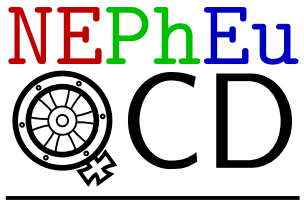
Lattice QCD estimate of the quark-gluon plasma photon emission rate

Arianna Toniato

Johannes Gutenberg Universität, Mainz

PRISMA+



JOHANNES GUTENBERG UNIVERSITÄT MAINZ

In collaboration with:

Bastian B. Brandt, Marco Cè, Anthony Francis, Tim Harris, Harvey Meyer, Aman Steinberg

- 1710.07050 (LAT2017)

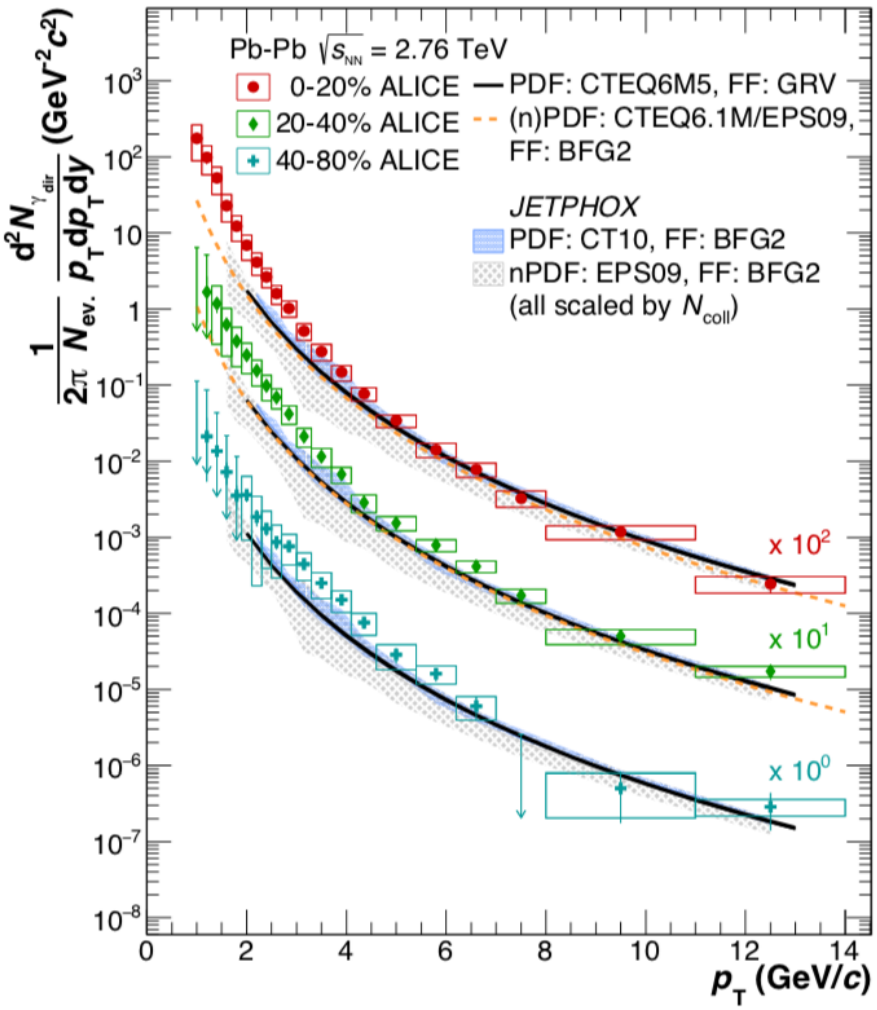
Contents

Photon rate and spectral function

Photon emission in heavy ion collisions

Direct photon spectrum (not originating from hadron decays)

- Prompt photons from hard scattering of partons
- Thermal photons from the quark-gluon plasma. They carry information on the temperature, collective behavior and time evolution of the medium



Phys. Lett. B 754 (2016) (ALICE)

The thermal photon emission rate

- Differential photon emission rate per unit volume:

$$\mathrm{d}\Gamma(\vec{k}) = e^2 \frac{\mathrm{d}^3 k}{(2\pi)^3 2k} \frac{-\rho^{\mu}{}_{\mu}(k,k)}{e^{\beta k} - 1} \,, \qquad k = \left|\vec{k}\right|$$

McLerran, Toimela, Phys.Rev. D31 (1985)

- Vector current correlator:

$$G^{\mu\nu}(t,\vec{k}) = \int \mathrm{d}^3 x \; e^{-i\vec{k}\cdot\vec{x}} \langle j^{\mu}(t,\vec{x}) j^{\nu}(0,\vec{y}) \rangle$$

$$j^{\mu} = \sum_f Q_f \; \bar{\psi}_f \gamma^{\mu} \psi_f$$

- Spectral representation:

$$G^{\mu\nu}(t,\vec{k}) = \int_0^{\infty} \frac{\mathrm{d}\omega}{2\pi} \; \rho^{\mu\nu}(\omega,\vec{k}) \; \frac{\cosh[\omega(\beta/2 - t)]}{\sinh(\omega\beta/2)} \,, \qquad \beta = \frac{1}{T}$$

- Define the linear combination:

$$\rho(\omega, \vec{k}, \lambda) = (\delta^{ij} - \hat{k}^i \hat{k}^j)\rho^{ij} + \lambda(\hat{k}^i \hat{k}^j \rho^{ij} - \rho^{00}) \,, \qquad \hat{k}^i = k^i/k$$

for example: $\rho(\omega, k, 1) = \rho^{ii}(\omega, k) - \rho^{00}(\omega, k) = -\rho^{\mu}{}_{\mu}(\omega, k)$

- The photon rate can be expressed in terms of $\rho(\omega, k, \lambda)$:

$$\mathrm{d}\Gamma_\lambda(\vec{k}) = e^2 \frac{\mathrm{d}^3 k}{(2\pi)^3 2k} \frac{\rho(k, k, \lambda)}{e^{\beta k} - 1}$$

this expression is independent of λ, due to current conservation:

$$\omega^2 \rho^{00}(\omega, k) = k^i k^j \rho^{ij}(\omega, k)$$

- From now on we focus on $\underline{\lambda = -2}$

Properties of $\rho(\omega, k) \equiv \rho(\omega, k, \lambda = -2)$

- Non-negative for $\omega \leq k$
- Due to Lorentz invariance and transversality, ρ vanishes identically in vacuum $\Rightarrow$ ρ is UV-finite at $T > 0$
- OPE:

$$\boxed{\tilde{G}(\omega_n, k) \underset{\omega_n \to \infty}{\sim} k^2 \langle \mathcal{O}_4 \rangle / \omega_n^4}$$

- Sum rule:

$$\tilde{G}(\omega_n, k) \underset{\omega_n \to \infty}{\sim} \frac{1}{\pi \omega_n^2} \int_0^\infty \mathrm{d}\omega \, \omega \rho(\omega, k) + O(\omega_n^{-4})$$

$$\boxed{\int_0^\infty \mathrm{d}\omega \, \omega \rho(\omega, k) = 0}$$

Brandt, Francis, Harris, Meyer, Steinberg, 1710.07050

Lattice setup

- $N_f = 2$, $O(a)$-improved Wilson fermions

T (MeV)	T/T_c	β_{LAT}	β/a	L/a	$m_{\overline{\mathrm{MS}}(2\,\mathrm{GeV})}$ (MeV)	N_{meas}
250	1.2	5.3	12	48	12	8256
"	"	5.5	16	64	"	4880
"	"	5.83	24	96	"	9600
500	2.4	6.04	16	64	"	8064

- Continuum limit at $T = 250$ MeV
- Four independent discretizations of the isovector vector correlator $G^{\lambda=-2}(t, \vec{k})$ are considered
  - local or exactly-conserved lattice vector current
  - in the local-conserved case, two different definitions: conserved current defined on the site or in the midpoint of the link
- Projection to all spatial momenta, on- and off-axis, such that $k\beta \leq 2\pi$

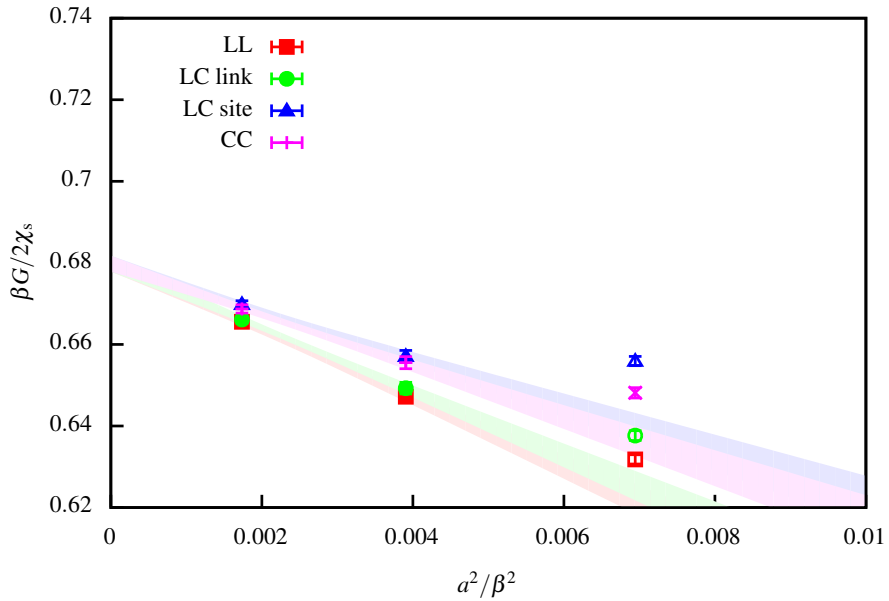
Brandt, Francis, Harris, Meyer, Steinberg, 1710.07050

Continuum limit

- Tree-level improvement of the correlator: $G(t,\vec{k}) \rightarrow \frac{G_{cont.t.l}(t,\vec{k})}{G_{lat.t.l}(t,\vec{k})} G(t,\vec{k})$
- A piecewise spline interpolation of the correlators is performed before taking the combined continuum limit of the four discretizations

- The coarsest ensemble $\beta/a = 12$ is not included in the continuum extrapolation
- In the subsequent analysis we use the continuum-extrapolated correlator with $t \geq \beta/4$

$k\beta = \pi,\ t = \beta/3$



Brandt, Francis, Harris, Meyer, Steinberg, 1710.07050

Padé ansatz and fitting strategy

Padé ansatz for the spectral function

$$\frac{\rho(\omega,k)}{\tanh(\omega\beta/2)} = \frac{A(1+B\omega^2)}{(\omega^2+a^2)[(\omega+\omega_0)^2+b^2][(\omega-\omega_0)^2+b^2]}$$

- $\rho(\omega,k) \underset{\omega\to\infty}{\sim} 1/\omega^4$, consistent with OPE
- At small k, expect $a \sim Dk^2$ and $\omega_0, b \sim \mathrm{O}(T)$
- Physical constraints on a and b:

$$\boxed{\min(a,b) > \min(D_{AdS/CFT}\cdot k^2, D_{PT}^{-1})}$$

  - $D_{AdS/CFT}\cdot k^2$: diffusion of electric charge, $D_{AdS/CFT} = 1/(2\pi T)$
  - D_{PT}^{-1}: damping of static current, $D_{PT}^{-1} = O(\alpha_S^2)\cdot T$
- To increase the number of degrees of freedom we fit simultaneously data at different momenta

Momentum dependence

$$\frac{\rho(\omega,k)}{\tanh(\omega\beta/2)} = \frac{A(1+B\omega^2)}{(\omega^2+a^2)[(\omega+\omega_0)^2+b^2][(\omega-\omega_0)^2+b^2]}$$

Introduce a k-dependence for the nonlinear parameters

- Quadratic ansatz:
  $a(k) = a_0 + a_2 k^2\,, \quad b(k) = b_0 + b_2 k^2\,, \quad \omega_0(k) = W_0 + W_2 k^2$
- Linear ansatz:
  $a(k) = a_0 + a_2 k\,, \quad b(k) = b_0 + b_2 k\,, \quad \omega_0(k) = W_0 + W_2 k$

Fitting strategy

- N_k momentum values are included in the fit

Fitting strategy

- N_k momentum values are included in the fit
- Scan in the non-linear parameters $(a_0, a_2, b_0, b_2, W_0, W_2)$

Fitting strategy

- N_k momentum values are included in the fit
- Scan in the non-linear parameters $(a_0, a_2, b_0, b_2, W_0, W_2)$
- At each k, B is determined by imposing the sum rule $\Rightarrow$ $B = B(a_0, a_2, b_0, b_2, W_0, W_2; k)$

Fitting strategy

- N_k momentum values are included in the fit
- Scan in the non-linear parameters $(a_0, a_2, b_0, b_2, W_0, W_2)$
- At each k, B is determined by imposing the sum rule $\Rightarrow$ $B = B(a_0, a_2, b_0, b_2, W_0, W_2; k)$
- N_k values of A are determined by χ^2 minimization, at fixed k

Fitting strategy

- N_k momentum values are included in the fit
- Scan in the non-linear parameters $(a_0, a_2, b_0, b_2, W_0, W_2)$
- At each k, B is determined by imposing the sum rule $\Rightarrow$ $B = B(a_0, a_2, b_0, b_2, W_0, W_2; k)$
- N_k values of A are determined by χ^2 minimization, at fixed k
  - Regularized correlated fit (the off-diagonal elements of the covariance matrix are multiplied by $x = 0.85$)

Fitting strategy

- N_k momentum values are included in the fit
- Scan in the non-linear parameters $(a_0, a_2, b_0, b_2, W_0, W_2)$
- At each k, B is determined by imposing the sum rule $\Rightarrow$ $B = B(a_0, a_2, b_0, b_2, W_0, W_2; k)$
- N_k values of A are determined by χ^2 minimization, at fixed k
  - Regularized correlated fit (the off-diagonal elements of the covariance matrix are multiplied by $x = 0.85$)
  - The regularization is necessary not because the covariance matrix is poorly determined, but because the most accurate modes still suffer from cutoff effects

Fitting strategy

- N_k momentum values are included in the fit
- Scan in the non-linear parameters $(a_0, a_2, b_0, b_2, W_0, W_2)$
- At each k, B is determined by imposing the sum rule $\Rightarrow$ $B = B(a_0, a_2, b_0, b_2, W_0, W_2; k)$
- N_k values of A are determined by χ^2 minimization, at fixed k
  - Regularized correlated fit (the off-diagonal elements of the covariance matrix are multiplied by $x = 0.85$)
  - The regularization is necessary not because the covariance matrix is poorly determined, but because the most accurate modes still suffer from cutoff effects
- Global χ^2: $\boxed{\chi^2_{gl} = \sum^{N_k} \chi^2(k)}$

Fitting strategy

- N_k momentum values are included in the fit
- Scan in the non-linear parameters $(a_0, a_2, b_0, b_2, W_0, W_2)$
- At each k, B is determined by imposing the sum rule $\Rightarrow$ $B = B(a_0, a_2, b_0, b_2, W_0, W_2; k)$
- N_k values of A are determined by χ^2 minimization, at fixed k
  - Regularized correlated fit (the off-diagonal elements of the covariance matrix are multiplied by $x = 0.85$)
  - The regularization is necessary not because the covariance matrix is poorly determined, but because the most accurate modes still suffer from cutoff effects
- Global χ^2: $\boxed{\chi^2_{gl} = \sum^{N_k} \chi^2(k)}$
  - At this stage, we neglect the correlation between data at different k

Degrees of freedom

- Group 1: $(N_k = 3,\ k\beta = \pi/2, \ldots, \pi\sqrt{3}/2)$ 12 d.o.f
- Group 2: $(N_k = 5,\ k\beta = \pi\sqrt{3}/2, \ldots, \pi\sqrt{2})$ 24 d.o.f
- Group 3: $(N_k = 7,\ k\beta = \pi\sqrt{2}, \ldots, \pi\sqrt{7/2})$ 36 d.o.f

Computational speed-up

$$G(t, \vec{k}) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\, \rho(\omega, \vec{k})\, \frac{\cosh[\omega(\beta/2 - t)]}{\sinh(\omega\beta/2)}$$

- insert Padé ansatz for the spectral function
- the integral can be computed analytically and be expressed in terms of Lerch transcendent functions $\phi(z, s, a) = \sum_{n=0}^{\infty} \frac{z^n}{(n+a)^s}$
- this approach is computationally much cheaper than the numerical integration of the Padé ansatz

Results

Effective diffusion coefficient

Definition

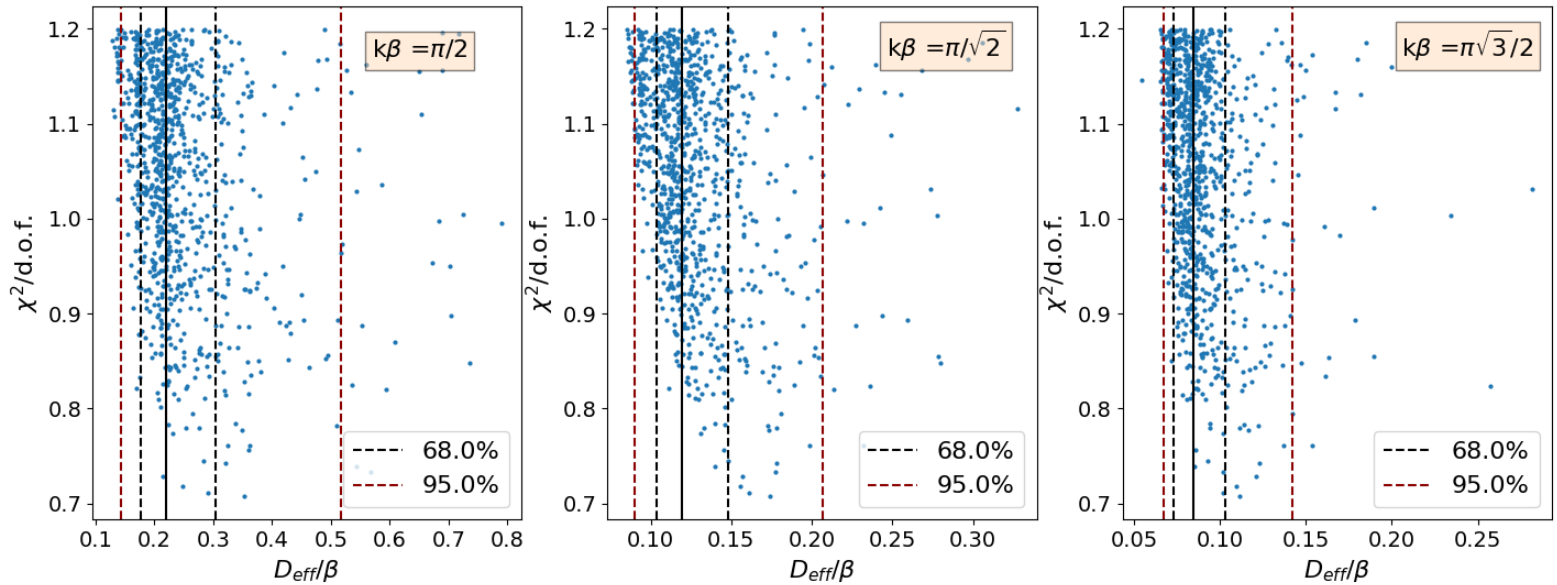
$$D_{eff}(k) = \frac{\rho(k,k)}{4k\chi_S}$$

- $\chi_S = \int \mathrm{d}^4x \langle j^0(x) j^0(0) \rangle$
- D_{eff} is proportional to the photon rate

Presentation of results

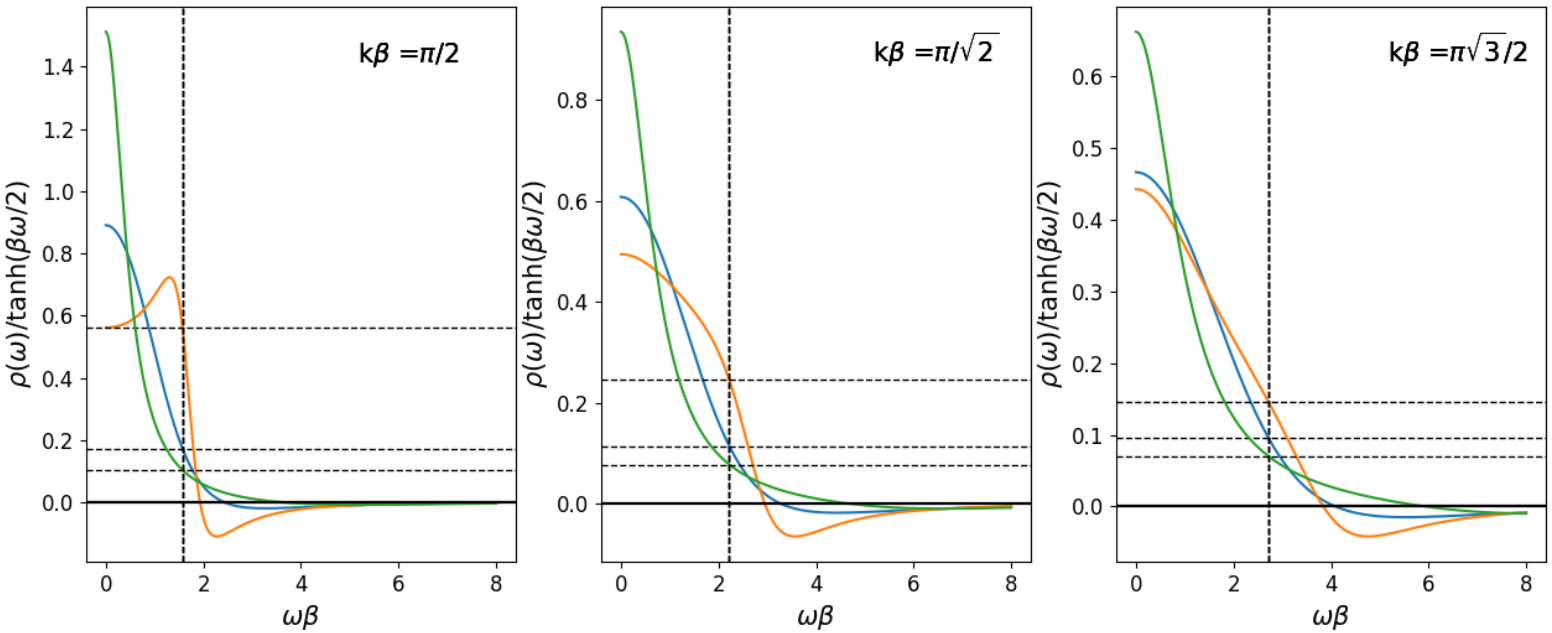
- many "equivalent" local minima in the χ^2 landscape
- consider all $\chi^2/\mathrm{d.o.f.} \leq 1.2$ measured in the scan (68% confidence level)
- select results satisfying the physical constraints
  - on the non-linear parameters a and b
  - $D_{eff} \geq 0$

D_{eff} distribution



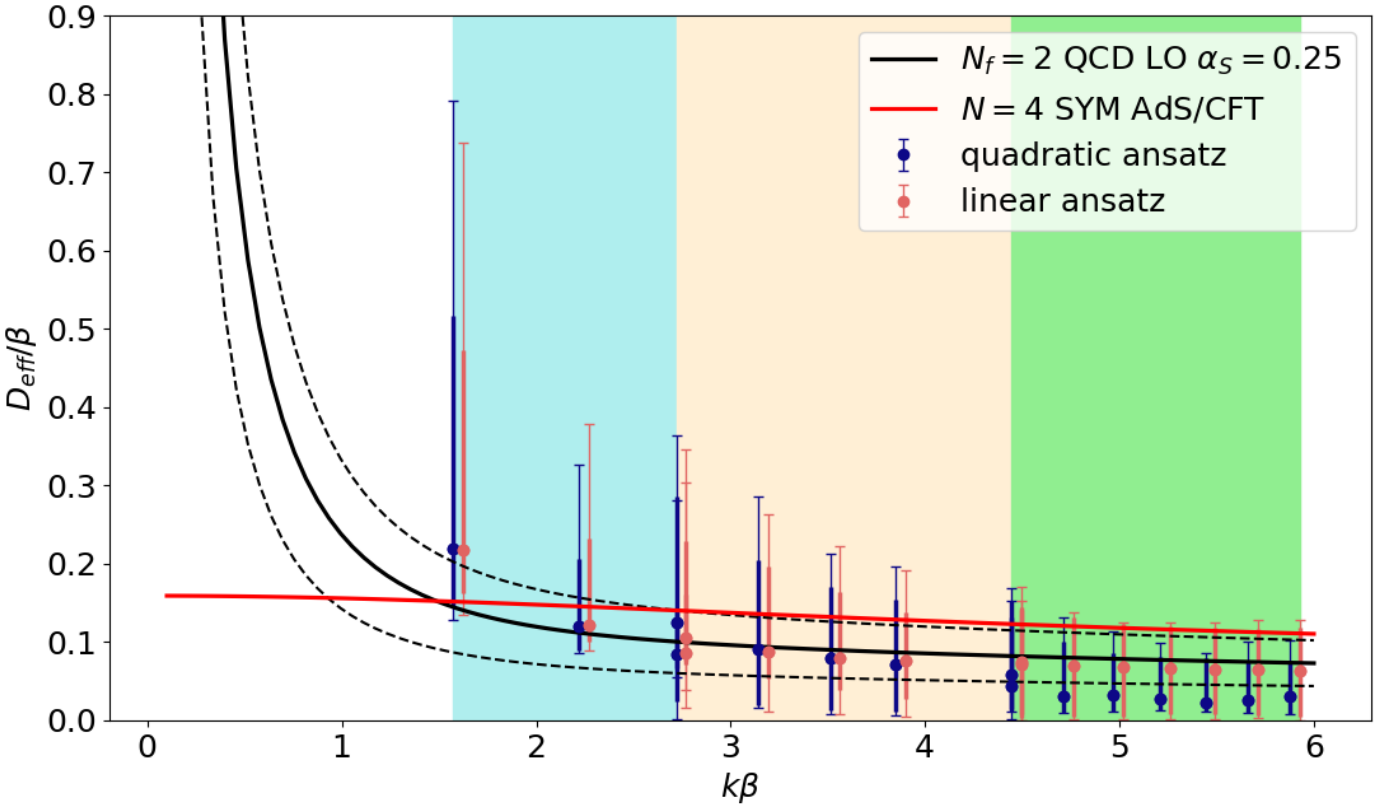
- Acceptable D_{eff} values for the first group of momenta ($N_k = 3$)
- Each point in parameter space $(a_0, a_2, b_0, b_2, W_0, W_2)$ corresponds to three values of D_{eff}, one for each momentum included in the fit
- Solid line: median of the distribution
- Dashed lines highlight intervals containing 68% and 95% of the points

Spectral function



- Examples of spectral function for the first group of momenta
- Three illustrative points in parameter space are chosen:
  - one giving large values of the photon rate (orange lines)
  - one close to the median of the D_{eff} distribution (blue lines)
  - one in the part of the distribution with smaller values of D_{eff} (green lines)

D_{eff}: momentum dependence



PQCD from Arnold, Moore, Yaffe, hep-ph/0111107 (JHEP)

AdS/CFT from Huot, Kovtun, Moore, Starinets, Yaffe, hep-th/0607237 (JHEP)

- Final results for D_{eff} as a function of momentum, together with theoretical predictions computed in the weak- and strong-coupling limit
- Shaded areas highlight the three momentum groups that are fitted simultaneously
- Thicker lines denote intervals containing 95% of acceptable results

Conclusions and outlook

- Non-perturbative computation of the quark-gluon plasma photon emission rate
- Padé ansatz approach to the inverse-problem regularization
- No clear global minimum in the χ^2 landscape $\Rightarrow$ consider results corresponding to all acceptable χ^2, in combination with exclusion criteria based on physical constraints

Coming soon...

- Improve the quality of data by including a new ensemble in the continuum extrapolation
- A Euclidean correlator at zero virtuality ($\rightarrow$ imaginary spatial momentum) can be used to exclusively probe the photon rate, rather than the full (ω, k) dependence

Meyer, 1807.00781 (Eur. Phys. J.)

Backup

Properties of $\rho(\omega, k) \equiv \rho(\omega, k, \lambda = -2)$

- Non-negative for $\omega \leq k$
- Due to Lorentz invariance and transversality, ρ vanishes identically in vacuum $\Rightarrow$ ρ is UV-finite at $T > 0$
- OPE:
  - Power counting: $\tilde{G}(\omega_n, k) \underset{\omega_n \to \infty}{\sim} \langle \mathcal{O}_4 \rangle / \omega_n^2 + \dots$
  - Charge conservation: $\tilde{G}(\omega_n, k) \underset{k \to 0}{\to} 0$, for $\omega_n \neq 0$

$$\boxed{\tilde{G}(\omega_n, k) \underset{\omega_n \to \infty}{\sim} k^2 \langle \mathcal{O}_4 \rangle / \omega_n^4}$$

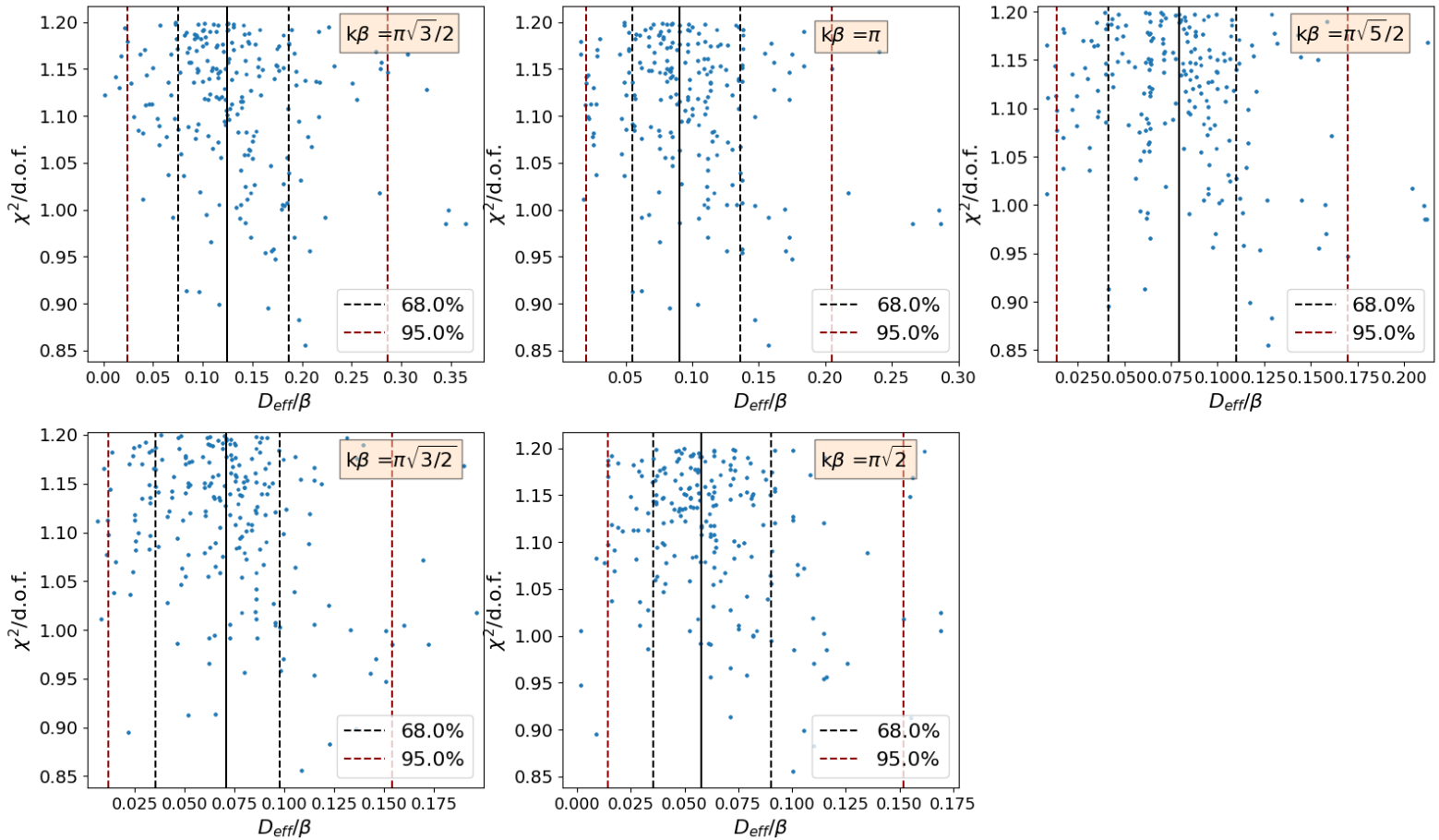
- Sum rule:

$$\tilde{G}(\omega_n, k) \underset{\omega_n \to \infty}{\sim} \frac{1}{\pi \omega_n^2} \int_0^\infty \mathrm{d}\omega \, \omega \rho(\omega, k) + O(\omega_n^{-4})$$

$$\boxed{\int_0^\infty \mathrm{d}\omega \, \omega \rho(\omega, k) = 0}$$

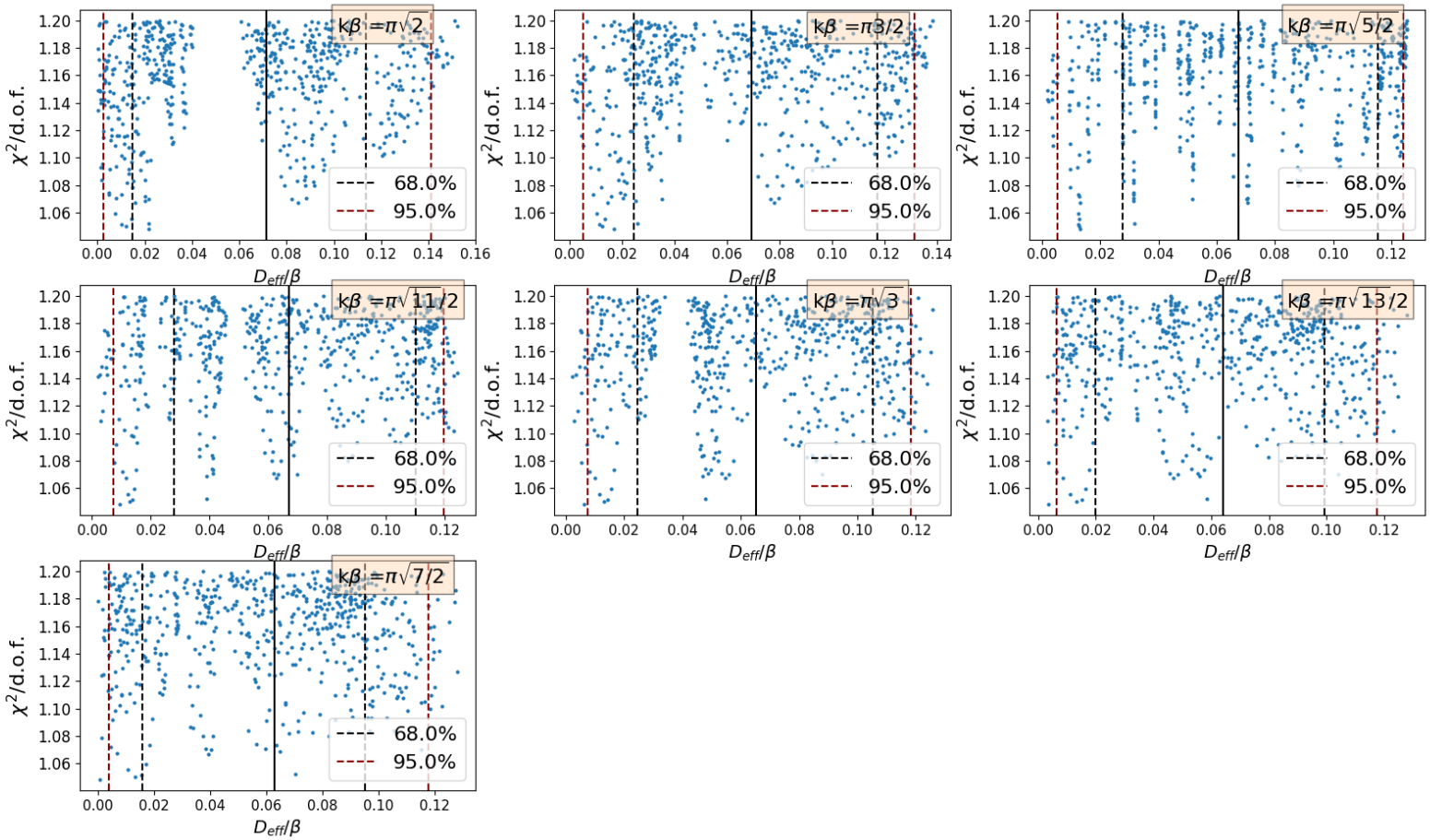
D_{eff} distribution

Group 2, quadratic ansatz



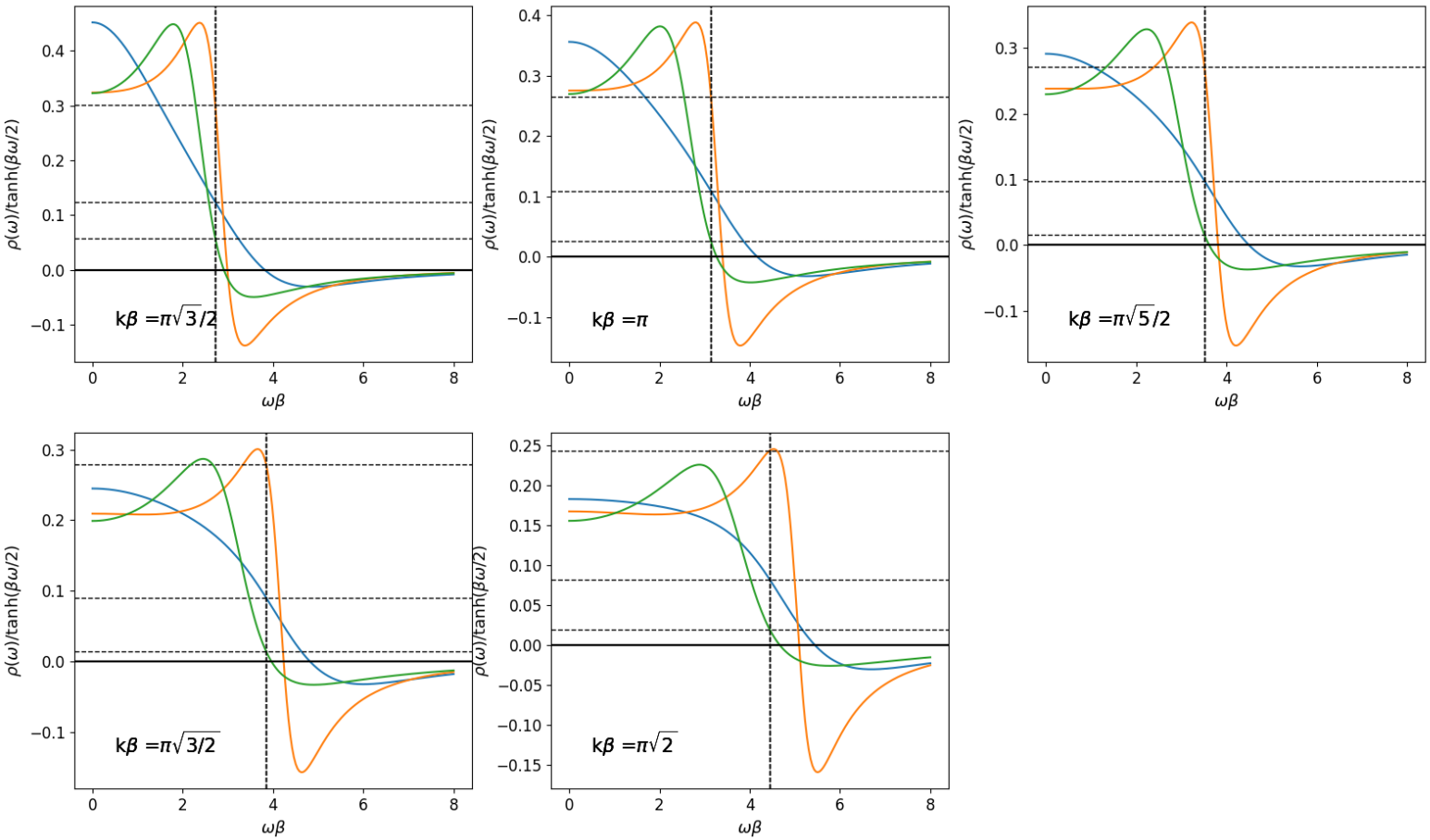
D_{eff} distribution

Group 3, linear ansatz



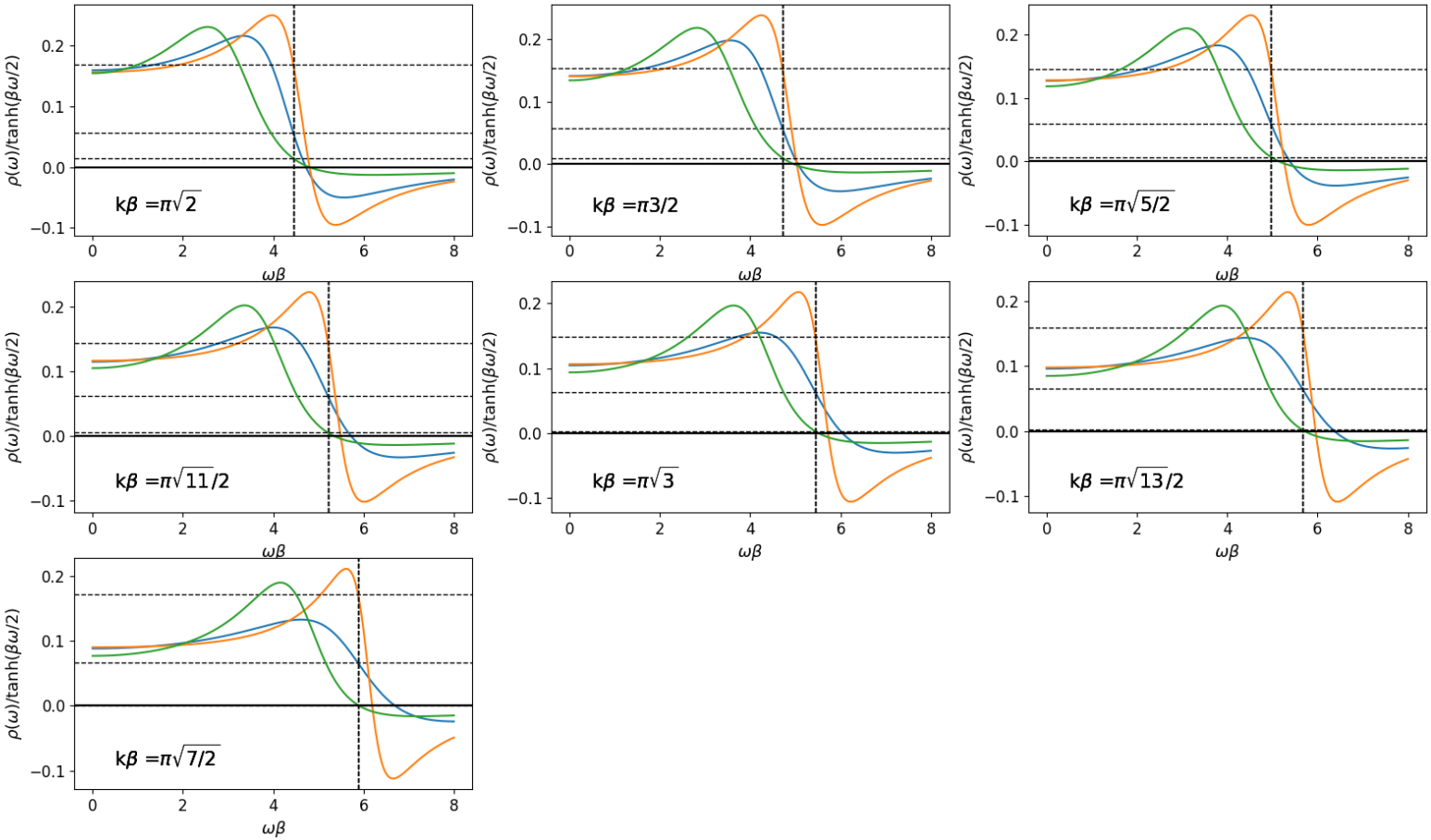
Spectral function

Group 2



Spectral function

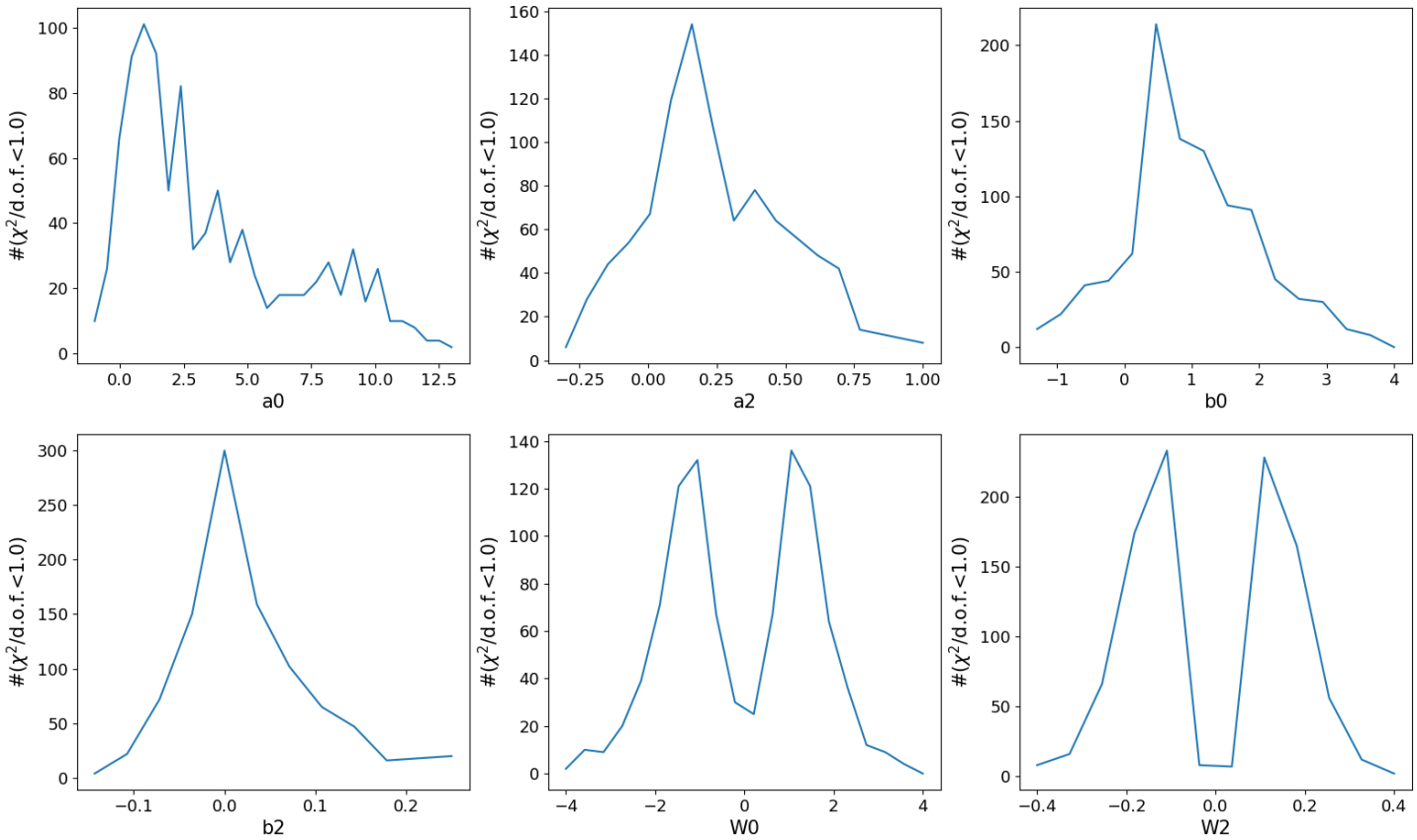
Group 3



χ^2 distribution

Group 2, quadratic ansatz

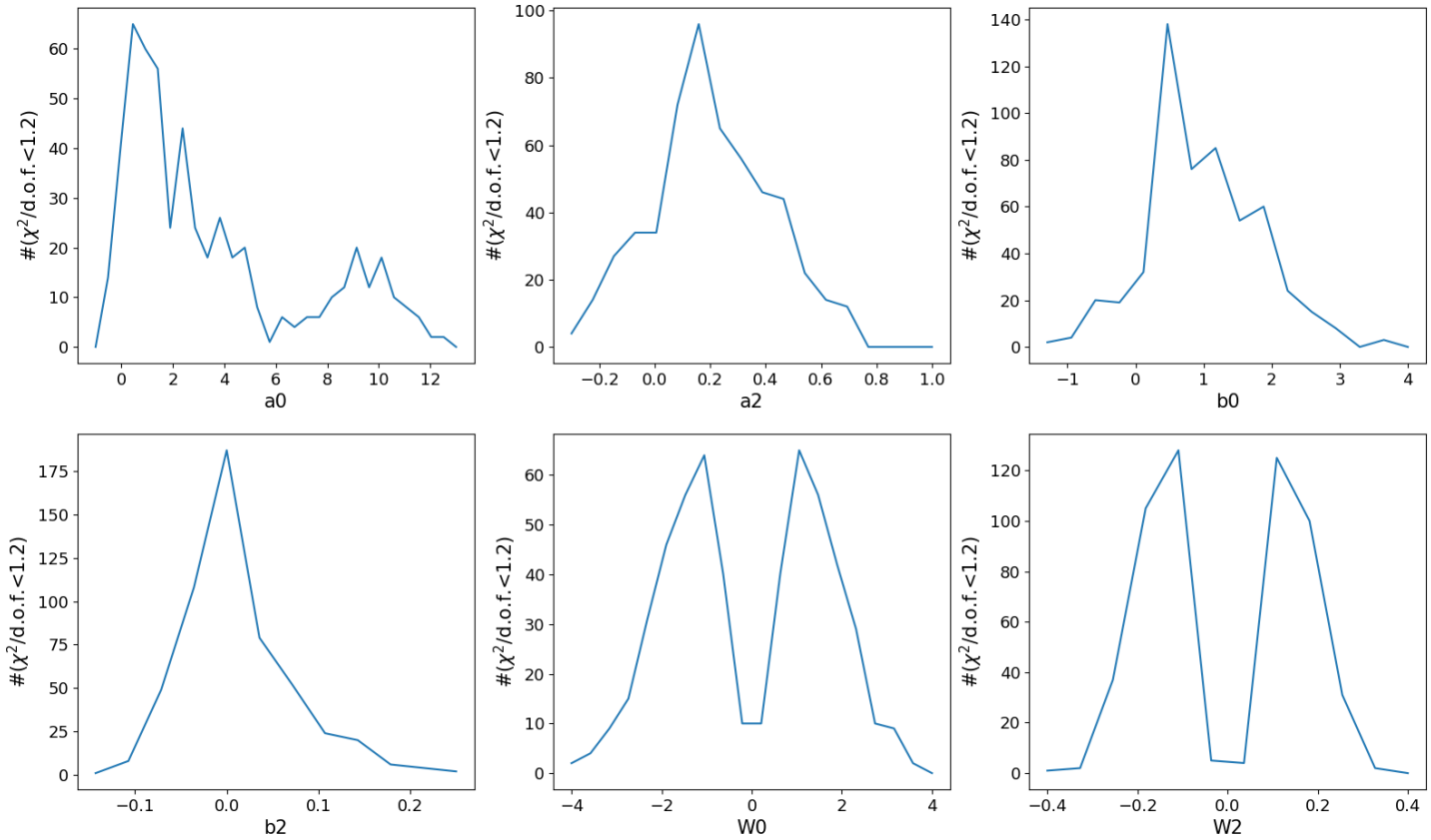
$$x = 0.8$$



χ^2 distribution

Group 2, quadratic ansatz

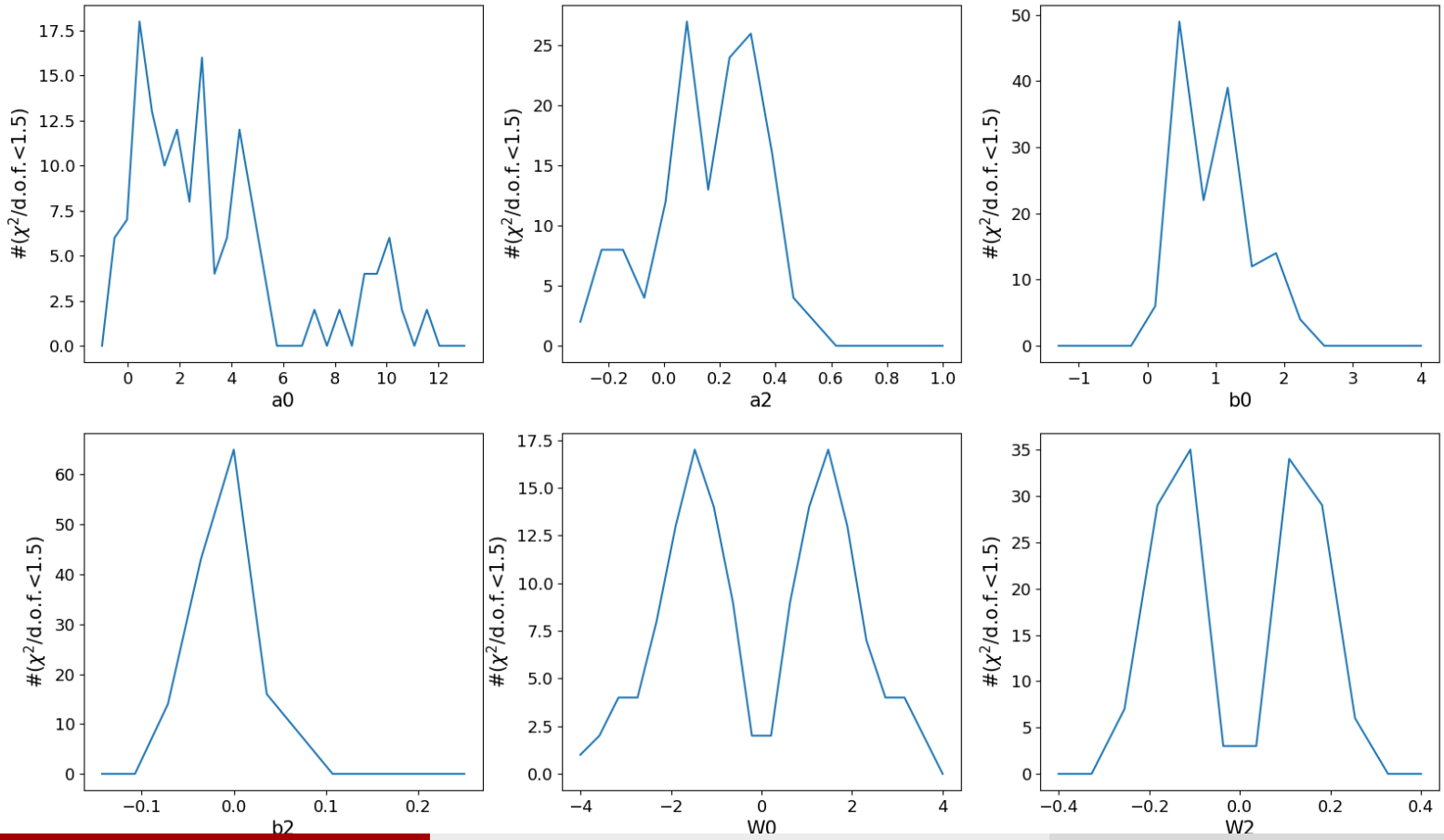
$$x = 0.85$$



χ^2 distribution

Group 2, quadratic ansatz

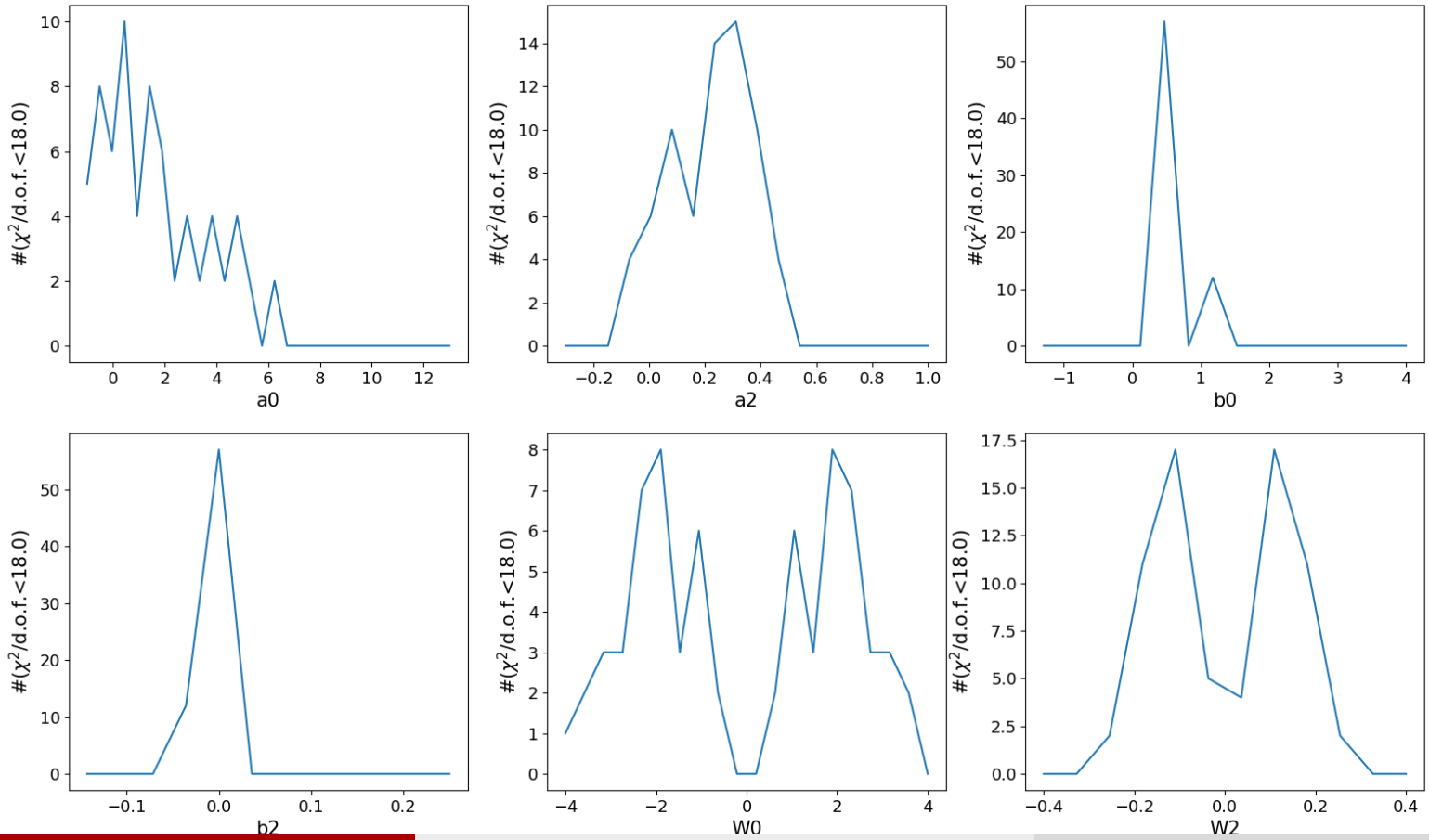
$$x = 0.9$$



χ^2 distribution

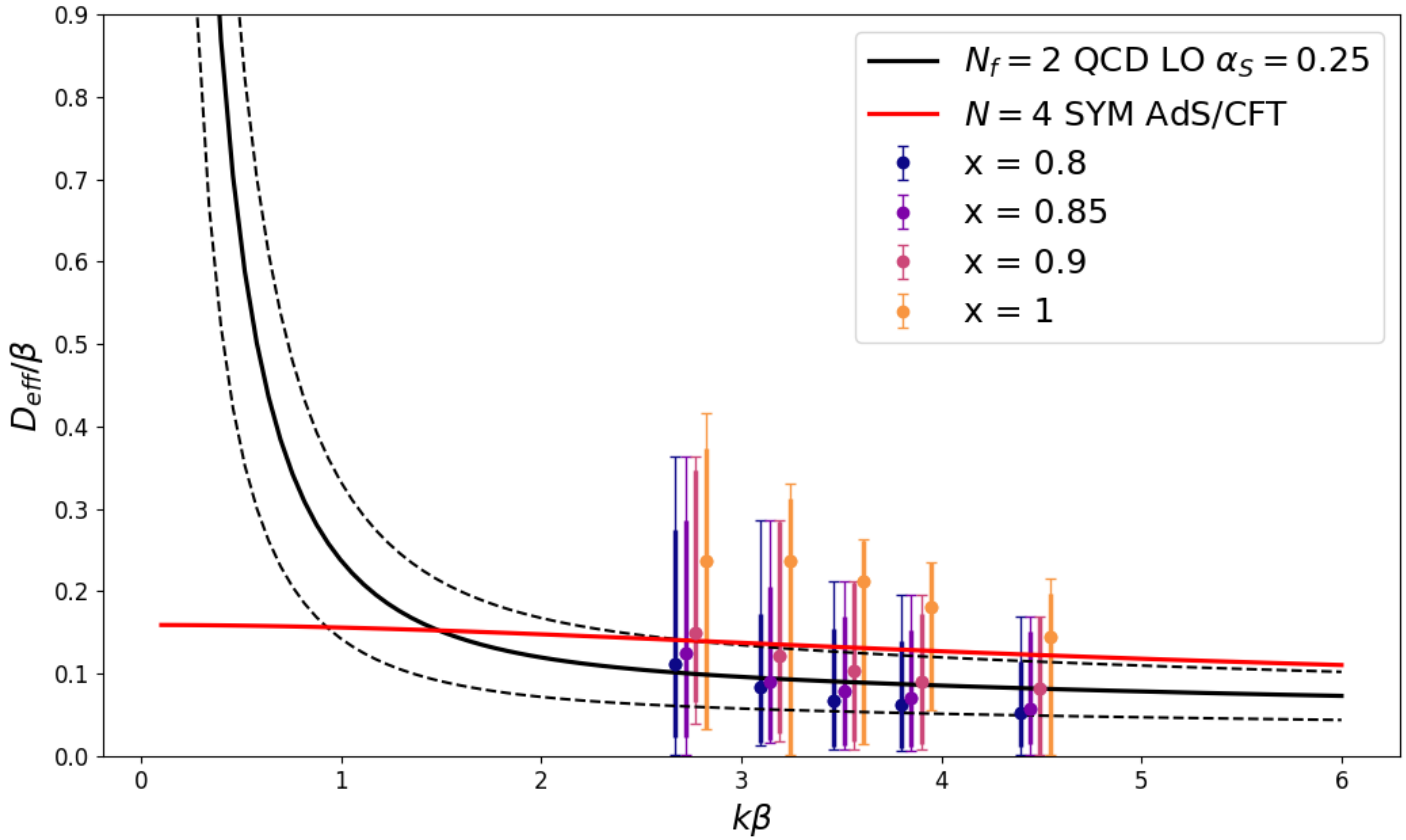
Group 2, quadratic ansatz

$$x = 1$$



x-dependence

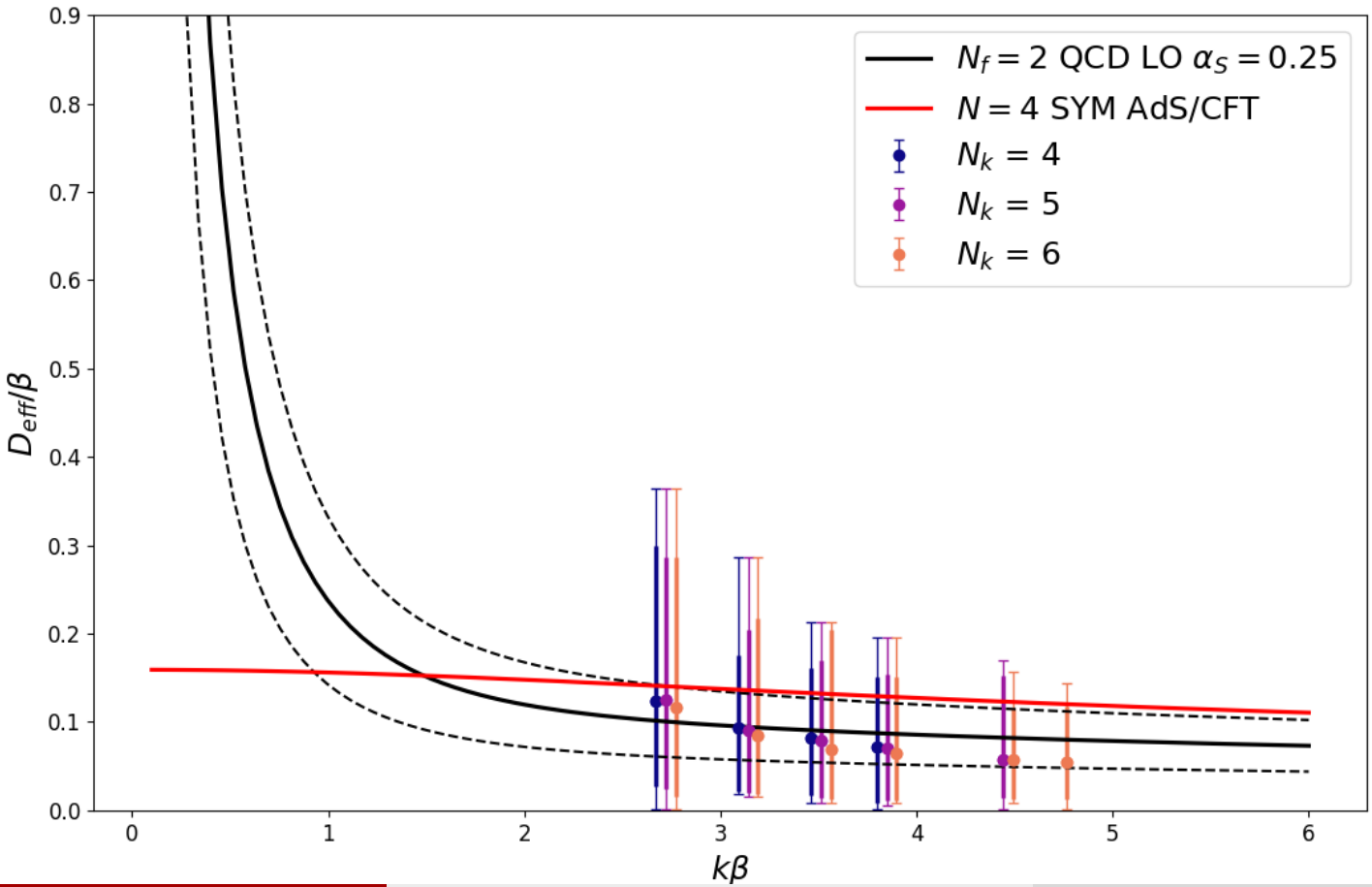
x	0.8	0.85	0.9	1
$\chi^2_{max}/\text{d.o.f.}$	1	1.2	1.5	18



N_k-dependence

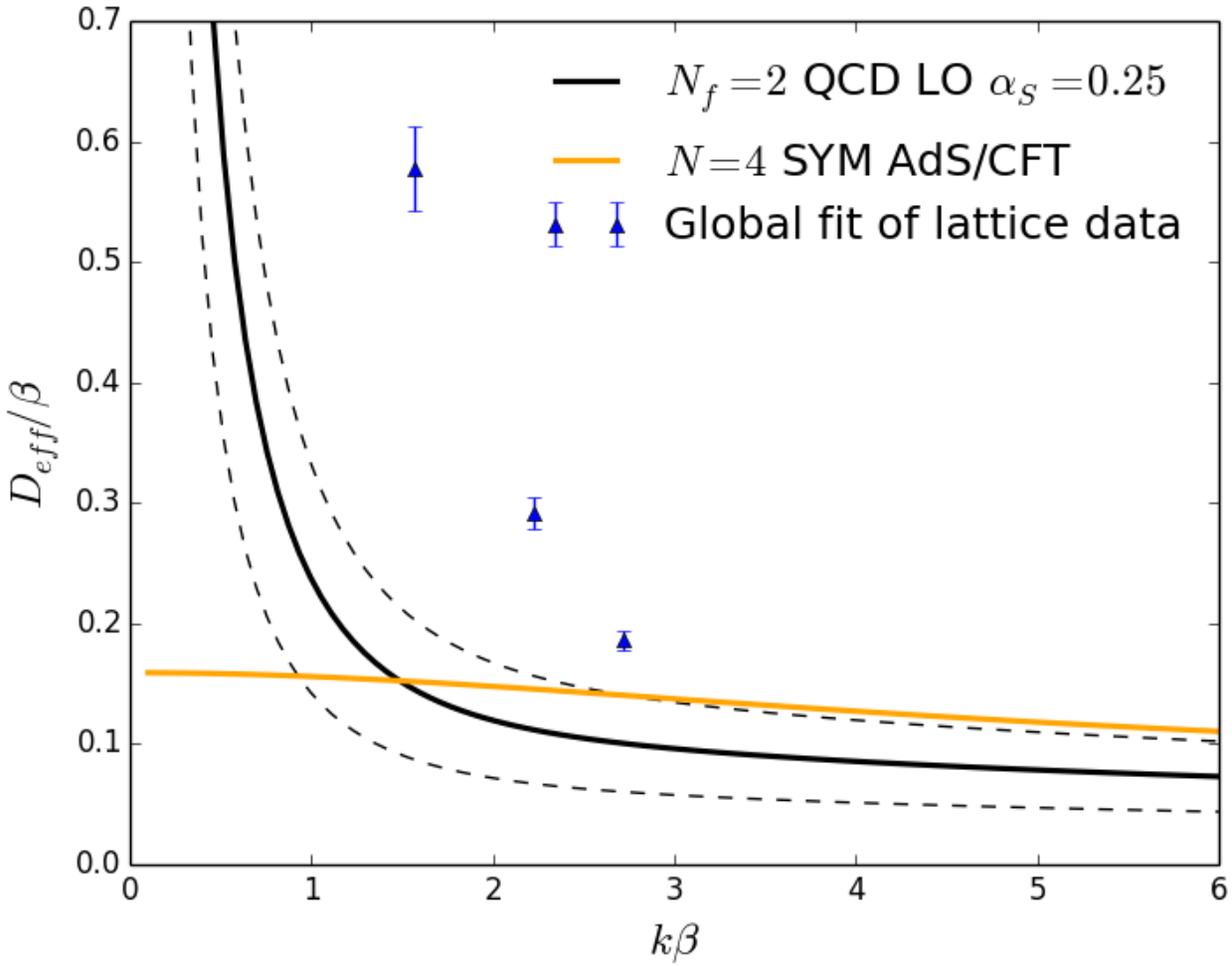
Group 2, quadratic ansatz

$$x = 0.85$$

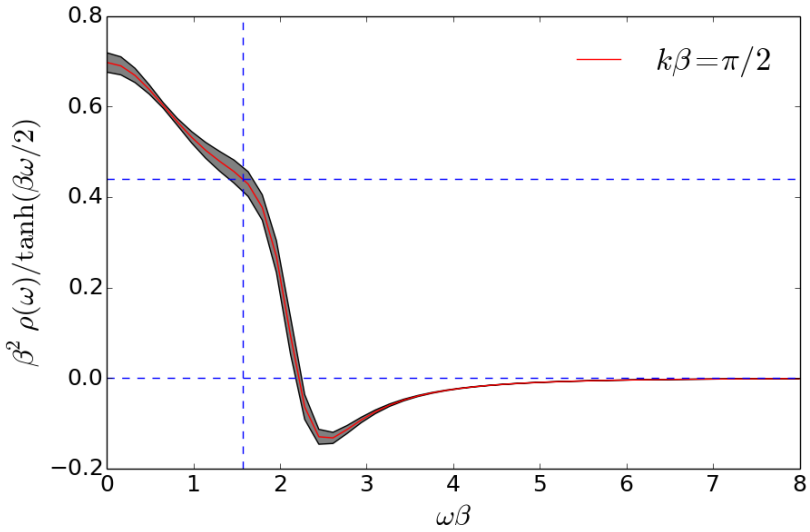


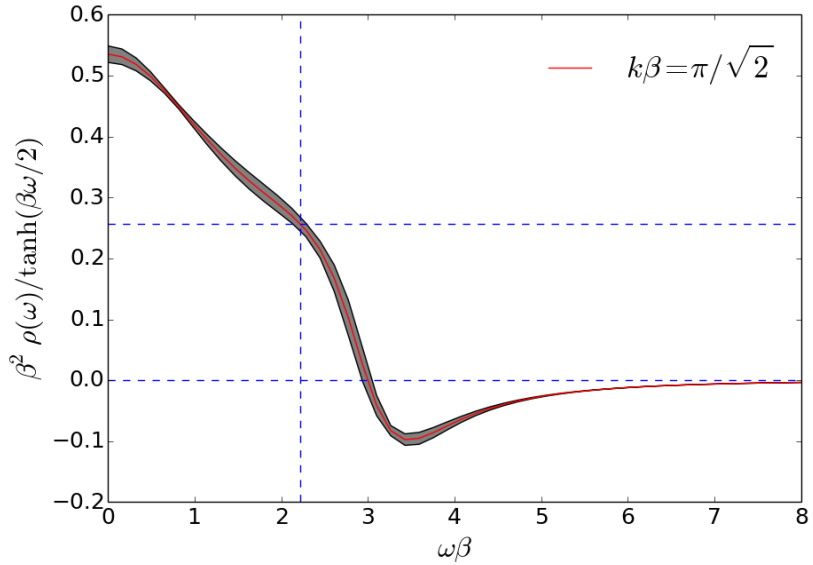
Results from the "global" (?) minimum

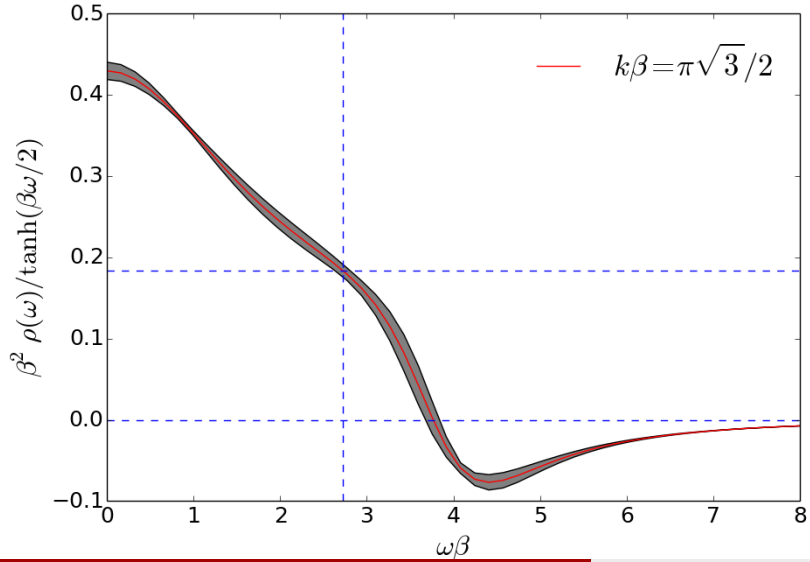
$\chi^2/\text{d.o.f.} = 0.63$



Results from the "global" (?) minimum







$$\chi^2/\text{d.o.f.} = 0.63$$