The order of phase transition in three flavor QCD with background magnetic field in crossover regime

RBRC
RIKEN BNL Research Center

Akio Tomiya (RIKEN-BNL)

Heng-Tong Ding (Central China Normal University)
Swagato Mukherjee (Brookhaven National Laboratory)
Christian Schmidt (Bielefeld University)

Outline

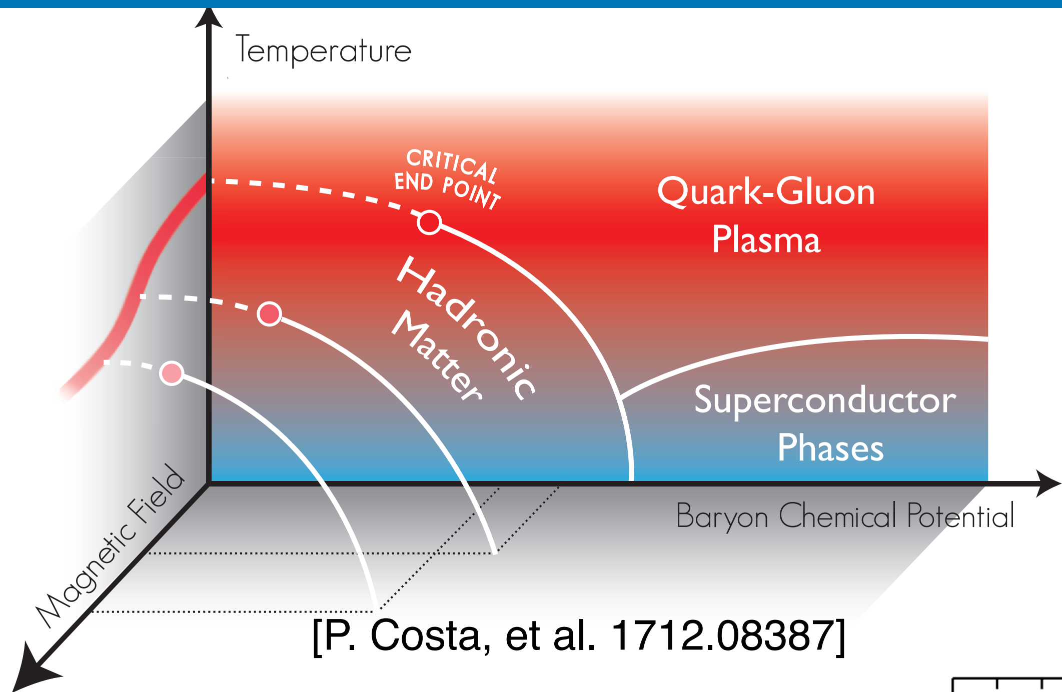
1. QCD phase transition with background magnetic field
2. Our numerical setup
3. Results
4. Summary

QCD phase transition with background magnetic field

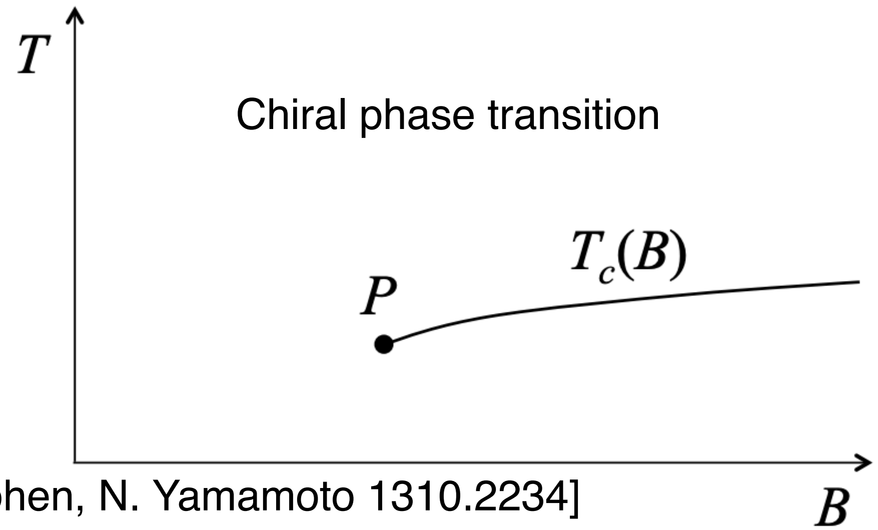
QCD phase transition?

Akio Tomiya

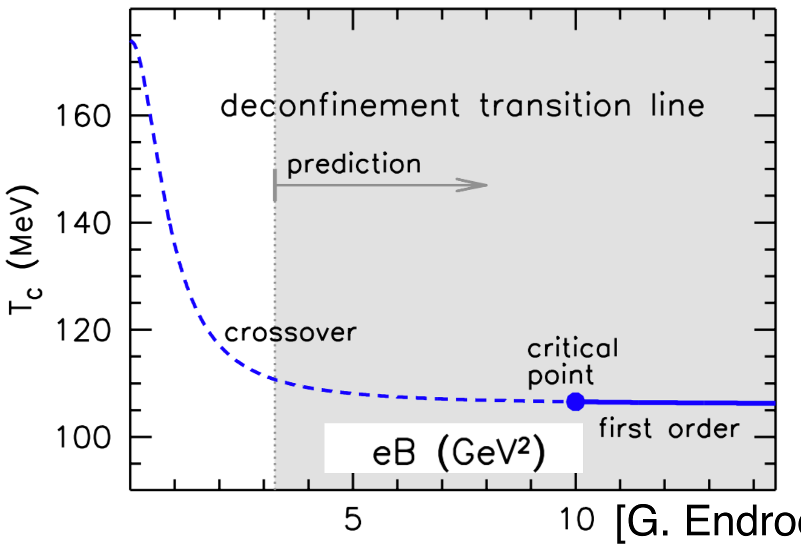
Background magnetic field could change the order



[P. Costa, et al. 1712.08387]



[T. Cohen, N. Yamamoto 1310.2234]



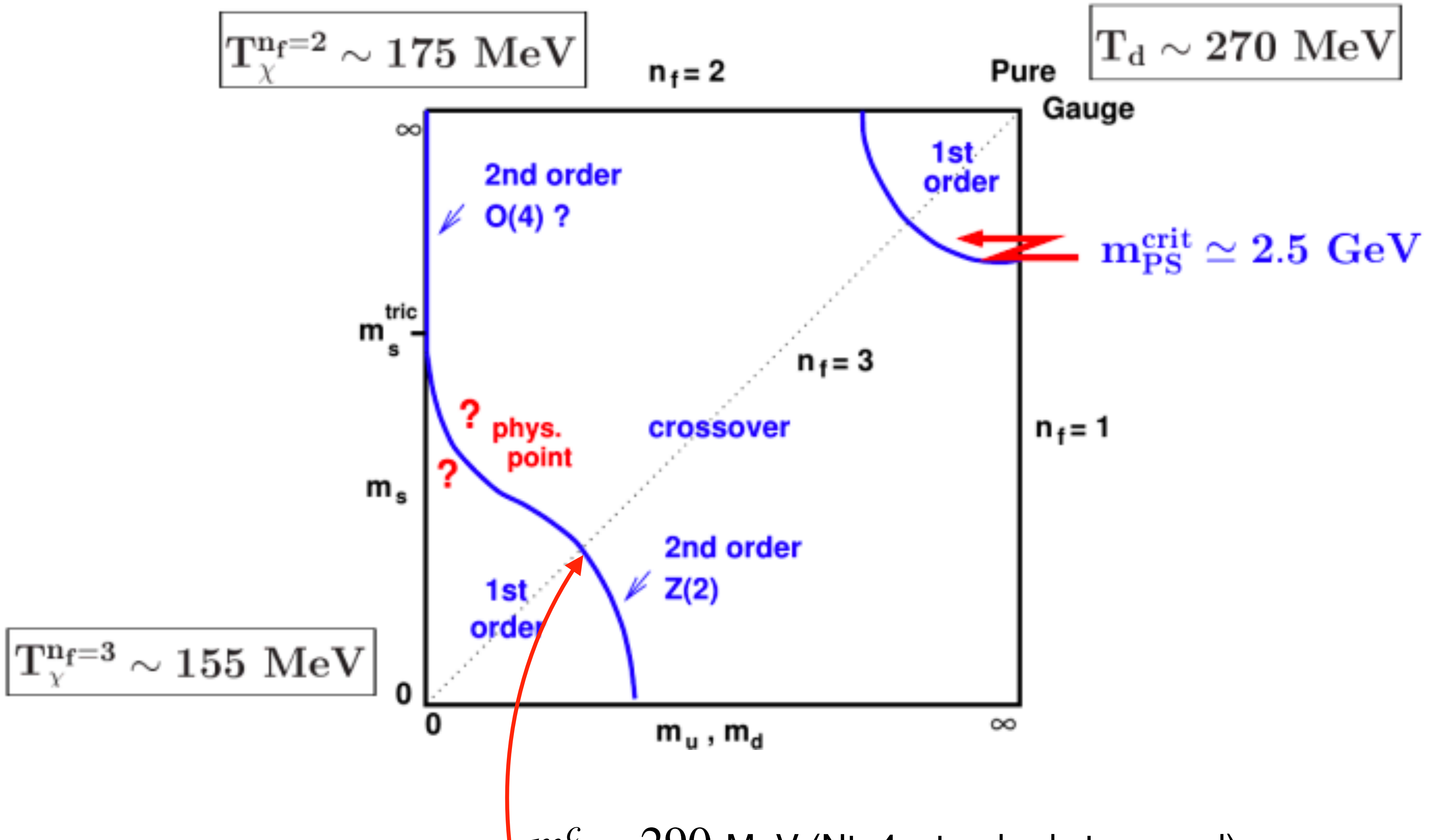
[G. Endrodi 1504.08280]

1. Magnetic field is a parameter of phase diagram
2. It does not have the sign problem but changes system
3. New Critical end point has been suggested for magnetic field

P. Klevansky, et al. PRD39 (1989)

QCD phase transition?

It depends on the number of light quarks & mass



$m_\pi^c \sim 290$ MeV (Nt=4, standard staggered)

$\Leftrightarrow am^c \sim 0.0273(4)$

Starting from slightly heavier than m^c for three flavor, does phase transition become stronger for eB>>0?

F. R. Brown, et al. Phys.Rev.Lett. 65 (1990), F. Karsch arXiv:hep-ph/0103314,P. Forcrand, et al. 1702.00330, D. Smith, et al. 1109.6729

QCD phase transition?

Akio Tomiya

Chiral condensates characterizes the transition for light quarks

At $m_{ud} = m_s = m \ll \Lambda_{QCD}$ **, QCD is characterized by,**

$$\langle \bar{\psi}\psi \rangle = \frac{\partial}{\partial m} \log Z(m)$$

It is a function of the inverse coupling β (~temperature). At m → 0 limit,

$$\begin{cases} |\langle \bar{\psi}\psi \rangle| = 0 & \text{Chiral symmetric phase (high temperature)} \\ |\langle \bar{\psi}\psi \rangle| \gg 0 & \text{Chiral symmetry breaking phase (low temperature)} \end{cases}$$

Binder cumulant = indicator of the <u>order</u> of phase transition

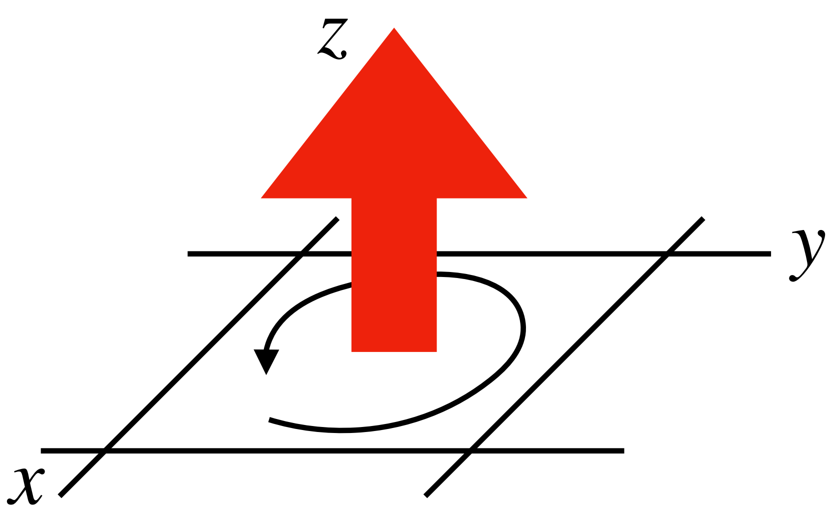
$$B_4^{\bar{\psi}\psi} = \frac{\langle \delta\Sigma^4 \rangle}{\langle \delta\Sigma^2 \rangle^2} \qquad \delta\Sigma = \bar{\psi}\psi - \langle \bar{\psi}\psi \rangle$$

1. At a critical point, B4 becomes scale invariant (independent to box size)
2. The value of B4 at a critical point indicates order of phase transition
    1. B4 = 3 Crossover (not critical)
    2. 1<B4<3 Second order phase transition, value depends on universality
    3. B4 = 1 First order phase transition

Magnetic field implementation

It is described by integer and it has maximum

Constant-uniform magnetic field along with z direction



$$u_x(n_x, n_y, n_z, n_t) = \begin{cases} \exp[-iqBN_x n_y] & (n_x = N_x - 1) \\ 1 & (\text{Otherwise}) \end{cases}$$

$$u_y(n_x, n_y, n_z, n_t) = \exp[iqBn_x],$$

$$u_z(n_x, n_y, n_z, n_t) = 1,$$

$$u_t(n_x, n_y, n_z, n_t) = 1.$$

$$S = \bar{\psi}(\not{D}[U] + m)\psi \rightarrow S(B) = \bar{\psi}(\not{D}[uU] + m)\psi$$

$$qB = a^{-2}\frac{2\pi N_b}{N_x N_y} \quad \text{with} \quad \begin{cases} 0 \leq N_b \leq \frac{N_x N_y}{4} \\ N_b \in \mathbb{Z} \end{cases}$$

We investigate the chiral phase & confinement transition with magnetic field, via the chiral condensate & Polyakov loops

Our numerical setup

Setup

3 volumes, several magnitude of magnetic field

Wilson plaquette gauge action + standard staggered fermions

3 mass degenerate quarks, mass = just above critical (ma=0.03)

Charge of quarks is assigned as same as the standard model, Q_u = 2/3, Q_d = Q_s =-1/3

$N_\sigma^3 x N_\tau$	$a\sqrt{eB}$	β	Statistics
8^3x4	0.0	5.130 - 5.160	~50k
8^3x4	0.9	5.160 - 5.180	~50k
8^3x4	1.2	5.170 - 5.195	~50k
16^3x4	0.0	5.130 - 5.160	~40k
16^3x4	0.9	5.160 - 5.180	~30k
16^3x4	1.2	5.170 - 5.190	~50k
24^3x4	0.0	5.140 - 5.160	~30k
24^3x4	0.44	5.130 - 5.160	~50k
24^3x4	0.9	5.165 - 5.175	30k-100k

Observables $\langle\bar\psi\psi\rangle$ $\chi_{disc}^{\bar\psi\psi}$ $B_4^{\bar\psi\psi}$ $B_4^{Polyakov}$

Machine: Fermilab NSC Nuclear Science Computing Center at CCNU

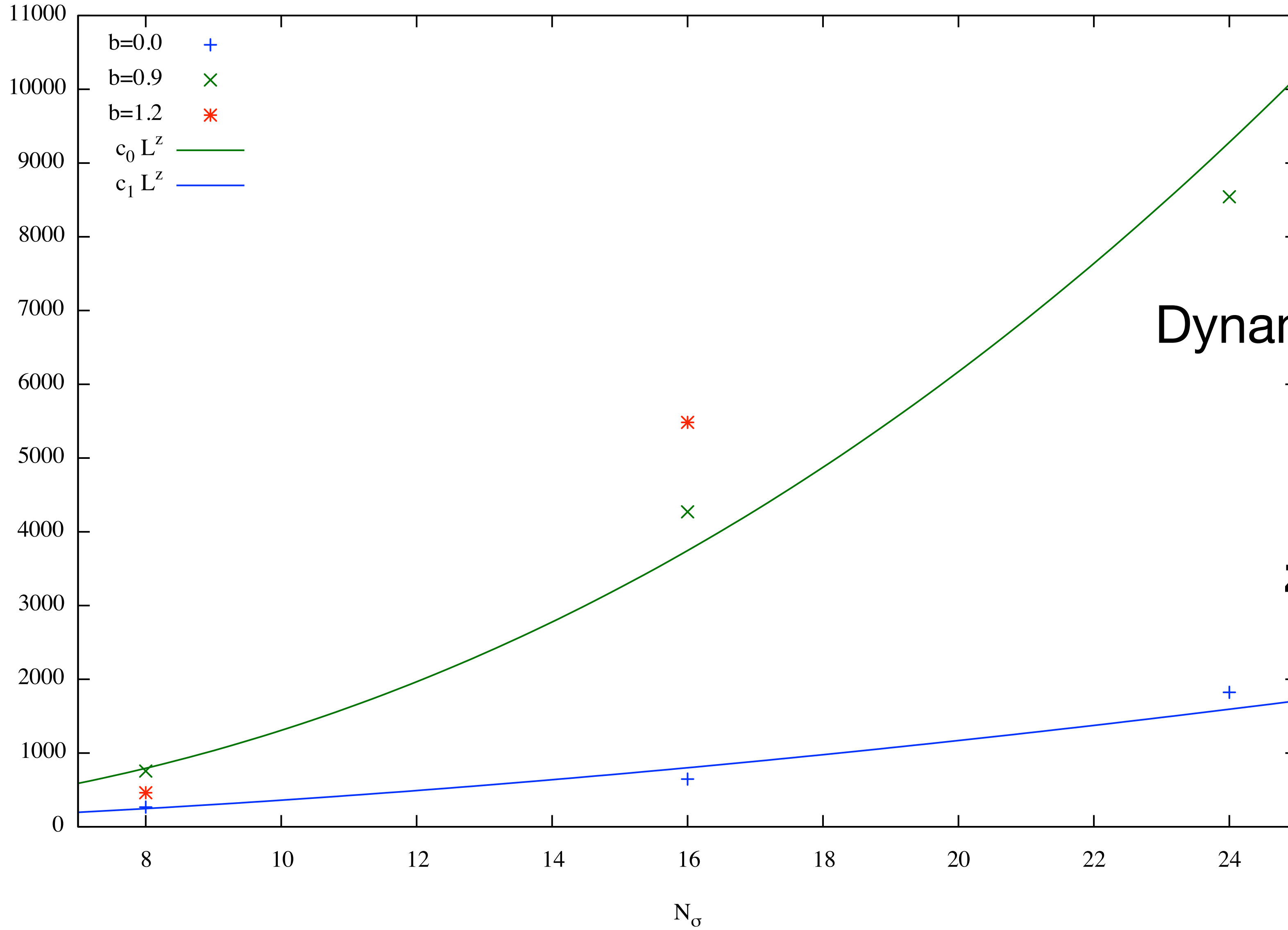
(New supercomputer in Wuhan)

Critical slowing down for eB > 0

We need more statistics, take care the Jackknife bin size

τ

(autocorrelation time at critical regime)



Dynamic critical exponent

$$\tau \sim \xi^z \sim L^z$$

z ~ 1.7 for √eB = 0.0

z ~ 2.2 for √eB = 0.9

Binning size N_{block} for Jackknife analysis is taken to be twice larger than the autocorrelation τ (if data available)

$$N_{block} \gtrsim 2\tau$$

$\tau(N_\sigma = 24) \sim 10$**k**

$\tau(N_\sigma = 16) \sim 5$**k**

We omit data for $N_\sigma = 24$ in final analysis

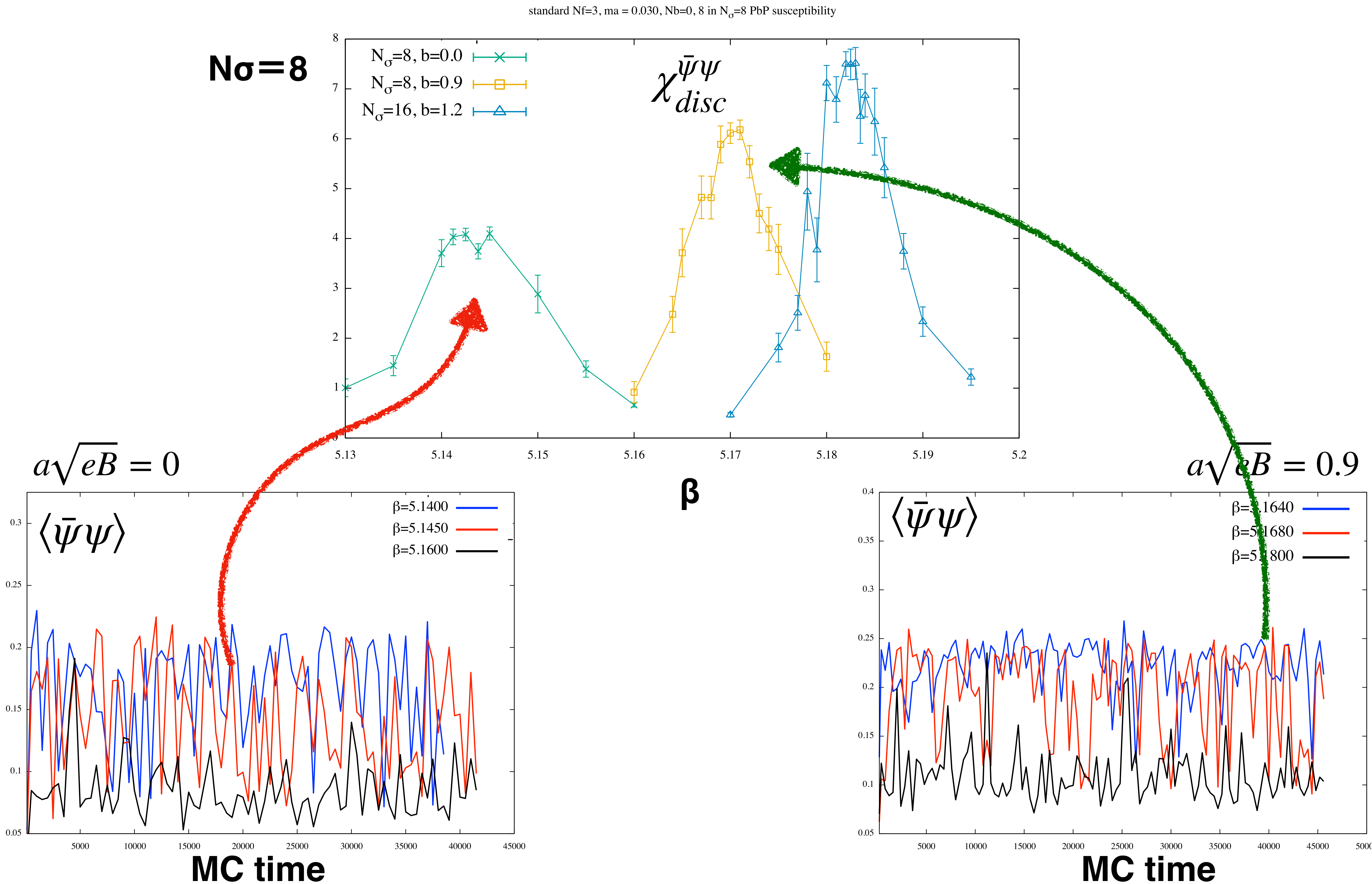
∵ Error bar is not reliable, $N_{indep}=N_{conf}/2\tau < O(10)$

Results

Akio Tomiya

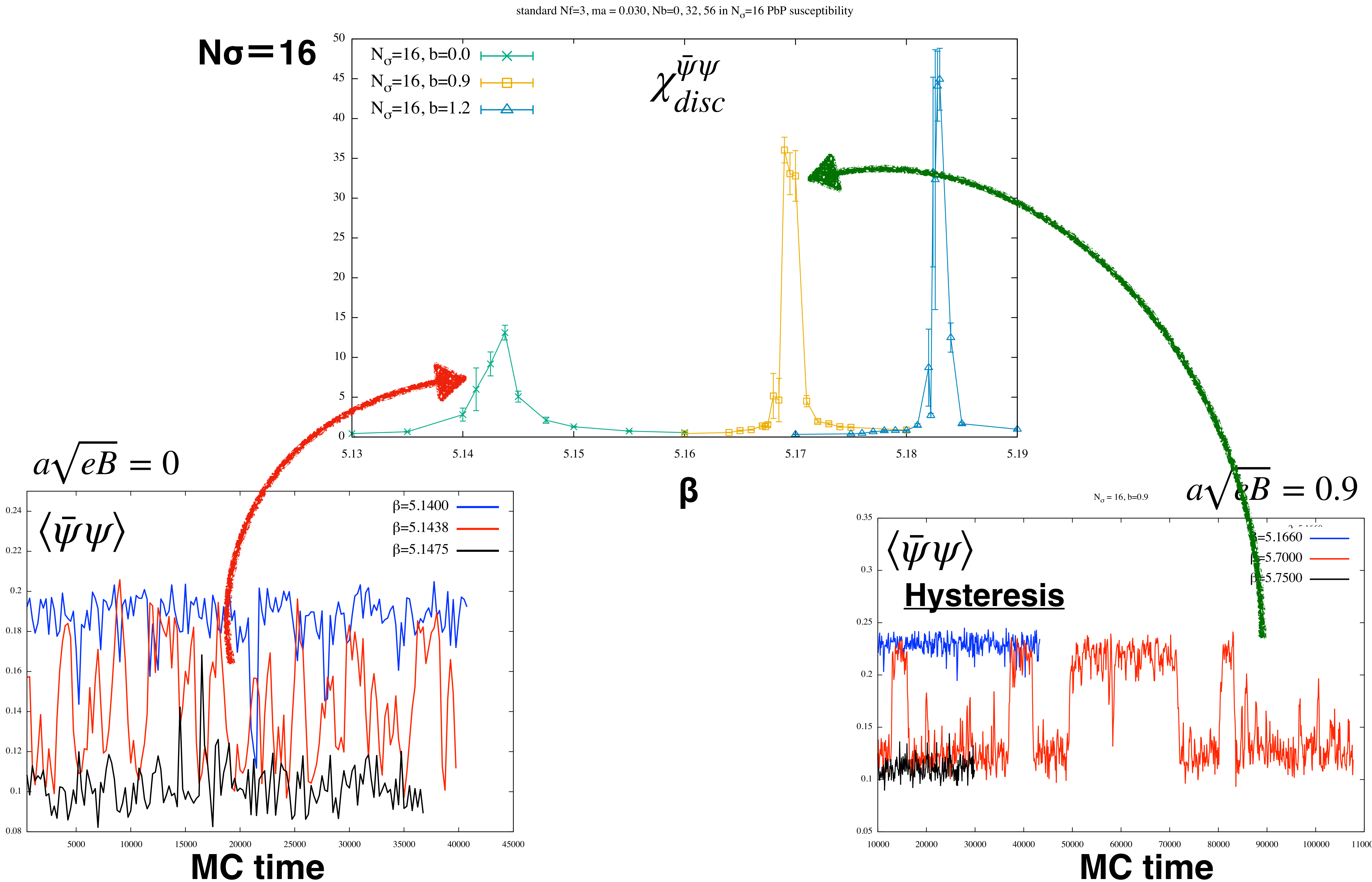
Results for Nσ=8

It shows hysteresis for eB>0.9 but it has finite volume effects



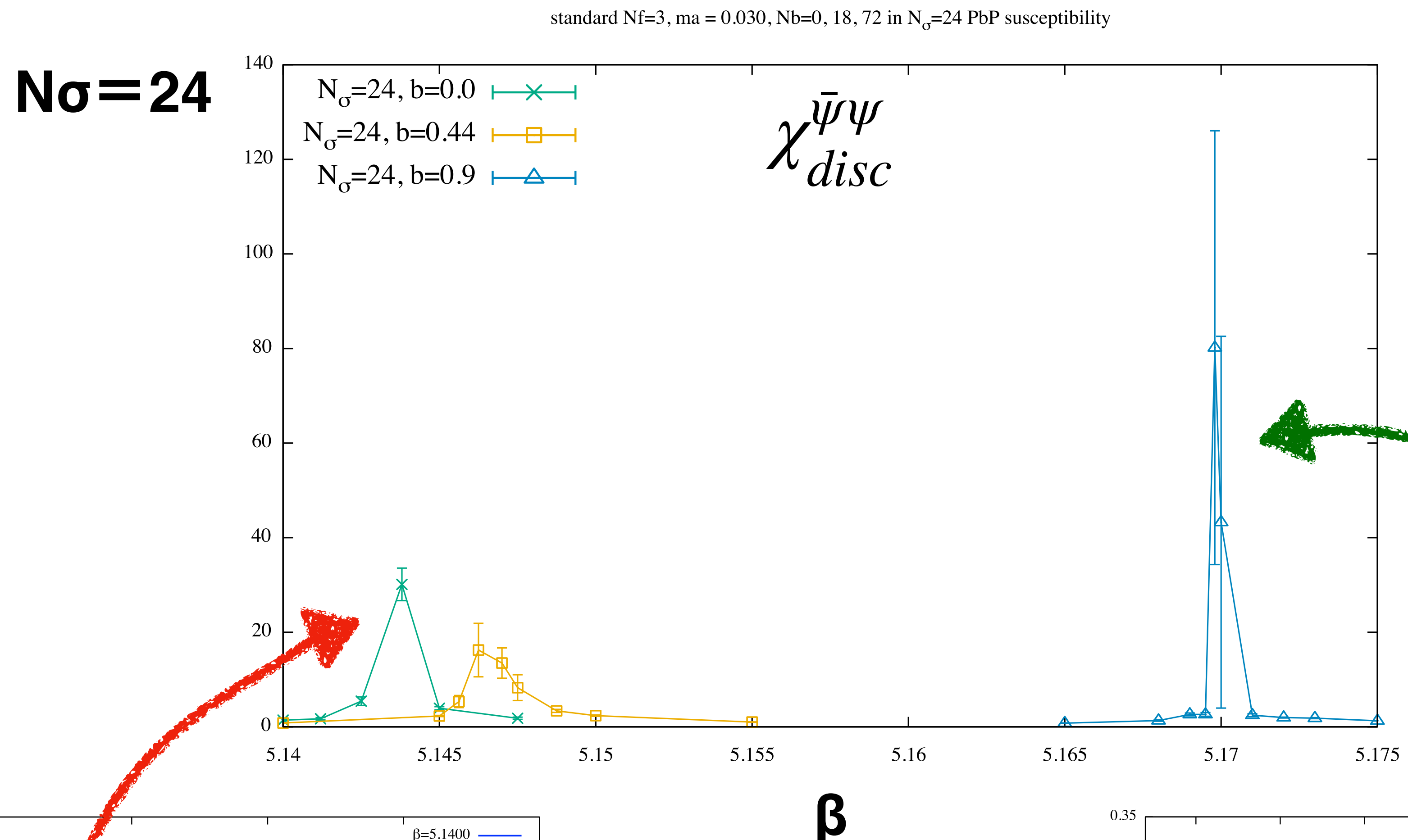
Results for Nσ=16

History for √eB>0.9 suggest first order like transition

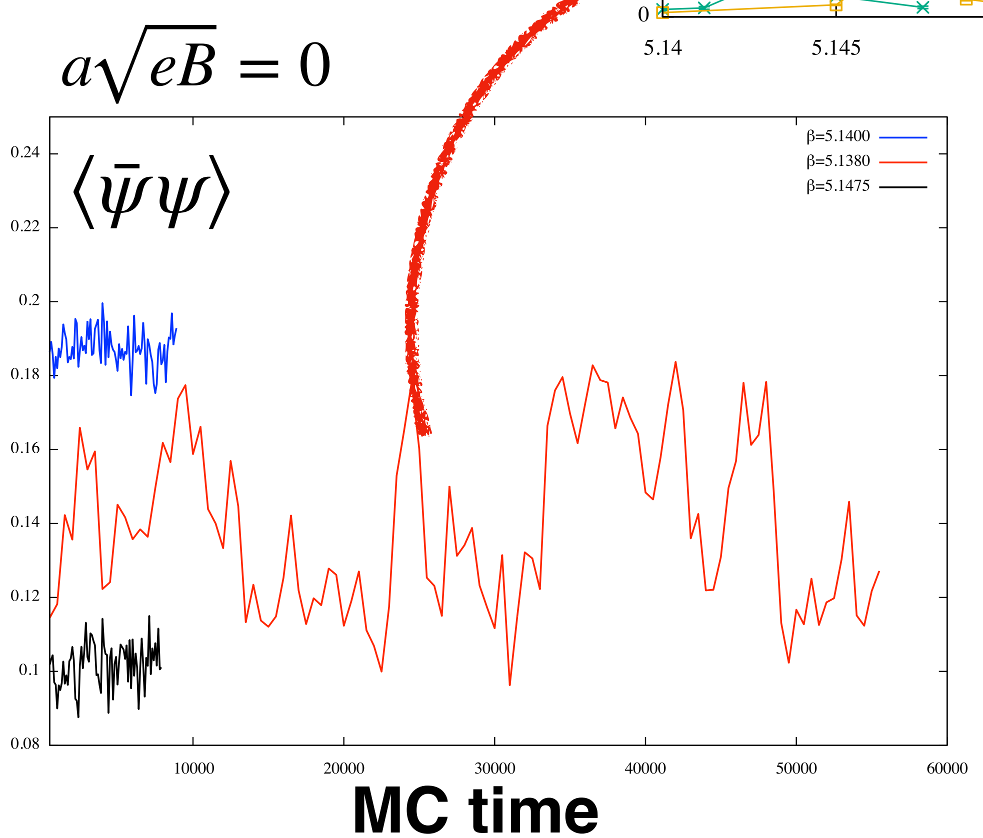


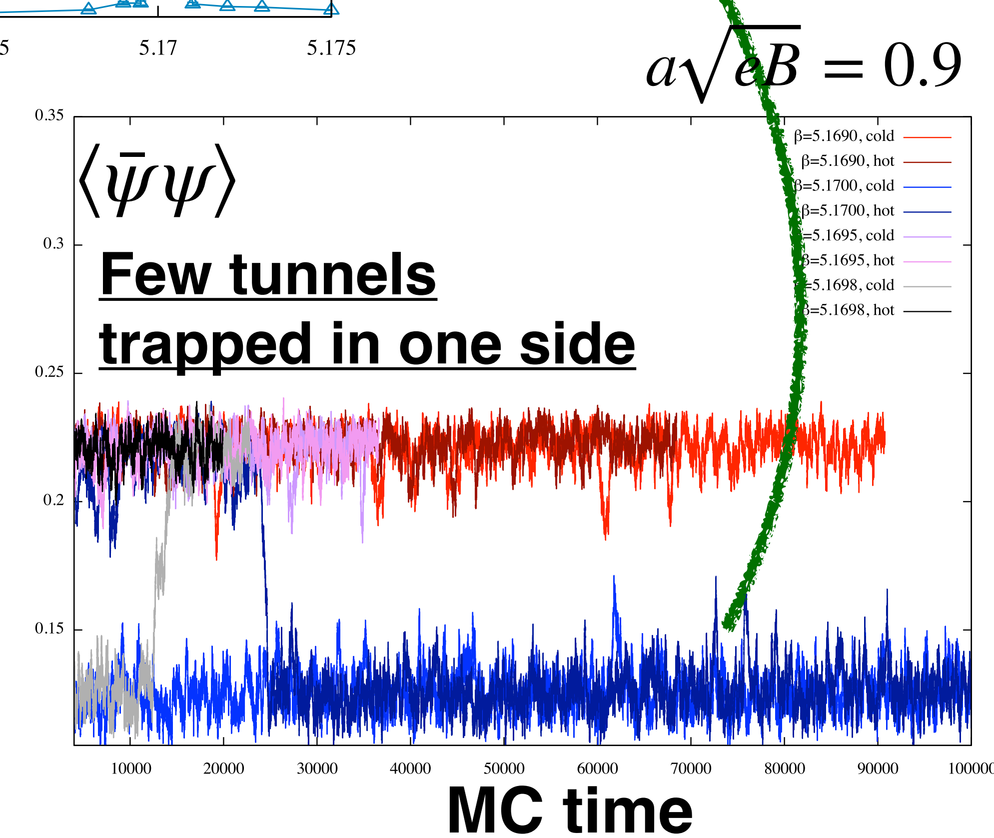
Results for Nσ=24

It has huge critical slowing down, short of statistics



Long autocorrelation deteriorates the results
→ <u>Omit from analysis</u>





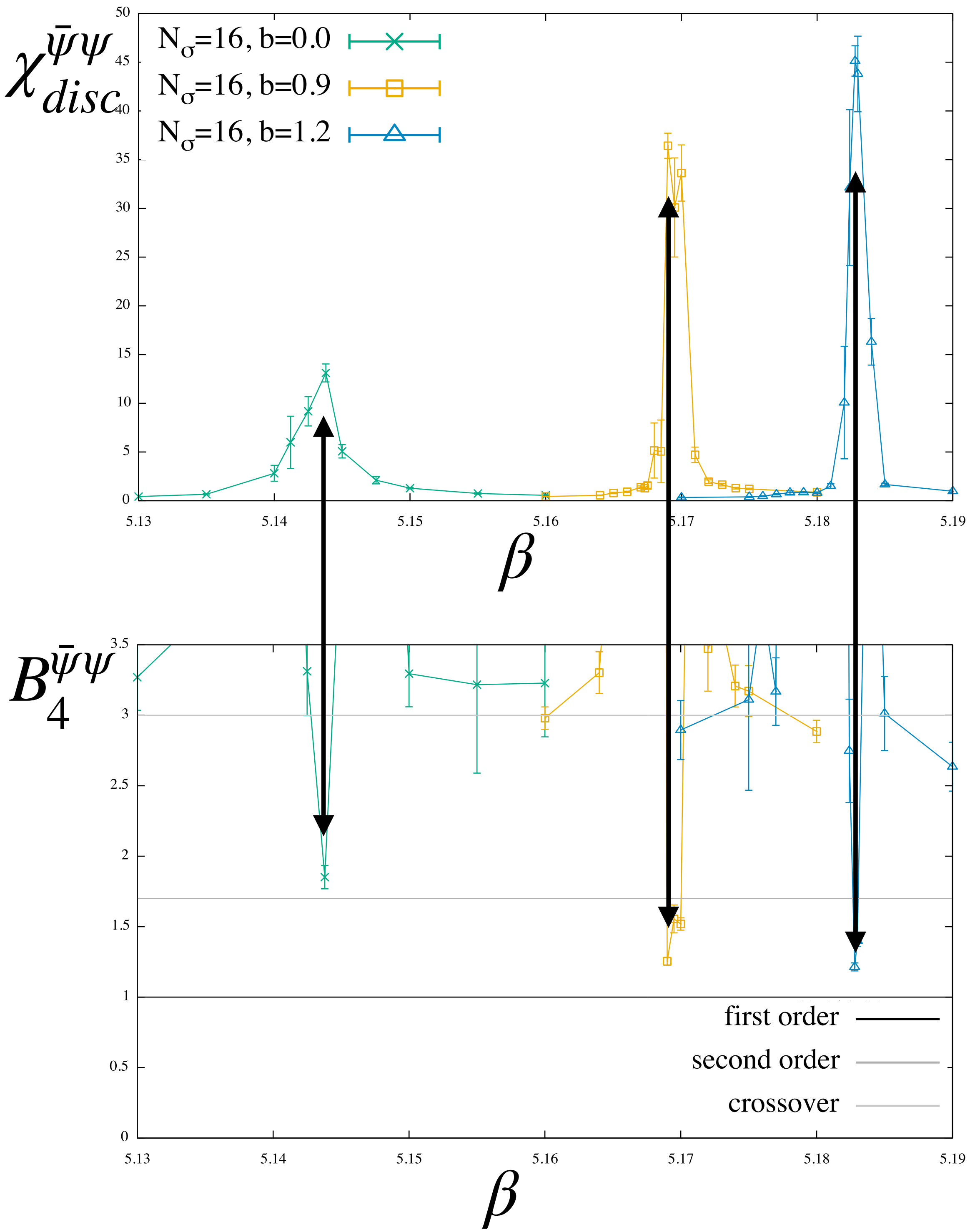
Binder cumulant analysis

Akio Tomiya

B4 is scale invariant at the critical regime

E.g. Nσ=16

$$\langle\bar{\psi}\psi\rangle = \langle\bar{\psi}\psi\rangle(N_\sigma, \beta, e\hat{B})$$



Analysis procedure, for one lattice size

1. Find a peak in susceptibility for (β, eB)
2. Calculate B4 for the corresponding point

$$B_4^{\bar{\psi}\psi} = \frac{\langle\delta\Sigma^4\rangle}{\langle\delta\Sigma^2\rangle^2} \qquad \delta\Sigma = \bar{\psi}\psi - \langle\bar{\psi}\psi\rangle$$

Repeat this analysis for all lattice size

→ Make a plot: B4(β*,Nσ, eB) v.s. eB for Nσ

Note: B4 is scale invariant at critical point

Following eq. determines critical value for the magnetic field = eB*

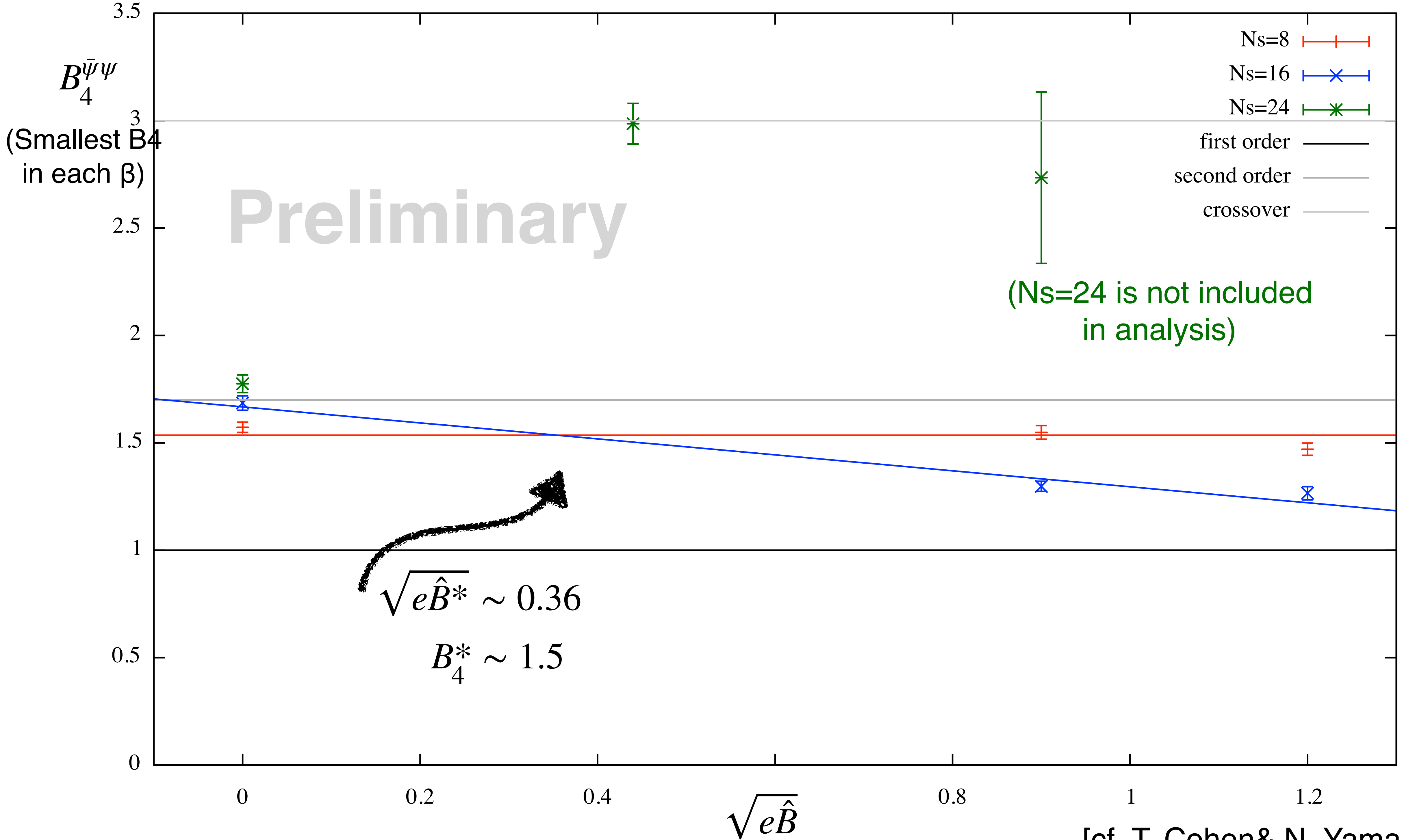
$$\begin{cases} B_4(\beta^*_{N_\sigma=8}, N_\sigma = 8, a\sqrt{eB}) \\ B_4(\beta^*_{N_\sigma=16}, N_\sigma = 16, a\sqrt{eB}) \end{cases}$$

(We omit Nσ = 24 data for analysis)

Binder cumulant for chiral cond.

It suggests stronger transition than eB = 0. a√eB* ~ 0.36

$am_c^{2nd} \lesssim am = 0.03$ (crossover for chiral, eB=0, V→∞)



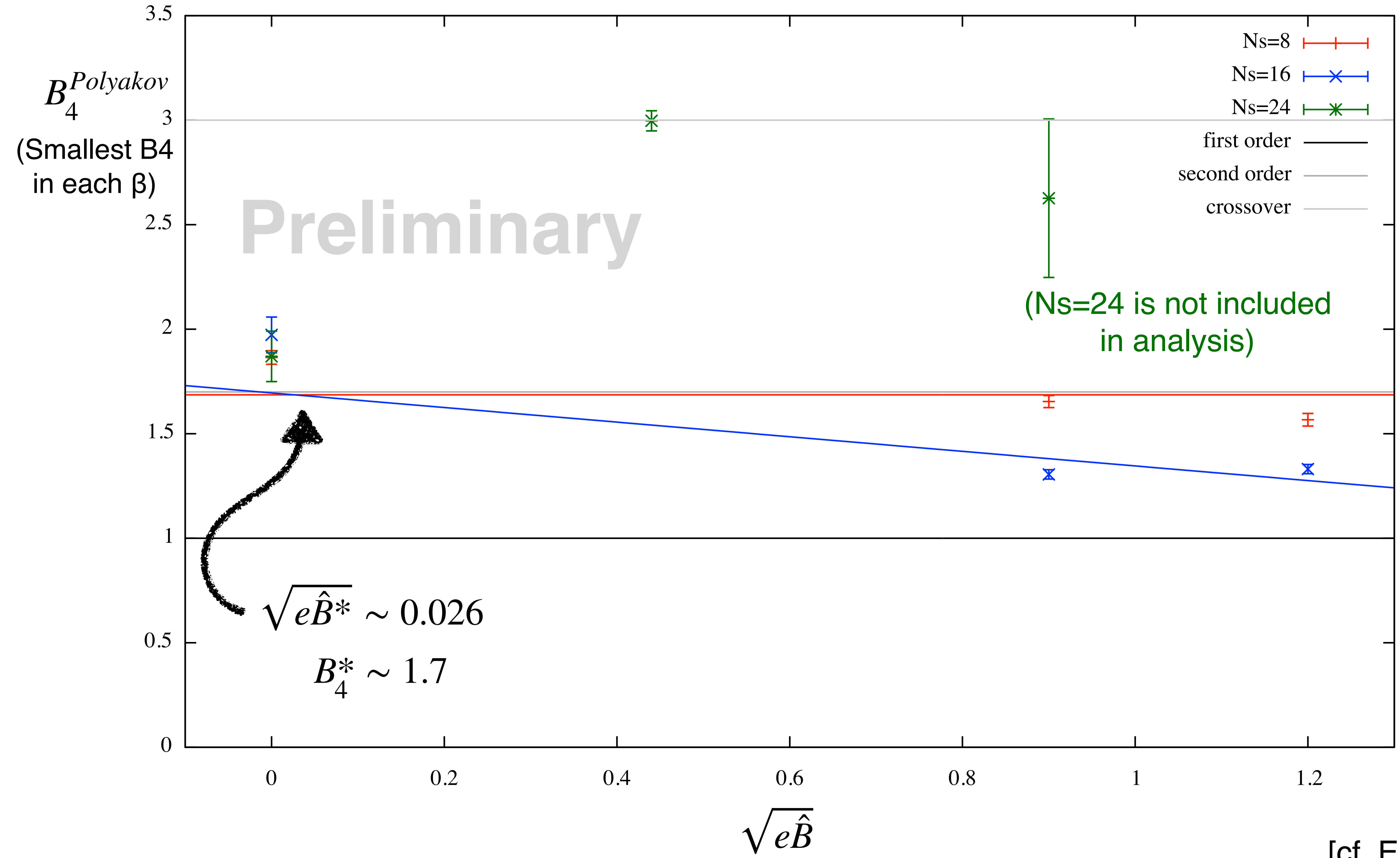
[cf. T. Cohen& N. Yamamoto 2013]

Binder cumulant for Polyakov loop

Akio Tomiya

It suggests stronger conf/deconf. transition than eB = 0. a√eB* ~ 0.026

$am_c^{2nd} \lesssim am = 0.03$ (crossover for chiral, eB=0, V→∞)



[cf. Endrodi 2017]

Summary

1. Standard staggered, Wilson plaquette in 3 flavor crossover mass regime.
2. Systematic study:
   Dependence of QCD phase transition on eB and Volume
3. Longer autocorrelation for eB>>0 → We are in a critical regime
4. Binder cumulant suggests, eB direction has a critical endpoint
   →This needs to confirm by collecting ~ 300k trajectories(?)
5. N_σ=24 has huge critical slowing down → N_σ=20 instead?
   → We can improve the Binder cumulant analysis
6. Does first order survive for the continuum limit?
   → Further studies are needed (by HISQ or see N_τ dep.)
7. To overcome critical slowing down, multi-canonical algorithm might be useful. Multipoint reweighting helps to find critical point

Backup