UNIVERSITÄT BIELEFELD
Faculty of Physics

CRC-TR 211
Strong-interaction matter under extreme conditions

Euclidean correlation functions of the topological charge density

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Motivation

- The classical QCD vacuum is not unique. QCD possesses infinitely many classical vacua which can be classified by their topological properties.
- The change of the topological charge is given by the sphaleron rate.
- It is related to the fermion number non-conservation through axial anomaly.
- Perturbative calculations exist but are not reliable in the relevant coupling region.
- Non-perturbative calculations required from lattice QCD.

Sphaleron rate

- The sphaleron rate is defined in Minkowski time by the correlation function: [Moore,Tassler,2010]

$$\Gamma_{sphal} = \int d^4x \left\langle \frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}(x) \frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}(0) \right\rangle$$

- Euclidean correlation function has same spectral function and is defined by

$$G_q(\tau, \vec{p}) = \int d^3x \, e^{i\vec{p}\vec{x}} \langle q(\vec{x}, \tau) q(\vec{0}, 0) \rangle$$

$$G_q(\tau, \vec{p}) = \int_0^\infty \frac{d\omega}{\pi} \rho_q(\omega, \vec{p}) \frac{\cosh \omega \left(\frac{1}{2T} - \tau \right)}{\sinh \frac{\omega}{2T}}$$

$$\Gamma_{sphal} = \lim_{\omega \to 0} \frac{2T \rho_q(\omega, 0)}{\omega}.$$

- Extracting ρ_q is ill-posed inversion problem. See talk of Anna-Lena Kruse and Hiroshi Ohno.

Topological charge

- Gluonic definition of the topological charge density:

$$q(x) = \frac{g^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}\left\{F_{\mu\nu}(x)F_{\rho\sigma}(x)\right\}.$$

- We use Highly-improved field-strength tensor on the lattice.

  [Bilson-Thompson,Leinweber,Williams,2002]

- Topolocial charge:

$$Q = \int d^4x\, q(x).$$

- Topological susceptibility:

$$\chi_{\mathrm{top}} = \lim_{V\to\infty}\frac{\langle Q^2\rangle}{V} = \int d^4x\,\langle q(x)q(0)\rangle.$$

The gradient flow

- All gluonic definitions valid only on smooth configurations.
- Introduce extra coordinate t (flowtime) and define a t-dependent gauge field $B_\mu(x,t)$ [Lüscher,2014]

$$\frac{d}{dt}B_\mu(x,t) = D_\nu G_{\nu\mu}(x,t),$$

$$D_\mu = \partial_\mu + [B_\mu(x,t),\ \cdot\ ],$$

$$G_{\mu\nu}(x,t) = \partial_\mu B_\nu(x,t) - \partial_\nu B_\mu(x,t) + [B_\mu(x,t), B_\nu(x,t)]$$

- with the initial condition: $B_\mu(x,t)|_{t=0} = A_\mu(x)$.
- The flow smoothens the fields over a region of radius $\sqrt{8t}$.
- We use Symanzik improved gradient flow on the lattice.

  [Ramos, Sint,2015]

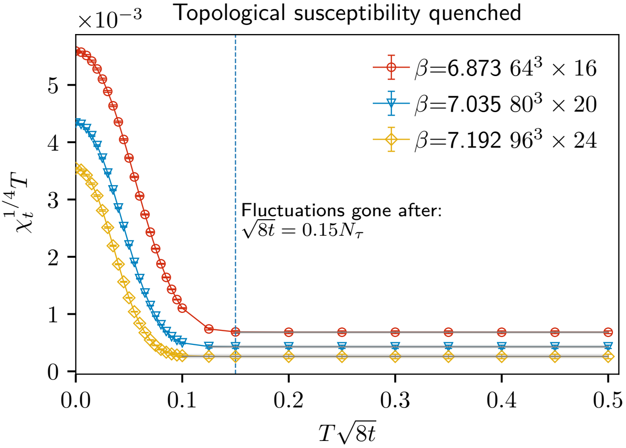
Setup

- SU(3) pure gauge configurations were generated using the quenched approximation.
- One sweep consists of one heat bath and four overrelaxation steps.
- Temperature $T/T_c = 1.5$.
- All configurations are separated by 500 sweeps.

N_s	N_τ	β	a [fm]	#conf
64	16	6.873	0.026045	10000
80	20	7.035	0.021342	10000
96	24	7.192	0.017650	7892

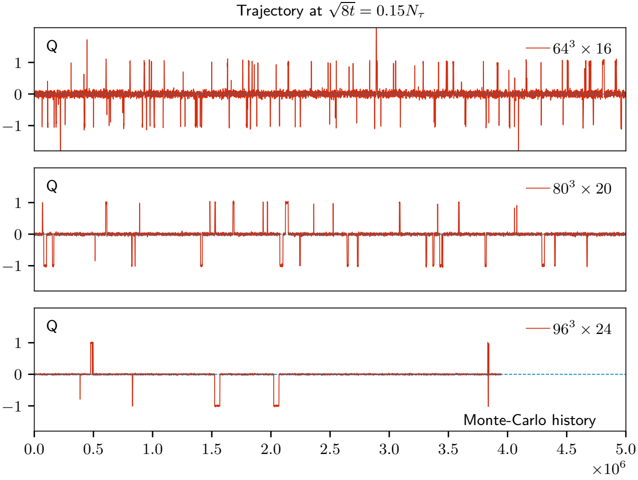
At what flowtime should be start?

- q only valid on sufficiently smoothed configurations.
- χ_t shows when UV-fluctuations are smoothed out.

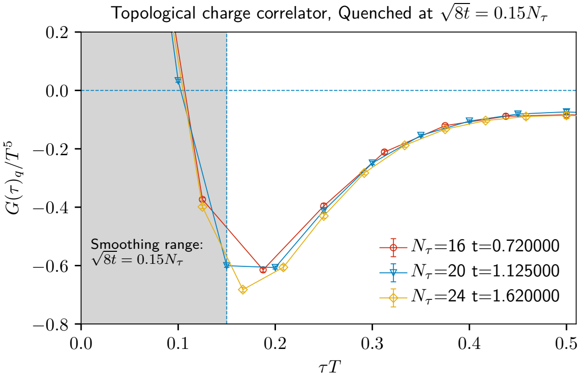


Is topological tunneling sufficient?

- Q gets frozen with finer lattices $\Rightarrow$ Does that affect $q(x)$?



Topological charge density correlator



- At $\tau T = 0 \Rightarrow G(\tau) > 0$ and at $\tau T > 0 \Rightarrow G(\tau) < 0$.
- Small cut-off effects are seen.

Is the $G(\tau)$ stable under flow?

Expectation:

- UV-part of correlation function $0 \leq \tau \leq \sqrt{8t}/N_\tau$ gets destroyed.
- Too much smoothing destroys the IR-part as well.

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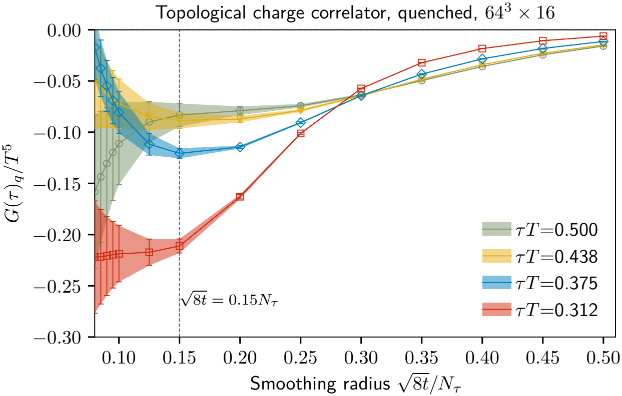
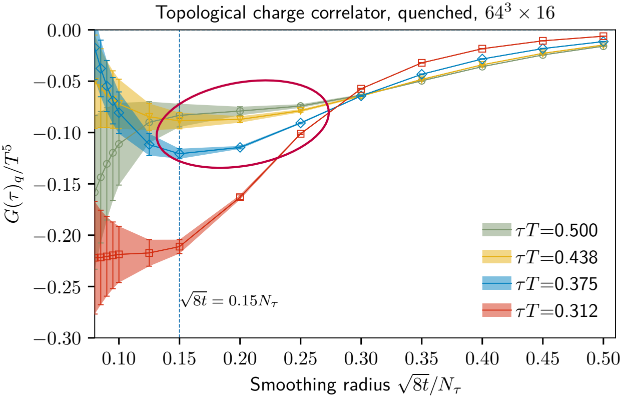
A plateau should be seen in the flowtime dependence of the correlation function $G_\tau(t)$.

Topological charge density correlator, flowtime dependence



- Low flowtime area $< 0.15N_\tau$ affected by UV fluctuation.
- Signal in high flowtime area $> 0.3N_\tau$ gets destroyed.
- Plateaus are seen in between.

Topological charge density correlator, flowtime dependence



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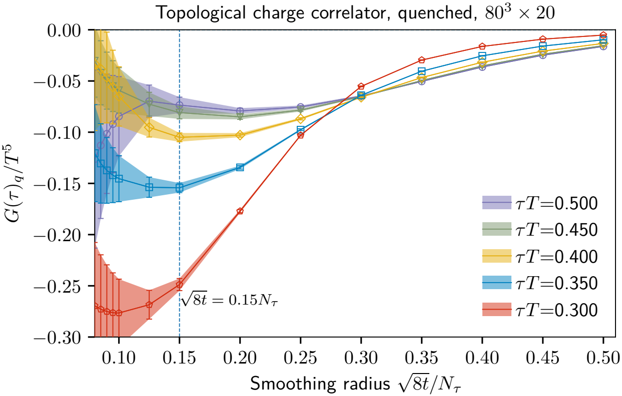

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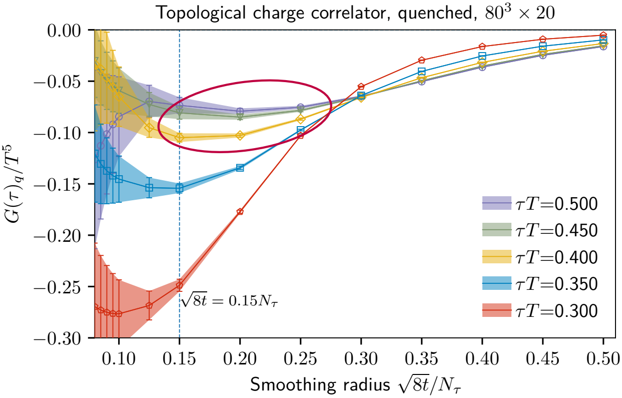

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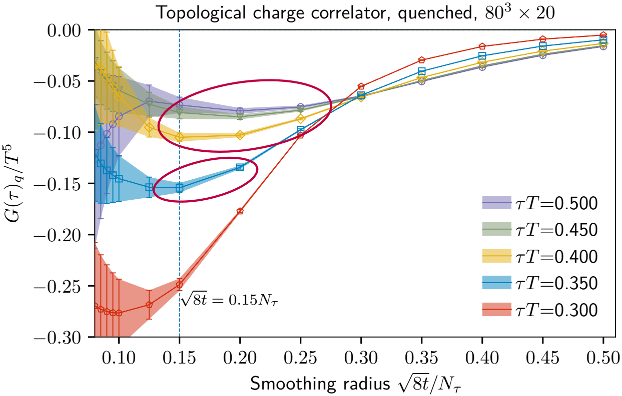

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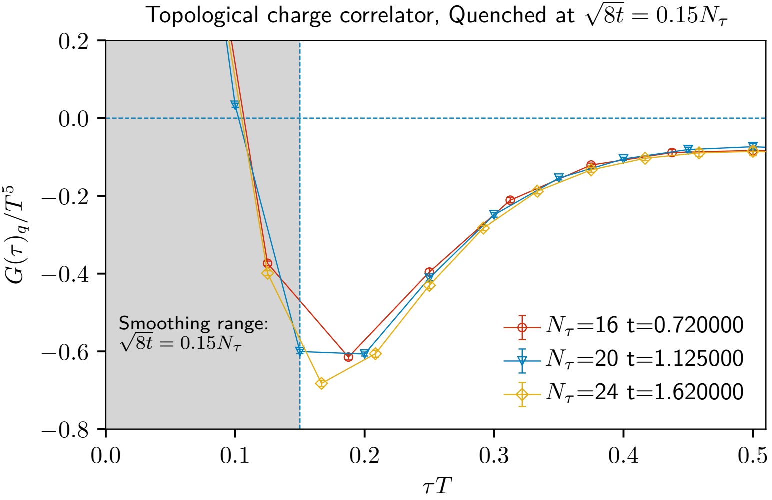

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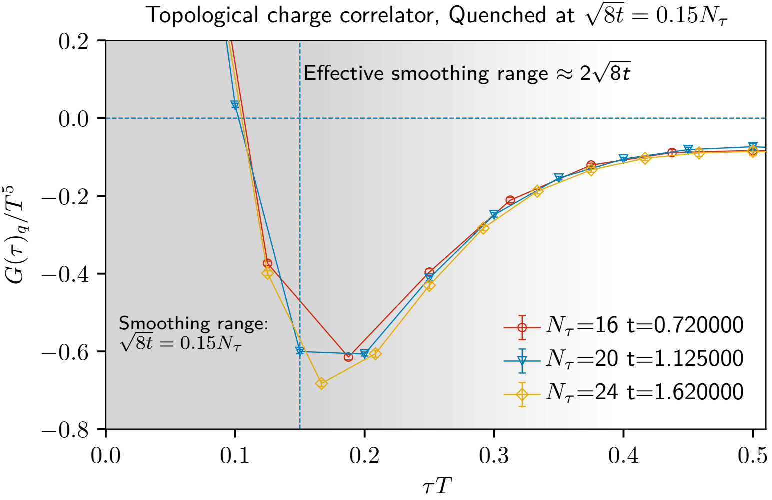

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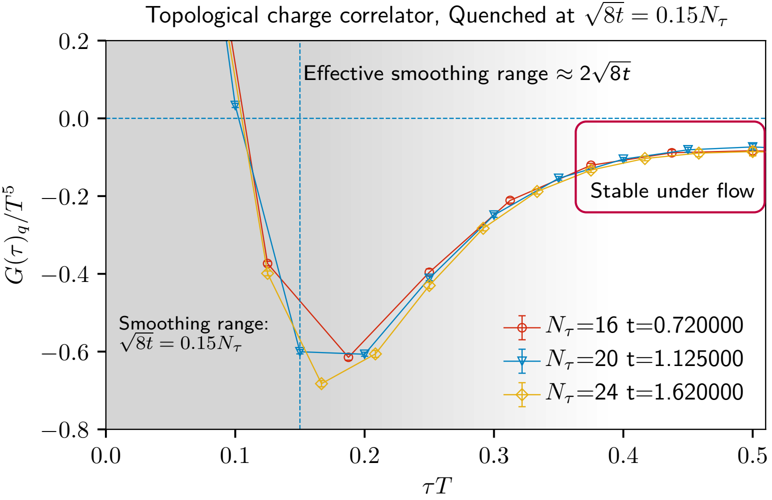

Conclusion

- Gradient flow reveals $\langle q(x)q(0)\rangle$ correlation function.
- Correlation function shows small cut-off effects.
- Flow smoothens out $\approx 2\sqrt{8t}$ of the correlation function.
  $\Rightarrow$ Large N_τ are needed.

Outlook:

- Continuum limit of correlation function. This might allow us to use even more distances.
- Compare with perturbation theory.
- Obtain correlation function from full QCD (HISQ).