Gauge Corrections to Strong Coupling LQCD on Anisotropic Lattices

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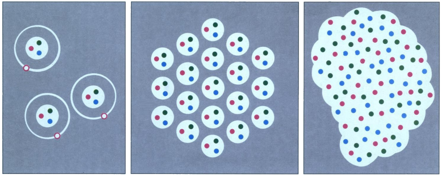

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Overview

Motivation:

1. **Anisotropic lattices:** necessary to study thermodynamics in strong coupling regime (β fixed)
2. Anisotropy has been studied in a dual formulation **in the strong coupling limit**, both for zero and non-zero quark mass
3. We extend these results to **finite β** (here: chiral limit)

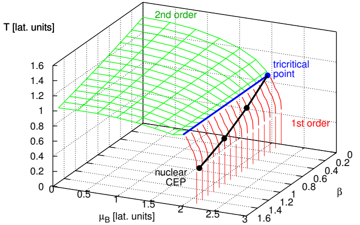
Content:

1. Lattice QCD in the **Dual Representation**, Role of Anisotropy
2. Results on Anisotropy and Continuous Time Limit
3. Some results on the **phase diagram**

Goal: What does the Phase Diagram including β look like?

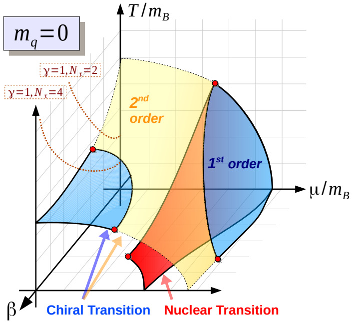
Phase Diagram in the Strong Coupling Regime:

- obtained via reweigthing
  [Langelage, de Forcrand, Philipsen & U., *PRL* **113** (2014)]
- important question: what happens to the chiral (tri)-critical point?



One of several **possible scenarios** for the extension to the continuum:

- back plane: strong coupling phase diagram ($\beta = 0$, a large), $N_f = 1$ (doublers decouple)
- front plane: continuum phase diagram ($\beta = \infty$, $a = 0$), $N_f = 4$ (no rooting)
- number of Goldstone bosons matter, between 1 at strong coupling and 15 in continuum.
- **anisotropy $\gamma \neq 1$ crucial**



Why Lattice QCD in a Dual Formulation?

1. **Dual representation**: color singlets (integers) as dual variables
   - all gauge fields $U_\mu(x)$ are integrated out
   - at strong coupling: link states are **mesons** and **baryons**
     [Kawamoto & Smit '81], [Rossi & Wolff' 84], [Karsch & Mütter '89]
   - at $\beta > 0$: color singlets, triplets, ... which can include **gluon contributions**
2. "Solution" to a sign problem:
   - $Z = \sum_{\{C\}} w(C)$ has negative/complex weights $w(C)$,
   - sampling with "wrong" weight exponentially hard $\left\langle e^{i\phi} \right\rangle_{||} = e^{-\frac{V}{T}\Delta f}$
   - find a **new representation** $\{C'\}$ such hat $\tilde{w}(C') \geq 0$
     (or a representation $\{C'\}$ where Δf small enough for practical purposes)
3. Sign problem in strong coupling regime $\beta = \frac{6}{g^2} \lesssim 1$ **mild enough**
   to study full phase diagram:
   - baryons are heavy: $\Delta_f \simeq 10^{-5}$ $\rightarrow$ **reweighting** of the sign feasible
   - **resummations** of world-lines possible to reduce the sign problem further
   - color singlets (hadrons) closer to **physical states** than colored gauge links

Note: Dual/worldline formulations also useful in many other lattice field theories,
see also talks by [Christopf Gattringer],[Maria Anosova], [Daniel Göschl], [Oliver Orasch]

Chiral Transition and Nuclear Transition

Chiral symmetry in SC-LQCD with **staggered fermions** for $N_\mathrm{f} = 1$:

$$U(1)_V \times U(1)_{55}: \qquad \chi(x) \mapsto e^{i\epsilon(x)\theta_A + i\theta_V}\chi(x), \quad \epsilon(x) = (-1)^{x_1+x_2+x_3+x_4}$$

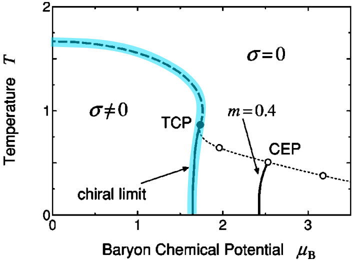
- $U(1)_V$ baryon number conserved
- $U(1)_{55}$ chiral symmetry spontaneously broken at low temperatures/densities
- expected to be O(2) 2nd order ($\mu = 0$)
- note: no chiral anomaly at $\beta = 0$

Nuclear Transition (T=0):

- **baryon crystal** forms (Pauli saturation)
- chiral symmetry restored
- expected to be 1st order

Existence of a **Tricritical Point:**

- chiral TCP and nuclear CEP coincide



Strong coupling phase diagram via mean field: [Nishida, PRD 69 (2004)]
For gauge corrections in MF see also [Nakano, Miura, Ohnishi, PRD 83 (2010)]

Dual Formulation: monomers+dimers+worldlines+worldsheets

$$Z(m_q,\mu) = \sum_{\{k,n,\ell,n_p\}} \underbrace{\prod_{b=(x,\mu)} \frac{(N_c-k_b)!}{N_c!(k_b-|f_b|)!}}_{\text{singlet hoppings } M_x M_y} \underbrace{\prod_x \frac{N_c!}{n_x!}(2am_q)^{n_x}}_{\text{chiral condensate } \bar\psi\psi} \underbrace{\prod_{\ell_3} w(\ell_3,\mu)}_{\text{triplet hoppings } \bar B_x B_y} \underbrace{\prod_{\ell_f} \tilde w(\ell_f,\mu)}_{\text{weight modifications}} \underbrace{\prod_P \frac{\left(\frac{\beta}{2N_c}\right)^{n_P+\bar n_P}}{n_P!\bar n_P!}}_{\text{gluon propagation}}$$

$k_b \in \{0,\ldots N_c\}$, $n_x \in \{0,\ldots N_c\}$, $\ell_b \in \{0,\pm 1\}$, $f_b = \partial n_p$, $f_x = \frac{1}{2}\sum_b |f_b|$

[G. Gagliardi, Kim & U. arXiv:1710.07564]

- color constraint:

$$n_x + \sum_{\hat\nu=\pm\hat 0,\ldots\pm\hat d} \left(k_{\hat\nu}(x) + \frac{N_c}{2}|\ell_{\hat\nu}(x)|\right) = N_c + f_x$$

- 3-flux weight $\tilde w(\ell_f)$ involves additional **site weights** v_x and **link weights** $w_b(B)$

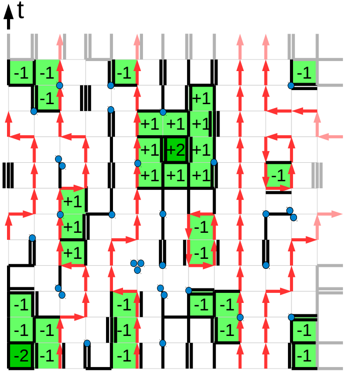
Sign of a configuration factorizes in **3-flux sign** and **gauge flux sign**!

$$\sigma(C) = \prod_{\ell_f} \sigma(\ell_f) \prod_{\ell_3} \sigma(\ell_3), \quad \sigma(\ell) = (-1)^{1+w(\ell)+N_-(\ell)} \prod_{\tilde\ell} \eta_\mu(x)$$

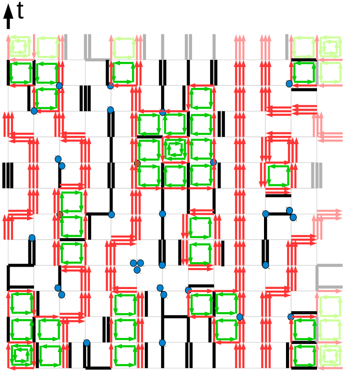
- -1 for each fermion loop, for each backward hopping (spatial or temporal) for each winding number (antiperiodic bc), and product of staggered phases

Typical Configurations

- 2-dimensional illustration at finite temperature, finite chemical potential, finite quark mass, finite gauge coupling
- simulations run in 3+1 dimensions



Dual variables used for Monte Carlo



Representation in terms of quark lines

Strong Coupling LQCD at Finite Temperature

How to vary the temperature?

- $aT = 1/N_t$ is discrete with N_t even
- $aT_c \simeq 1.5 \quad \Rightarrow \quad$ we cannot address the phase transition!

Solution: introduce an **anisotropy** γ in the Dirac couplings such that $a_t \neq a_s = a$:

$$\mathcal{L}_{\mathrm{F}} = \sum_{\mu} \frac{\gamma^{\delta_{\mu 0}}}{2} \eta_\nu(x) \left(e^{\mu \delta_{\mu 0}} \bar{\chi}(x) U_\nu(x) \chi(x+\hat{\mu}) - e^{-\mu \delta_{\mu 0}} \bar{\chi}(x+\hat{\mu}) U_\mu^\dagger(x) \chi(x) \right)$$

$$Z_F(m_q, \mu, \gamma) = \sum_{\{k,n,\ell\}} \prod_{b=(x,\mu)} \frac{(N_{\mathrm{c}} - k_b)!}{N_{\mathrm{c}}! k_b!} \gamma^{2 k_b \delta_{\mu 0}} \prod_x \frac{N_{\mathrm{c}}!}{n_x!} (2am_q)^{n_x} \prod_\ell w(\ell, \mu)$$

- Meanfield at strong coupling: $\frac{a_s}{a_t} \equiv \xi(\gamma) = \gamma^2$, since $\gamma_c^2 = N_t \frac{(d-1)(N_{\mathrm{c}}+1)(N_{\mathrm{c}}+2)}{6(N_{\mathrm{c}}+3)}$

  $\Rightarrow$ definition of the temperature: $aT = \frac{\xi(\gamma)}{N_t}$

However: Need to know the **precise correspondence between $\xi \equiv a_s/a_t$ and γ**

- Nonperturbative result: $\xi(\gamma) \approx \kappa \gamma^2 + \frac{\gamma^2}{1+\lambda\gamma^4}, \quad \kappa = 0.781(1)$

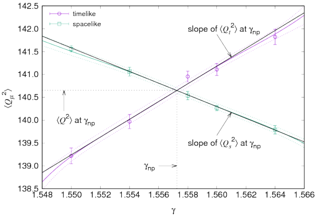
Status of SC-QCD on Anisotropic Lattices

Anisotropic Lattices: $\xi \equiv \frac{a_s}{a_t} > 1$

- idea of **anisotropy callibration**: determine γ such that $N_s a_s \stackrel{!}{=} N_t a_t \quad \Rightarrow \quad \xi = \frac{N_t}{N_s}$
- use conserved current related to pion:

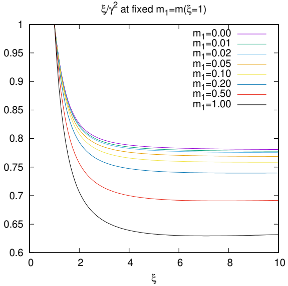
$$j_\mu(x) = \sigma(x)\left(k_\mu(x) - \tfrac{3}{2}|b_{x,\mu}|\right)$$

[Chandrasekharan & Jiang '03]



[de Forcrand, U., Vairinhos *PRD* **97** (2018)]

- relation $\xi \leftrightarrow \gamma$ has been studied for $m_q = 0$, $\beta = 0$, defines unambigously the **continuous time limit:** $a_t \to 0$ $(N_t \to \infty,\ \xi \to \infty$ at fixed $aT = \frac{\xi}{N_t})$
- callibration of γ extended to finite m_q, **non-perturbative correction factor** $\kappa(m_q) = \lim_{\xi\to\infty} \frac{\xi}{\gamma^2}$ has simple mass dependence
- temperature and chemical potential defined as $aT = \kappa(m_1)[aT]_{\rm mf}$, $a\mu_B = \kappa(\hat{m})[a\mu_B]_{\rm mf}$

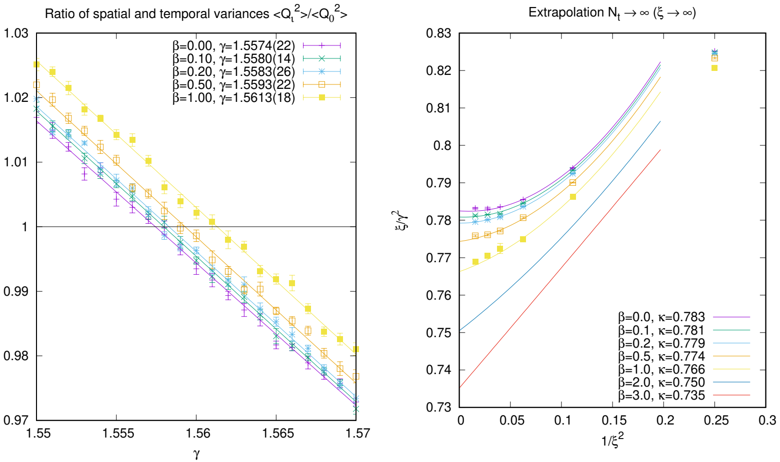


[LATTICE 2018]

To be addressed here:

- callibration of γ extended to **finite β:** $\xi(\gamma, \beta)$

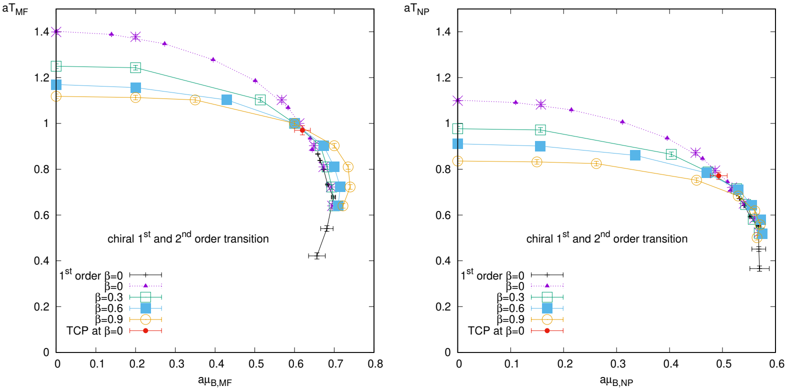
Anisotropy Callibration at finite β



- *Left:* determination of γ for various β by requiring the ratio of charge fluctuations to be the same. *Right:* extrapoltaion of the correction factor ξ/γ^2 towards continuous time.

Phase Diagram in the Strong Coupling Regime

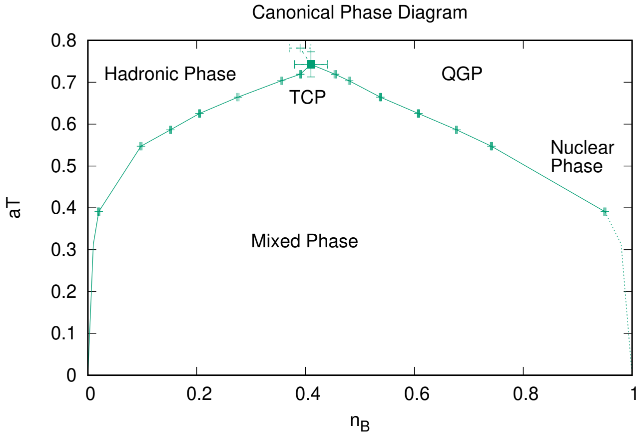
Preliminary result before (left) and after (right) taking into account the non-perturbative anisotropy $\xi(\gamma, \beta)$:



- **backbending vanished**
- Tri-critical point no longer at intersection
- need detailed analysis of whether the TCP is β-dependent

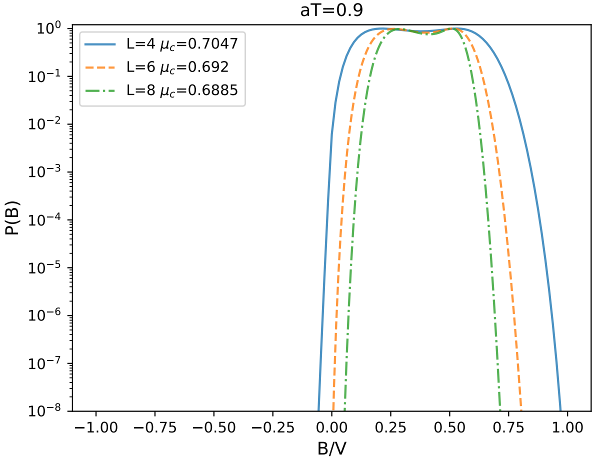
Canonical Phase Diagram

- canonical phase diagram: $n_B - T$ plane instead $\mu_B - T$ plane
- obtained via the **Wang-Landau method**
- mixed phase ends in tri-critical point (chiral limit)
- to do: extended to $\beta > 0$



Probability Distribution: volume dependence

- Wang-Landau method: probability distribution to high accuracy
- allows precise determination of μ_c



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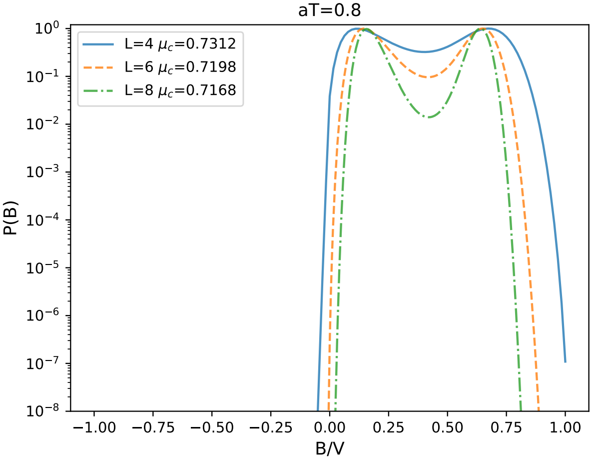

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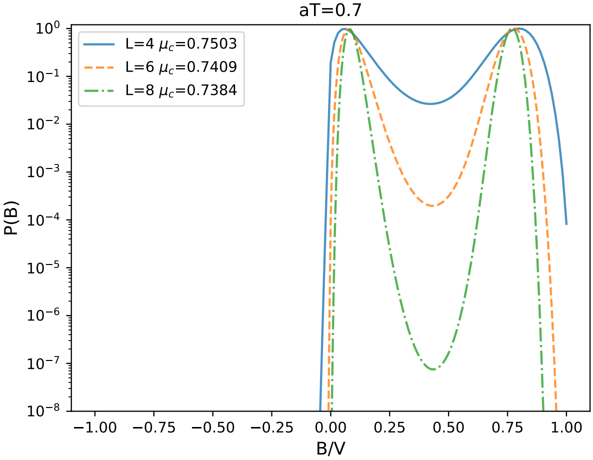
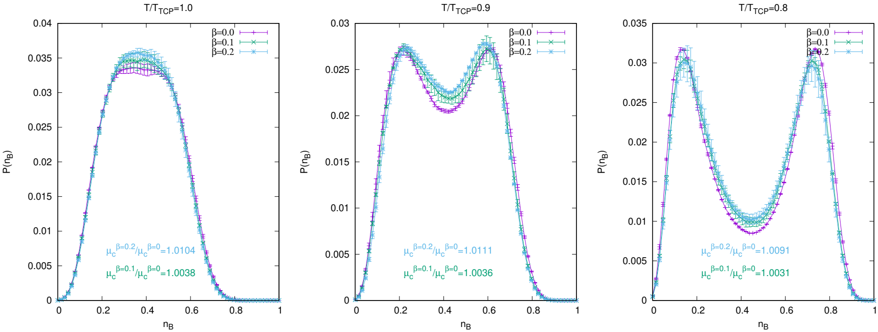


Probability Distribution: β-dependence

- Wang-Landau method can be extended to finite β
- the first order transition weakens with β
- the critical chemical potential increases with β



Conclusions

Results:

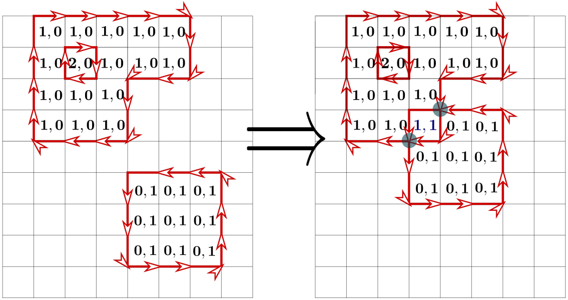
- anisotropy $\frac{a}{a_t} \equiv \xi(\gamma, \beta)$ determined non-perturbatively for $m_q = 0$
- allows to measure thermodynamical observables correctly
- allows **unambigous identification of phase boundary**
- simulations in continuous time limit $\xi \to \infty$ also well defined at $\beta > 0$
  $\rightarrow$ last talk by **Marc Klegrewe**:
  "Continuous Time Simulations of SC-LQCD at Finite Baryon Density"
- extension to finite quark mass:
  $\rightarrow$ talk by **Jangho Kim**:
  "β Dependence of the Nuclear Transition Points at Finite Quark Mass"
- first results for the **canonical phase diagram** for $\beta > 0$

Goals: **extend range of validity** to $\beta > 1$

- character expansion needed to reduce the sign problem further
- requires to solve more complicated Gauge- and Grassmann integrals
  $\rightarrow$ tensor network, computationally demanding

Higher order plaquette excitations

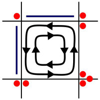
- So far simulations in the dual formulation **limited to plaquette worldsheets bounded by disconnected quark loops**.
- Intersection between surfaces generates contributions that require more sophisticated calculations (non-trivial polynomial integration over $SU(N_c)$ + Grassman integration).



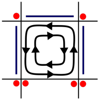
We developed analytic techniques to evaluate these **NLO** β-contributions.

Higher order β-corrections

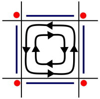
- depending on the quark content circulating on the lattice and the plaquette excitations, **several diagrams must be computed**
- example: for a single plaquette-antiplaquette excitation with $n_p = \bar{n}_p = 1$ (genuine $O(\beta^2)$ correction) we have several monomer-dimer coverings:



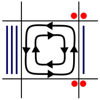
$$\longrightarrow \frac{\beta^2}{(2N_c)^2}\frac{(2am_q)^8}{2!2!3!}(N_c!)^3(N_c-1)!$$



$$\longrightarrow \frac{\beta^2}{(2N_c)^2}\frac{(2am_q)^6}{2!2!}(N_c!)^2((N_c-1)!)^2$$



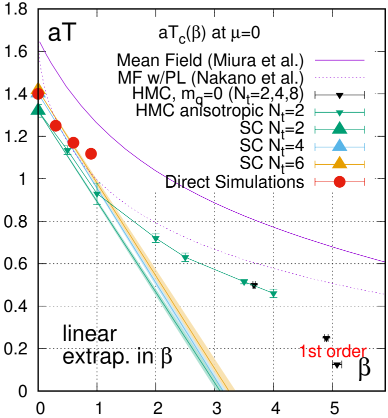
$$\longrightarrow \frac{\beta^2}{(2N_c)^2}(2am_q)^4\frac{N_c^8-4N_c^6+4N_c^4+2}{(N_c^2-1)^4}(N_c-1)!^4$$



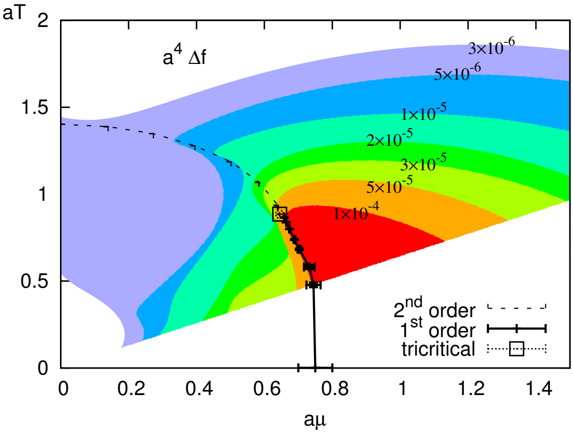
$$\longrightarrow \frac{\beta^2}{(2N_c)^2}\frac{(2am_q)^4}{2!2!3!}\frac{N_c^2+3N_c+2}{(N_c^2-1)(N_c^2-4)}N_c!^2(N_c-1)!^2$$

- however: **difficult to sample** (non-local weights)

Hybrid Monte Carlo Crosschecks



Residual Sign Problem in Dual Representation



- average sign: $\langle \mathrm{sign} \rangle \simeq e^{-\frac{V}{T}\Delta_f}$
- volumes $32^3 \times N_t$ can be easily simulated at tricritical point
- sign problem more **severe at low temperatures**