Topological component of Yang-Mills fields: from lattice to collider

Jinfeng Liao
The QCD Vacuum: Confinement

The missing particles: quarks & gluons (in the QCD lagrangian) are not seen in physically observed states.

Free Quark Searches

All searches since 1977 have had negative results.

QCD vacuum as “dual superconductor”?! [’t Hooft, Mandelstam, Nambu, Polyakov, …]
What are the most important/relevant configurations/DoFs for enforcing confinement?

Two strategies:
1. Use real computers with brute force
2. Effective models that start with the right DoFs

What are the most important/relevant configurations/DoFs for enforcing confinement?

\[ Z = \int D[A_\mu] e^{-S} \]
The Mystery of Yang-Mills Fields

* Gluon fields hold the key — studying pure Yang-Mills first

* Topological configurations are important — instantons?

* Magnetic objects — easy to identify with adjoint scalars, but not QCD or pure YM

* E-M duality — demonstrated in e.g. Seiberg-Witten

New developments: instantons with non-trivial holonomy
[Krann-von-Baal; Lee-Lu: KvBLL]
— promising for confinement in pure YM (and QCD)
Confinement from Correlated Instanton-Dyon Ensemble

Pioneering works from:
Diakonov et al; Shuryak, Zahed, Larsen, et al

Formulating the Problem: Polyakov Loop

Polyakov loop:

\[ \mathcal{L}[A_\mu] = \hat{\mathcal{P}} \exp \left( i \int_0^{1/T} \! \! dx_4 \ A_4(\vec{x}, x_4) \right) \]

\[ L \equiv \left\langle \frac{1}{N_c} \ Tr \ \mathcal{L} \right\rangle \]

Here we focus on SU(2) pure YM

\[ \text{Deconfined} \]

\[ \text{Confined} \]

Digal, Fortunato, Petreczky, PRD68(034008)2003
Formulating the Problem: Holonomy

**Introducing holonomy:**

$$L_\infty \equiv \mathcal{L}[A_\mu] \bigg|_{|\vec{x}| \to \infty}$$

$$\mathcal{L}_\infty = \text{diag}(e^{-i\pi h}, e^{i\pi h}) \quad h \in [0, 1]$$

$$L_\infty = \frac{1}{2} \text{Tr} \mathcal{L}_\infty = \cos(\pi h)$$

$$L_\infty = 0 \iff h = \frac{1}{2}$$

$$L_\infty = 1 \iff h = 0$$

**Confining holonomy**

**Trivial holonomy**

In pure YM/QCD it is holonomy that can play a role like the adjoint scalar.
Formulating the Problem: Holonomy Potential

\[ Z = \int [DA_\mu] e^{-S_E} \]

\[ \rightarrow \int dh \left\{ \int [DA_\mu^h] e^{-S_E} \right\} = \int dh \ e^{-U[h]V/T} \]

\[ U[h] \text{ or } U[L_\infty] \]

Holonomy potential

The holonomy value that minimizes holonomy potential is physically realized.
Perturbative Holonomy Potential

\[ \mathcal{U}_{G_{\text{PY}}}^I = \frac{4\pi^2}{3} T^4 \ h^2 \bar{h}^2 \]

\[ \bar{h} \equiv 1 - h \]

It must be nonperturbative, topological configurations that drive the system toward confining nontrivial holonomy!
Ensemble of Topological Objects

Classifying gauge field configurations according to holonomy:

\[ Z = \int [\mathcal{D}A_\mu] e^{-S_E} \rightarrow \int dh \left\{ \int [\mathcal{D}A_\mu^h] e^{-S_E} \right\} \]

Nonperturbative contributions from topological sector

\[ \sim \sum_{N_{topo}} e^{-S_{N_{topo}}} \]

[But, what types of topological objects???]

\[ \rightarrow \int dh e^{-U[h]V/T} \]

which would correctly enforce confining holonomy
What Are the Right DoFs?

* **Topological object**
  (—nonperturbative, important at strong coupling)

* **Magnetically charged**

* **Sensitive to holonomy**

Instantons with nontrivial holonomy
— KvBLL calorons! (constructed ~1998)
KvBLL Calorons

\[ S_{YM} = \frac{1}{2g^2} \int d^4x \text{Tr} \, F_{\mu\nu} F_{\mu\nu} \quad \text{where} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu] \]

\[ Q_T = \frac{1}{16\pi^2} \int d^4x \text{Tr} \, F_{\mu\nu} F_{\mu\nu} = \pm 1 \]

\[ S = \frac{8\pi^2}{g^2} \]

\[ A_{KvBLL}^\mu = \delta_{\mu4} \tau^3 \frac{\tau_3}{2} + \frac{\tau^3}{2} \eta_{\mu\nu} \partial_\nu \log \Phi + \frac{\Phi}{2} \text{Re} \left[ (\bar{\eta}^{1}_{\mu\nu} - i\bar{\eta}^{2}_{\mu\nu}) (\tau^1 + i\tau^2) (\partial_\nu + iv\delta_{\nu4}) \tilde{\chi} \right] \]

\[ v = \left( 2\pi T \right) h \]

**Most interestingly:**

this object is made of

\text{Nc constituent dyons (monopoles)}!!!
## Properties of Instanton-Dyons

<table>
<thead>
<tr>
<th></th>
<th>( M )</th>
<th>( \bar{M} )</th>
<th>( L )</th>
<th>( \bar{L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric charge</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>Magnetic charge</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>( h )-charge</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>Action</td>
<td>( h \frac{8\pi^2}{g^2} )</td>
<td>( h \frac{8\pi^2}{g^2} )</td>
<td>( \bar{h} \frac{8\pi^2}{g^2} )</td>
<td>( \bar{h} \frac{8\pi^2}{g^2} )</td>
</tr>
<tr>
<td>Size</td>
<td>((2\pi Th)^{-1})</td>
<td>((2\pi Th)^{-1})</td>
<td>((2\pi T\bar{h})^{-1})</td>
<td>((2\pi T\bar{h})^{-1})</td>
</tr>
</tbody>
</table>

\[ h = \frac{1}{2} \]

\[ h = \frac{1}{4} \]

**Instanton-dyons are sensitive to holonomy!**

**Holonomy acts as effective “Higgsing”**.
Building Ensemble of Instanton-Dyons

\[ Z = \int [DA_\mu] e^{-S_E} \]

\[ \rightarrow \int dh \left\{ \int [DA_\mu^h] e^{-S_E} \right\} \]

\[ Z_{h}^{\text{dyon}} = e^{-U_{G_{PY}}^{II}(h)} \frac{V}{T} \sum_{N_M, N_L, N_L, N_M} \frac{1}{N_L!N_M!N_L!N_M!} \int \prod_{l=1}^{N_L} f_L T^3 \, d^3r_{L_l} \prod_{m=1}^{N_M} f_M T^3 \, d^3r_{M_m} \]

\[ \times \prod_{\tilde{l}=1}^{N_L} f_{\tilde{l}} T^3 \, d^3r_{\tilde{L}_{\tilde{l}}} \prod_{\tilde{m}=1}^{N_M} f_{\tilde{M}} T^3 \, d^3r_{\tilde{M}_{\tilde{m}}} \det(G_D) \det(G_D^\dagger) e^{-V_{\Sigma_{\Sigma}}} \]

In short: sum over a statistical ensemble of many L & M Instanton-dyons with interactions (with 2-loop perturbative quantum fluctuations included)
Dyon-anti-Dyon Correlations

“Gas” ensemble: negligible correlations;
“Liquid” ensemble: significant short range correlations

$\Rightarrow \Rightarrow$ Properties of the ensemble crucially depend on such correlations!

\[
\mathcal{Z}_{h}^{dyon} = e^{-U_{GPY}^{IJ}(h)} \frac{V}{T} \sum_{N_M, N_L, N_M} \frac{1}{N_M!N_M!N_L!N_M!} \int \prod_{l=1}^{N_L} f_L T^3 \, d^3 r_{L_l} \prod_{m=1}^{N_M} f_M T^3 \, d^3 r_{M_m} \times \prod_{l=1}^{N_L} f_L T^3 \, d^3 r_{L_l} \prod_{m=1}^{N_M} f_M T^3 \, d^3 r_{M_m} \det(G_D) \det(G_D) e^{-V_{DD}},
\]

Implemented via interaction potential energy term
The Correlation Potential

Long range correlations are dictated by the charges of the objects:

\[ V_{\text{long}} = (e_i e_j + m_i m_j - 2h_i h_j) \frac{S}{2\pi T} \frac{e^{-M_D r_{ij}}}{r_{ij}}. \]

Short range correlations are mimicked with repulsive core:

\[ V_{\text{short}} = \frac{c_h V_c}{1 + e^{(2\pi T)r_{ij}c_h - \zeta_c}}, \quad \text{for} \quad r_{ij} < \frac{\zeta_c}{(2\pi T)c_h}, \]

Two key parameters:
Strength \( V_c \), and force range \( \zeta_c \)

A repulsive core potential is crucial for enforcing confinement!
[first shown by Shuryak and collaborators]
The Holonomy Potential

A change of shape from high to low $T$!

Denser $\rightarrow$ Short range correlations become really important!

Weaker coupling

Stronger coupling

Diluter
Confinement dynamics is sensitive to the short range correlations.

Key parameter is the range parameter of the core.

Quantitatively viable for describing lattice data.
Density of the Topo Component

\[ \rho T^{-3} \]

[Graph showing data points and curves labeled \( \zeta_c = 1.8 \), \( \zeta_c = 2.2 \), and \( \zeta_c = 2.4 \).]
Topological Component:
Implications for Collider Phenomenology
The Big Machines

Boiling a quark-gluon plasma in lab routinely

A nearly perfect liquid — strongly coupled, sQGP
So, what are the right degrees of freedom?

**The old belief**

- Tc
- Strongly coupled confined phase
- Asymptotically free QGP

**The new paradigm thanks to discoveries at RHIC and LHC (1~3Tc):**

- Tc
- ~3Tc
- Strongly coupled confined phase
- QGP (sQGP)
- Asymptotically free QGP

The matter just above confinement (in 1~3Tc), is more closely related to the confined world, rather than to the asymptotic QGP!

A “postconfinement” regime?!

What are the DoFs???
Liberation of Color? Missing DoF?

Degrees of freedom

A region around \( T_c \) with liberated degrees of freedom but only partially liberated color-electric objects. (Pisarski & collaborators: semi-QGP)

Then what are the “extra” dominant DoF here???

Thermal monopoles evaporated from vacuum condensate!
Condensate monopoles → dense thermal monopoles near $T_c$: thermal monopoles play key role in this regime.

PHYSICAL REVIEW C 75, 054907 (2007)

Strongly coupled plasma with electric and magnetic charges

Jinfeng Liao and Edward Shuryak
The magnetic component significantly enhances the scattering, needed for understanding the experimentally observed QGP transport properties.

\[
\alpha_E \times \alpha_M = 1.
\]
Magnetic Quenching of (Electric) $q/g$ Jets

**Magnetic component helps resolve a puzzle in jet energy loss!**

In-Plane

Out-of-Plane

Compilation of J. Jia, ~2008

\[ R_{aa}(\phi) \]

\[ I_{in} < I_{out} \Rightarrow (R_{aa})_{in} > (R_{aa})_{out} \]

Angular Dependence of Jet Quenching Indicates Its Strong Enhancement near the QCD Phase Transition

Jinfeng Liao$^{1,2,*}$ and Edward Shuryak$^{1,**}$

$^{1}$Department of Physics and Astronomy, State University of New York, Stony Brook, New York 11794, USA

$^{2}$Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

(Received 22 October 2008; revised manuscript received 19 February 2009; published 22 May 2009)
Magnetic Quenching of (Electric) q/g Jets

Magnetic component helps resolve a puzzle in jet energy loss!

In the paper PRL(2009) we concluded:

“In relativistic heavy ion collisions the jets are quenched about 2--5 times stronger in the near-Tc region than the higher-T QGP phase.”

— Evidence for Magnetic DoFs!
An entirely new era of jet energy loss modeling: CUJET3 based on semi-Quark-Gluon-Monopole Plasma (sQGMP)

— quantitatively describe single hadron jet energy loss data for $R_{AA}$ and $v_2$ @ RHIC+LHC, for light/heavy flavors.

— magnetic component is crucial for phenomenology.

From Gluon Topology to Quark Chirality

\[ Q_w = \frac{1}{32\pi^2} \int d^4x \left( gG_{\mu\nu}^a \right) \cdot \left( g\tilde{G}_{\mu\nu}^a \right) \]

**QCD anomaly: gluon topology \( \rightarrow \) chirality imbalance**

\[ N_R - N_L = N_5 = 2Q_w \]

**Chiral Magnetic Effect (CME)**

\[ \vec{J} = \frac{Q^2}{2\pi^2} \mu_5 \vec{B} \]

**In heavy ion collisions: extremely strong B field \( \rightarrow \) observable signal from CME \( \rightarrow \) charge-dependent azimuthal correlations**
Search for CME in Heavy Ion Collisions

Evidences are accumulating, but not conclusive (yet).

[STAR PRL2014]

Exciting opportunity of discovery: Isobaric Collision @ RHIC 2018 Run — stay tuned!
Summary
Summary

— Topological component could quantitatively explain YM confinement.

— Topological component is crucial for understanding transport properties and jet energy loss observables in heavy ion collisions.

— Search for topological component via Chiral Magnetic Effect.