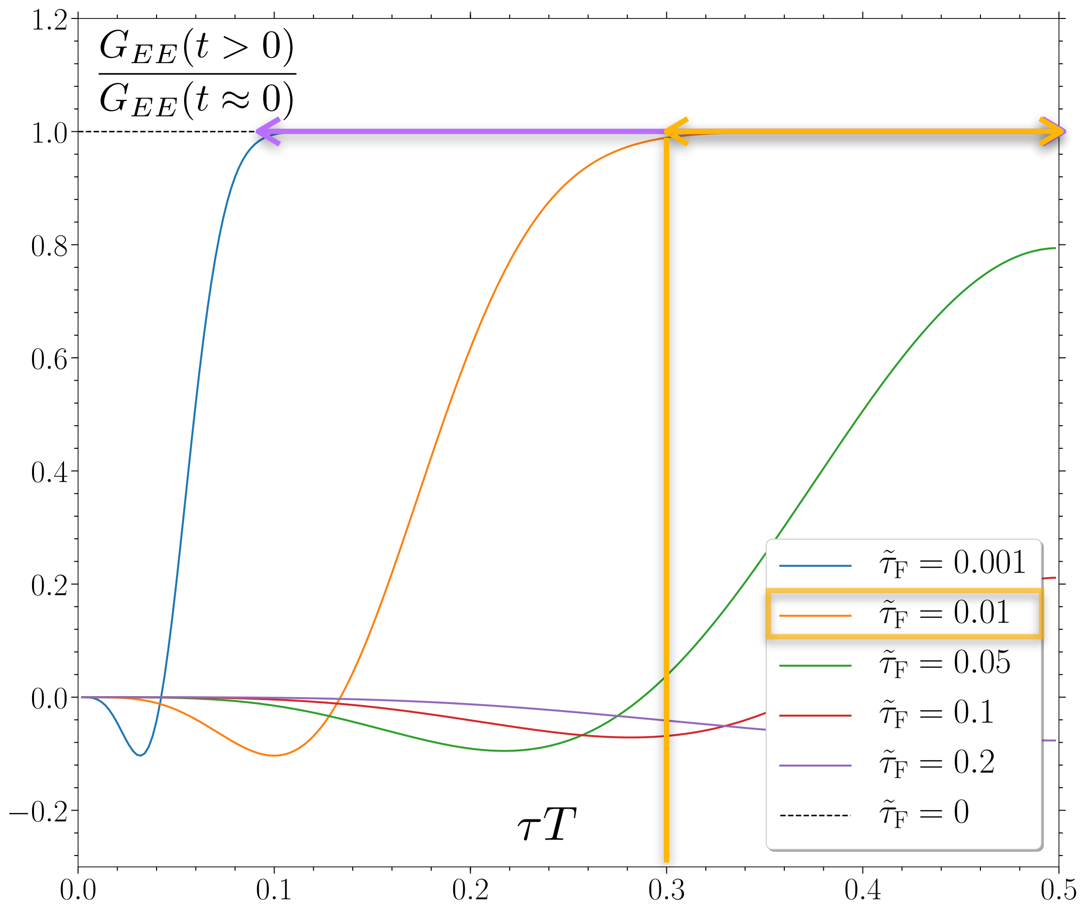
currently correlators are measured at $1.5T_c$
on 3 different quenched lattices (and beta)

G_{EE} under flow (64^3x16)

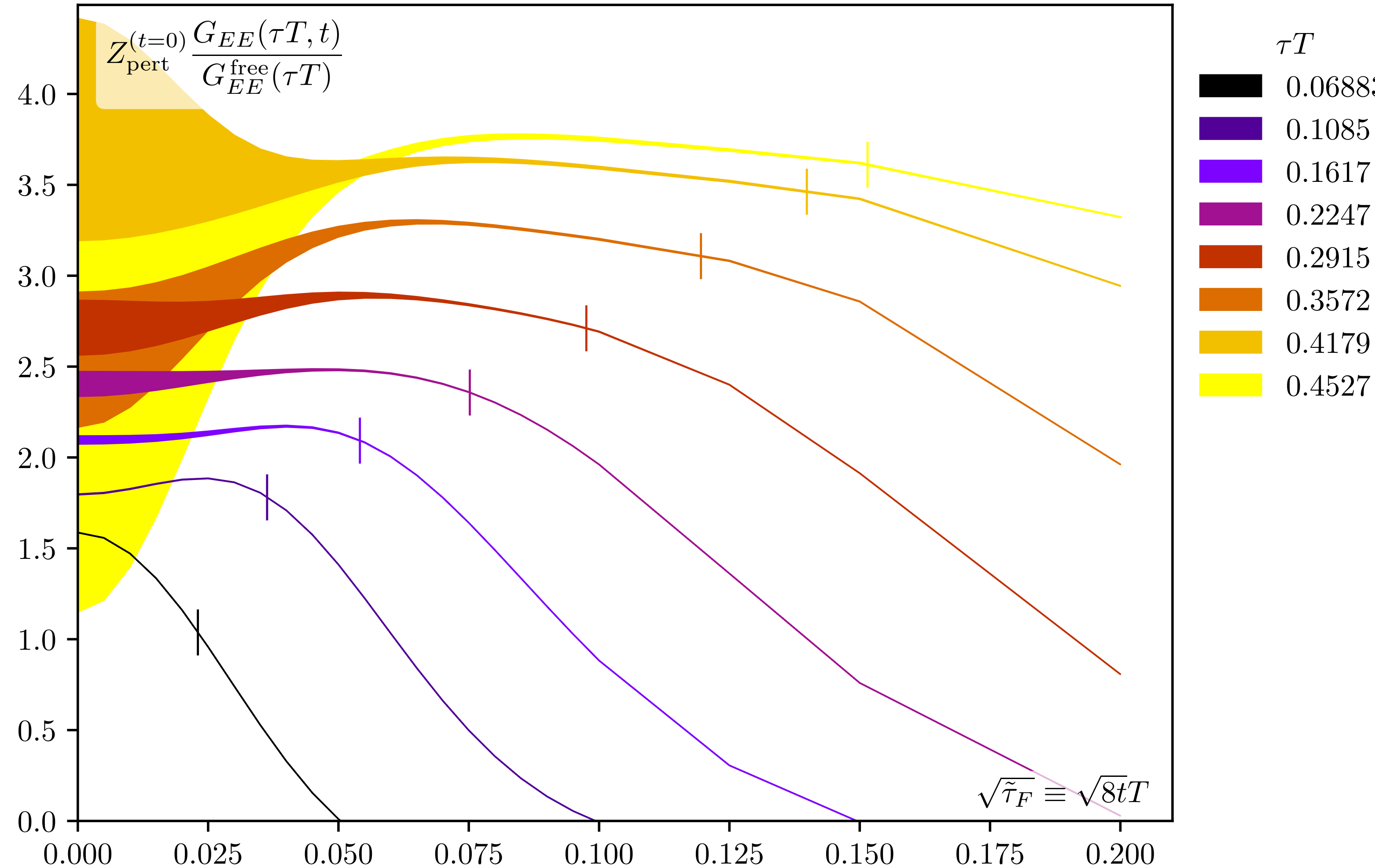


- Gradient flow reduces the error
- How much can we flow ?

Flow time limit for G_{EE} (64^3x16)



[A M. Eller and G. Moore, PRD97 (2018) 11, 114507]

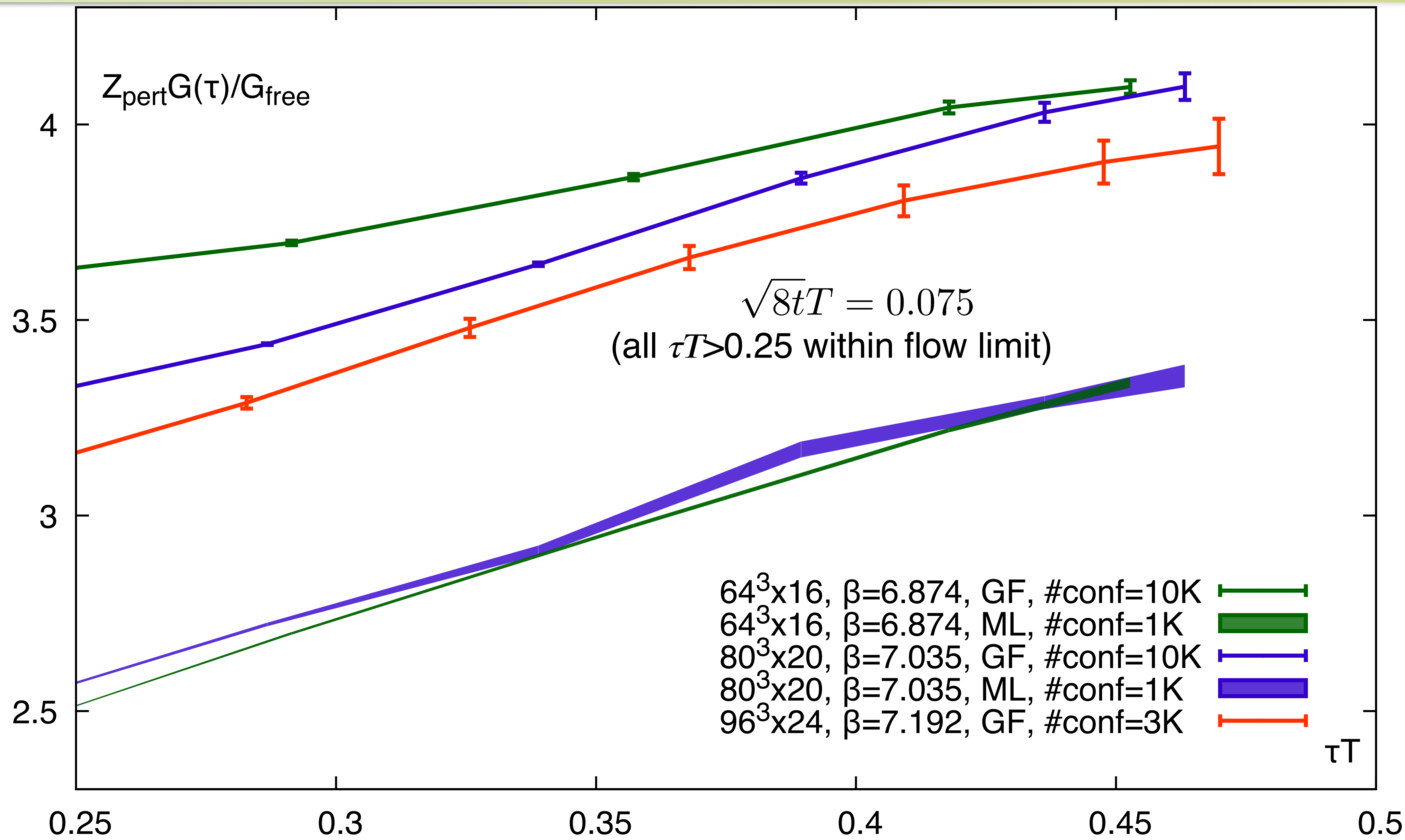


LO perturbative limit for flow time:

$$\tilde{\tau}_F < 0.1136(\tau T)^2$$

- Good signal within flow limit
- Future steps: continuum limit & $t \rightarrow 0$ limit

Gradient Flow v.s. Multi-Level (1)



- Comparable errors in both methods
- Almost continuum limit in ML

Gradient Flow v.s. Multi-Level (2)

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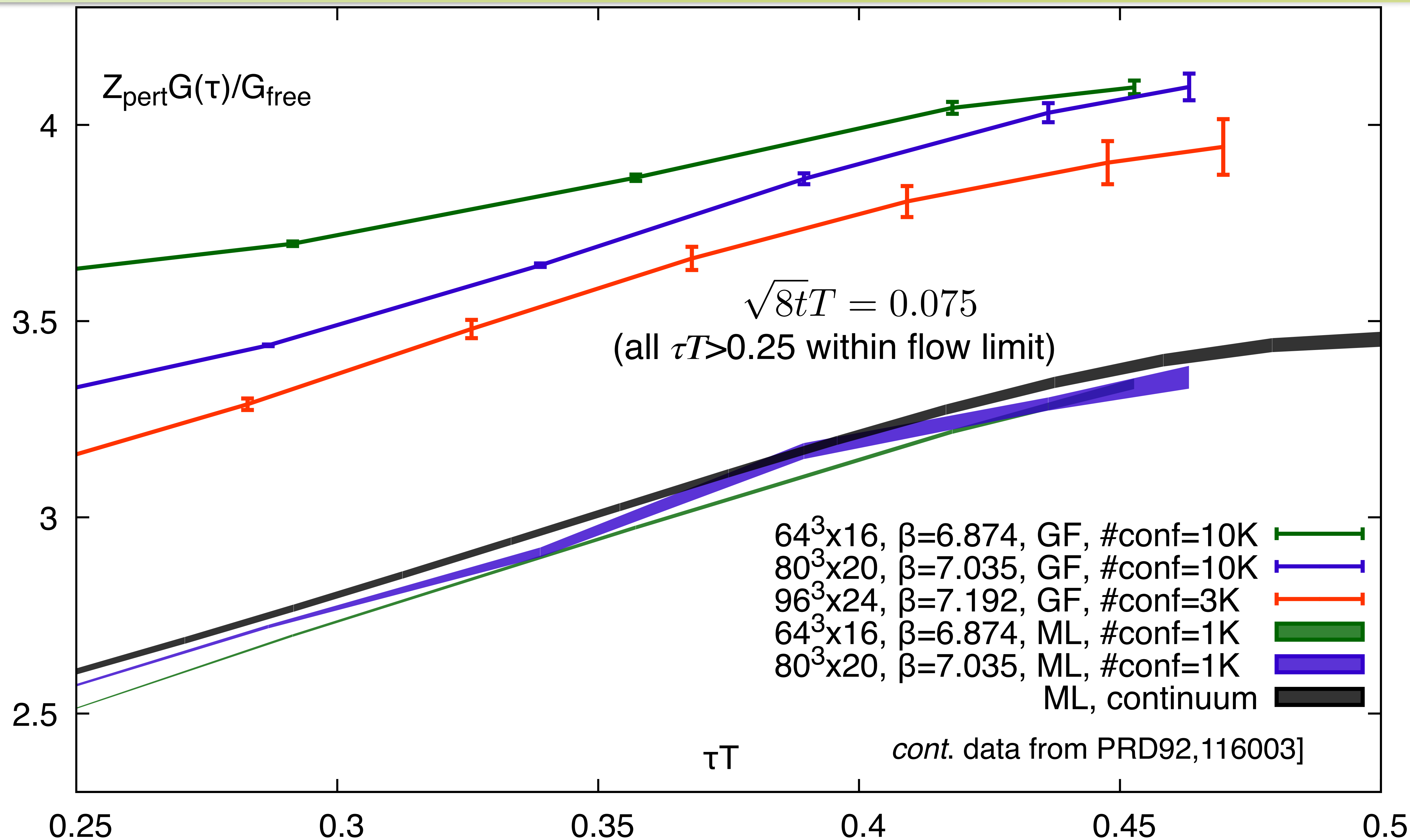
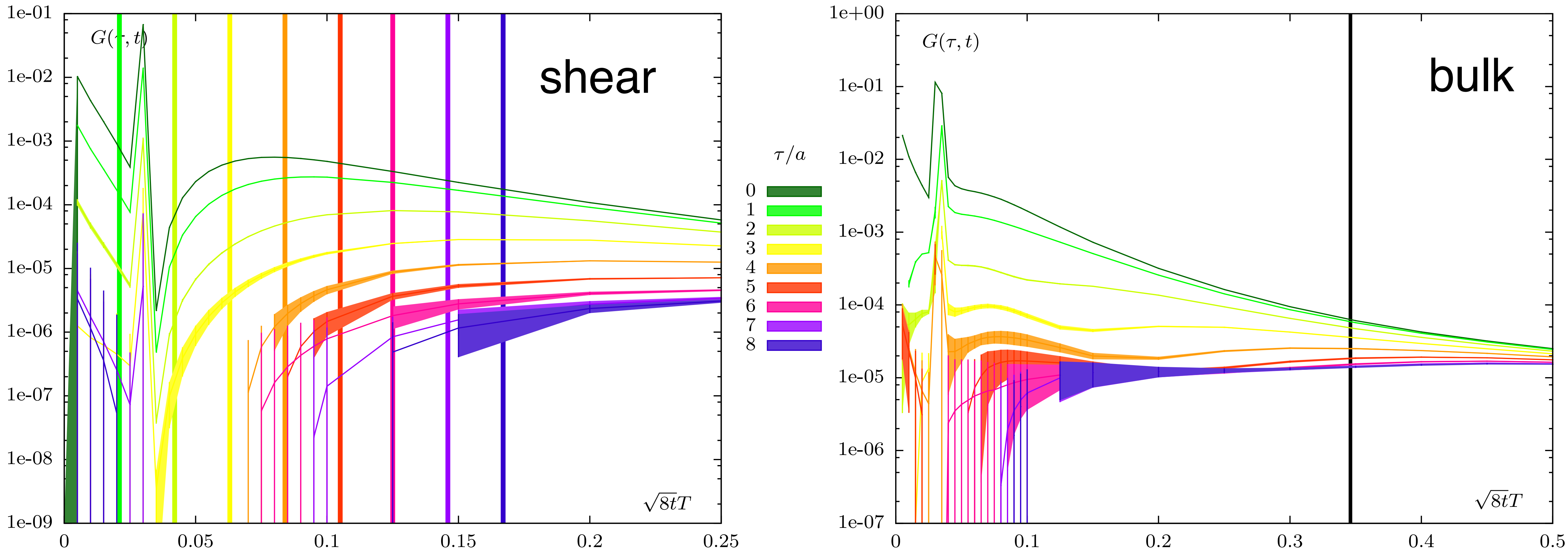


- Lattice effects in GF can be seen
- Data points under flow move in “correct” way
- Continuum-extrapolation at fixed flow time is anticipated

G_{TT} under flow (64^3x16)



with one-loop Suzuki coefficients

[Hiroshi Suzuki, PTEP 2013 (2013) 083B03]

- Flow effects can also be seen in energy-momentum tensor correlators
- Need more statistics in shear channel
- Understand the behavior of correlators within & beyond the flow time limit

Summary & Outlook

- G_{EE} and G_{TT} are measured at $1.5T_c$ on 3 different lattices under GF
- Good signal for G_{EE} under GF, comparable to those from ML algorithm
- Need more statistics for G_{TT}
- Confident in GF for dynamic quarks

✳ Move on to finer & larger lattices and different temperatures

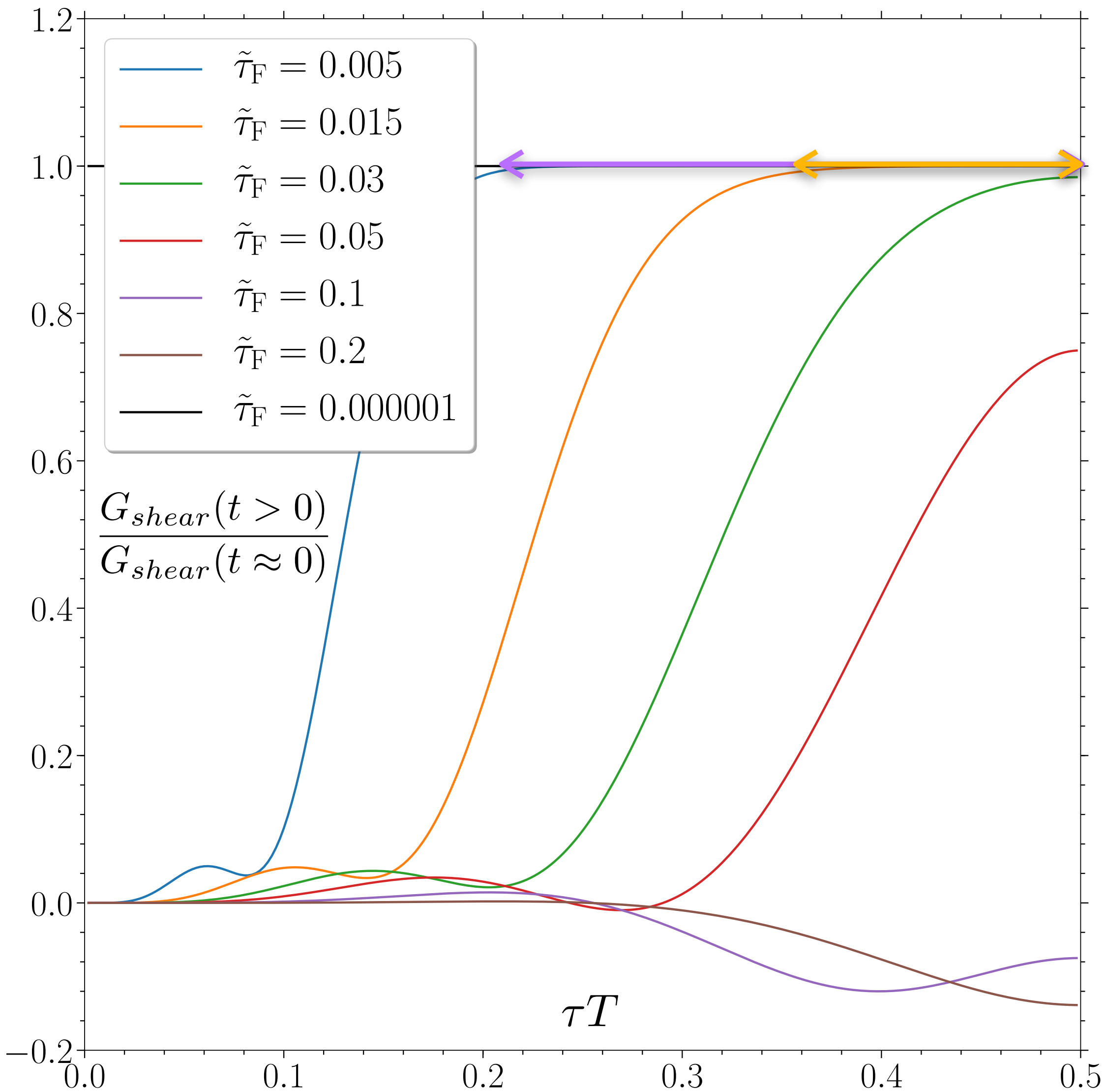
✳ Perform continuum & $t \to 0$ extrapolation for the correlators

✳ Extract spectral functions from correlators and estimate η, ζ, κ (and γ) accordingly (consider perturbative constraints)

✳ Extend to full QCD using large and fine 2+1-flavor HISQ lattices

Thanks!

Backup: perturbative flow time limit



[S. Eller and G. Moore, PRD97 (2018) 11, 114507]