Nuclear Science Computing Center at CCNU

Meson masses in external magnetic fields with HISQ fermions

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in collaboration with

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2019.06.20

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Lattice 19 Wuhan • China | June 16-22

Outline

- Motivation
- Lattice Setup
- Results
- Summary and Outlook

Motivation

- NJL model and GL approach:

  - magnetic field led to a vacuum instability → superconductivity

  - superconducting states signaled by condensation of charged ρ-meson.

M. N. Chernodub Phys. Rev. Lett. **106**, 142003 (2011)

M. N. Chernodub, Phys. Rev. D 82, 085011 (2010)

- Vafa-Witten theorem and QCD inequality:

  - charged ρ-meson cannot condensate in QCD

  - $m_{\rho^\pm} \geq \left(m_{\pi^0_u} + m_{\pi^0_d}\right)/2$

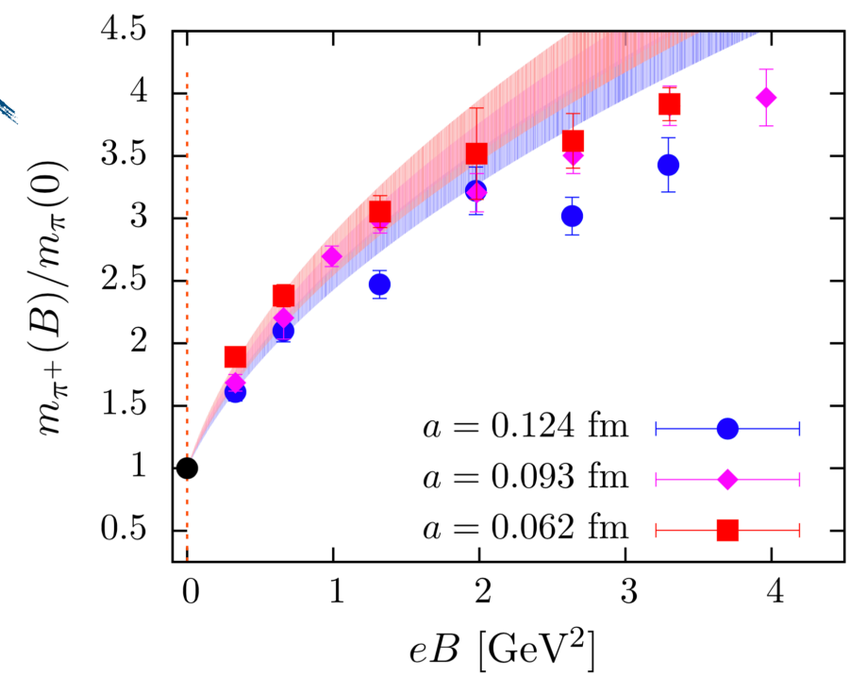
Y. Hidaka and A.Yamamoto Phys. Rev. D 87, 094502 (2013)

Motivation

- lattice study using quenched Wilson fermion

Bali, Brandt et al., PHYS. REV. D 97, 034505 (2018)

  - m_{π^+} increases ↑ as magnetic field increases ↑
  - m_{π^0} decreases ↓ as magnetic field increases ↑



- full QCD with external magnetic field

  - inverse magnetic catalysis observed in $\Delta\Sigma^{\mathrm{sea}}$

Bruckmann, Endrődi, Kovács, J. High Energ. Phys. (2013)

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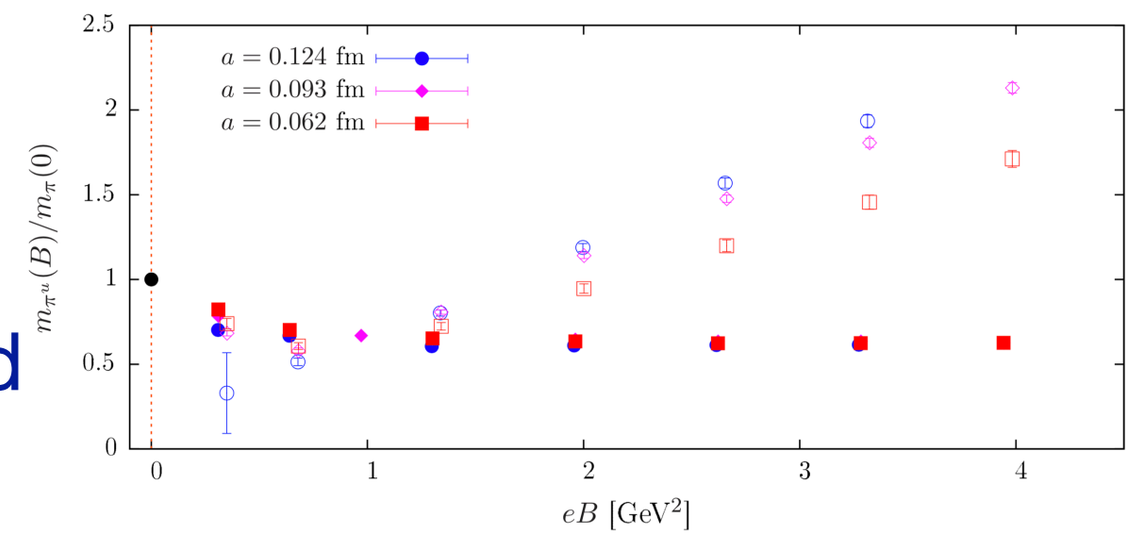

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Lattice Setup

- (2+1) flavor Dynamical HISQ fermion at T=0
- Lattice size: $32^3 \times 96$, $a^{-1} = 1.6852$ GeV
- $m_\pi = 230$ MeV
- $$qB = a^{-2}\frac{2\pi N_b}{N_x N_y} \quad \text{with} \quad \begin{cases} 0 \leq N_b \leq \frac{N_x N_y}{4} \\ N_b \in \mathbb{Z} \end{cases}$$

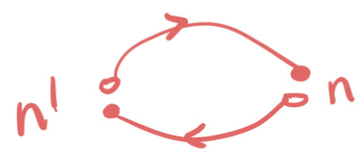
$$N_b = 0, 4, 8, 16, 24, 32, 48, 64$$ ($0 < |eB| \lesssim 3$ GeV^2)

Correlator

take π_u^0 and π^+ as example

- Correlators:

π^+ $\quad \langle O_{\pi^+}(n)\overline{O}_{\pi^+}(n')\rangle_F$



connected part

π^0 $\quad \langle O_{\pi^0}(n)\overline{O}_{\pi^0}(n')\rangle_F$

disconnected part

- Staggered correlators:

$$G(\tau) = -\sum_n \zeta(\vec{n})\,\mathrm{Tr}\left[M^{-1\dagger}(n,0)M^{-1}(n,0)\right]$$

$\zeta(\vec{n})$ is the phase factor characterizing Dirac and flavor index of staggered fermion

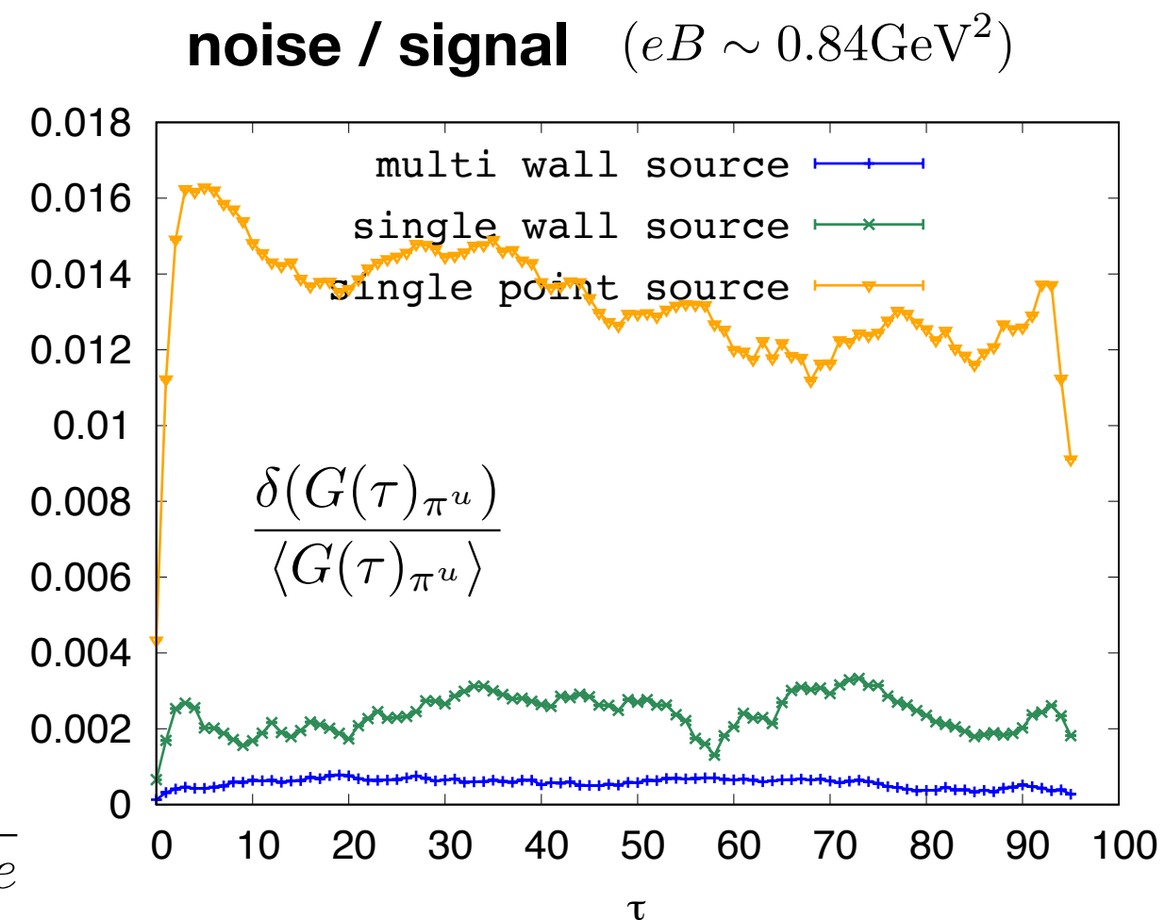
Reduction of noise ratio

- point, corner wall, multiple corner wall sources

➡ corner wall source : source at the origin of every 2^4 cube on particular time slice (improved the quality of signal by around 20% compared to single point source)

➡ multiple corner wall sources : put 12 sources at (0,0,0,0) , (0,0,0,8), (0,0,0,16)... (0,0,0,88)

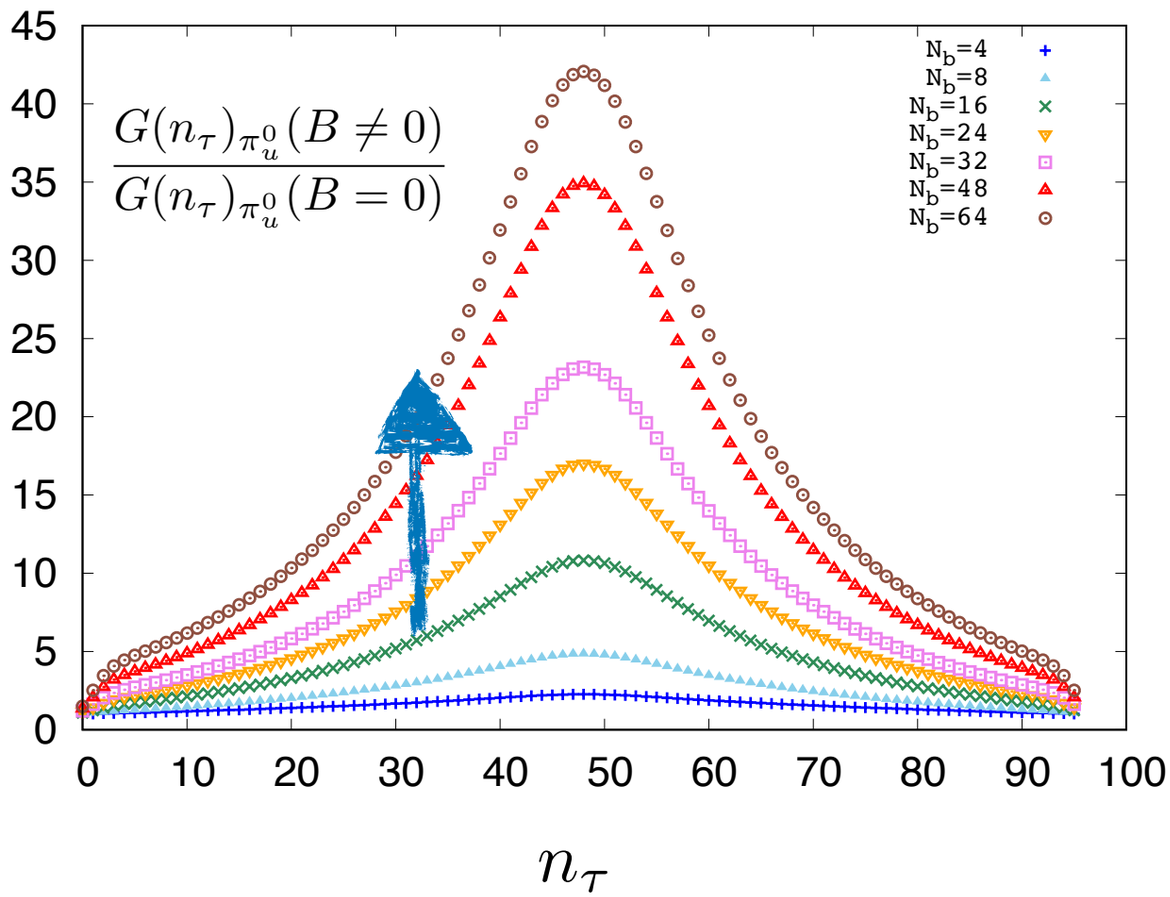
➡ improved the signal around $\sqrt{\#\ of\ source}$ times better than the single corner wall source

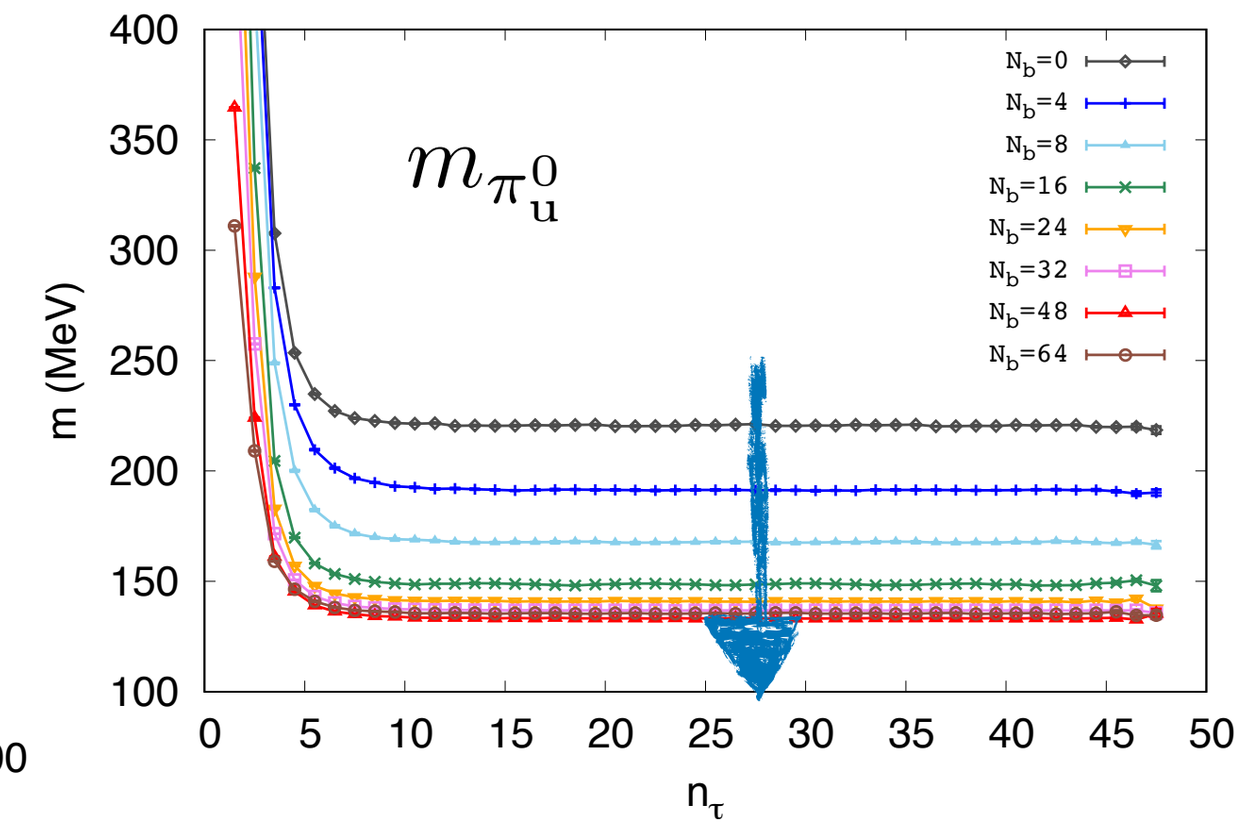


Neutral Pseudo-scalar Correlators

(take π_u^0 as example)

$$\frac{G(n_\tau)}{G(n_\tau+1)} = \frac{\cosh\left(m_{\rm eff}\left(n_\tau - N_\tau/2\right)\right)}{\cosh\left(m_{\rm eff}\left(n_\tau + 1 - N_\tau/2\right)\right)}$$

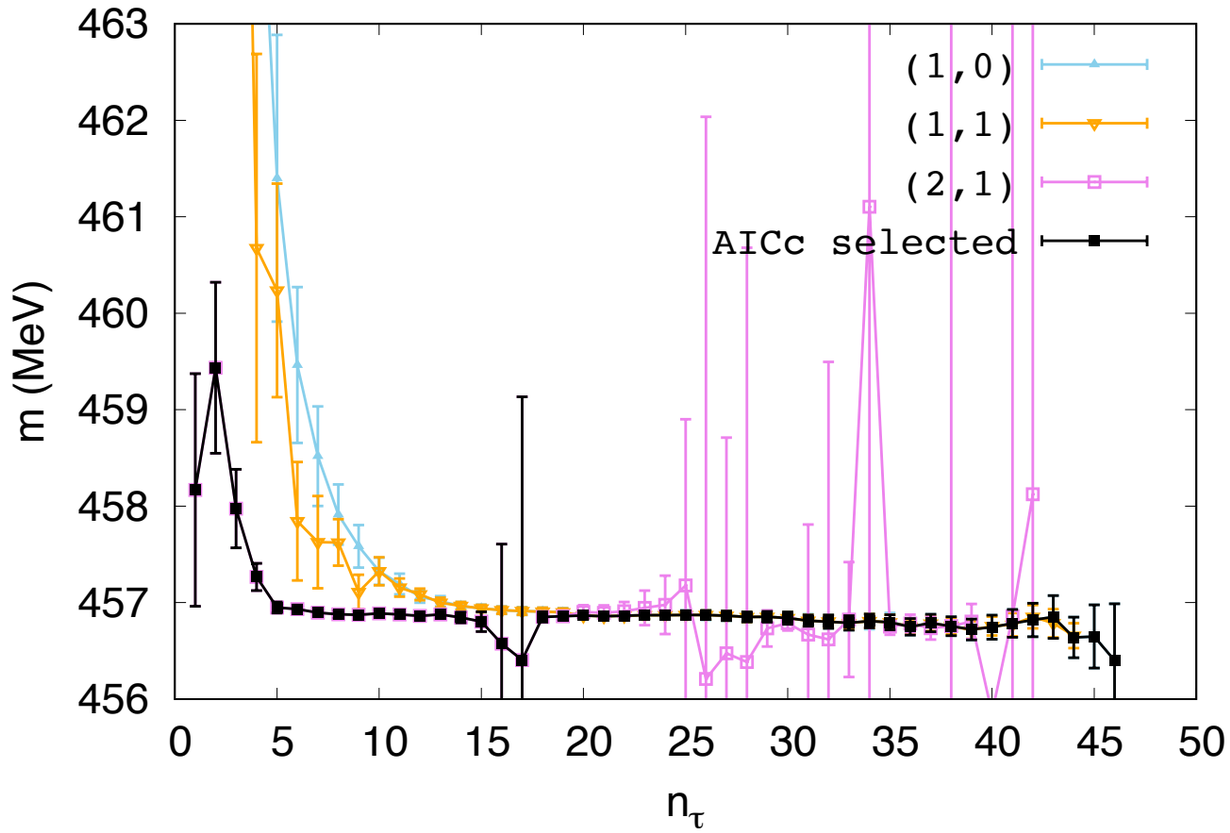




- $G(\tau)_{\pi_u^0}$ increases as the magnetic field grows
- ground state effective mass tends to decrease as magnetic field grows

Correlator Fitting

with (# of non-oscillating states, # of oscillating states)



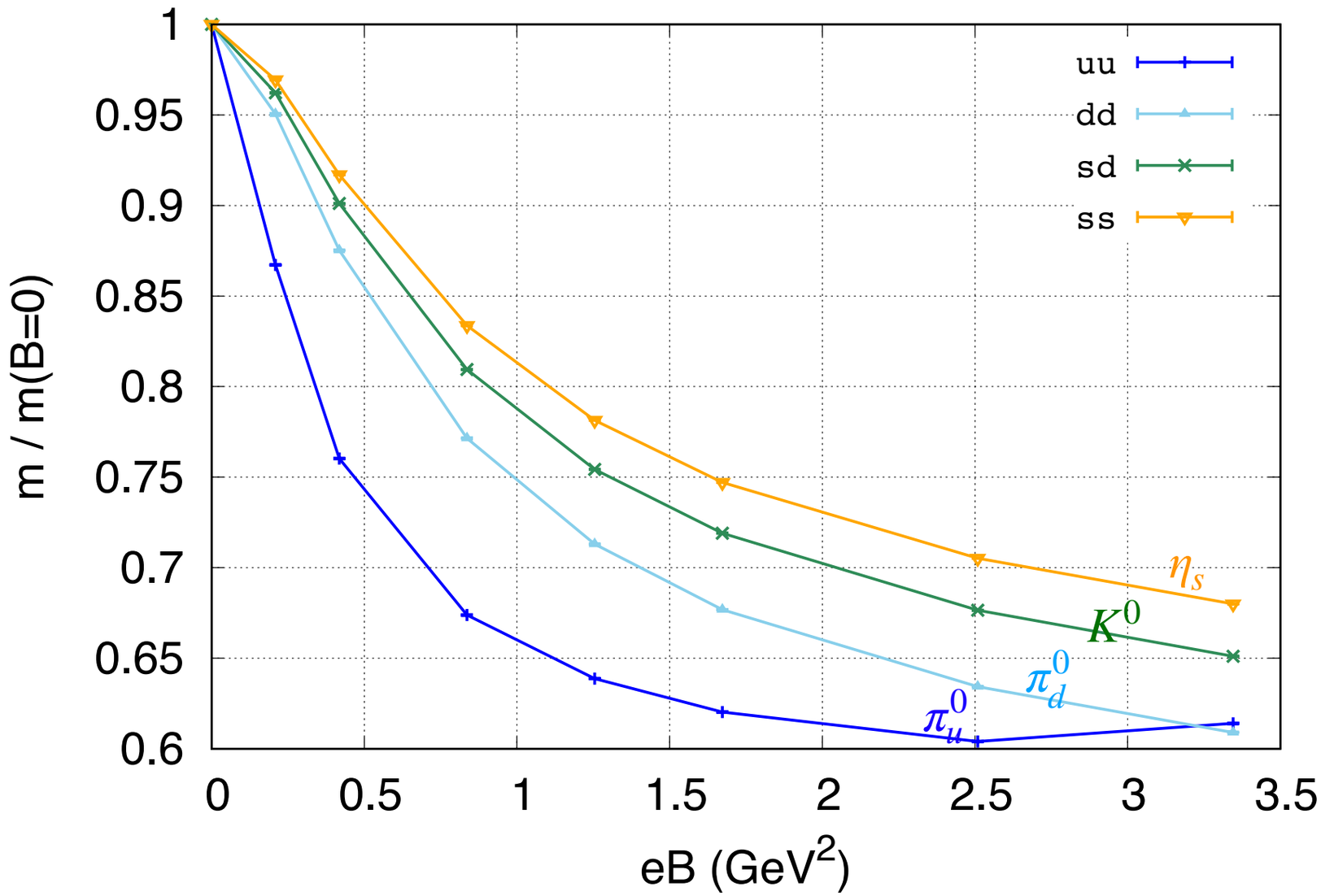
- fit ansatz for different number of states

$$G_{\text{fit}}(n_\tau) = \sum_{i=1}^{N_s,no} A_{no,i} \exp(-m_{no,i} n_\tau) - (-1)^{n_\tau} \sum_{i=0}^{N_s,osc} A_{osc,i} \exp(-m_{osc,i} n_\tau)$$

- select the fit with lowest AICc Akaike 1974 Cavanaugh 1997

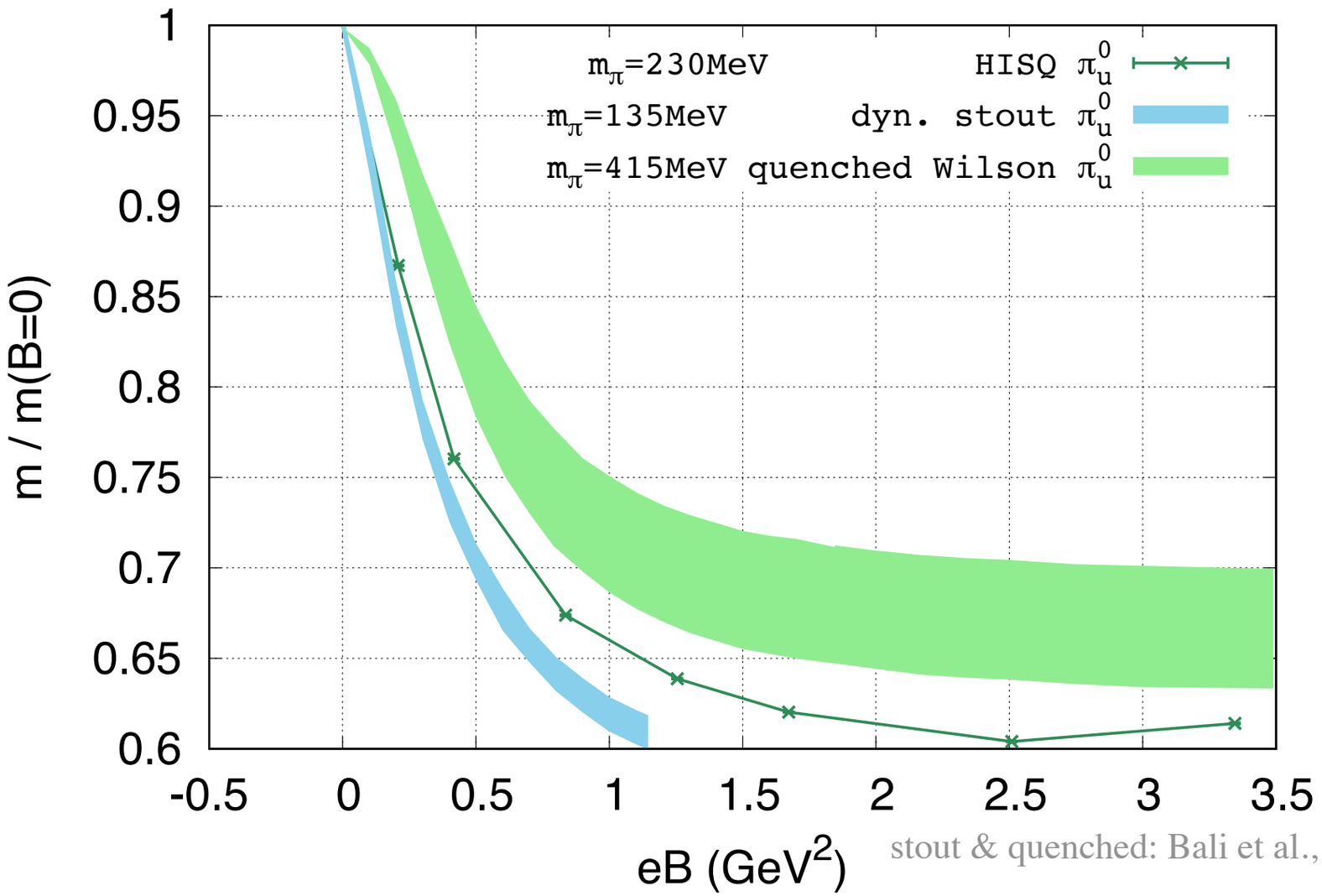
- use Gaussian bootstrapping to estimate final value and error

Summary of Neutral PS Particle Mass



- Neutral PS particles' masses decrease as eB grows
- Neutral PS particles' masses saturate at large eB
- the lighter quark has stronger influence of eB

Neutral Pseudo-scalar Particle Mass



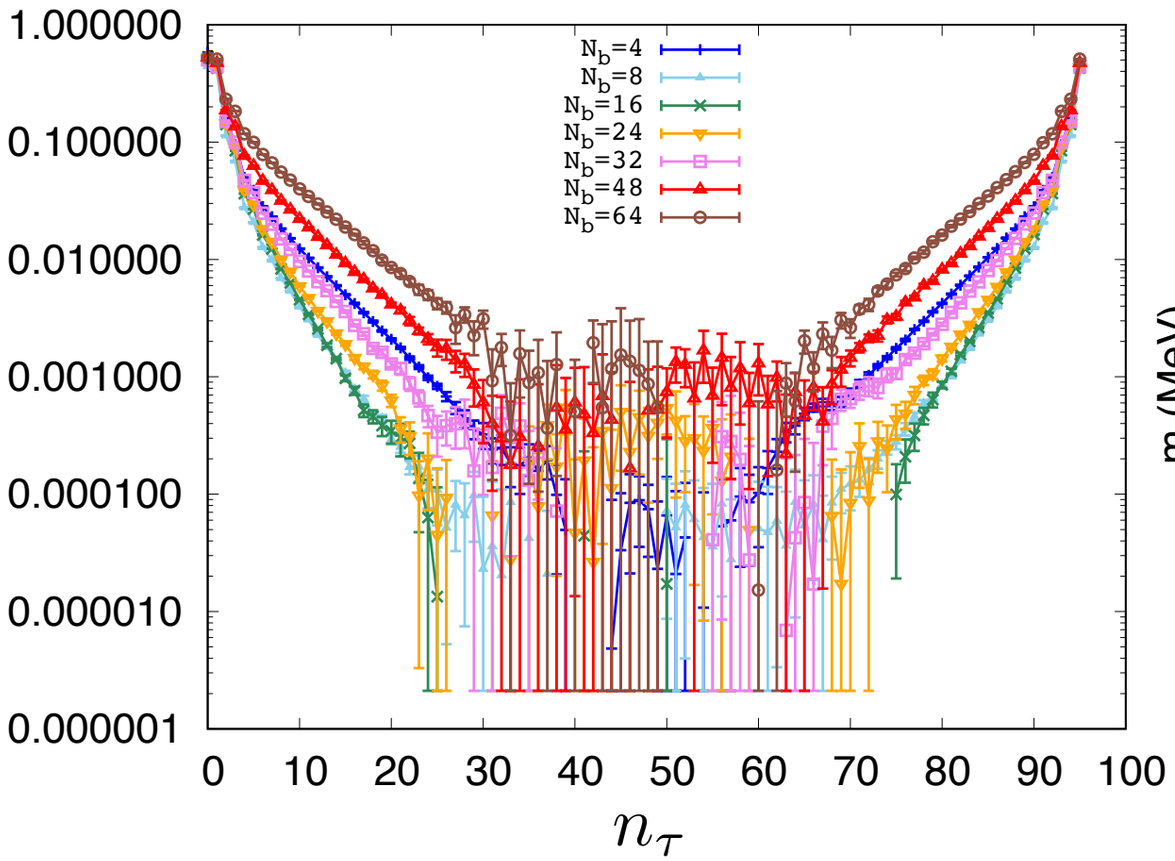
stout & quenched: Bali et al., PHYS. REV. D 97, 034505 (2018)

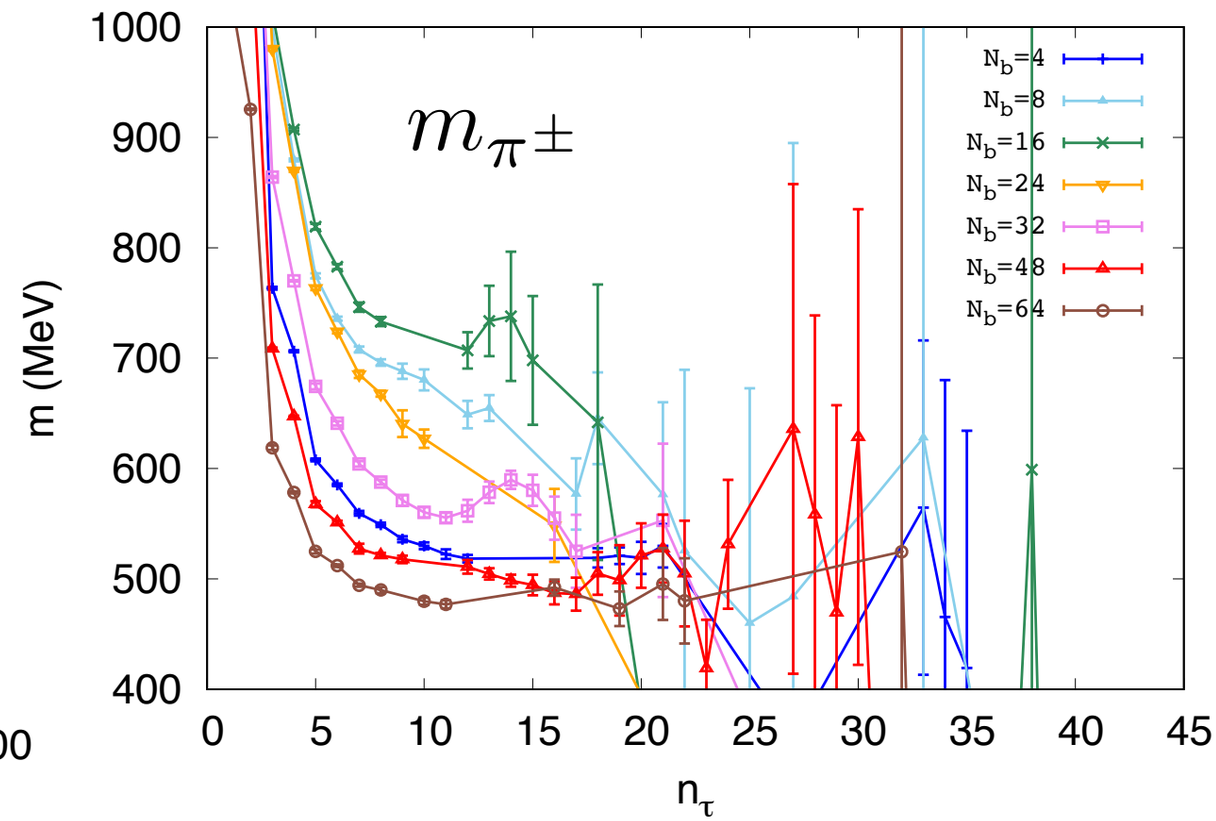
- general tendency agreed with each other
- quark loop contribution

Charged Pseudo-scalar Correlators

$$\frac{G(n_\tau)_{\pi^\pm}(B \neq 0)}{G(n_\tau)_{\pi^\pm}(B = 0)}$$

$$G(n_\tau) \approx A_{no} \exp(-m_{no} n_\tau) - (-1)^{n_\tau} A_{osc} \exp(-m_{osc} n_\tau)$$





- non-monotonous behavior
- fit correlator instead of mass plateau

Charged Pseudo-scalar Correlators

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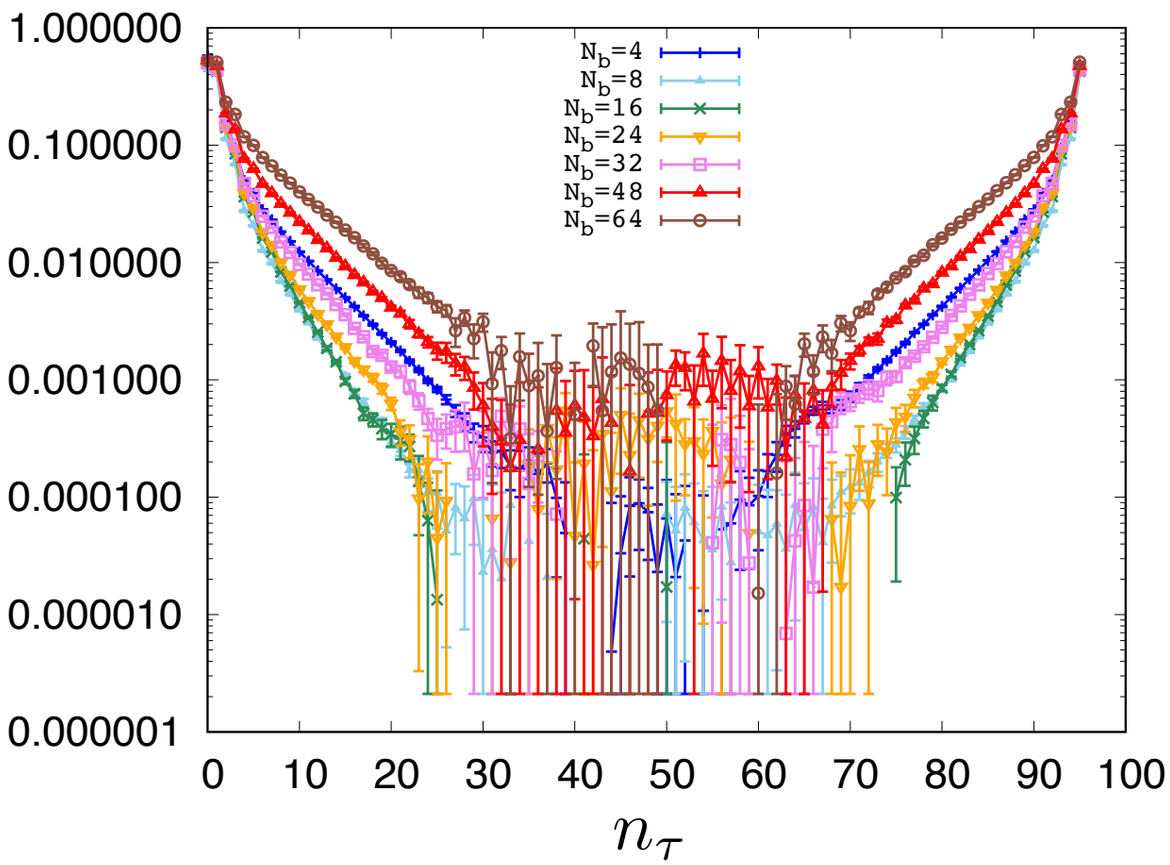
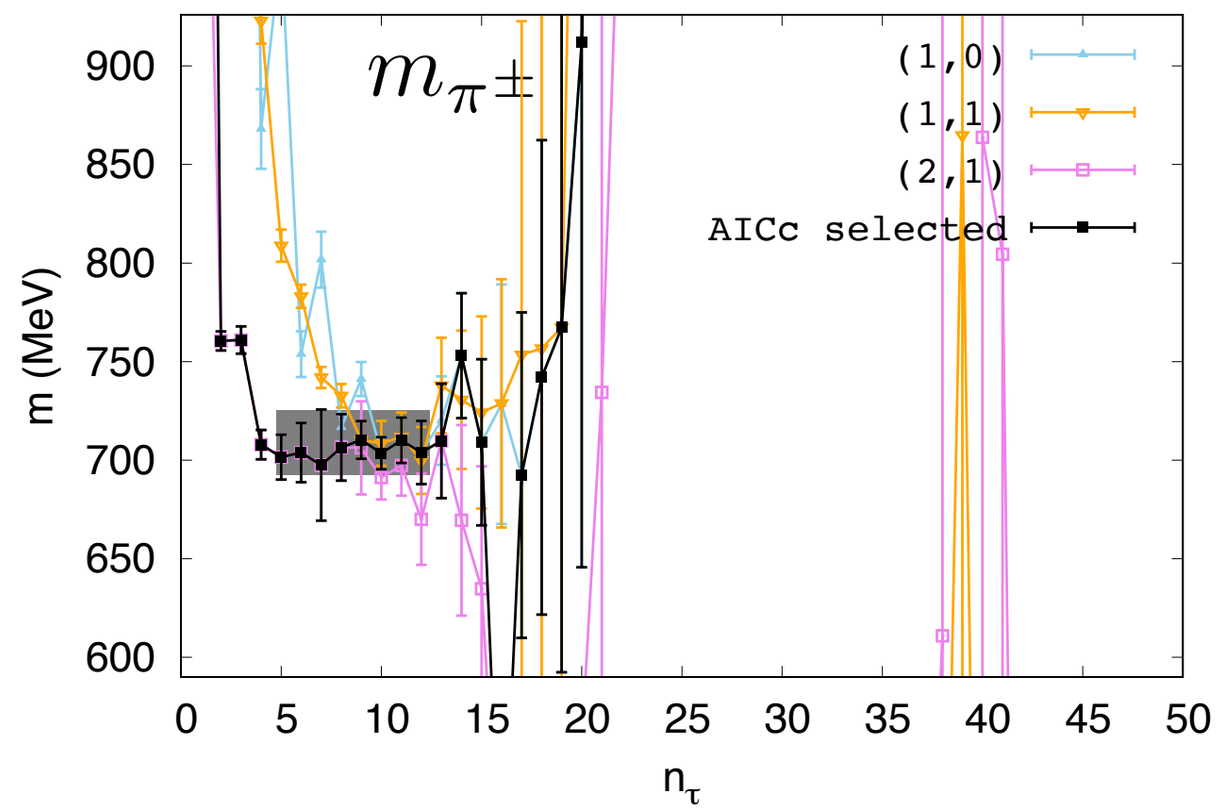


correlator fitting result



- non-monotonous behavior
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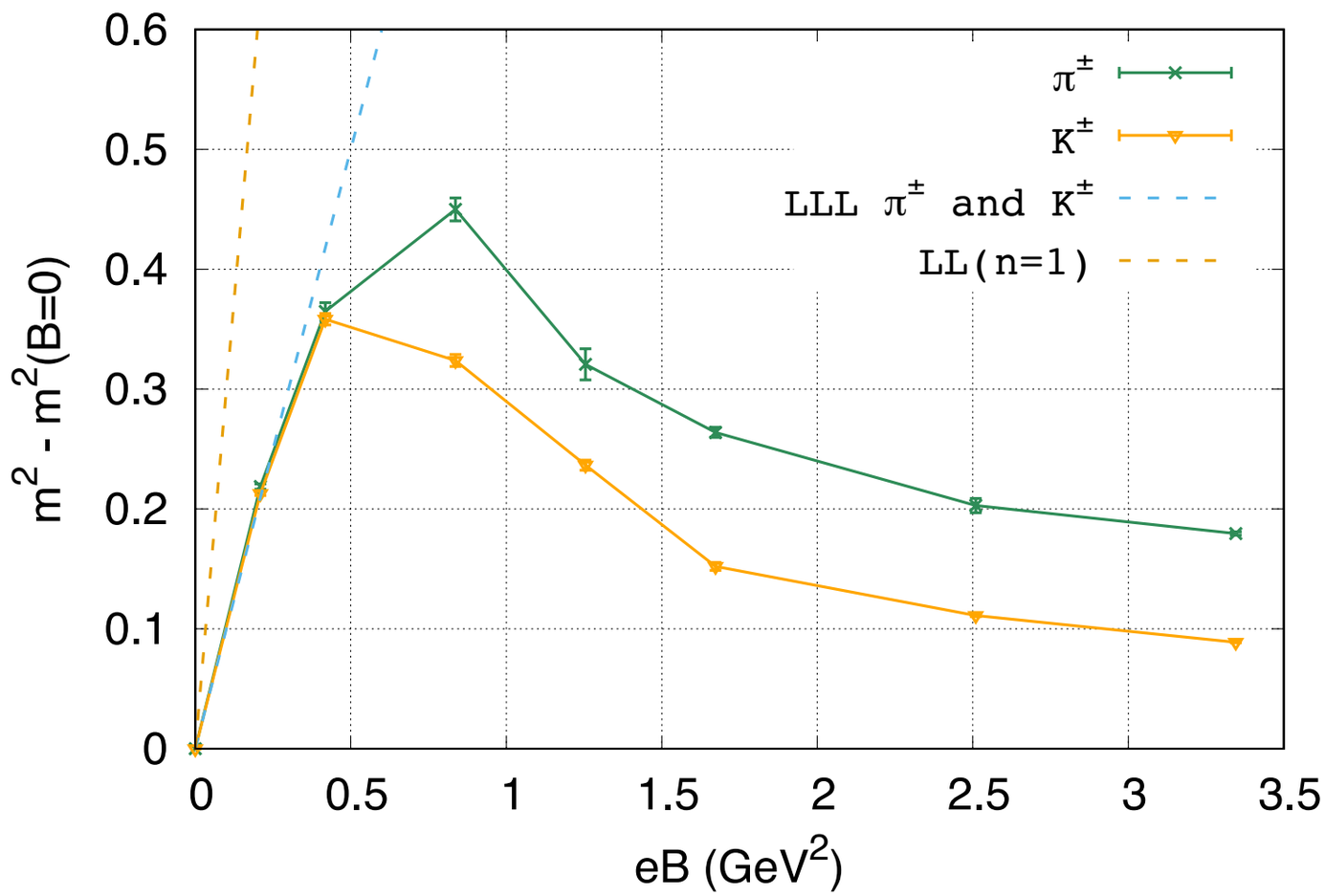
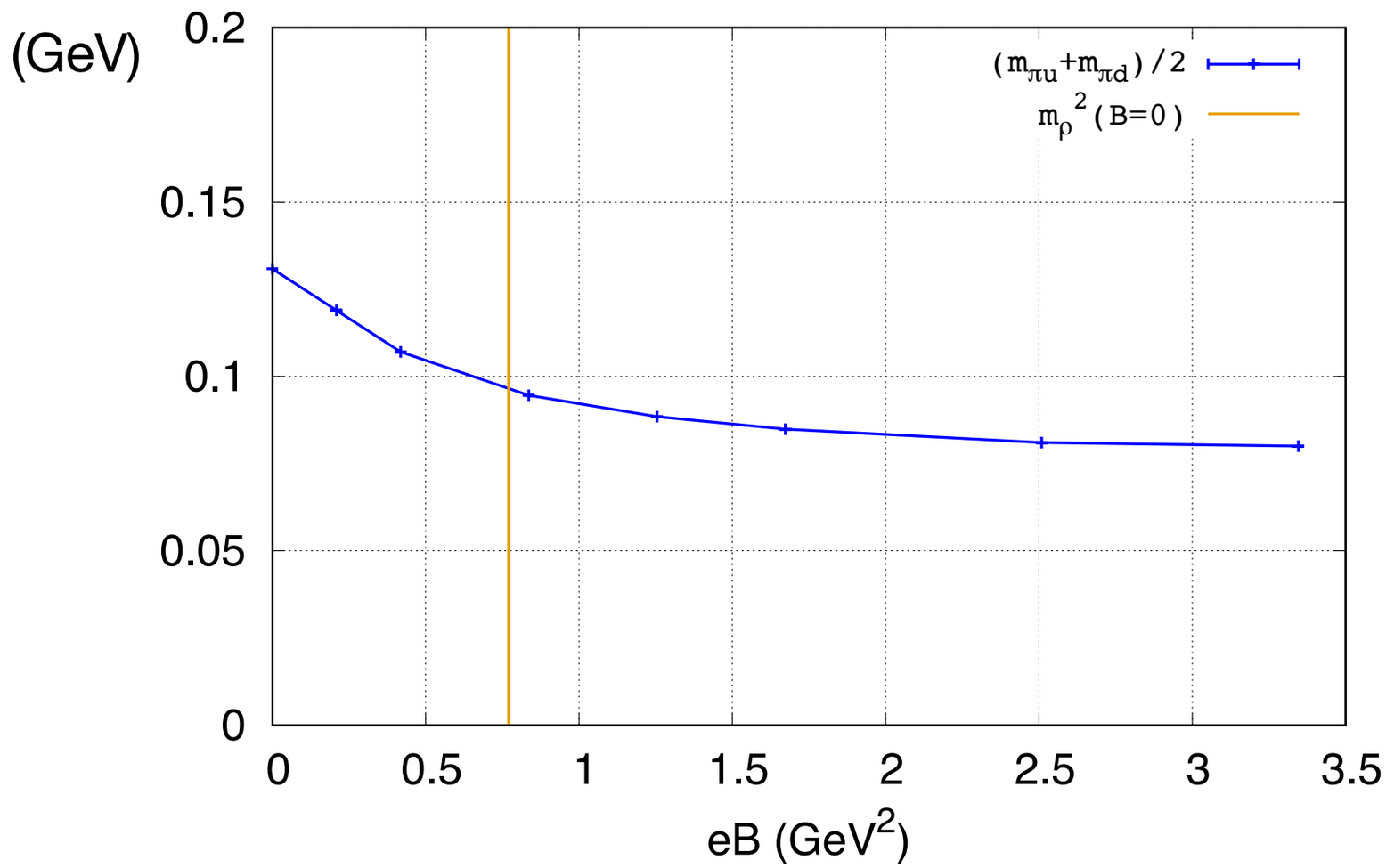
Charged Pseudo-scalar Particle Mass

$$LLL\ approx: E^2 = p_z^2 + 2n|qB| + m^2 \longrightarrow m^2_{\pi^\pm}(B) = m^2_{\pi^\pm}(B=0) + eB$$



- In very small eB regime, the mass of charged PS particle roughly agreed with the mesonic LLL approximation.
- As magnetic field grows larger, the mass of charged PS particle start decreasing.

ϱ-meson condensation



- QCD inequality

$$m_{\rho^\pm} \geq \left(m_{\pi_u^0} + m_{\pi_d^0}\right)/2$$

- NJL model

$$eB_c \sim m_\rho^2(B=0)$$

no ρ-meson condensation observed

Summary and Outlook

- Masses of neutral pseudo-scalar particles decrease as the magnetic field grows but saturate at large eB.

  ➡ Neutral particles interact with magnetic field.

- Charged PS particle showed non-monotonous behavior as eB increases.

- According to the mass relation between $\rho^{\pm}$ - meson with π_u^0 and π_d^0 , $\rho^{\pm}$meson cannot condensate.

  ➡ Superconductivity was not observed in current simulations.

- Try Z(2) Wall source to further improve the signal.
- Investigate vector particle masses and mixing of π and ρ.

Phase factor $\zeta(\vec{n})$ for staggered fermion

$$G(\tau) = -\sum_n \zeta(\vec{n}) \, \mathrm{Tr}\left[D^{-1\dagger}(n,0) D^{-1}(n,0)\right]$$

$\zeta(\vec{n})$ is the phase factor characterized Dirac and flavor index of staggered fermion

Phasefactor $\zeta(x)$	Γ non-osc.	Γ osc.	$\mathcal{J}^{\mathcal{PC}}$ non-osc.	$\mathcal{J}^{\mathcal{PC}}$ osc.	Particles non-osc.	Particles osc.	Channel
$(-1)^{x+y+z}$	$\gamma_4\gamma_5$	$\mathbb{1}$	0^{-+}	0^{++}	π	a_0	M_1
$+1$	γ_5	γ_4	0^{-+}	0^{+-}	π	-	M_2
$(-1)^{y+z}$	$\gamma_1\gamma_4$	$\gamma_1\gamma_5$	1^{--}	1^{++}	ρ_2	a_1	M_3
$(-1)^{x+z}$	$\gamma_2\gamma_4$	$\gamma_2\gamma_5$	1^{--}	1^{++}	ρ_2	a_1	M_4
$(-1)^{x+y}$	$\gamma_3\gamma_4$	$\gamma_3\gamma_5$	1^{--}	1^{++}	ρ_2	a_1	M_5
$(-1)^{x}$	γ_1	$\gamma_2\gamma_3$	1^{--}	1^{+-}	ρ_1	b_1	M_6
$(-1)^{y}$	γ_2	$\gamma_1\gamma_3$	1^{--}	1^{+-}	ρ_1	b_1	M_7
$(-1)^{z}$	γ_3	$\gamma_1\gamma_2$	1^{--}	1^{+-}	ρ_1	b_1	M_8