Canonical partition functions in lattice QCD at high temperature

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Lattice 2019, Wuhan, 19 Jun 2019

Plan of the Talk

1. Introduction
   How/What shall Lattice QCD contribute to Experiments at finite baryon density ?

2. Brief Summary of Canonical Approach

3. Analyses of Experimental data with Lattice QCD

4. Summary

1. Introduction

Now we can handle finite density QCD using the Canonical approach.

Question:

How we can contribute Experiments ?

Our Answer:

Estimate Chemical Potential,Volume and Temperature by combining Lattice + Experimental data.

2. Brief Summary of Canonical Approach

$$\mathrm{Tr}\, e^{-(\hat{H}-\mu\hat{N})/T}$$

$$= \boxed{Z(\textcolor{blue}{\mu},\textcolor{red}{T})} = \sum_n \boxed{z_n(\textcolor{red}{T})}\xi^n$$

Grand Canonical Partition Function

Canonical Partition Function

$\textcolor{blue}{\mu}$: Chemical Potential

$\textcolor{red}{T}$: Temperature

$\xi \equiv e^{\textcolor{blue}{\mu}/\textcolor{red}{T}}$ **Fugacity**

Z_N are related with experimental data

$$Z(\mu, T) = \sum_n Z_n (e^{\mu/T})^n$$
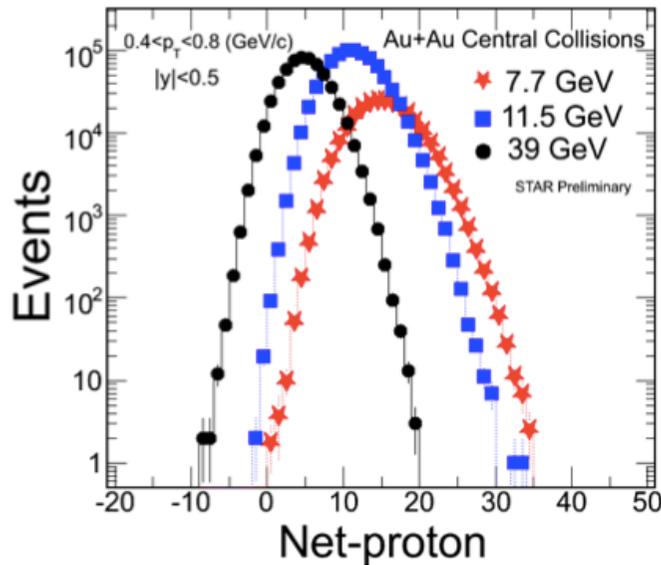
STAR@RHIC
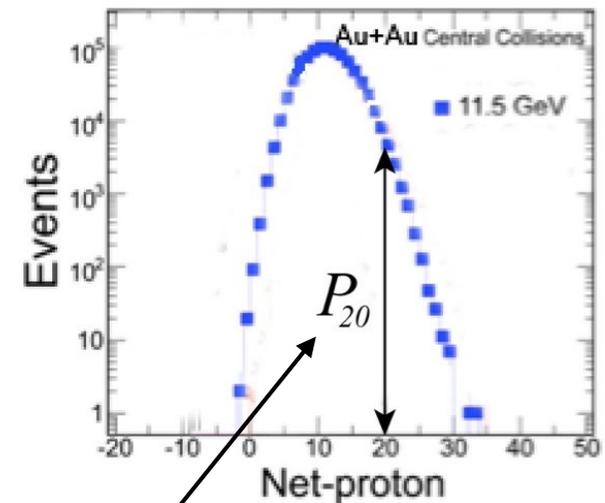
$$P_{20} \sim Z_{20} (e^{\mu/t})^{20}$$

$$[\det D(\mu)]^* = \det D(-\mu^*)$$

For Pure Imaginary μ ➡ $\det D$ **real**

A.Hasenfratz and Toussant, 1992

$$Z_n = \int \frac{d\theta}{2\pi} e^{i\theta n} Z_{GC}\left(\theta \equiv \frac{\mathrm{Im}\,\mu}{T}, T\right)$$

All information is in Imaginary Chemical Potential regions!

Great Idea ! But practically it did not work.

For making the method workable, we need
the two additional ingredients.

1) Multi-Precision Calculations
2) Integration method

Integration Method

n_B *in imaginary* μ ➡️ Z_N

$$n_B = \frac{1}{3V} T \frac{\partial}{\partial \mu} \log Z_G$$

$$= \frac{N_f}{3 N_s^3 N_t} \int \mathcal{D}U e^{-S_G} \mathrm{Tr}\, \Delta^{-1} \frac{\partial \Delta}{\partial \mu} \det \Delta$$

(For pure imaginary μ, n_B is also imaginary)

Then, for fixed T

$$Z(\theta \equiv \frac{\mu}{T}) = \exp\left(V \int_0^\theta n_B d\theta'\right)$$

$$Z_k = \frac{3}{2\pi} \int_{-\pi/3}^{+\pi/3} d\theta \exp\left(i\, k\theta + \int_0^\theta n_B d\theta'\right)$$

We map Information
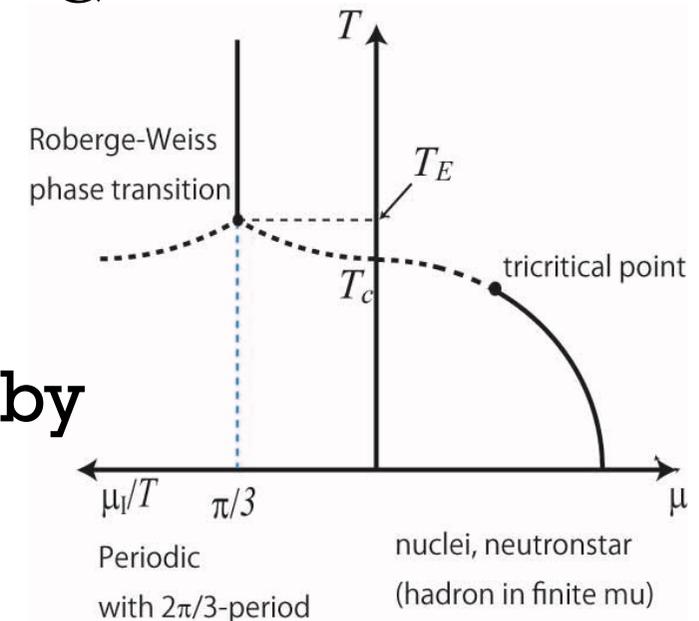# in Pure Imaginary Chemical Potential
to Real ones.

- We measure the number density at many pure imaginary chemical potential $n_B(\mu_I)$.

- We construct Grand Partition Function Z_G, by integrating $n_B(\mu_I)$

- By Fourier transformation, we get z_n

- Then we can calculate Real μ regions by

$$Z(\xi, T) = \sum_n z_n(T)\xi^n$$

T

Roberge-Weiss phase transition

T_E

T_c

tricritical point

μ_I/T $\pi/3$ μ

Periodic with $2\pi/3$-period

nuclei, neutronstar (hadron in finite mu)

$\xi \equiv e^{\mu/T}$ Fugacity

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2. Brief Summary of Canonical Approach

3. **Analyses of Experimental data with Lattice QCD**

4. Summary

3. Analyses of Experimental data with Lattice QCD

$n_B(\mu_I)$ is well approximated by sine function at $T<Tc$.
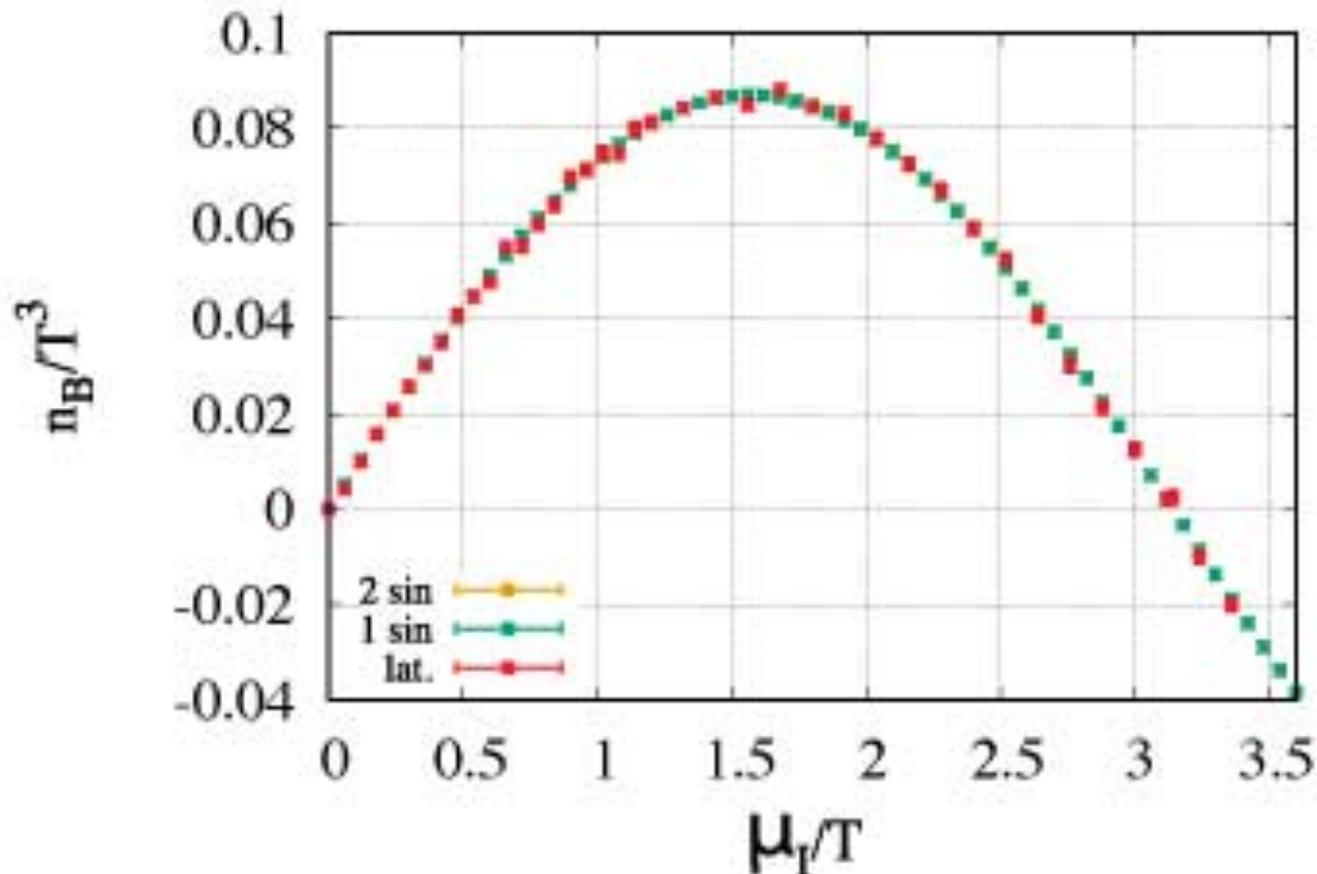
* D'Elia, M. Lombardo, Phys. Rev. D70 (2004) 074509
* Takahashi et al. Phy. Rev. D91 (1) (2015) 014501.
* Bornyakov et al., Phys.Rev. D95, 094506 (2017)

Proton multiplicity: Lattice data

Probability interpretation:

$$1 = \sum_n \frac{Z_n \xi^n}{Z_{GC}(\mu)}, \quad \xi = e^{\mu/T} \qquad \frac{N(n)}{N(0)} = \frac{N Z_n \xi^n / Z_{GC}(\mu)}{N Z_0 \xi^0 / Z_{GC}(\mu)} = Z_n \xi^n / Z_0$$

$$\Rightarrow \text{Proton multiplicity: } \frac{P_n}{P_0} = \boxed{\frac{Z_n}{Z_0}} e^{n\mu/T}$$

Lattice QCD

Lattice data μ/T = 0.4

T/T_c = 0.84	T/T_c = 1.08
T/T_c = 0.93	T/T_c = 1.20
T/T_c = 0.99	T/T_c = 1.35

Lattice data T/T_c = 0.93

μ/T = 0.0
μ/T = 0.5
μ/T = 1.0
μ/T = 1.5
μ/T = 2.0
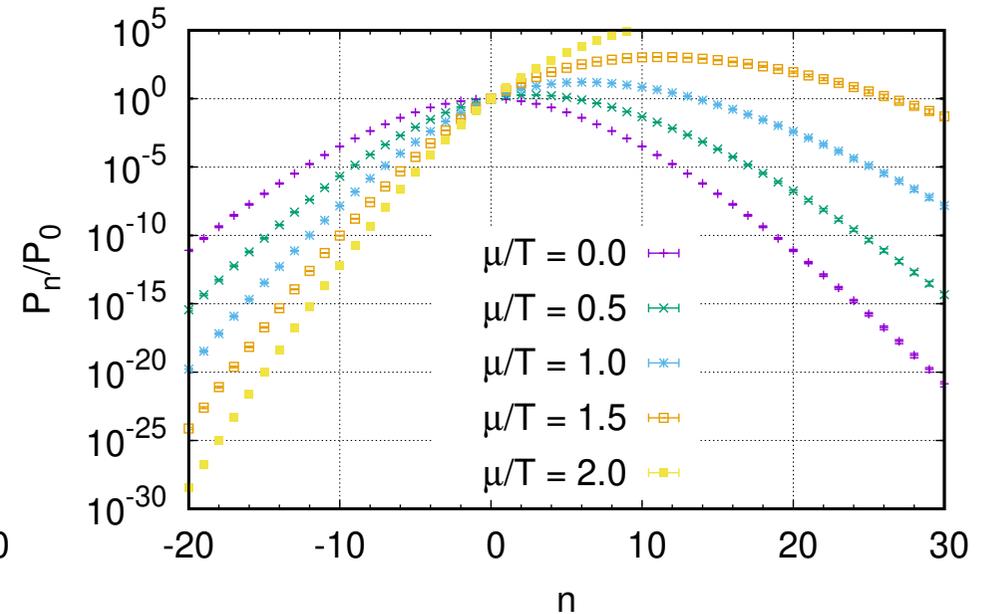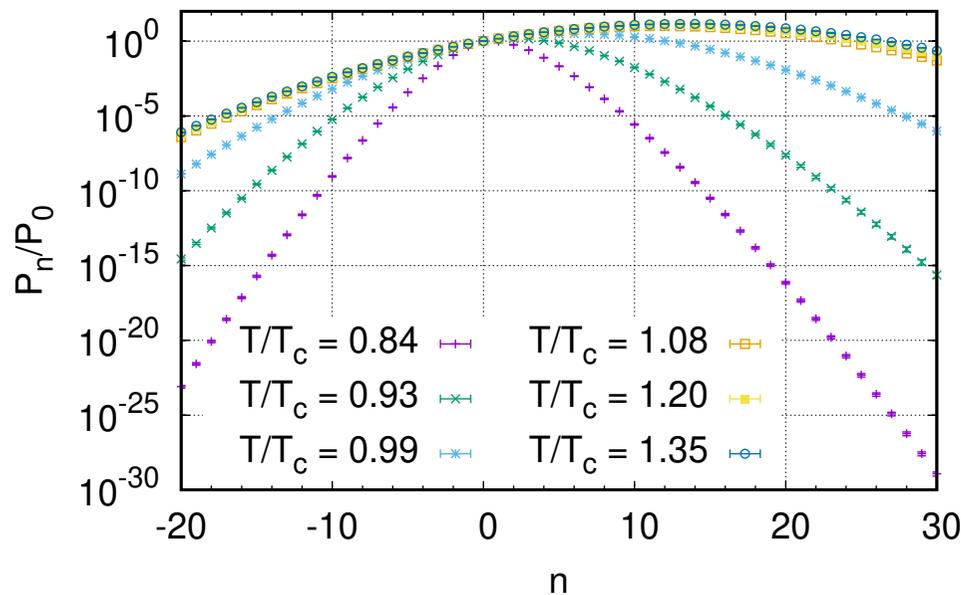
3. Analyses of Experimental data with Lattice QCD

$$\boxed{\frac{P_n}{P_0}} = \frac{Z_n}{Z_0} e^{n\mu/T} = \underbrace{\frac{\int d\theta e^{in\theta} e^{-V\int_0^\theta n(T,\tilde{\theta})d\tilde{\theta}}}{\int d\theta e^{i\theta} e^{-V\int_0^\theta n(T,\tilde{\theta})d\tilde{\theta}}}}_{} \times e^{n\mu/T}$$

Experiment

Lattice with μ, T and V as parameters

$$n_B = \frac{T}{V}\frac{\partial}{\partial\mu}\log Z_{GC}$$

$$= \sum_k f_k(T)\cos 3k\left(\frac{\mu}{T}\right)$$

We use

⭐ Boyda et al., arXiv/1704.03980

    Wilson, heavy quark mass

⭐ Vovchenko, et al., arXiv/1708.02852

    Staggered, physical quark masses

RHIC Energy Scan Data

Fitting RHIC multiplicity
data using nB of
1) Boyda et al.
2) Vovchenko et al.
with paramters, μ, T
and V

1) $\mu = 0.15, T/T_c = 0.95, V^{1/3} = 3.0$
   or
   $\mu = 0.15, T/T_c = 0.94, V^{1/3} = 6.5$

2) $\mu = 0.15, T/T_c = 0.95, V^{1/3} = 5.2$
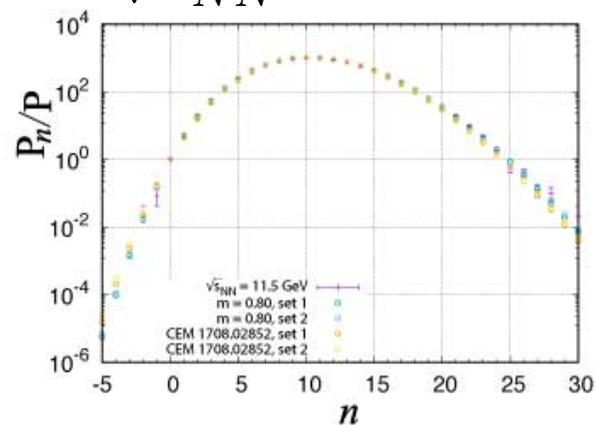   or
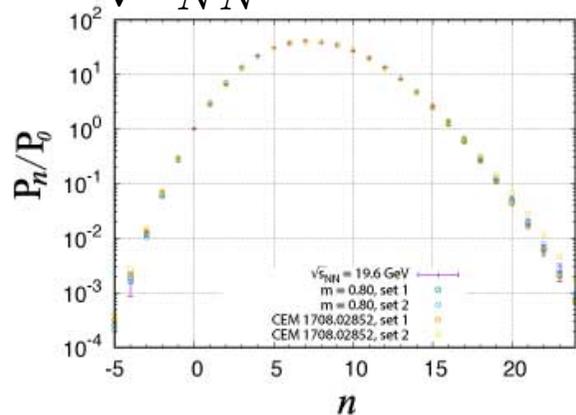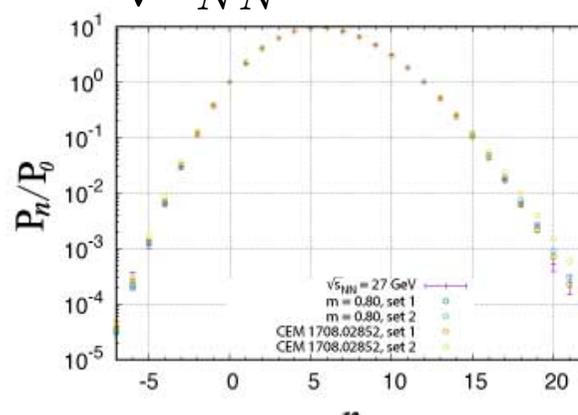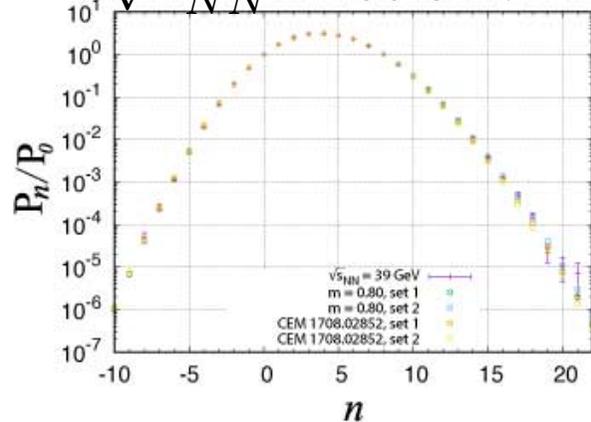 $\mu = 0.15, T/T_c = 0.90, V^{1/3} = 3.0$

$$\sqrt{s}_{NN} = 200\text{GeV}$$
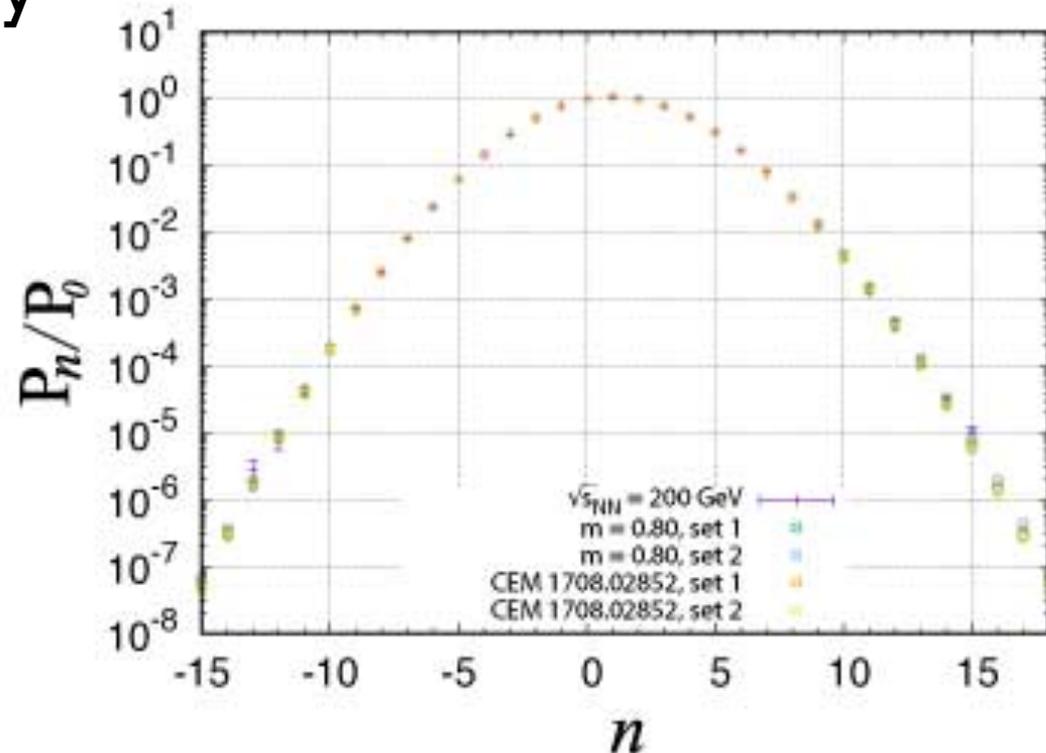
$\sqrt{s_{NN}} = 7.7\text{GeV}$
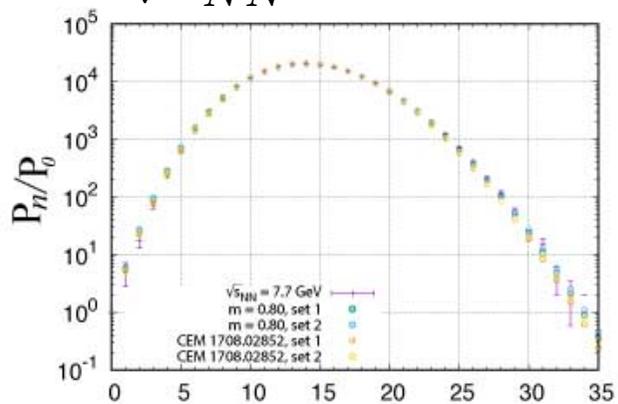
$\sqrt{s_{NN}} = 11.5\text{GeV}$

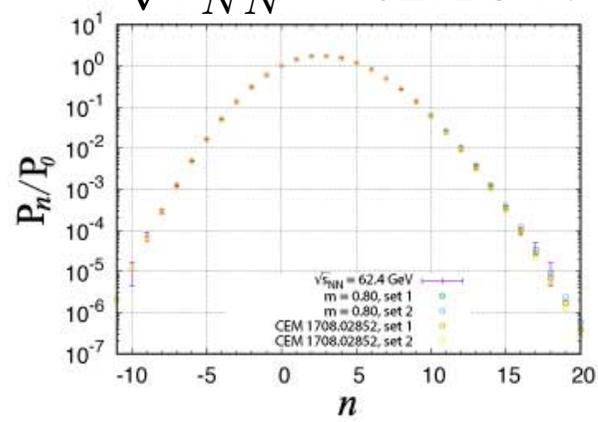$\sqrt{s_{NN}} = 19.6\text{GeV}$

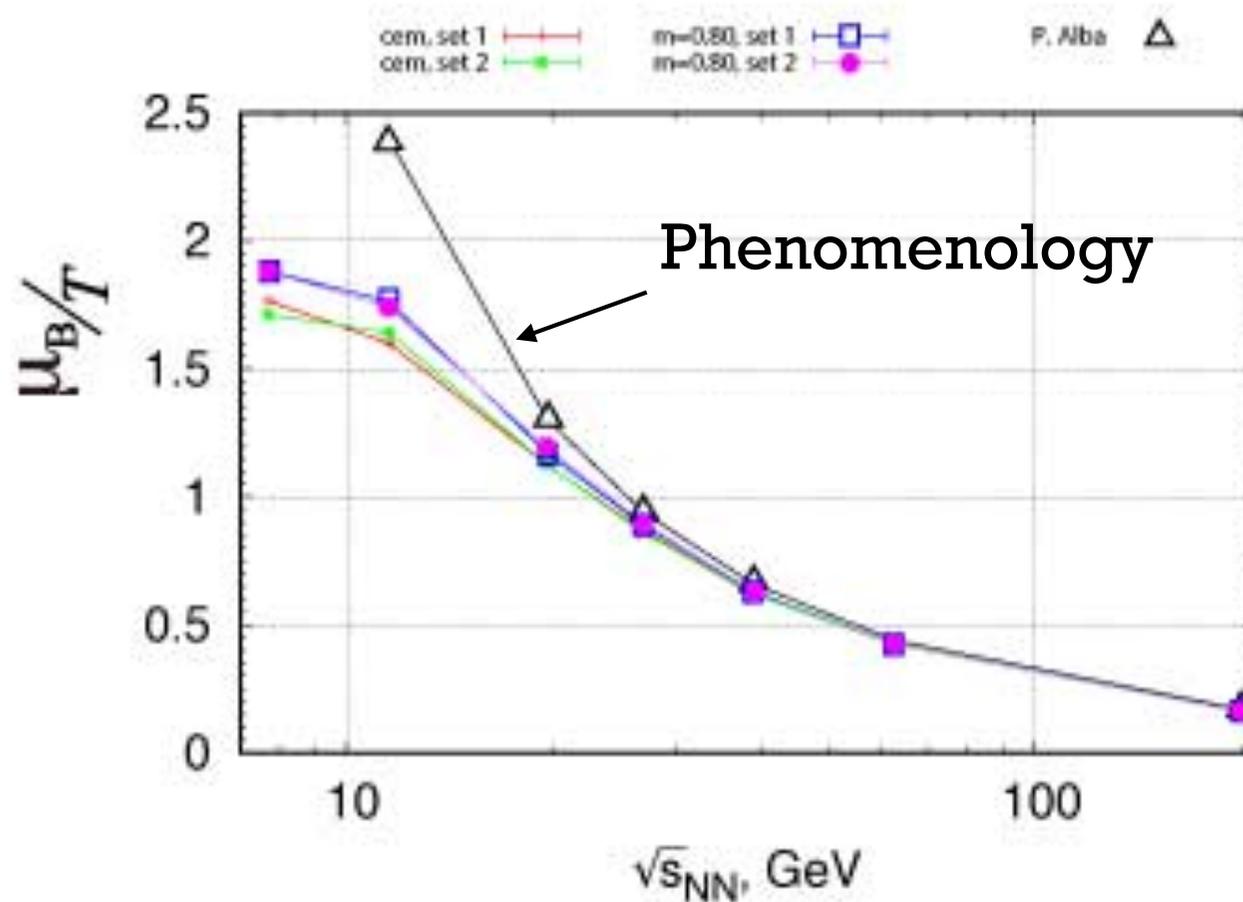$\sqrt{s_{NN}} = 27\text{GeV}$

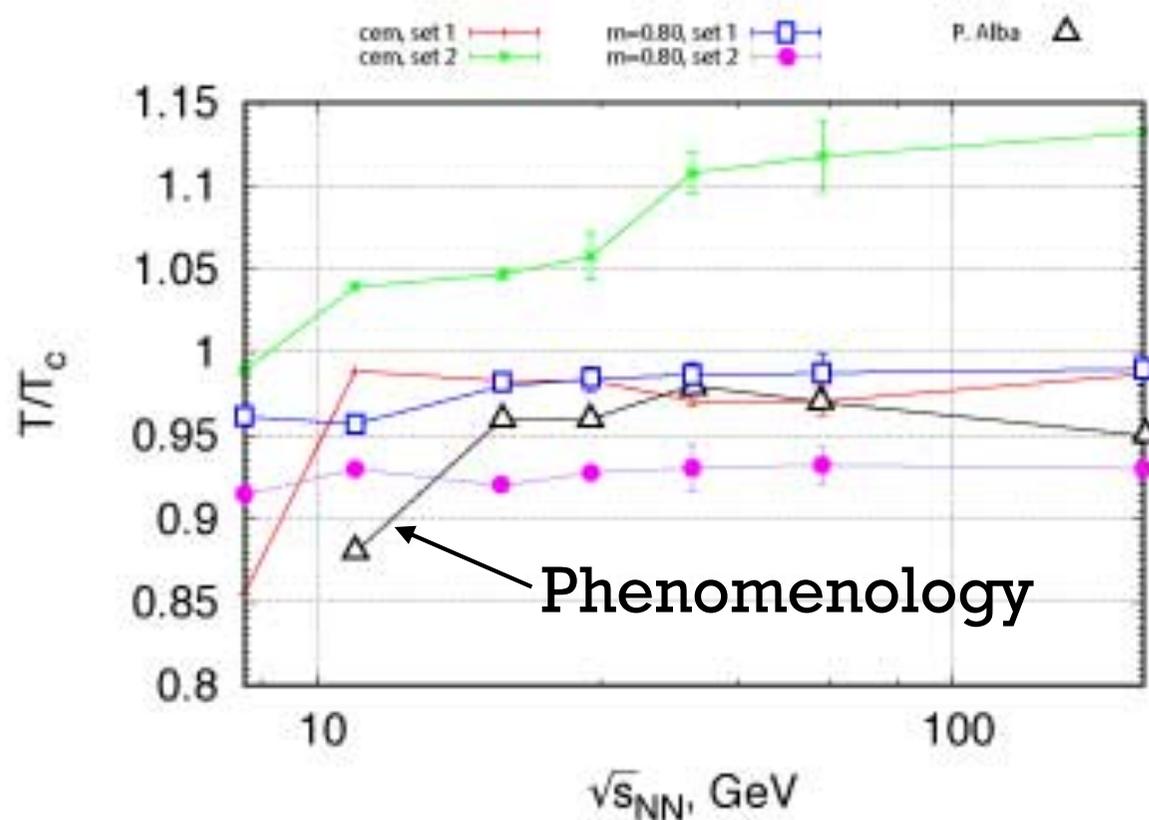$\sqrt{s_{NN}} = 39\text{GeV}$

$\sqrt{s_{NN}} = 62.4\text{GeV}$

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Obtained fitting Parameters
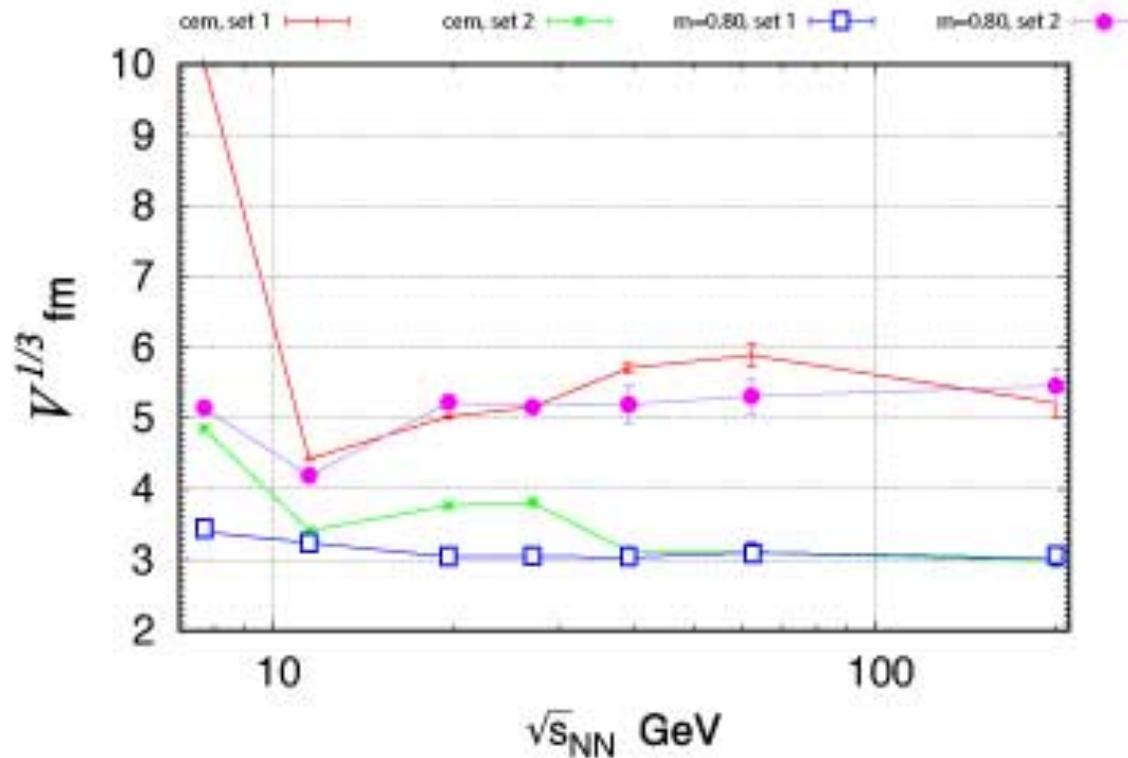# i) Chemical Potential

Obtained fitting Parameters
ii)Temperature

Obtained fitting Parameters
iii)Volume

Summary

🔴 Canonical approach can study finite density QCD at T>0 especially RHIC energy scan regions.

🔴 This lattice study results + RHIC multiplicity data

We can estimate μ, T and V of the created fire ball

📌 Results are very reasonable.

🔴 But still large ambiguity. This may be improved by using not only the multiplicity, P_n/P_0 but also higher moment