The 37th International Symposium on **Lattice Field Theory**
June 17th, **2019**

Symmetries of the Light Hadron Spectrum in High Temperature QCD

Christian Rohrhofer (Osaka Univ.)

Y. Aoki, G. Cossu, H. Fukaya, C. Gattringer,
L. Glozman, S. Hashimoto, C.B. Lang, K. Suzuki

Overview

High-temperature phase of QCD (previous talk by K. Suzuki):

- topological susceptibility consistent with zero (∃ critical quark mass?)
- $U(1)_A$ susceptibility strongly suppressed

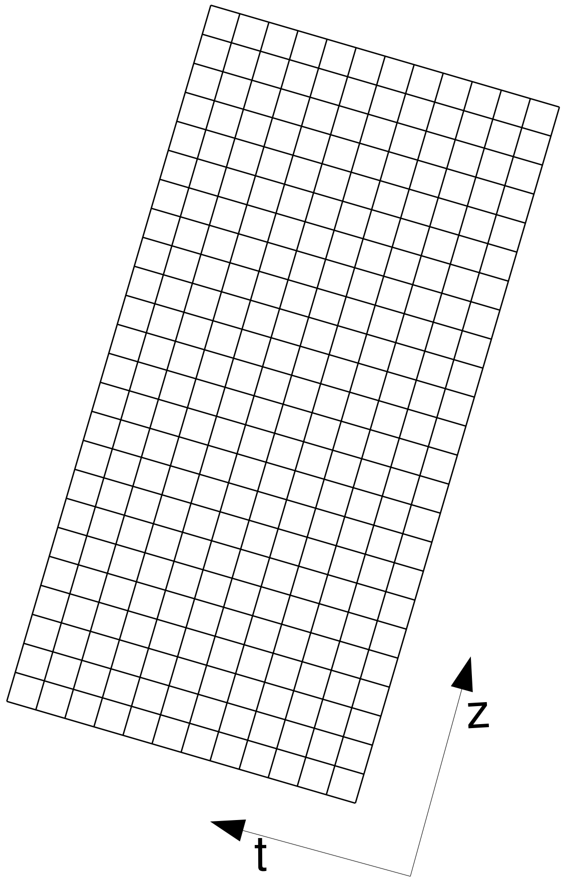
Outline:

- at 1.2 T_c: $U(1)_A$ and $SU(2)_L \times SU(2)_R$ symmetries
  - for mesonic screening spectrum
  - for parity doubling in baryon spectrum
  - for baryonic screening spectrum
- at higher temperature:
  - $SU(2)_{CS}$ chiral spin and $SU(4)$ symmetries

Simulation Setup

- **n_f=2** flavor QCD
- **2.6 GeV** cutoff (1/a)
- **domain wall fermions** with m_{res}< 1 MeV
- quark masses from **m_{ud}= 2.6 MeV** to 26 MeV
- temperatures from **T = 220 MeV** to 1 GeV
  - pseudo-critical temperature: 175 MeV
  - point sources for quark propagators

$N_s^3 \times N_t$	β	T [MeV]	T/T_c
$24^3 \times 12$	4.30	220	1.2
$32^3 \times 12$	4.30	220	1.2
$40^3 \times 12$	4.30	220	1.2
$32^3 \times 8$	4.30	330	1.8
$32^3 \times 8$	4.37	220	2.2
$32^3 \times 6$	4.30	440	2.5
$32^3 \times 8$	4.50	480	2.7
$32^3 \times 4$	4.30	660	3.8
$32^3 \times 4$	4.50	960	5.5



Chiral Symmetries at 1.2 T_c

Meson Operators

- local isovector operators: $O_\Gamma(x) = \bar{q}(x)(\vec{\tau} \otimes \Gamma)q(x)$

$$\langle O(t)\bar{O}(0)\rangle \quad \sim \quad e^{-mt} + e^{-m(N_t - t)}$$

- extract effective mass: $m_{eff}(t) = \ln \left| \frac{C(t)}{C(t+1)} \right|$ (or cosh to respect periodicity)

- for screening spectrum: $t \to z, \quad C(n_z) = \sum_{n_x, n_y, n_t} \langle O(n_x, n_y, n_z, n_t)\bar{O}(\mathbf{0})\rangle$

Meson Operators

- local isovector operators: $O_\Gamma(x) = \bar{q}(x)(\vec{\tau} \otimes \Gamma)q(x)$

$$\langle O(t)\bar{O}(0)\rangle \quad \sim \quad e^{-mt} + e^{-m(N_t - t)}$$

- extract effective mass: $m_{eff}(t) = \ln \left| \frac{C(t)}{C(t+1)} \right|$

(or cosh to respect periodicity)

- for screening spectrum: $t \to z, \quad C(n_z) = \sum_{n_x, n_y, n_t} \langle O(n_x, n_y, n_z, n_t)\bar{O}(\mathbf{0})\rangle$

Chiral-Parity Group Rep.	Γ	Abbreviation	Symmetries
$(\frac{1}{2}, \frac{1}{2})_a$	γ_5	PS	] $U(1)_A$
$(\frac{1}{2}, \frac{1}{2})_b$	$\mathbb{1}$	S	
$[(0,1)+(1,0)]_a$	$\gamma_k\gamma_5$	$\mathbf{A}$	] $SU(2)_A$
$[(0,1)+(1,0)]_a$	γ_k	$\mathbf{V}$	
$(\frac{1}{2}, \frac{1}{2})_a$	$\gamma_k\gamma_3$	$\mathbf{T}$	] $U(1)_A$
$(\frac{1}{2}, \frac{1}{2})_b$	$\gamma_k\gamma_3\gamma_5$	$\mathbf{X}$	

Meson Operators

- local isovector operators: $O_\Gamma(x) = \bar{q}(x)(\vec{\tau} \otimes \Gamma) q(x)$

$$\langle O(t)\bar{O}(0)\rangle \quad \sim \quad e^{-mt} + e^{-m(N_t - t)}$$

- extract effective mass: $m_{eff}(t) = \ln \left| \frac{C(t)}{C(t+1)} \right|$

(or cosh to respect periodicity)

- for screening spectrum: $t \to z, \quad C(n_z) = \sum_{n_x, n_y, n_t} \langle O(n_x, n_y, n_z, n_t)\bar{O}(\mathbf{0})\rangle$

Chiral-Parity Group Rep.	Γ	Abbreviation	Symmetries
$(\frac{1}{2}, \frac{1}{2})_a$	γ_5	PS	] $U(1)_A$
$(\frac{1}{2}, \frac{1}{2})_b$	$\mathbb{1}$	S	
$[(0,1)+(1,0)]_a$	$\gamma_k\gamma_5$	**A**	] $SU(2)_A$
$[(0,1)+(1,0)]_a$	γ_k	**V**	
$(\frac{1}{2}, \frac{1}{2})_a$	$\gamma_k\gamma_3$	**T**	] $U(1)_A$
$(\frac{1}{2}, \frac{1}{2})_b$	$\gamma_k\gamma_3\gamma_5$	**X**	

Chiral Symmetry of the Mesons Spectrum

@ 1.2 T_c

→ *extract symmetry violation by comparing*
screening masses *of related interpolating fields*

$$U(1)_A : \quad \Delta m_{eff} = |m^{PS}_{eff} - m^{S}_{eff}|$$

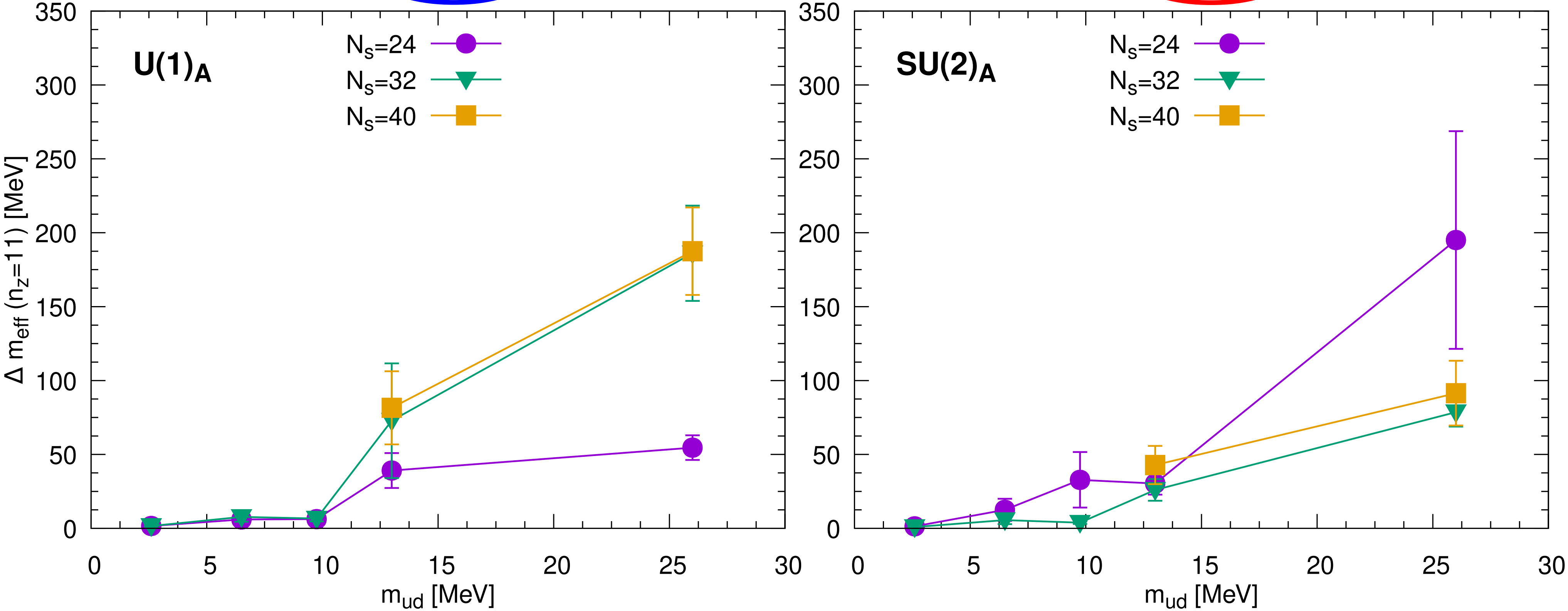
Chiral Symmetry of the Mesons Spectrum

→ *extract symmetry violation by comparing* ***screening masses*** *of related interpolating fields*

@ 1.2 T_c

$$U(1)_A : \quad \Delta m_{eff} = |m^{PS}_{eff} - m^{S}_{eff}|$$

$$PS \longleftarrow e^{i(\gamma_5 \otimes \mathbb{1})\vec{\theta}} \longrightarrow S \qquad V_x \longleftarrow e^{i(\gamma_5 \otimes \vec{\tau})\vec{\theta}} \longrightarrow A_x$$



Baryon Operators

- local nucleon operators (isospin=1/2, spin=1/2)
- parity projection: $N_{\pm} = \frac{(1 \pm \gamma_4)}{2} N$

$$\langle N^{\pm}(t)\bar{N}^{\pm}(0)\rangle \quad \sim \quad e^{-m_{\pm}t} + e^{-m_{\mp}(N_t - t)}$$

positive parity forward — **negative parity backward**

- for screening spectrum: $t \to z, \quad C(n_z) = \sum_{n_x, n_y, n_t} e^{in_t\omega_0}\langle N^{\pm}(n_x, n_y, n_z, n_t)\bar{N}^{\pm}(\mathbf{0})\rangle$

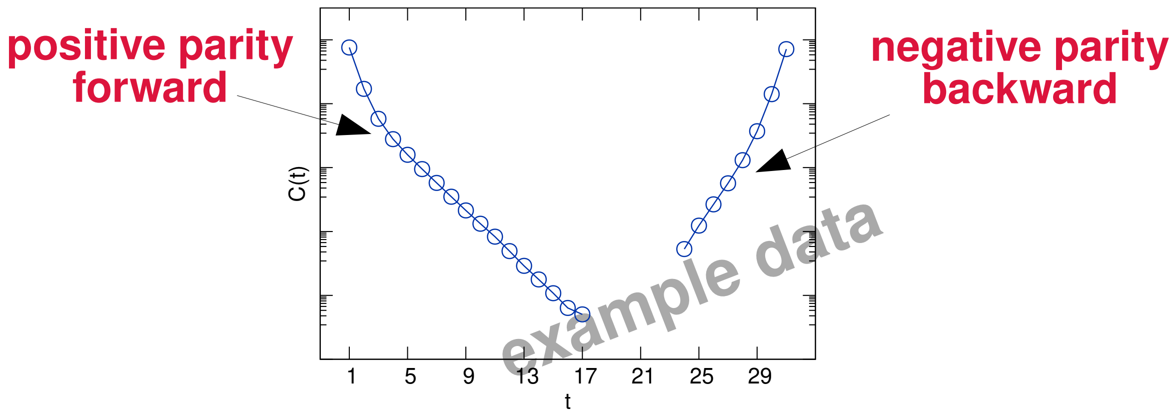
Chiral-Parity Group Rep.	Operator	Abbreviation	Symmetries
$[(0, \frac{1}{2}) + (\frac{1}{2}, 0)]_a$	$(\tilde{q}q)q$	N_1	$]\ U(1)_A$
$[(0, \frac{1}{2}) + (\frac{1}{2}, 0)]_b$	$(\tilde{q}\gamma_5 q)\gamma_5 q$	N_2	
$(\frac{1}{2}, 1) + (1, \frac{1}{2})$	$(\tilde{q}\gamma_\mu q)\gamma^\mu q$	N_3	$]\ SU(2)_A$
$(\frac{1}{2}, 1) + (1, \frac{1}{2})$	$(\tilde{q}\gamma_\mu\gamma_5\tau^i q)\gamma^\mu\gamma_5\tau^i q$	N_4	

$$\tilde{q} = q^T C\gamma_5(i\tau_2)$$

Parity Doubling of Nucleons

→ *extract symmetry violation by comparing*
***positive and negative parity states** of same interpolating field*

@ 1.2 T_c



'R-parameter':

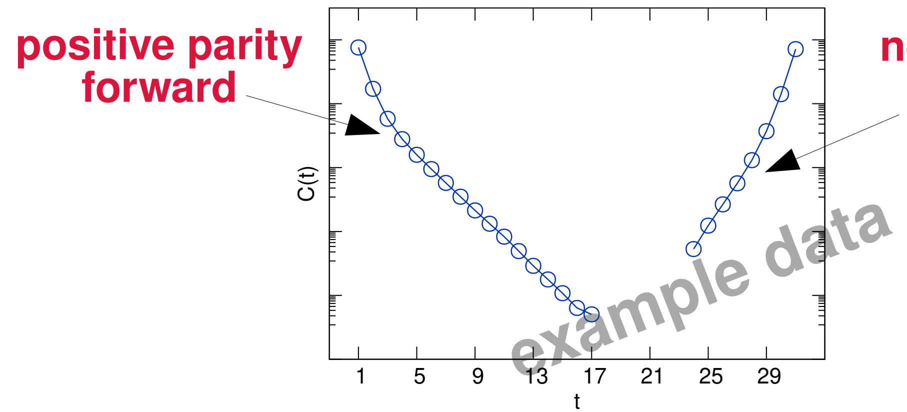
$$R(t) = \frac{N_1(t) - N_1(N_t - t)}{N_1(t) + N_1(N_t - t)}$$

compare e.g. S.Datta *et al*, JHEP 1302 (2013) 145; G.Aarts *et al*, JHEP 1706 (2017) 34

Parity Doubling of Nucleons

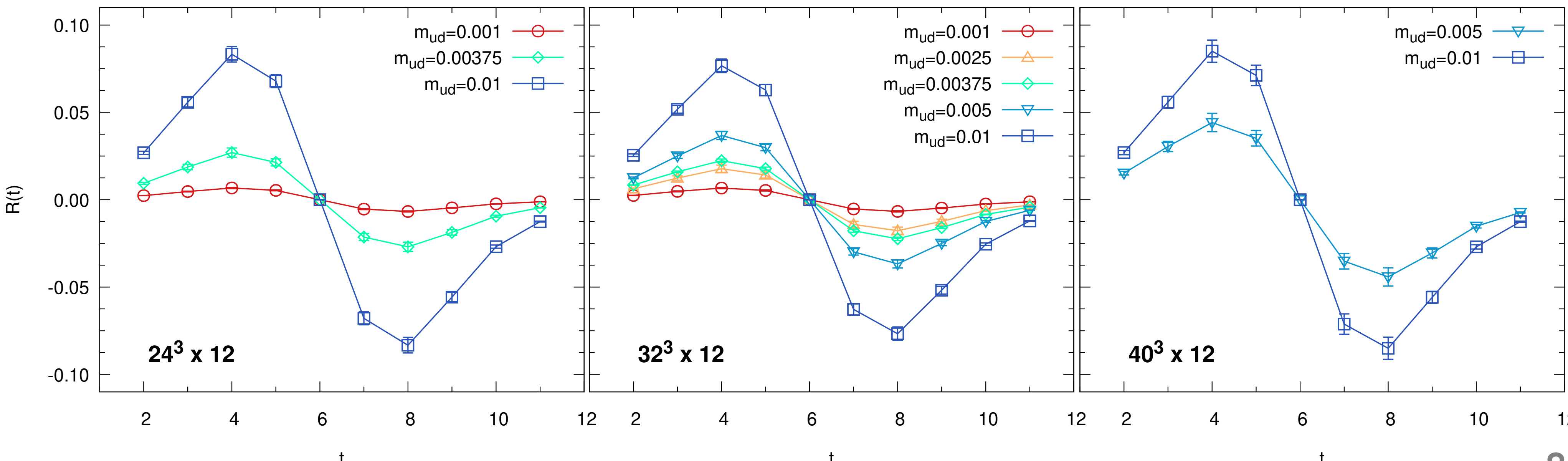
→ *extract symmetry violation by comparing*
positive and negative parity states *of same interpolating field*

@ 1.2 T_c



'R-parameter':

$$R(t) = \frac{N_1(t) - N_1(N_t - t)}{N_1(t) + N_1(N_t - t)}$$



compare e.g. S.Datta *et al*, JHEP 1302 (2013) 145; G.Aarts *et al*, JHEP 1706 (2017) 34

Parity Doubling of Nucleons

→ *extract symmetry violation by comparing*
positive and negative parity states *of same interpolating field*

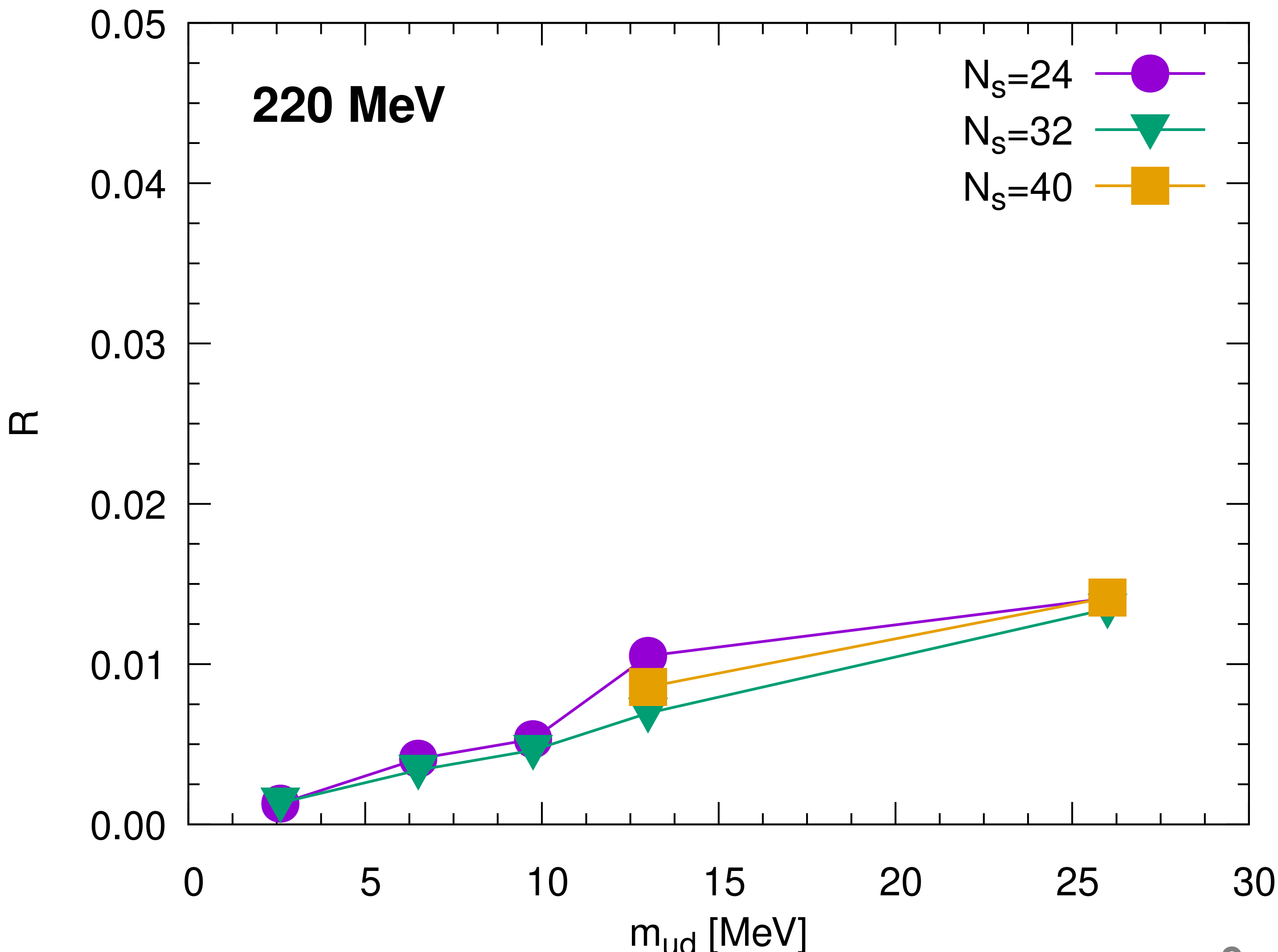
@ 1.2 T_c

integrated *'R-parameter'*:

$$R = \frac{\sum_{t=1}^{N_t/2} R(t)/\sigma^2(t)}{\sum_{t=1}^{N_t/2} 1/\sigma^2(t)}$$

$R \neq 0$
no parity doubling

$R = 0$
parity doubling



compare e.g. S.Datta *et al*, JHEP 1302 (2013) 145; G.Aarts *et al*, JHEP 1706 (2017) 34

Parity Doubling of Nucleons

→ *extract symmetry violation by comparing*
positive and negative parity states *of same interpolating field*

@ 1.2 T_c

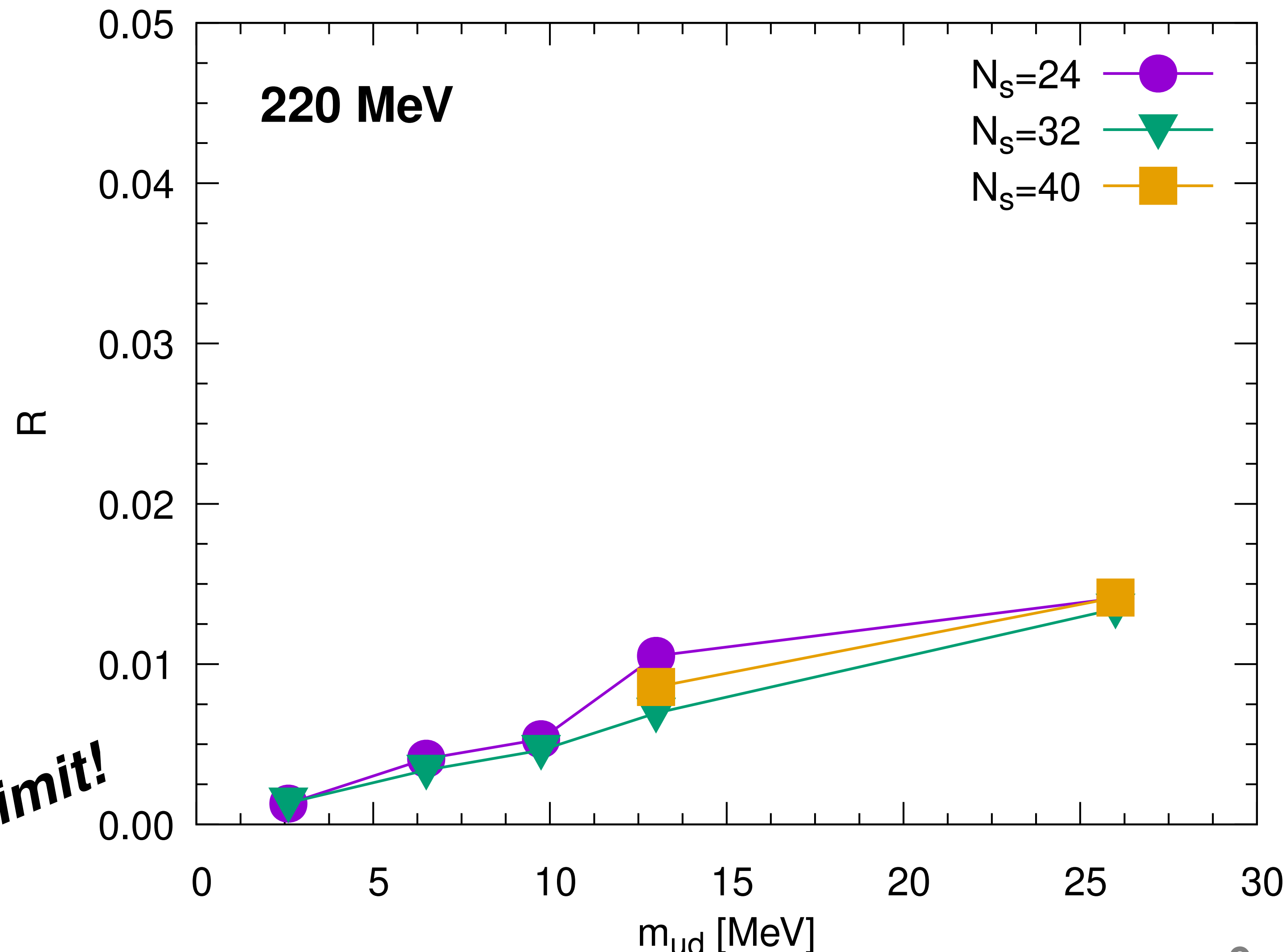
integrated *'R-parameter'*:

$$R = \frac{\sum_{t=1}^{N_t/2} R(t)/\sigma^2(t)}{\sum_{t=1}^{N_t/2} 1/\sigma^2(t)}$$

$R \neq 0$
no parity doubling

$R = 0$
parity doubling

in chiral limit!



compare e.g. S.Datta *et al*, JHEP 1302 (2013) 145; G.Aarts *et al*, JHEP 1706 (2017) 34

Chiral Symmetry of the Baryon Spectrum

@ 1.2 T_c

→ *extract symmetry violation by comparing*
screening masses *of related interpolating fields*

$$U(1)_A : \quad \Delta m_{eff} = |m_{eff}^{N_1^+} - m_{eff}^{N_2^+}|$$

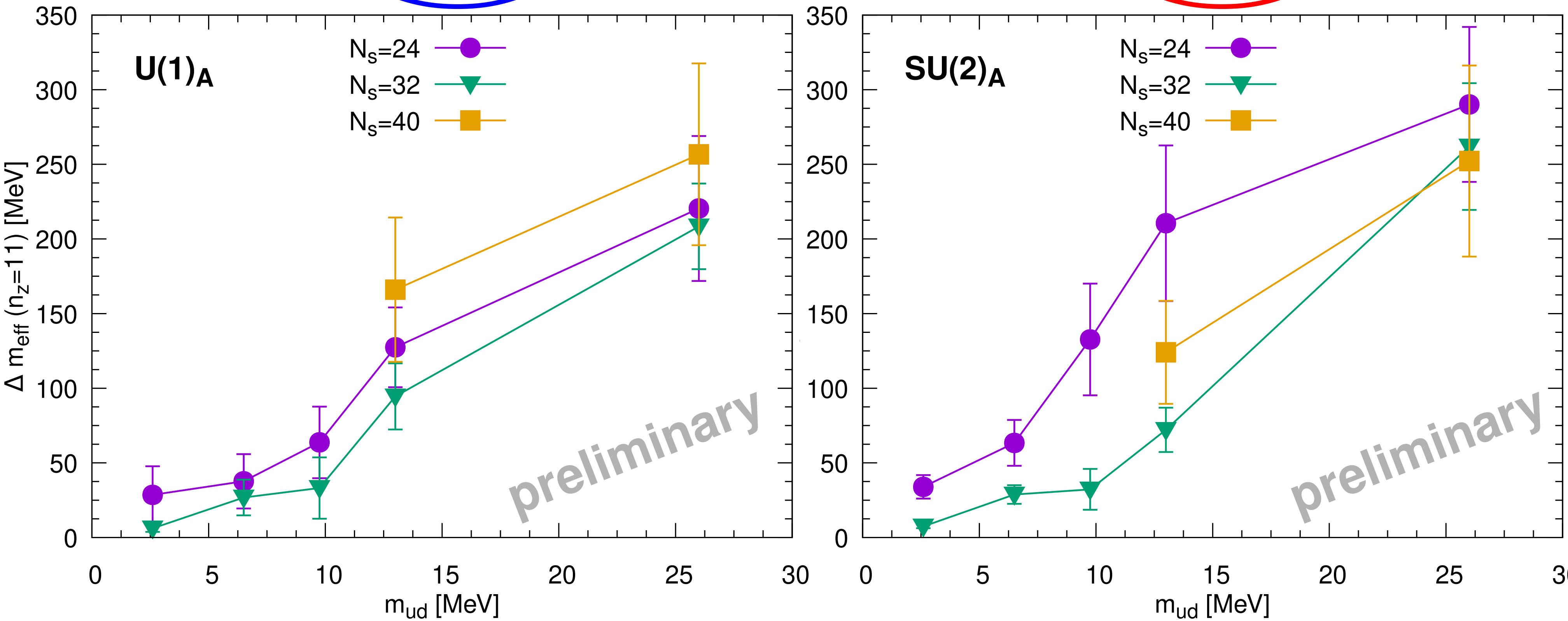
Chiral Symmetry of the Baryon Spectrum

@ 1.2 T_c

→ *extract symmetry violation by comparing* ***screening masses*** *of related interpolating fields*

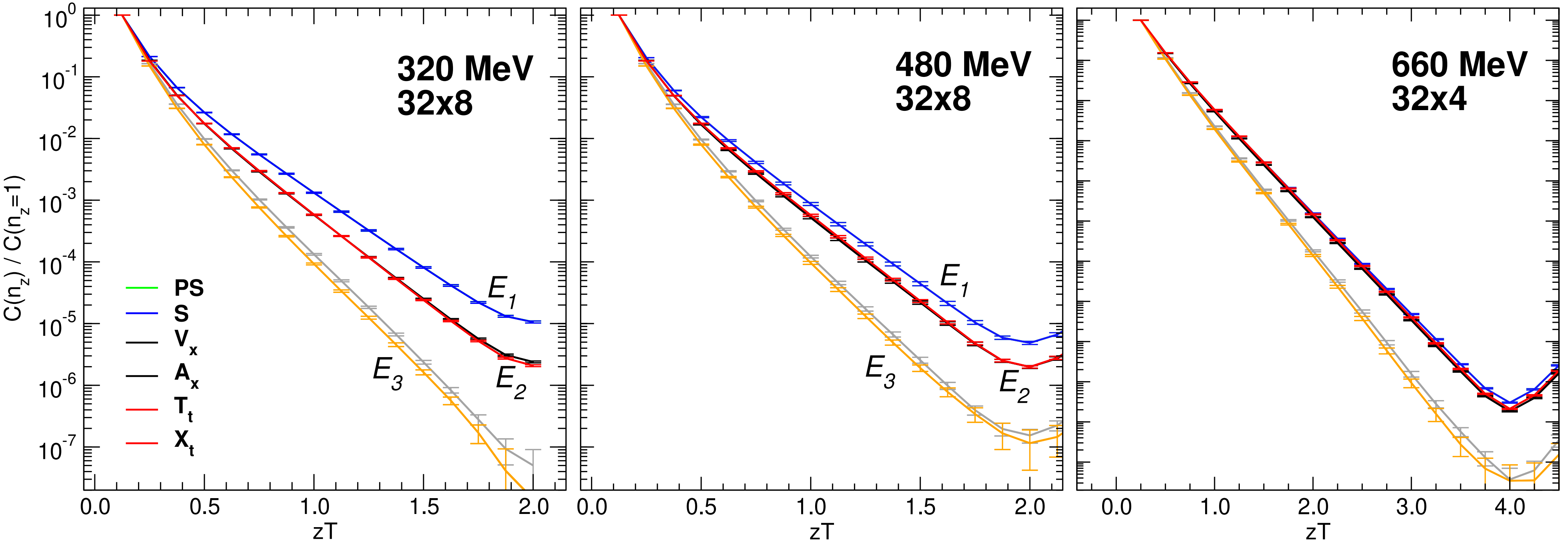
$$U(1)_A : \quad \Delta m_{eff} = |m_{eff}^{N_1^+} - m_{eff}^{N_2^+}|$$

$$N_1^+ \xleftarrow{\quad} e^{i(\gamma_5 \otimes \mathbb{1})\vec{\theta}} \xrightarrow{\quad} N_2^+ \qquad N_3^+ \xleftarrow{\quad} e^{i(\gamma_5 \otimes \vec{\tau})\vec{\theta}} \xrightarrow{\quad} N_4^+$$



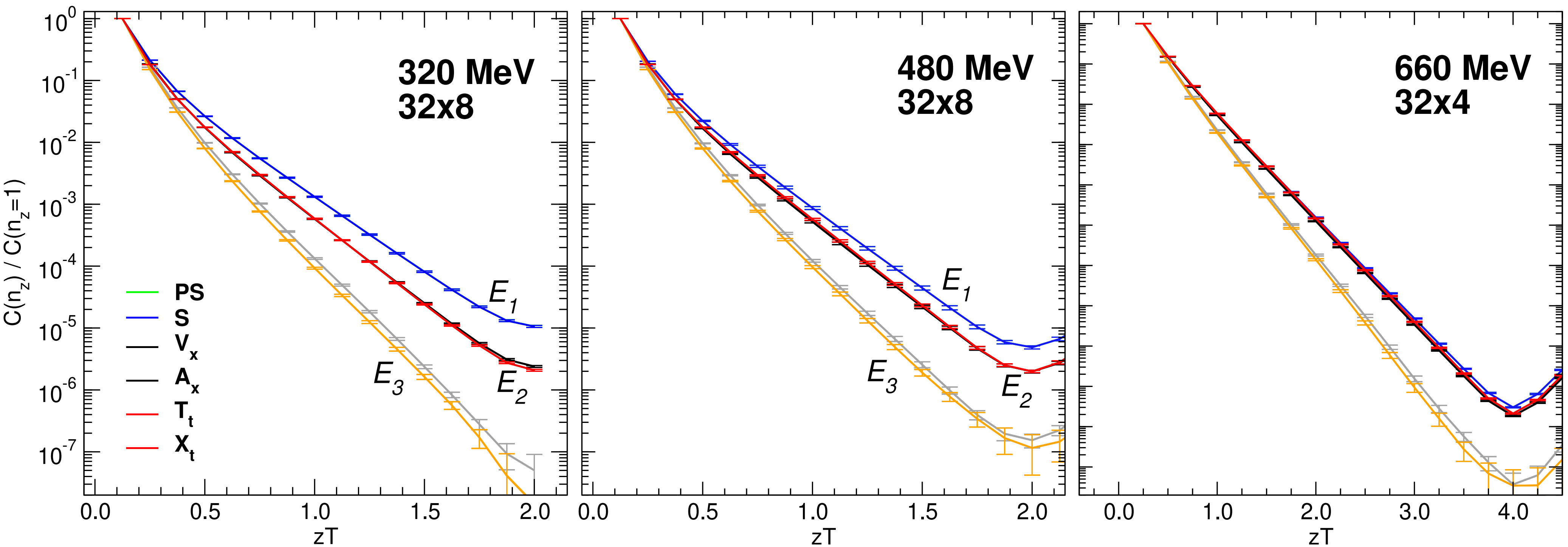
Symmetries at Higher Temperature

Temperature Evolution of Meson Spectrum



L.Glozman and M.Pak, Phys.Rev. D92 (2015) no.1, 016001
Phys.Rev. D96 (2017) 094501 [arxiv:1707.01881]

Temperature Evolution of Meson Spectrum



states of different chirality connected by **chiral spin**:

$$\mathrm{SU}(2)_{CS}$$

$$\{\gamma_2, -i\gamma_5\gamma_2, \gamma_5\} \Rightarrow A_x \leftrightarrow T_t \leftrightarrow X_t$$

minimal group containing **chiral spin** and **chiral symmetry**:

$$\mathrm{SU}(4)$$

$$V_x \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_x \quad \leftarrow E_2$$

$$V_t \leftrightarrow T_x \leftrightarrow X_x \leftrightarrow A_t \quad \leftarrow E_3$$

L.Glozman and M.Pak, Phys.Rev. D92 (2015) no.1, 016001
Phys.Rev. D96 (2017) 094501 [arxiv:1707.01881]

Is High Temperature QCD *'more'* symmetric?

E_1, E_2, E_3 *groups show **multiplet** structure..*

define '***kappa'*** parameter

$$\kappa = \frac{|C_{A_x} - C_{T_t}|}{|C_{A_x} - C_S|}$$

as intuitive symmetry measure!

(distance ***within*** a multiplet relative to the distance ***between*** multiplets)

Phys.Rev. D100 (2019) 01xxxx [arxiv:1902.03191]

Is High Temperature QCD *'more'* symmetric?

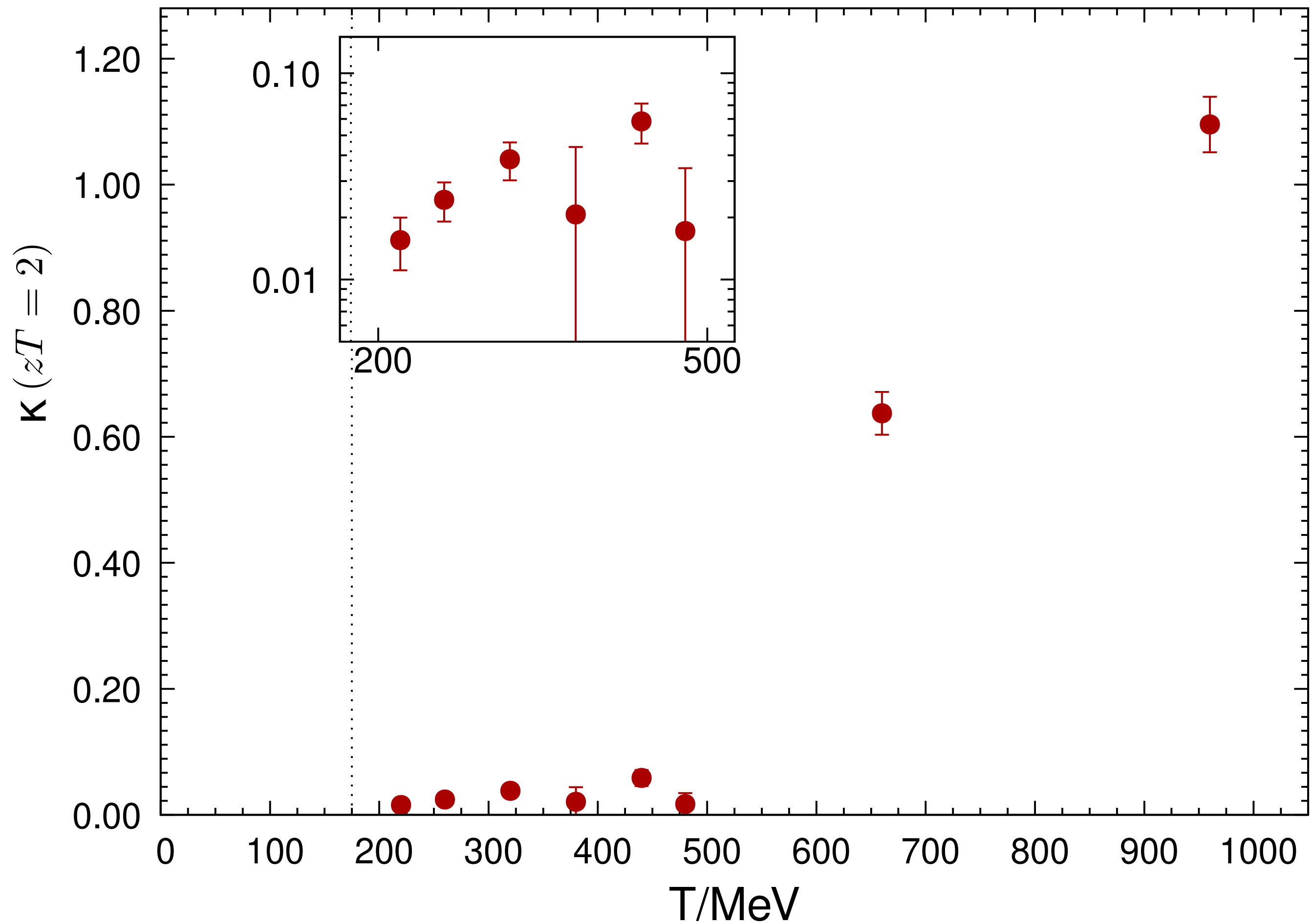
E_1, E_2, E_3 *groups show **multiplet** structure..*

define '***kappa'*** parameter

$$\kappa = \frac{|C_{A_x} - C_{T_t}|}{|C_{A_x} - C_S|}$$

as intuitive symmetry measure!

(distance ***within*** a multiplet relative to the distance ***between*** multiplets)



Phys.Rev. D100 (2019) 01xxxx [arxiv:1902.03191]

Is High Temperature QCD *'more'* symmetric?

E_1, E_2, E_3 *groups show* ***multiplet*** *structure..*

define '***kappa'*** parameter

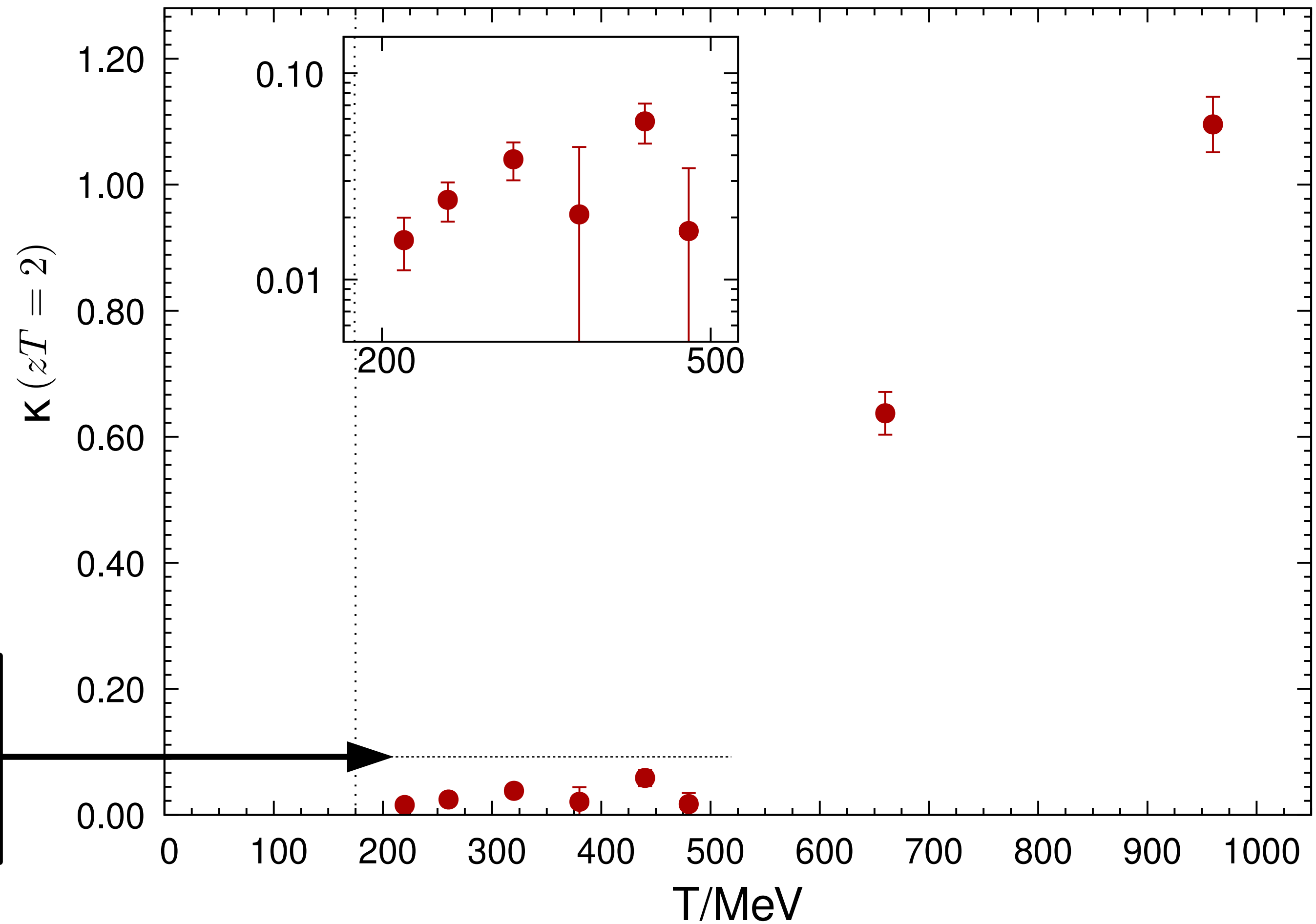
$$\kappa = \frac{|C_{A_x} - C_{T_t}|}{|C_{A_x} - C_S|}$$

as intuitive symmetry measure!

(distance ***within*** a multiplet relative to the distance ***between*** multiplets)

temperature range

$T \sim 220 - 500\ \text{MeV}$



Phys.Rev. D100 (2019) 01xxxx [arxiv:1902.03191]

Chiral Spin and the Lagrangian

$$\Psi \xrightarrow{\mathrm{SU}(2)_{CS}} e^{i\vec{\Sigma}\vec{\theta}/2}\Psi \qquad \vec{\Sigma} = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}$$

free, massless fermions: $\mathcal{L} = \bar{\Psi} i \partial\!\!\!/ \Psi$

covariant derivative: $D_\mu = \partial_\mu - igA_\mu$

interacting, massless fermions: $\mathcal{L} = \bar{\Psi} i D\!\!\!\!/ \Psi = \bar{\Psi} i\gamma^0 D_0 \Psi + \bar{\Psi} i\gamma^i D_i \Psi$

A *and* ***T*** *mix under chiral spin transformations:*
use ratio to measure breaking ***within*** *multiplet!*

Phys.Rev. D100 (2019) 01xxxx [arxiv:1902.03191]

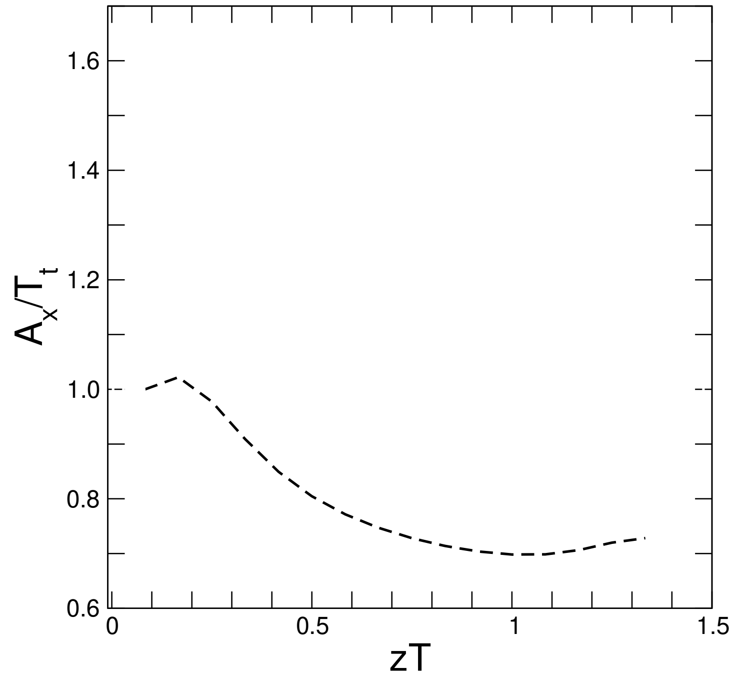
Chiral Spin and the Lagrangian

$$\Psi \xrightarrow{\mathrm{SU}(2)_{CS}} e^{i\vec{\Sigma}\vec{\theta}/2}\Psi \qquad \vec{\Sigma} = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}$$

free, massless fermions: $\mathcal{L} = \bar{\Psi} i \partial\!\!\!/ \Psi$

covariant derivative: $D_\mu = \partial_\mu - igA_\mu$

interacting, massless fermions: $\mathcal{L} = \bar{\Psi} i D\!\!\!\!/ \Psi = \bar{\Psi} i\gamma^0 D_0 \Psi + \bar{\Psi} i\gamma^i D_i \Psi$



- **kinetic** term breaks chiral spin

***A** and **T** mix under chiral spin transformations:*
*use ratio to measure breaking **within** multiplet!*

Phys.Rev. D100 (2019) 01xxxx [arxiv:1902.03191]

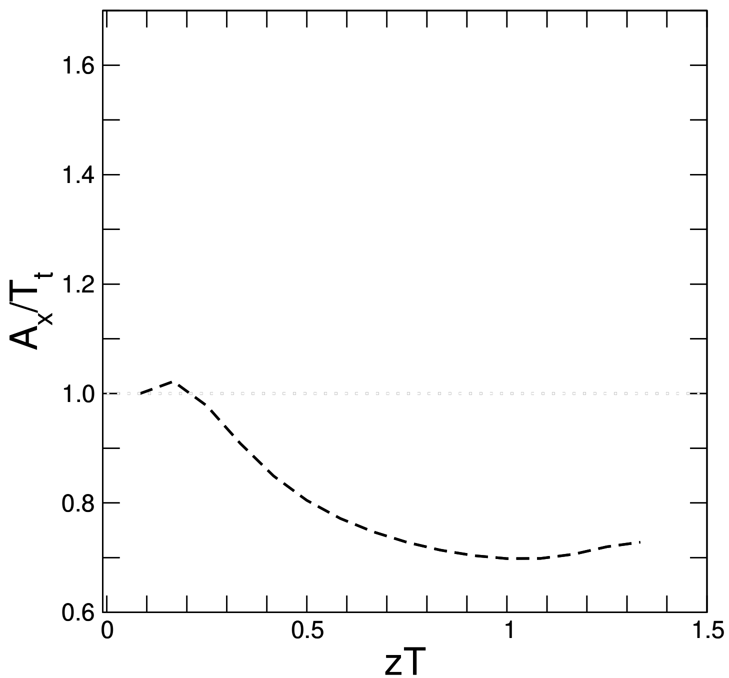
Chiral Spin and the Lagrangian

$$\Psi \xrightarrow{\mathrm{SU}(2)_{CS}} e^{i\vec{\Sigma}\vec{\theta}/2}\Psi \qquad \vec{\Sigma} = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}$$

free, massless fermions: $\mathcal{L} = \bar{\Psi} i \partial\!\!\!/ \Psi$

covariant derivative: $D_\mu = \partial_\mu - igA_\mu$

interacting, massless fermions: $\mathcal{L} = \bar{\Psi} i D\!\!\!\!/ \Psi = \bar{\Psi} i \gamma^0 D_0 \Psi + \bar{\Psi} i \gamma^i D_i \Psi$



- **kinetic** term breaks chiral spin
- **electric** term is invariant

A *and* ***T*** *mix under chiral spin transformations:*
use ratio to measure breaking ***within*** *multiplet!*

Phys.Rev. D100 (2019) 01xxxx [arxiv:1902.03191]

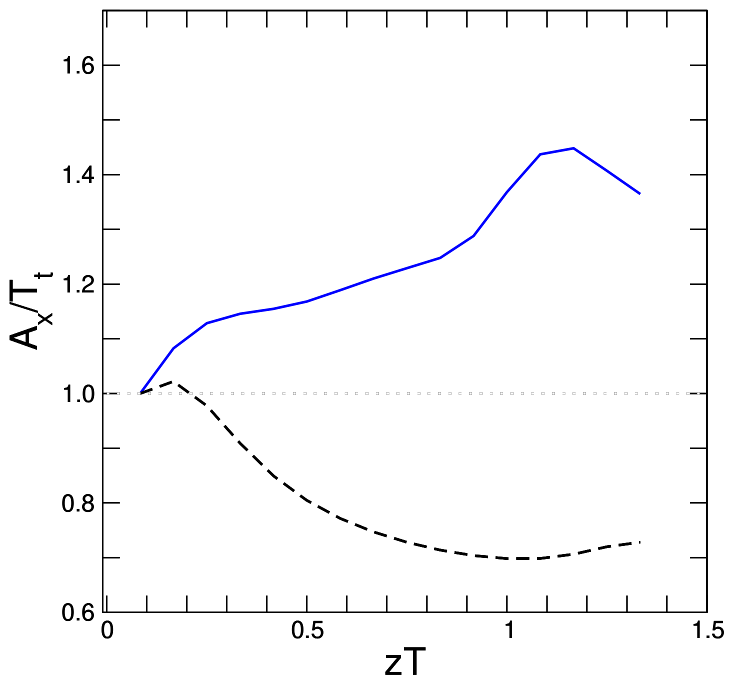
Chiral Spin and the Lagrangian

$$\Psi \xrightarrow{\mathrm{SU}(2)_{CS}} e^{i\vec{\Sigma}\vec{\theta}/2}\Psi \qquad \vec{\Sigma} = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}$$

free, massless fermions: $\mathcal{L} = \bar{\Psi} i \partial\!\!\!/ \Psi$

covariant derivative: $D_\mu = \partial_\mu - igA_\mu$

interacting, massless fermions: $\mathcal{L} = \bar{\Psi} i D\!\!\!\!/ \Psi = \bar{\Psi} i\gamma^0 D_0 \Psi + \bar{\Psi} i \gamma^i D_i \Psi$

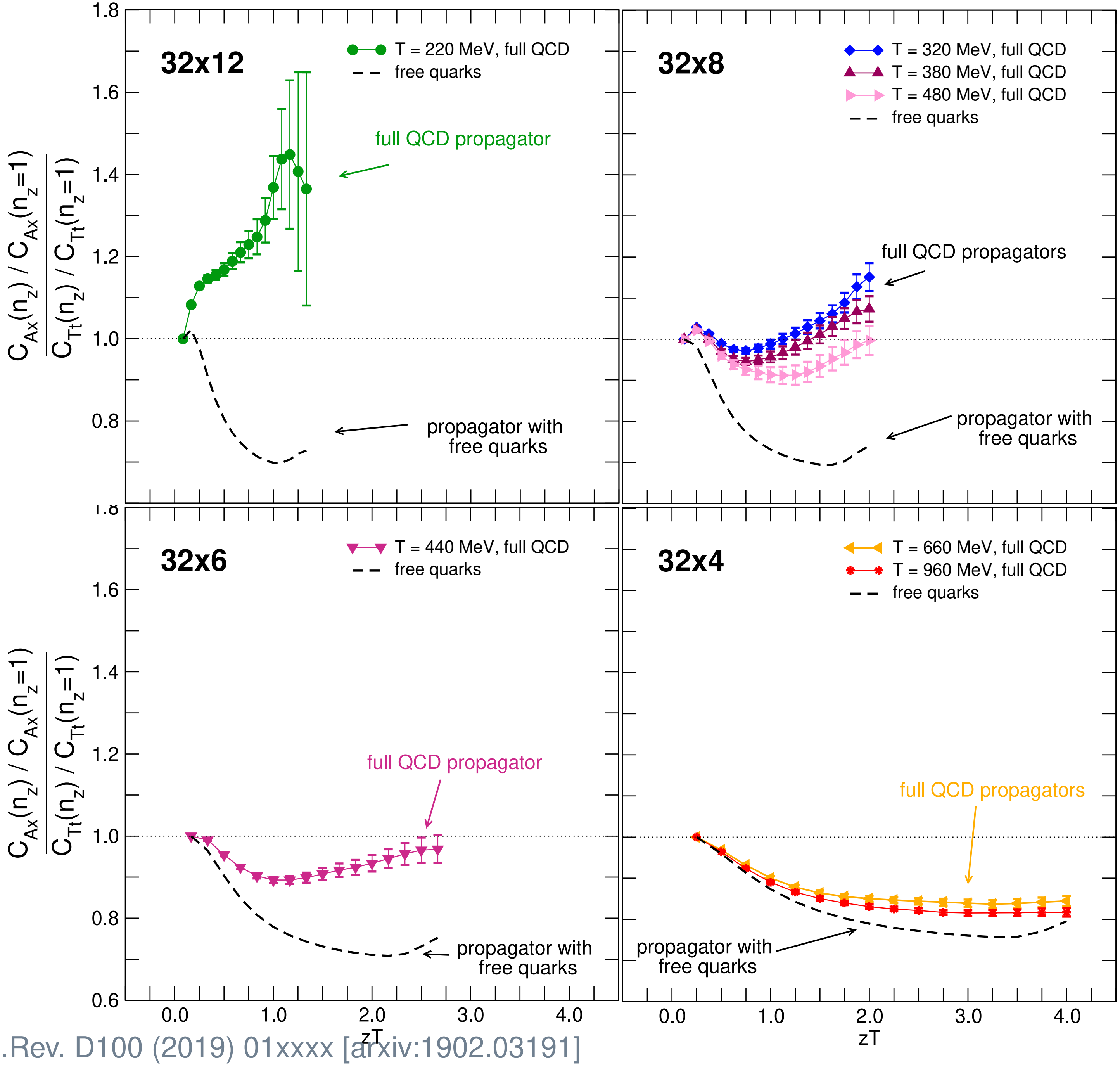


- **kinetic** term breaks chiral spin
- **electric** term is invariant
- **magnetic** term breaks chiral spin

__A__ and __T__ mix under chiral spin transformations: use ratio to measure breaking __within__ multiplet!

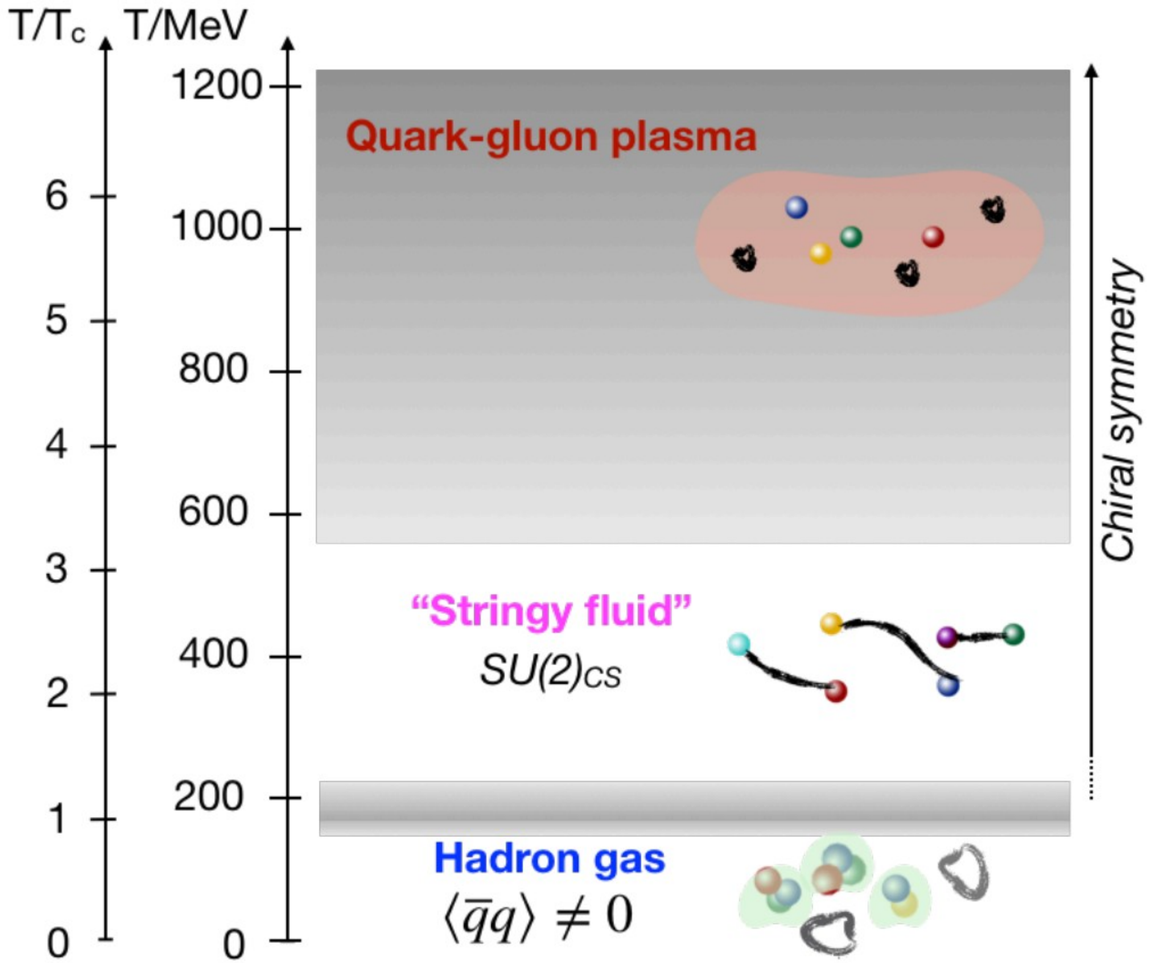
Phys.Rev. D100 (2019) 01xxxx [arxiv:1902.03191]

Interaction within SU(4) multiplets



Phys.Rev. D100 (2019) 01xxxx [arXiv:1902.03191]

Sketch of a ‘*new*’ Phase Diagram



strongly interacting matter

between

chiral transition

and

weakly interacting **QGP**

Chemical potential does not change picture:

$$S = \int_0^{1/T} \int d^3x \;\; \bar{\Psi}[\gamma_\mu D_\mu + \mu\gamma_4]\Psi$$

*‘**stringy fluid**’ regime at experimental accessible temperatures!*

Phys.Rev. D100 (2019) 01xxxx [arxiv:1902.03191]

Conclusions

Above chiral transition at 220 MeV:

- **$SU(2)_L$ x $SU(2)_R$** restored
- **$U(1)_A$** effectively restored
- **parity doubling** for baryons

Conclusions

Above chiral transition at 220 MeV:

- **$SU(2)_L$ x $SU(2)_R$** restored
- **$U(1)_A$** effectively restored
- **parity doubling** for baryons

At temperatures up to 500 MeV:

- QCD matter approximately **SU(4)** symmetric
- favors **color-electric** degrees of freedom
- chiral symmetry restoration ≠ deconfinement