Large N_c behaviour of an effective lattice theory for heavy dense QCD

The 37[th] International Symposium on Lattice Field Theory

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Introduction

Motivation

- At finite baryon density, lattice QCD has a sign problem, which prohibits direct simulation.
- Approximate methods: Taylor expansion, reweighting, imaginary potential.
  $\rightarrow$ Fail for $\mu/T \gtrapprox 1$.
- Need alternative methods to probe cold and dense QCD.

3d Effective Theory

Centre symmetric 3d effective actions for thermal SU(N) Yang-Mills from strong coupling series

Langelage, J.; Lottini, S. & Philipsen, O.
JHEP, 2011, 02, 057

Onset Transition to Cold Nuclear Matter from Lattice QCD with Heavy Quarks

Fromm, M.; Langelage, J.; Lottini, S.; Neuman, M. & Philipsen, O. Phys. Rev. Lett., 2013, 110, 122001

Equation of state for cold and dense heavy QCD

Glesaaen, J.; Neuman, M. & Philipsen, O.
JHEP, 2016, 03, 100

3d Effective Theory

Definition of effective theory ($U \in SU(N_c)$):

$$Z = \int \mathcal{D}U\mathcal{D}\Psi\mathcal{D}\bar{\Psi}\, e^{-S_G[U]-S_f^{(W)}[U,\Psi,\bar{\Psi}]}$$

$$=: \int \mathcal{D}U_0\, e^{-S_{\mathrm{eff}}} \overset{SU(3)}{=} \int \mathcal{D}L\, e^{-S_{\mathrm{eff}}[L]}$$

$$\Rightarrow S_{\mathrm{eff}}[U_0] = -\log\left(\int \mathcal{D}U_i\mathcal{D}\Psi\mathcal{D}\bar{\Psi}\, e^{-S_G[U]-S_f^{(W)}[U,\Psi,\bar{\Psi}]}\right)$$

Analytic determination using combined strong coupling (small $\beta = \frac{2N_c}{g^2}$) and hopping expansion (small $\kappa = \frac{1}{2am+8}$).

S_{eff} has a mild sign problem and weak couplings $\rightarrow$ pertubative treatment also possible.

Systematics of the hopping expansion

Effective Theory at Strong Coupling

At strong coupling, link integration factorizes [Rossi, Wolff 84]

$$\exp(-S_{\mathrm{eff}}[U_0]) = \int \mathcal{D}\Psi\mathcal{D}\bar{\Psi}\, e^{\bar{\Psi}(1+T[U_0])\Psi}$$
$$\times \prod_{n\in\Lambda}\prod_{i=1}^{3}\int \mathrm{d}U_i(n)\, e^{\kappa\,\mathrm{tr}\left(J_i(n)U_i(n)+U_i^\dagger(n)K_i(n)\right)}$$

$$J_i(n)_{ab} = \bar{\Psi}(n)^f_{\alpha,b}(1-\gamma_i)_{\alpha\beta}\Psi(n+\mathbf{e}_i)^f_{\beta,a}$$
$$K_i(n)_{ab} = \bar{\Psi}(n)^f_{\alpha,b}(1-\gamma_i)_{\alpha\beta}\Psi(n+\mathbf{e}_i)^f_{\beta,b}$$

Single Site integral

Known for U(N) [Bars 80], for $SU(N)$ use

$$\int_{SU(N)} \mathrm{d}U\, f(U) = \sum_{q\in\mathbb{Z}} \int_{U(N)} \mathrm{d}U\, \det(U)^q f(U)$$

to obtain

$$\int_{SU(N_c)} \mathrm{d}U\, e^{\kappa\, \mathrm{tr}\left(JU+U^\dagger K\right)} = \sum_{k=0}^{\infty} \left(1 - \frac{\delta_{k,0}}{2}\right)\left(\det(J)^k + \det(K)^k\right)$$

$$\times \sum_{r\in GL(N_c)\ \mathrm{irreps}} a_r(\kappa) b_{r,k}(\kappa) \frac{\chi_r(JK)}{d_r}.$$

Summands are of order $\mathcal{O}(\kappa^{kN_c+2\sum_{l=1}^{N_c}\lambda_l}) \Rightarrow$ spatial Baryon hoppings surpressed for large N_c.
Due to Grassmann constraint $kN_c + 2\sum_{l=1}^{N_c} \leq 4N_f N_c$.

Grassmann integration

After spatial link integration

$$e^{-S_{\text{eff}}} = \int \mathcal{D}\Psi \mathcal{D}\bar{\Psi}\ e^{\bar{\Psi}(1+T)\Psi}$$
$$\times \prod_{n\in\Lambda} \prod_{i=1}^{3} \left[1 + P(\Psi(n), \Psi(n+\mathbf{e}_i), \bar{\Psi}(n), \bar{\Psi}(n+\mathbf{e}_i))\right]$$

After expanding product: Grassmann integration using Wick's theorem.

Note: $(1+T)^{-1}(x,y) \sim \delta_{\mathbf{x},\mathbf{y}}$

$\Rightarrow$ Integration factorizes for Ψ's with different spatial coordinates

$\Rightarrow$ Can be organized as an expansion of clusters of connected graphs on Λ_s using the moment cumulant formalism. [Ruelle 69, Münster 81]

Free Energy to NLO

Perturbative treatment of effective theory for arbitrary N_c and N_f (degenerate) quark flavours:

Free energy

$$-f = \log(z_0) - 6N_f \frac{\kappa^2 N_\tau}{N_c}\left(\frac{z_{11}}{z_0}\right),$$

with the $SU(N_c)$ integrals

$$z_0 = \int\limits_{SU(N_c)} \mathrm{d}W \det(1 + h_1 W)^{2N_f},$$

$$z_{11} = \int\limits_{SU(N_c)} \mathrm{d}W \det(1 + h_1 W)^{2N_f} \operatorname{tr}\left(\frac{h_1 W}{1 + h_1 W}\right),$$

where $h_1 = (2\kappa e^{a\mu})^{N_\tau} = e^{\frac{\mu - m}{T}}$, $m = -\log(2\kappa)$.

Integration of temporal links

The integrands only depend on the eigenvalues of the group element W.
$\Rightarrow$ Use eigenvalues for parametrization (reduced Haar measure).

$$\int_{SU(N_c)} \mathrm{d}W \det(1 + h_1 W)^{2N_f}$$

$$= \frac{1}{(2\pi)^{N_c}} \sum_{q \in \mathbb{Z}} \det_{1 \le k,l \le N_c} \left[\int \mathrm{d}\varphi_i \, (1 + h_1 e^{i\varphi_k})^{2N_f} e^{i(l-k+q)\varphi_i} \right]$$

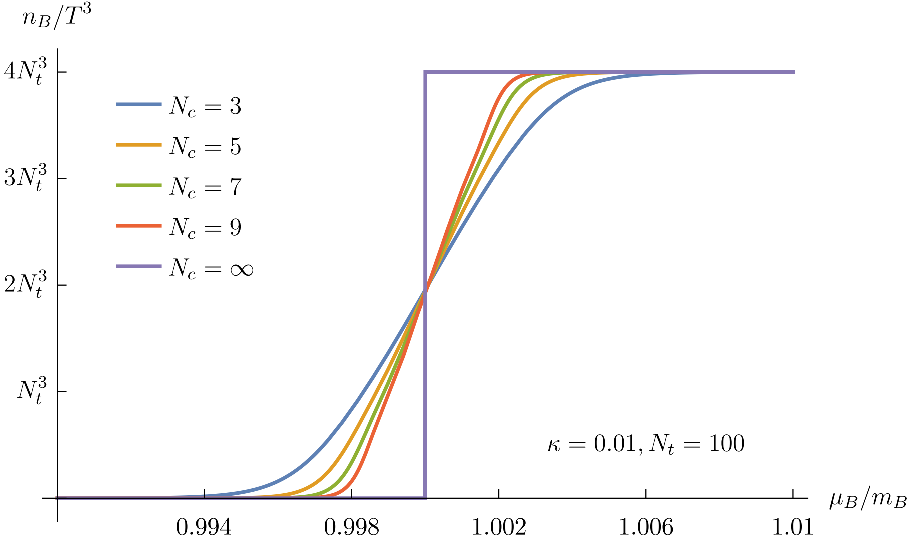
$$= \sum_{q=0}^{2N_f} \det_{1 \le i,j \le N_c} \left[\binom{2N_f}{i - j + p} \right] h_1^{pN_c}.$$ [Nishida 03]

Specifically for $N_f = 1$

$$z_0 = 1 + (N_c + 1) h_1^{N_c} + h_1^{2N_c}.$$

Large N_c limit

Nuclear liquid gas transition for general N_c



Asymptotic Analysis

For $N_f = 1$ the κ^2 correction to the pressure:

$$a^4 p_1 = -6\kappa^2 \frac{(\frac{1}{2}N_c(N_c+1)h_1^{N_c} + N_c h_1^{2N_c})^2}{N_c(1 + h_1^{N_c}(1+N_c) + h_1^{2N_c})^2}.$$

$h_1 < 1$, for $N_c \to \infty$ expand about $h_1^{N_c} = 0$

$$\begin{aligned} a^4 p_1 &= -\frac{3}{2}\kappa^2 N_c(N_c+1)^2 h_1^{2N_c} + \mathcal{O}(h_1^{3N_c}) \\ &\sim -\frac{3}{2}\kappa^2 N_c^3 h_1^{2N_c}. \end{aligned}$$

$h_1 > 1$, expand about $1/h_1^{N_c} = 0$

$$\begin{aligned} a^4 p_1 &= -6\kappa^2 N_c + \mathcal{O}(1/h_1^{N_c}) \\ &\sim -6\kappa^2 N_c. \end{aligned}$$

Asymptotic Analysis - results

$N_f = 2,\ h_1 < 1$:

$$p \sim \frac{1}{6a^4 N_\tau} N_c^3 h_1^{N_c} - \kappa^2 \frac{1}{48a^4} N_c^7 h_1^{2N_c} + \kappa^4 \frac{3N_\tau}{800a^4} N_c^8 h_1^{2N_c} + \mathcal{O}(\kappa^6)$$

$$n_B \sim \frac{1}{6a^3} N_c^3 h_1^{N_c} - \kappa^2 \frac{N_\tau}{24a^3} N_c^7 h_1^{2N_c} + \kappa^4 \frac{(9N_\tau + 1)N_\tau}{1200a^3} N_c^8 h_1^{2N_c} + \mathcal{O}(\kappa^6)$$

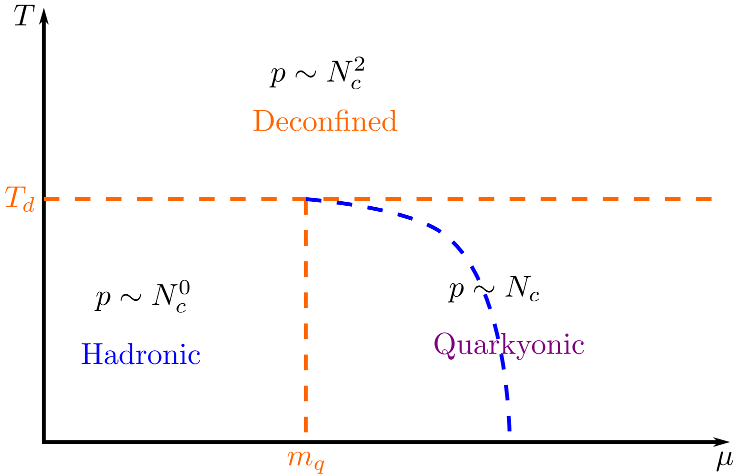
$h_1 > 1$:

$$p \sim \frac{4\log(h_1)}{N_\tau a^4} N_c - \kappa^2 \frac{12}{a^4} N_c + \kappa^4 \frac{198}{a^4} N_c + \mathcal{O}(\kappa^6)$$

$$n_B \sim \frac{4}{a^3} - \kappa^2 \frac{N_\tau}{a^3} \frac{N_c^4}{h_1^{N_c}} - \kappa^4 \frac{(59N_\tau - 19)N_\tau}{20a^3} \frac{N_c^5}{h_1^{N_c}} + \mathcal{O}(\kappa^6)$$

Conjecture large N_c phase diagram

't Hooft limit: $N_c \to \infty$, hold $\lambda = g^2 N_c$ fixed ['t Hooft] [McLerran, Pisarski 09]:



Gauge corrections

Include Gauge corrections using character expansion, to leading order:

$$-f = \log(z_0(h_{1,\mathrm{corr}})) + \frac{\kappa^2 N_t}{N}\left[1 + 2\frac{u_F - u_F^{N_t}}{1-u_F}\right](-6N_f)\frac{z_{11}(h_{1,\mathrm{corr}})}{z_0(h_{1,\mathrm{corr}})}^2$$

$$h_{1,\mathrm{corr}} = \exp\left[N_t\left(1 + 2\frac{u_F - u_F^{N_t}}{1-u_F}\right)\right].$$

In 't Hooft limit [Gross, Witten 1979]

$$u_F = \frac{1}{\lambda} \Rightarrow \text{qualitative results unchanged.}$$

Open questions:

- Higher order corrections?
- Interchange strong strong-coupling and large N_c limit?
- N_c-dependence of a?