Canonical partition functions in lattice QCD at high temperature

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Plan of the Talk

1. Introduction
   How/What shall Lattice QCD contribute to Experiments at finite baryon density ?
2. Brief Summary of Canonical Approach
3. Analyses of Experimental data with Lattice QCD
4. Summary

1. Introduction

Now we can handle finite density QCD using the Canonical approach.

Question:

How we can contribute Experiments ?

Our Answer:

Estimate Chemical Potential, Volume and Temperature by combining Lattice + Experimental data.

2. Brief Summary of Canonical Approach

$$\mathrm{Tr}\, e^{-(\hat{H}-\mu\hat{N})/T}$$

$$= Z(\mu, T) = \sum_n z_n(T)\xi^n$$

Grand Canonical Partition Function

Canonical Partition Function

μ : Chemical Potential

T : Temperature

$\xi \equiv e^{\mu/T}$ **Fugacity**

$$Z(\mu, T) = \sum_n z_n(T)\xi^n$$

$$\xi \equiv e^{\mu/T}$$

This is very useful relation because

we can calculate $z_n(T)$ at imaginary μ

where no sign problem,

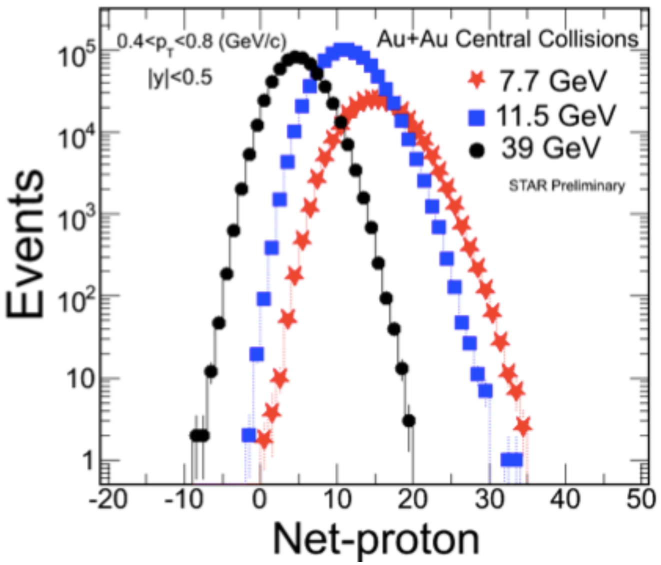
then

we know $Z(\mu, T)$ at any μ .

↑

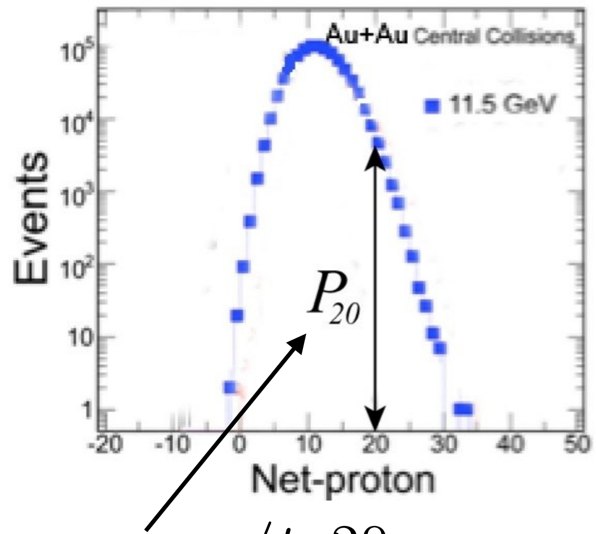
real, imaginary or even complex

Z_N are related with experimental data



STAR@RHIC

$$Z(\mu, T) = \sum_n Z_n (e^{\mu/T})^n$$



$$P_{20} \sim Z_{20} \, (e^{\mu/t})^{20}$$

$$[\det D(\mu)]^* = \det D(-\mu^*)$$

For Pure Imaginary μ ➜ $\det D$ **real**

A.Hasenfratz and Toussant, 1992

$$Z_n = \int \frac{d\theta}{2\pi} e^{i\theta n} Z_{GC}(\theta \equiv \frac{\mathrm{Im}\mu}{T}, T)$$

All information is in Imaginary Chemical Potential regions!

Great Idea ! But practically it did not work.

For making the method workable, we need the two additional ingredients.

1) Multi-Precision Calculations
2) Integration method

Integration Method

n_B *in imaginary* μ → Z_N

$$n_B = \frac{1}{3V} T \frac{\partial}{\partial \mu} \log Z_G$$

$$= \frac{N_f}{3N_s^3 N_t} \int \mathcal{D}U e^{-S_G} \mathrm{Tr}\Delta^{-1} \frac{\partial \Delta}{\partial \mu} \det \Delta$$

(For pure imaginary μ, n_B is also imaginary)

Then, for fixed *T*

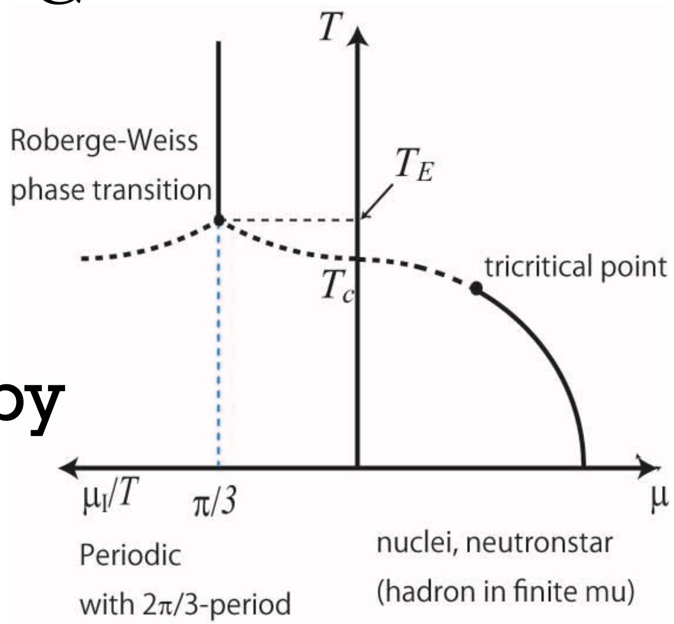
$$Z(\theta \equiv \frac{\mu}{T}) = \exp(V \int_0^{\theta} n_B d\theta')$$

$$Z_k = \frac{3}{2\pi} \int_{-\pi/3}^{+\pi/3} d\theta \exp\left(i\, k\theta + \int_0^{\theta} n_B d\theta' \right)$$

We map Information in Pure Imaginary Chemical Potential to Real ones.

- We measure the number density at many pure imaginary chemical potential $n_B(\mu_I)$.
- We construct Grand Partition Function Z_G, by integrating $n_B(\mu_I)$
- By Fourier transformation, we get z_n
- Then we can calculate Real μ regions by

$$Z(\xi, T) = \sum_n z_n(T)\xi^n$$



$$\xi \equiv e^{\mu/T}$$ Fugacity

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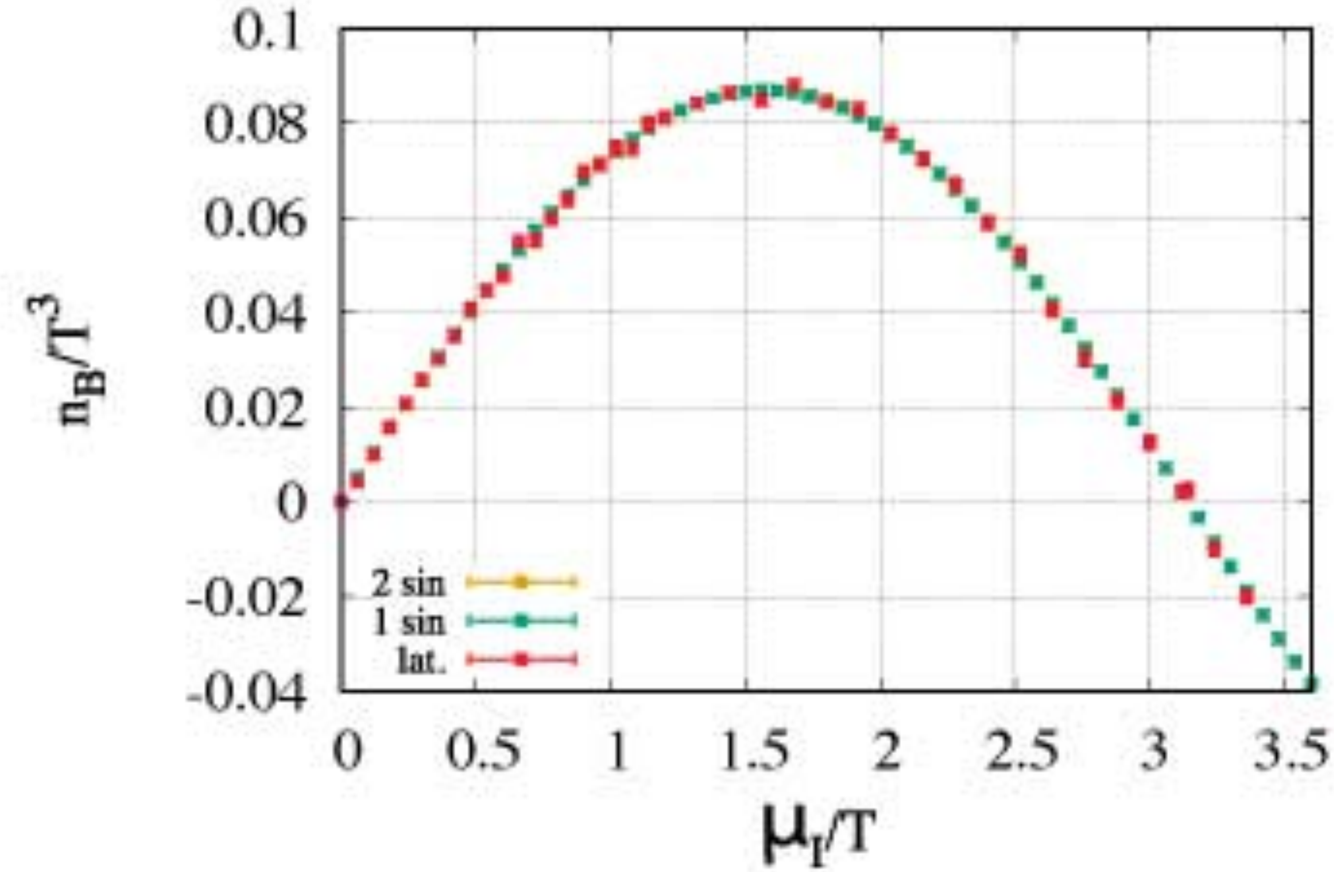
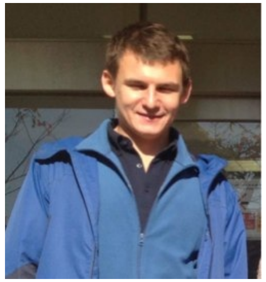
3. Analyses of Experimental data with Lattice QCD

$n_B(\mu_I)$ is well approximated by sine function at T<Tc.



* D'Elia, M. Lombardo, Phys. Rev. D70 (2004) 074509
* Takahashi et al. Phy. Rev. D91 (1) (2015) 014501.
* Bornyakov et al., Phys.Rev. D95, 094506 (2017)

Proton multiplicity: Lattice data

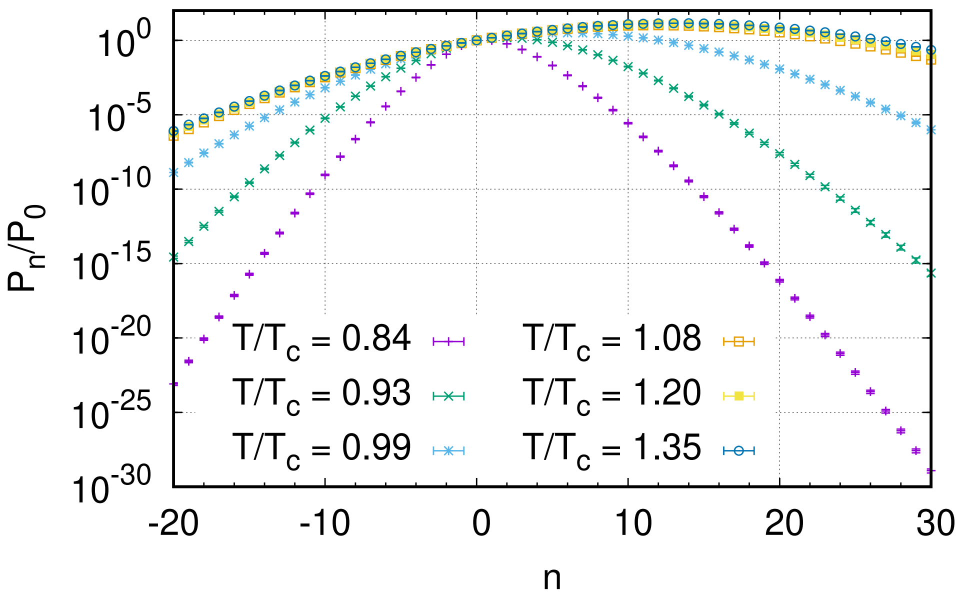


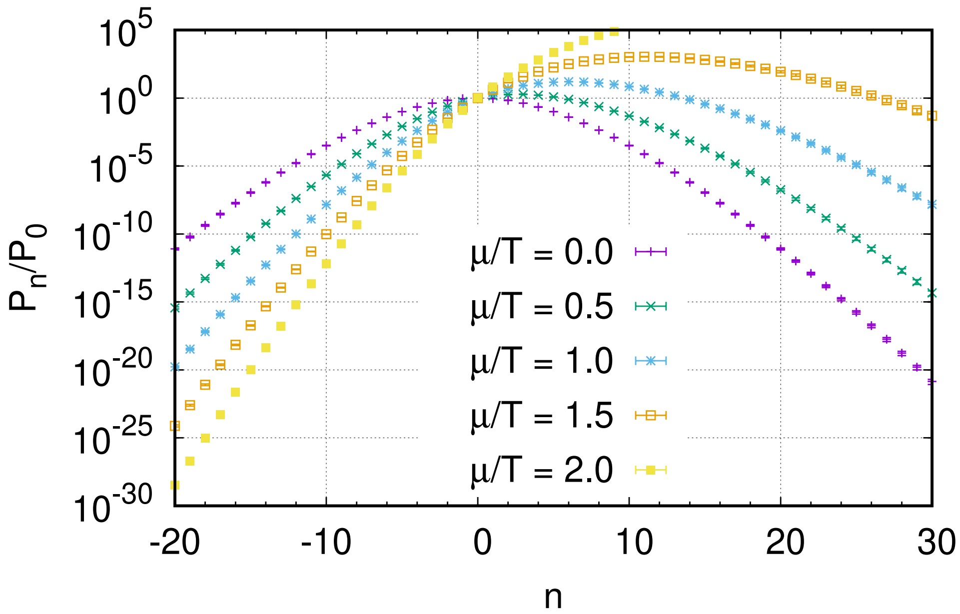
Probability interpretation:

$$1 = \sum_n \frac{Z_n \xi^n}{Z_{GC}(\mu)}, \quad \xi = e^{\mu/T} \qquad \frac{N(n)}{N(0)} = \frac{N\, Z_n \xi^n / Z_{GC}(\mu)}{N\, Z_0 \xi^0 / Z_{GC}(\mu)} = Z_n \xi^n / Z_0$$

$$\Rightarrow \text{Proton multiplicity: } \frac{P_n}{P_0} = \frac{Z_n}{Z_0} e^{n\mu/T}$$

Lattice QCD





$$\frac{P_n}{P_0} = \frac{Z_n}{Z_0} e^{n\mu/T} = \frac{\int d\theta e^{in\theta} e^{-V \int_0^\theta n_B(T,\tilde{\theta}) d\tilde{\theta}}}{\int d\theta e^{i\theta} e^{-V \int_0^\theta n_B(T,\tilde{\theta}) d\tilde{\theta}}} \times e^{n\mu/T}$$

Experiment

Lattice with μ, T and V as parameters

$$n_B = \frac{T}{V} \frac{\partial}{\partial \mu} \log Z_{GC}$$

$$= \sum_k f_k(T) \cos 3k \left(\frac{\mu}{T}\right)$$

We use

- Boyda et al., arXiv/1704.03980
  Wilson, heavy quark mass
- Vovchenko, et al., arXiv/1708.02852
  Staggered, physical quark masses

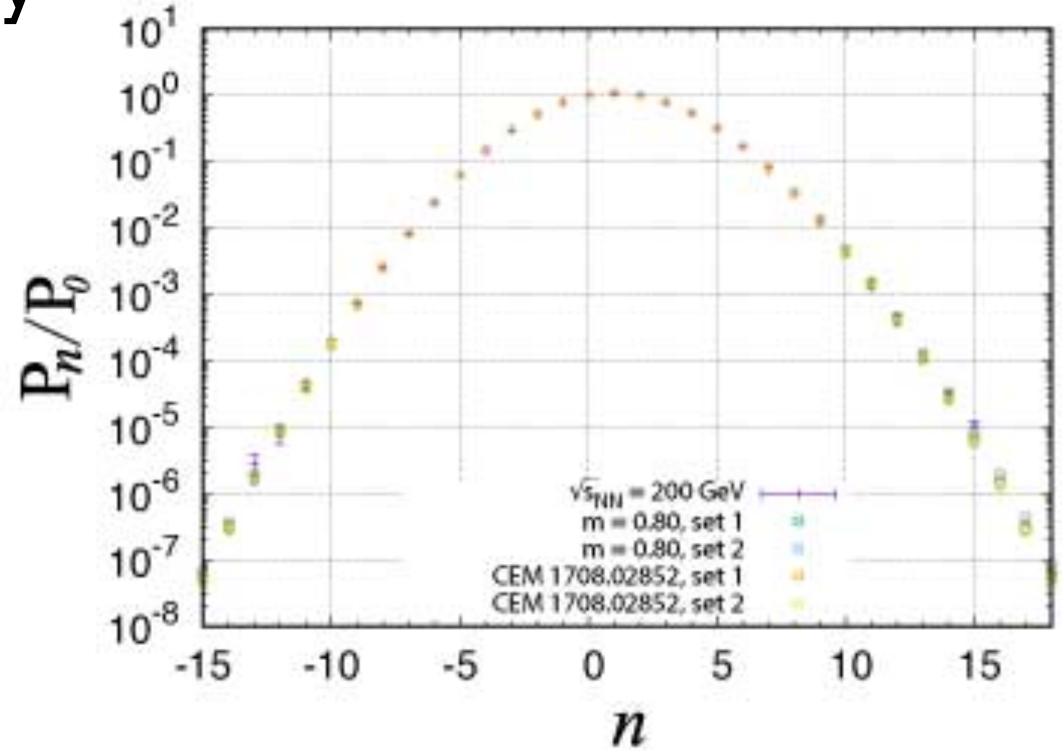
RHIC Energy Scan Data

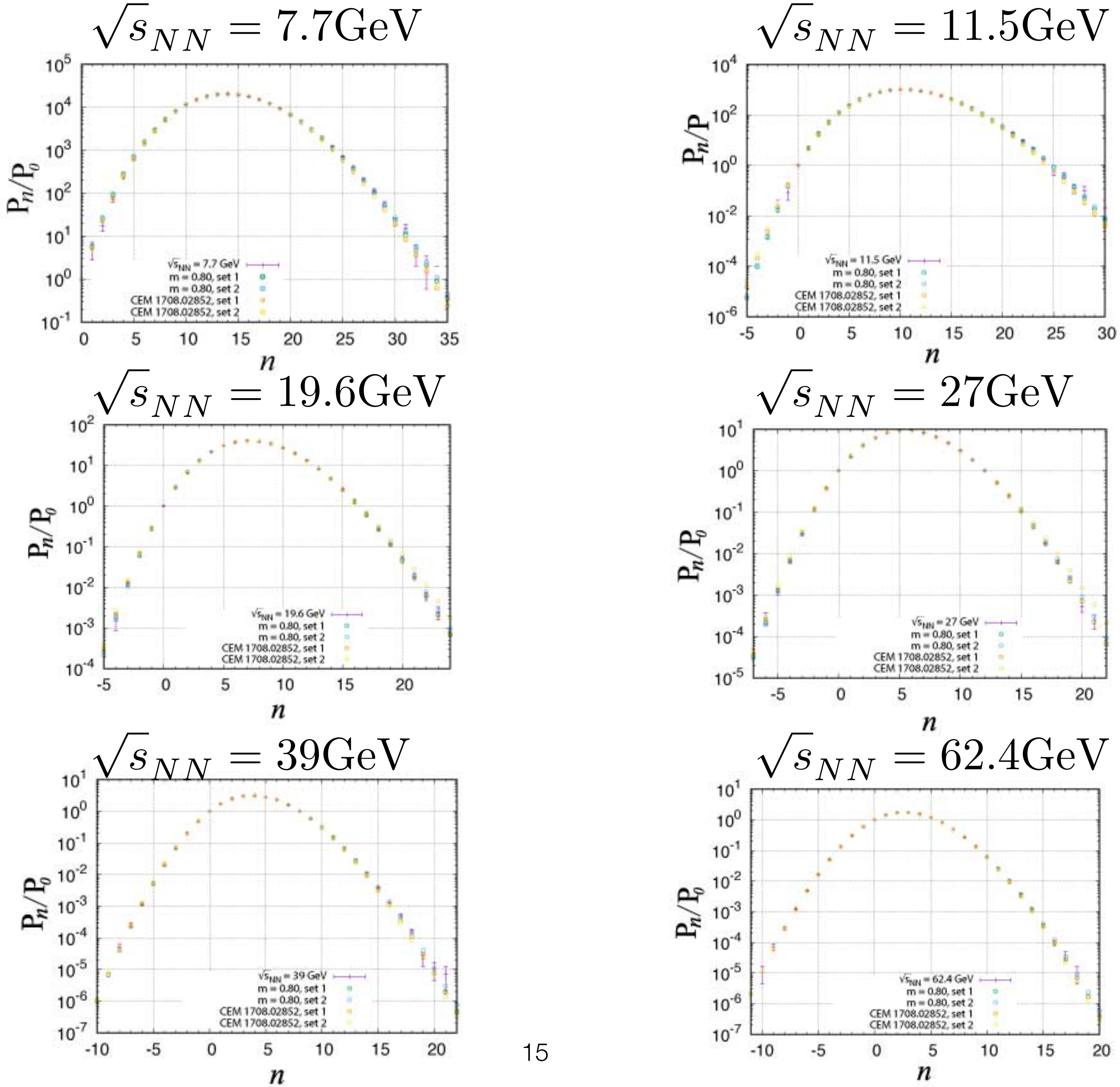
Fitting RHIC multiplicity data using nB of
1) Boyda et al.
2) Vovchenko et al.
with paramters, μ, T and V

1) $\mu = 0.15, T/T_c = 0.95, V^{1/3} = 3.0$
or
$\mu = 0.15, T/T_c = 0.94, V^{1/3} = 6.5$

2) $\mu = 0.15, T/T_c = 0.95, V^{1/3} = 5.2$
or
$\mu = 0.15, T/T_c = 0.90, V^{1/3} = 3.0$

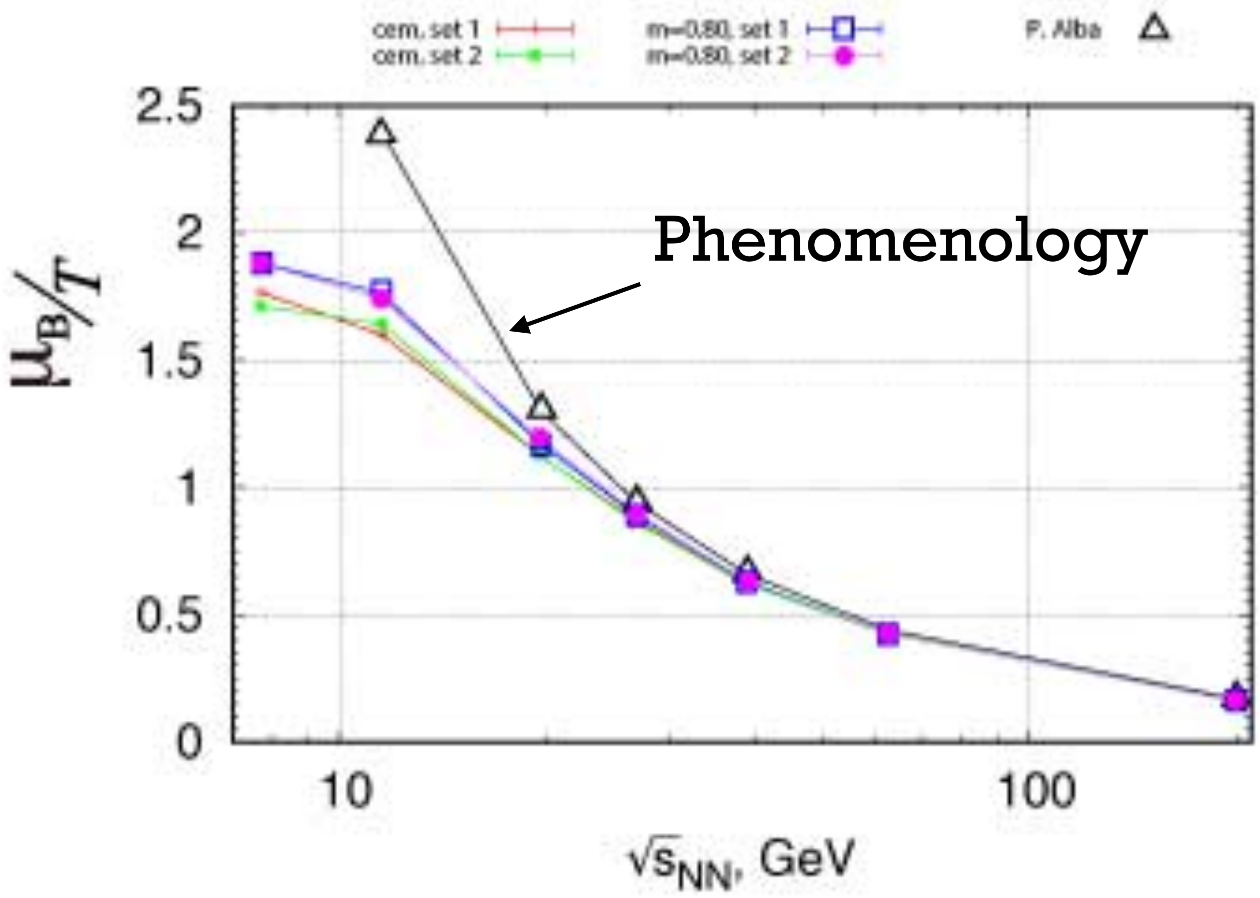
$\sqrt{s}_{NN} = 200\mathrm{GeV}$



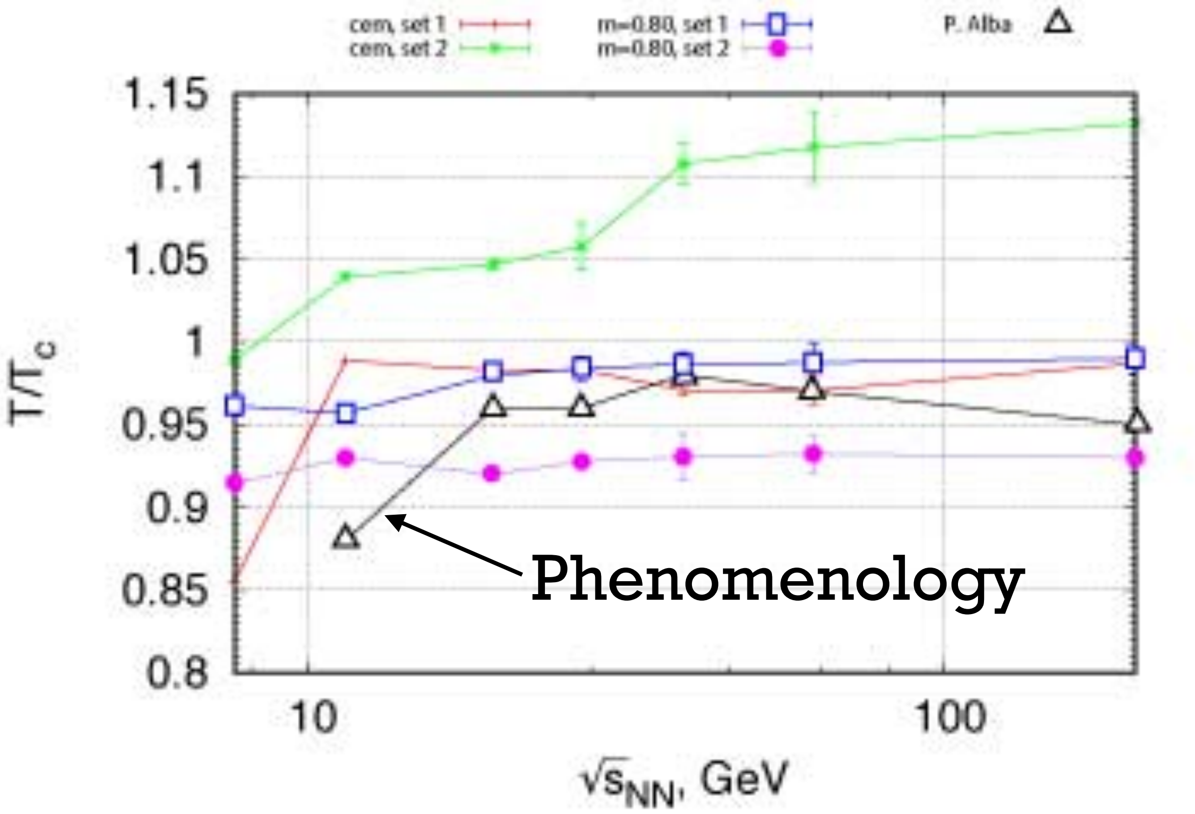


$\sqrt{s}_{NN} = 7.7\mathrm{GeV}$
$\sqrt{s}_{NN} = 11.5\mathrm{GeV}$
$\sqrt{s}_{NN} = 19.6\mathrm{GeV}$
$\sqrt{s}_{NN} = 27\mathrm{GeV}$
$\sqrt{s}_{NN} = 39\mathrm{GeV}$
$\sqrt{s}_{NN} = 62.4\mathrm{GeV}$
P_n/P_0
n
$\sqrt{s}_{NN}$ = 7.7 GeV
m = 0.80, set 1
m = 0.80, set 2
CEM 1708.02852, set 1
CEM 1708.02852, set 2
$\sqrt{s}_{NN}$ = 11.5 GeV
$\sqrt{s}_{NN}$ = 19.6 GeV
$\sqrt{s}_{NN}$ = 27 GeV
$\sqrt{s}_{NN}$ = 39 GeV
$\sqrt{s}_{NN}$ = 62.4 GeV

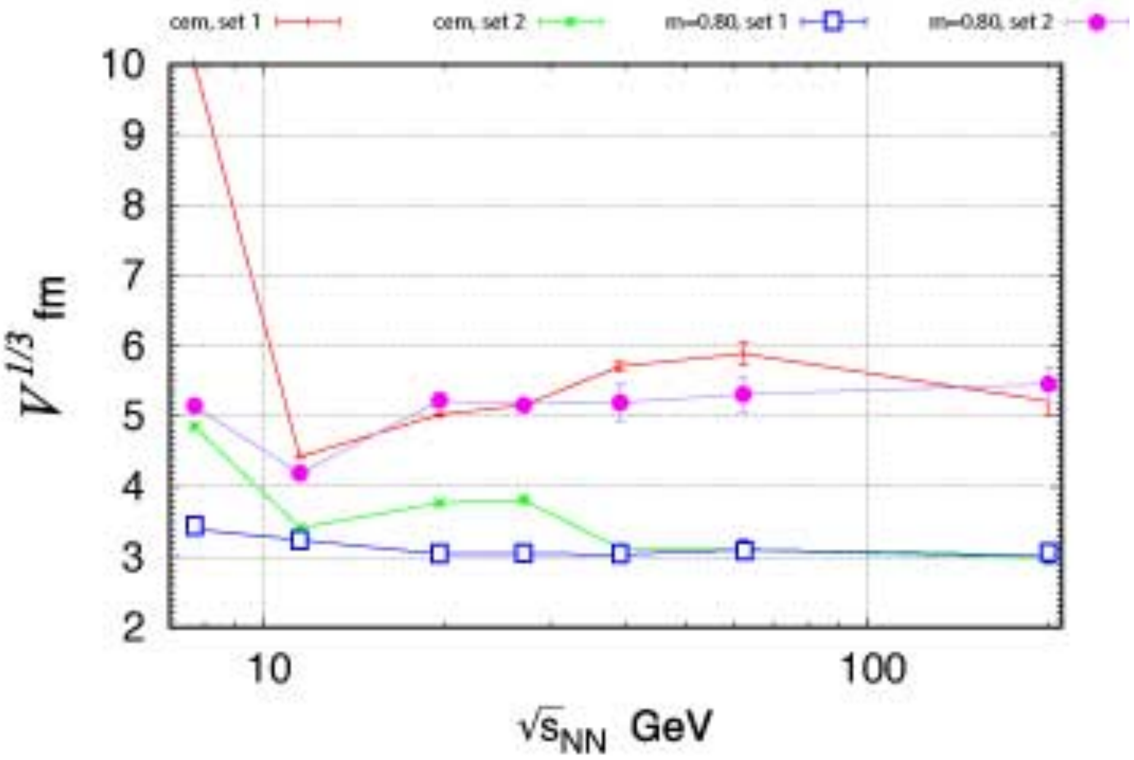
Obtained fitting Parameters
i) Chemical Potential



Obtained fitting Parameters
ii)Temperature



Obtained fitting Parameters
iii)Volume



Summary

- Canonical approach can study finite density QCD at T>0 especially RHIC energy scan regions.
- This lattice study results + RHIC multiplicity data

  →

  We can estimate $\mu, T \,\mathrm{and}\, V$ of the created fire ball

  - Results are very reasonable.
- But still large ambiguity. This may be improved by using not only the multiplicity, P_n/P_0 but also higher moment