

# Thermal modifications of quarkonia and heavy quark diffusion from a comparison of continuum-extrapolated lattice results to perturbative QCD

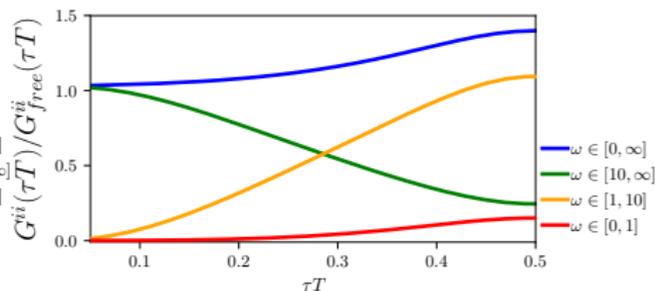
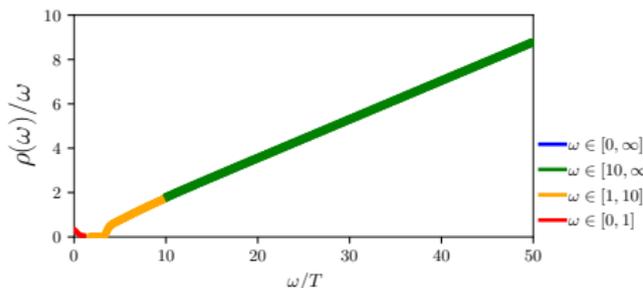
Anna-Lena Kruse

with H.-T. Ding, O. Kaczmarek, H. Ohno, H. Sandmeyer & H.-T. Shu



- Information about in-medium properties of quarkonia encoded in spectral function
- Extraction of spectral functions from correlators difficult

$$G_{ii}(\tau) = \int_0^{\infty} \frac{d\omega}{2\pi} \rho_{ii}(\omega) K(\omega, \tau), \quad K(\omega, \tau) = \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$$



- Many reconstruction methods available [see also plenary talk by H. Ohno]
- Here: Fit of perturbative spectral function and MEM

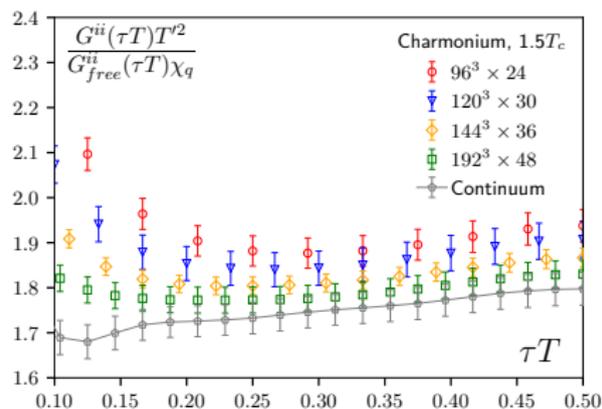
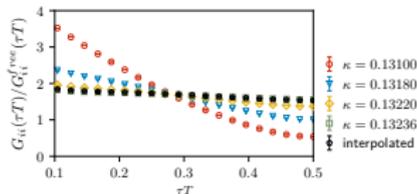
Combination of different energy regimes, matched to connect at  $\omega^{match}$

[Burnier et al., JHEP11(2017)206]

- High energy: Vacuum asymptotics [Burnier, Laine, Eur.Phys.J.C 72 (2012) 1902]
- Threshold region: pNRQCD [Laine, JHEP 0705:028,2007]
- Suppressed at low  $\omega$

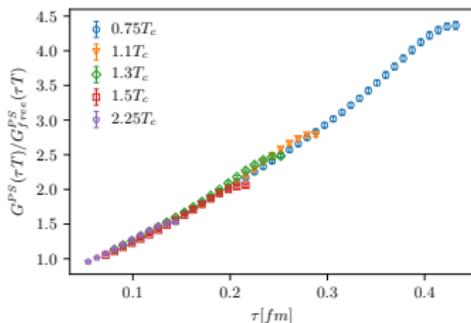
$$\rho^{pert}(\omega) = \underbrace{\rho^{vac}(\omega)}_{\text{Vacuum asymptotics}} \theta(\omega - \omega^{match}) + A^{match} \underbrace{\rho^{NRQCD}(\omega)}_{\text{pNRQCD}} \theta(\omega^{match} - \omega) \underbrace{\Phi(\omega)}_{\text{Suppression}}$$

- Large and fine lattices needed  
⇒ Quenched approximation
- Continuum extrapolated with a modified version of [Ding et al., EPJ WoC 175 (2018) 07010]
- Normalization with  $\chi_q/T^2$
- Mass interpolation to physical charmonium and bottomonium masses
- Continuum extrapolation with splines

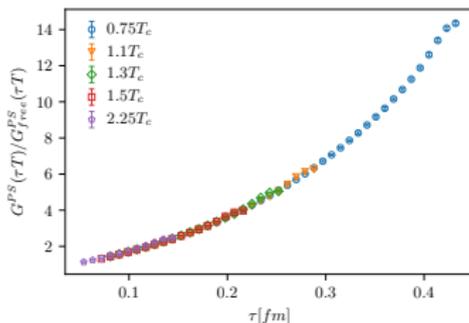


## Pseudoscalar

### Charmonium

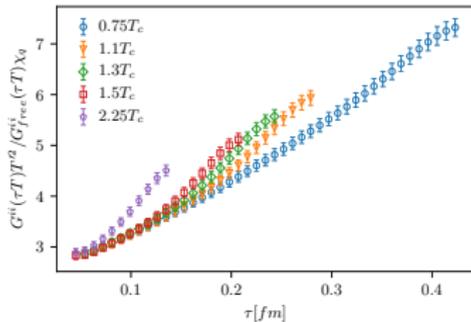


### Bottomonium

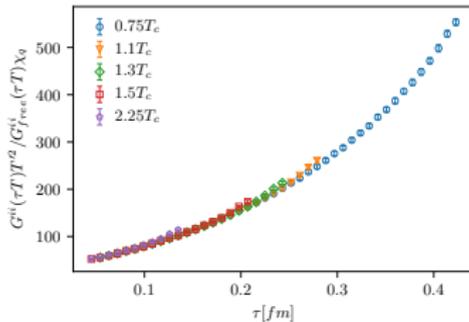


## Vector

### Charmonium



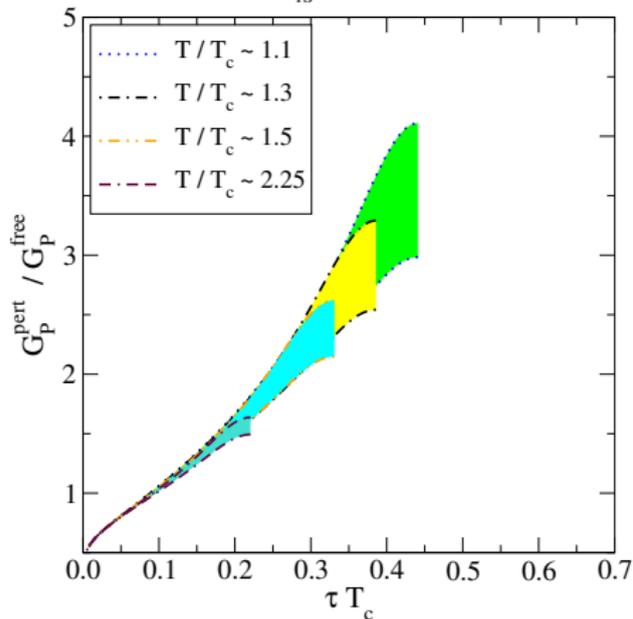
### Bottomonium



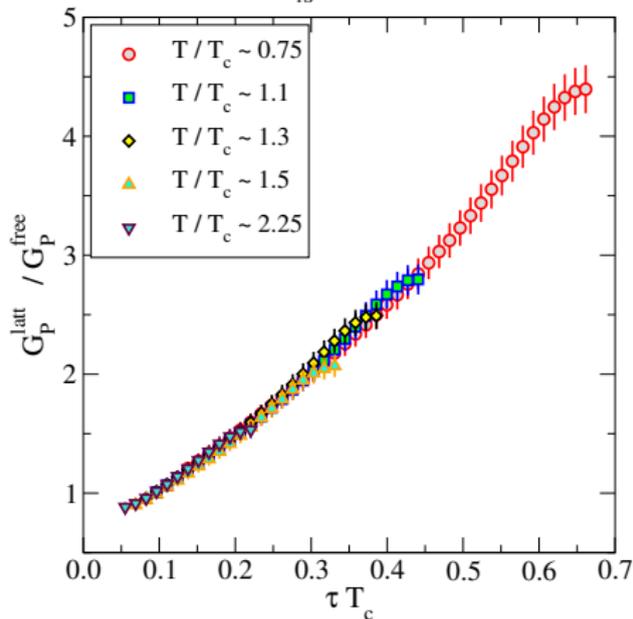
$$G(\tau) = \int_0^{\infty} d\omega K(\omega, \tau) \rho(\omega)$$

Charmonium - Pseudoscalar

$M_{1S} \sim 1.5 \text{ GeV}$



$M_{1S} \sim 1.5 \text{ GeV}$



A first look reveals qualitative agreement!

Systematic uncertainties may arise from

- a slightly incorrect renormalization on the lattice side  
⇒ Overall normalization  $A$
- the not exactly known relation between the pole mass and the  $\overline{MS}$  mass on the perturbative side  
⇒ Mass shift  $B$

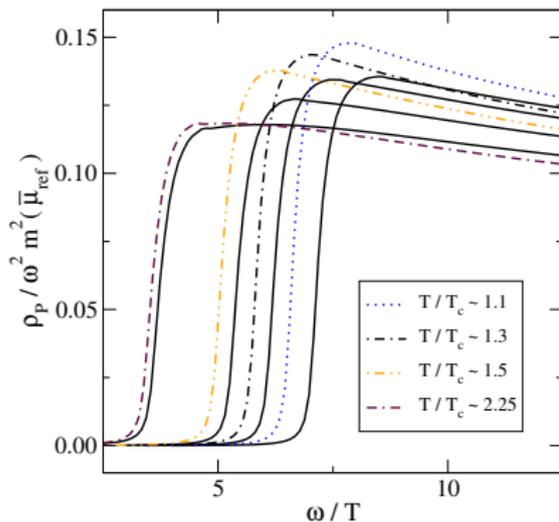
Resulting model spectral function

$$\rho^{\text{mod}}(\omega) = A \rho^{\text{pert}}(\omega - B)$$

- No transport peak  $\rightarrow$  easier
- Perturbative spectral function describes lattice data perfectly
- Only slight modifications needed ( $A \approx 1$ ,  $B$  small) [Burnier et al., JHEP11(2017)206]

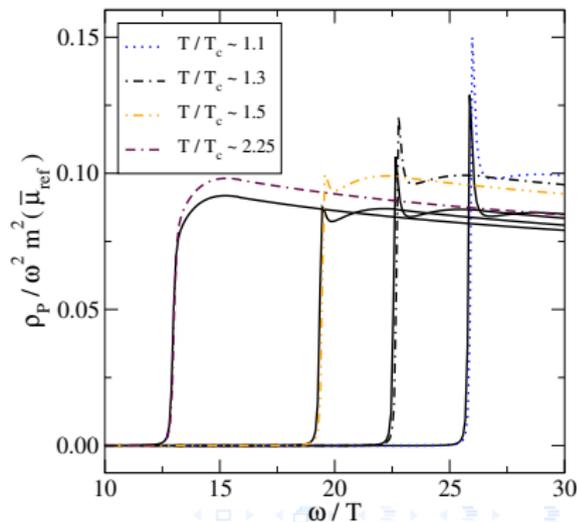
## Charmonium

$$m(\bar{\mu}_{\text{ref}}) = 1 \text{ GeV}$$



## Bottomonium

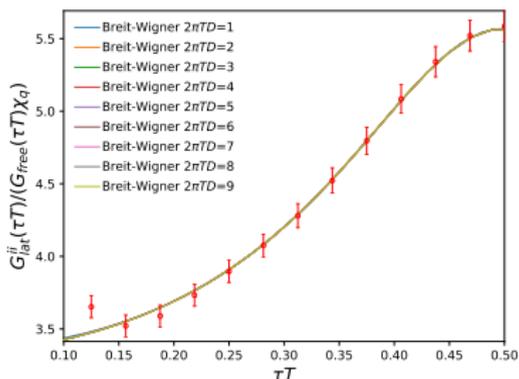
$$m(\bar{\mu}_{\text{ref}}) = 5 \text{ GeV}$$



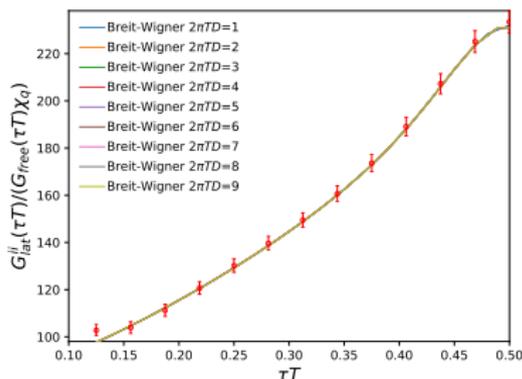
- Here: Additional transport peak  
 $\rho^{ii}(\omega) = A\rho^{pert}(\omega - B) + \rho^{trans}(\omega)$
- No perturbative description of transport peak
- Assume Breit-Wigner ansatz  
$$\rho^{trans}(\omega) = 3D\chi_q \frac{\omega\eta_D^2}{\omega^2 + \eta_D^2} \frac{1}{\cosh\left(\frac{\omega}{2\pi T}\right)}$$
- In total 4 parameters:  $A$ ,  $B$ ,  $D$ ,  $\eta_D \Rightarrow$  too many
- Try to fix some of the parameters

Model 1: Fix  $2\pi TD$  to values between 1 and 9, determine  $A$ ,  $B$ ,  $\eta_D$  via fit

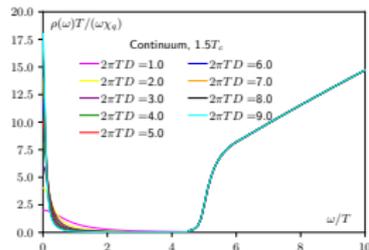
Charmonium  $2.25 T_c$



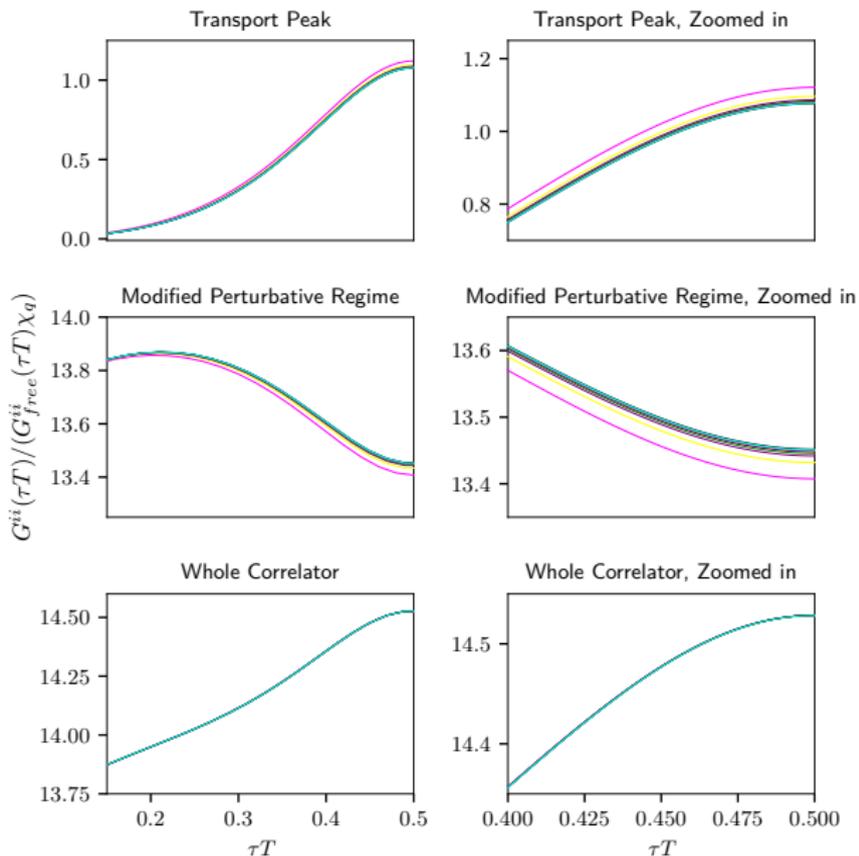
Bottomonium  $2.25 T_c$



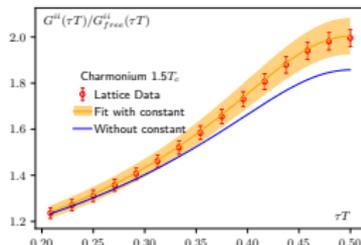
- Transport contribution can be described with Breit-Wigner
- All choices for  $2\pi TD$  reproduce the data equally well



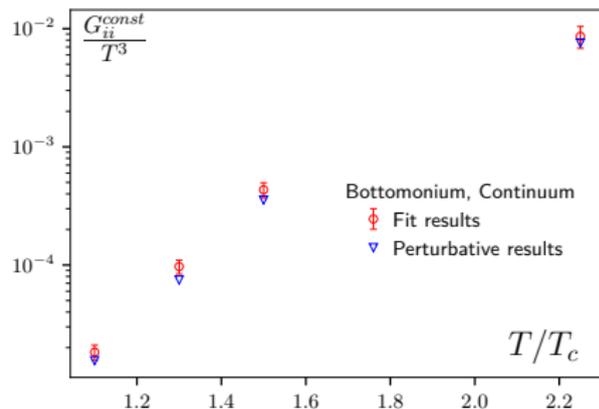
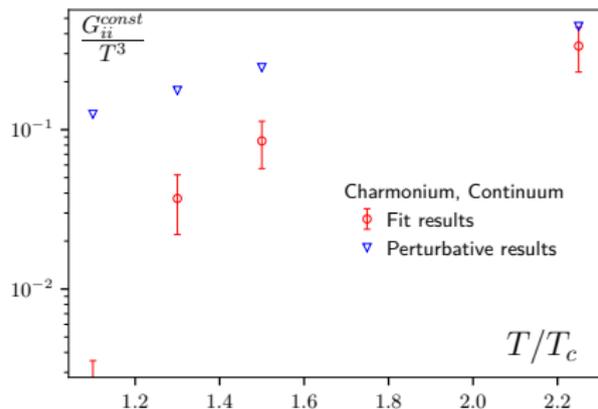
- Small changes in **A** and **B**
- Changes in different parts cancel each other



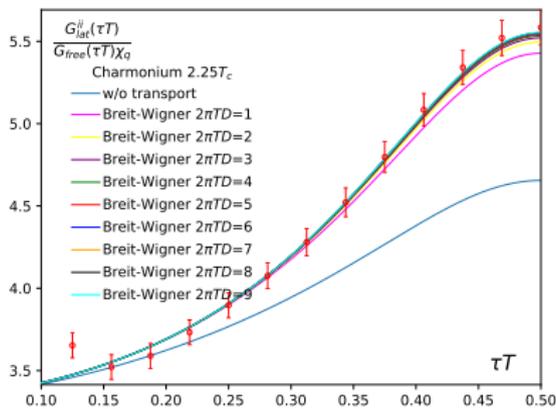
- Reminder:  $\rho^{ii}(\omega) = A\rho^{pert}(\omega - B) + \rho^{trans}(\omega, D, \eta_D)$   
In total 4 parameters:  $A, B, D, \eta_D$
- Fitting transport and bound state region simultaneously does not lead to results  
⇒ Separate the fit into transport and bound state region
- Investigation of bound state region: Assume constant contribution for transport peak (=  $\delta$ -peak in  $\rho$ )  
⇒  $G^{ii}(\tau T) = G^{pert.mod}(\tau T) + c$
- Analyze constant contribution to gain information on transport peak



- Constant contribution can be estimated perturbatively
- High agreement for bottomonium
- Decreasing agreement for decreasing temperature for charmonium



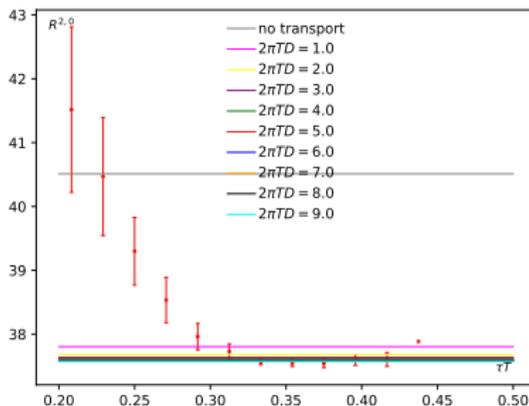
- $A$ ,  $B$  fixed by fit for high  $\omega$ , Transport so far described by constant
- Replace constant by Breit-Wigner
- $2\pi TD$  varied from 1 to 9
- $\eta_D$  solved to obtain the constant contribution from previous fit

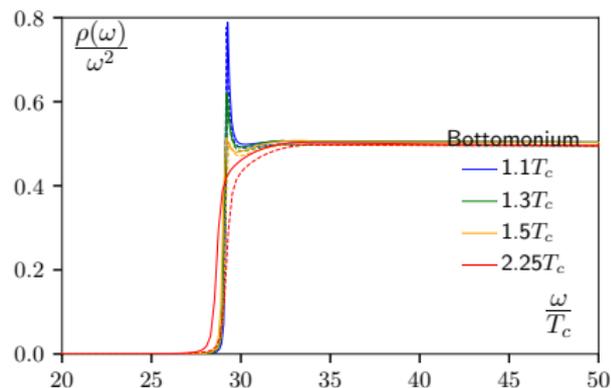
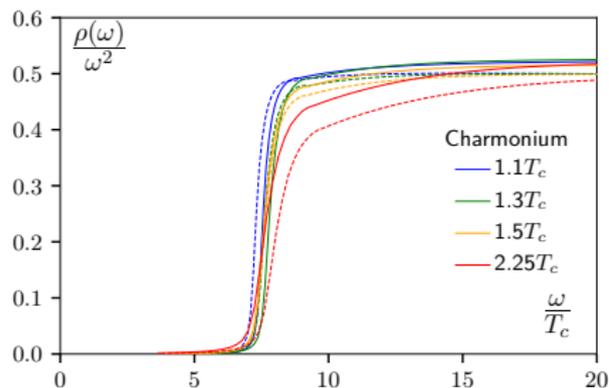


- Possible lower bound at  $2\pi TD = 2$
- Compare to thermal moments:

$$G^{(n)} = \frac{1}{n!} \left. \frac{d\omega^n G(\tau T)}{d\omega(\tau T)^n} \right|_{\tau T = \frac{1}{2}}, \quad R^{(n+2,n)} = \frac{G^{(n+2)}}{G^{(n)}}$$

- Thermal moments = Taylor coefficients of the correlator around the midpoint  $\tau T = 0.5 \rightarrow$  High influence of transport peak

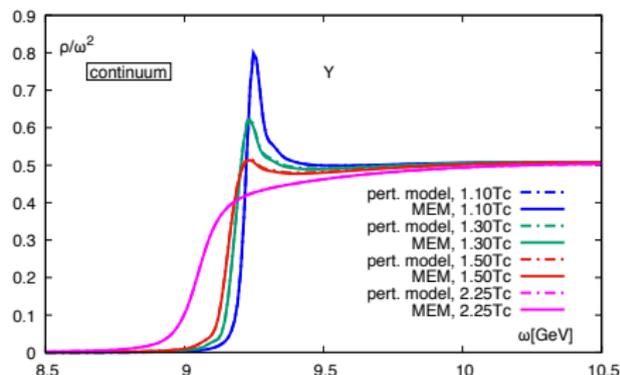
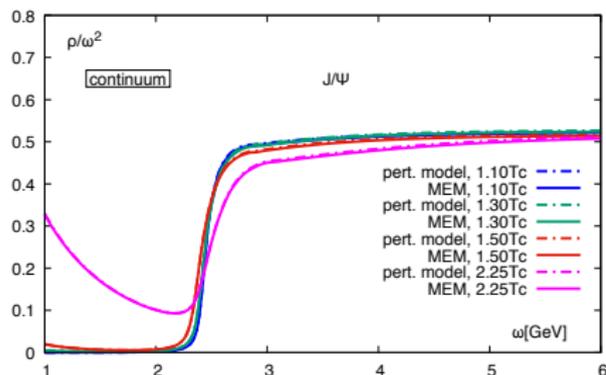




Dashed: Perturbative spectral function, Solid: Model spectral function

Like in pseudoscalar, the perturbative spectral function describes the data well with only slight modifications!

- Use Maximum Entropy Method to crosscheck spectral functions
- Use the results from fits as default model
- Default model remains almost unchanged  
⇒ MEM confirms the fit results



- Transport peak can be described with Breit-Wigner ansatz
- Solving for  $\eta_D$  hints to a lower bound of  $2\pi TD = 2$
- Correlators in pseudoscalar channel perfectly described by modified perturbative spectral function
- Bound state region in vector channel also perfectly matched by modified perturbative spectral function

In the future: Go to full QCD

- Transition temperature around 2 times smaller, no longer 1st order
- Effective couplings become larger

⇒ The physics may change, but the method remains the same!