[55] *Lattice study of meson properties at fine temperature using the truncated overlap fermions*

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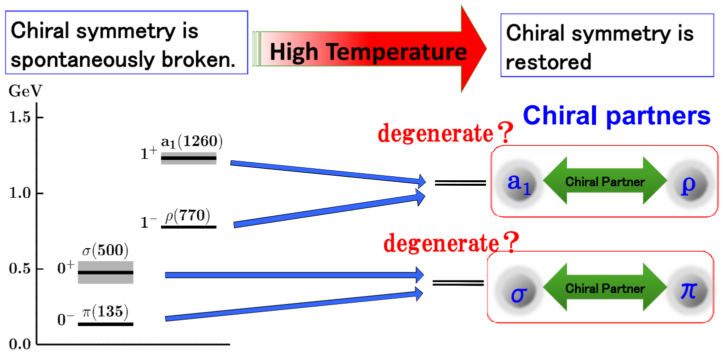
H. Wada (Kokushikan Univ.) , Y. Murakami (Research and Development Laboratory, Seikow Chemical Engineering & Machinery, LTD), A. Nakamura (Far Eastern Federal Univ., RCNP, Osaka Univ., Nishina Center, RIKEN), M. Sekiguchi (Kokushikan Univ.), and M. Wakayama (CENuM, Korea Univ., Pukyong National Univ. and RCNP, Osaka Univ.)

Abstract

We study the meson properties at finite temperature using the quenched lattice QCD simulations with the truncated overlap fermion. We calculate the screening masses of four types mesons, *i.e.* pseudo scalar, vector, scalar and axial vector meson, using two-flavor truncated overlap fermions. We explore rather heavy quark mass regions. But, we observe the tendency that the screening masses in all the channels degenerate, which is in accord with the effective restoration of $U_A(1)$ symmetry. In the low temperature region below pseudocritical temperature, the screening masses in all the channels are almost constant.

Temperature and Chiral Symmetry



Chiral Symmetry on the Lattice

In lattice QCD, we cannot describe a lattice Fermion action with the Chiral Symmetry because of the fermion doubling problem.

$$D\gamma_5 + \gamma_5 D = 0$$

- **Wilson Fermion action: explicitly breaks the Chiral symmetry**

There is the lattice chiral symmetry related to the Ginsparg-Wilson relation.

$$D\gamma_5 + \gamma_5 D = RaD\gamma_5 D$$

(P.H. Ginsparg & K.G. Wilson, Phys. Rev. D25 (1982))

Truncated overlap Fermion action

Domain wall fermion action

$$S_{DWF} = \bar{\psi}(x, x_5) D_{DWF}(x, y; x_5, y_5) \psi(y, y_5)$$

(4 dimensional cordinates x, y
+ 5th dimension cordinate $x_5, y_5 = (1, 2, \cdots, N_5)$)

Domain Wall Fermion operator

$$D_{DWF} = \begin{pmatrix} D_{WF}+1 & -P_L & 0 & \cdots & 0 & m_f P_R \\ -P_R & D_{WF}+1 & -P_L & \ddots & \vdots & 0 \\ 0 & -P_R & D_{WF}+1 & \ddots & 0 & \vdots \\ \vdots & 0 & \ddots & \ddots & -P_L & 0 \\ 0 & \vdots & \ddots & -P_R & D_{WF}+1 & -P_L \\ m_f P_L & 0 & \cdots & 0 & -P_R & D_{WF}+1 \end{pmatrix}$$

Fifth dimension

where

Wilson Fermion operator

$$D_{WF}(x,y) = (4 - M_5)\,\delta_{x,y} - \frac{1}{2}\sum_{\mu=1}^{4}\left[(1-\gamma_\mu)\,U_\mu(x)\delta_{y,x+\hat{\mu}} + (1+\gamma_\mu)\,U^\dagger_\mu(y)\delta_{y,x-\hat{\mu}}\right]$$

$P_L = \frac{1-\gamma_5}{2}$, $P_R = \frac{1+\gamma_5}{2}$ **: Chiral Projection operators**

m_f **: Fermion Mass ,** M_5 **: Hight of domain wall**

Fifth dimensional lattice size : N_5 → infinity
Quark mass : m_f → zero

=> DWF satisfies the GW relation.

Truncated overlap Fermion operator

A. Boriçi, Nucl. Phys. Proc. Suppl. 83,771(2000)

$$D_{TOF} = \epsilon^\dagger{}_s P^\dagger{}_{st} (D_{PV}^{-1})_{tu} (D_{DWF})_{uv} P_{vw} \epsilon_w$$

※ **Indices represent fifth dimensional sites.**

where

$$P = \begin{pmatrix} P_L & P_R & & \cdots & 0 & 0 \\ 0 & P_L & & & 0 & 0 \\ & \vdots & & \ddots & \vdots & \\ 0 & 0 & & & P_L & P_R \\ P_R & 0 & & \cdots & 0 & P_L \end{pmatrix}, \quad \epsilon = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}, \quad D_{PV}: \text{Domain Wall Fermion operator with } m_f = 1.0$$

Transform the expression

$$D_{TOF} = \frac{1+m_f}{2} + \frac{1-m_f}{2}\gamma_5 \frac{(1+H_w)^{N_5} - (1-H_w)^{N_5}}{(1+H_w)^{N_5} + (1-H_w)^{N_5}}$$

where $H_w = \gamma_5 \dfrac{D_{WF}}{D_{WF}+2}$

We can solve the inverse of D_{TOF} with 4-dimensional Wilson fermion.

Simulation

- Iwasaki gauge action
- Quenched 2 flavor Truncated overlap fermion action
- fifth dimension length N_5=32
- Hight of domain wall M_5 = 1.65
- The number of values of β is eight.

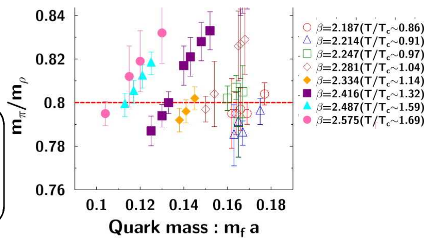
(We calculated the mass of the a_1 meson using the same parameter values of N_5 and M_5 .
M. Wakayama et al. JPS Conf. Proc. QNP18-077 (2019))

The simulation was performed on an NEC SX-ACE at RCNP. ,Osaka Univ.

Step 1. Determination of quark masses for each β value

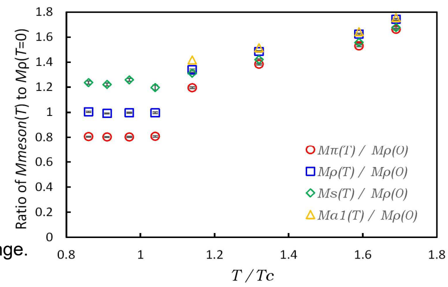
- Lattice size : N_t= 16, N_s = 16
- 12-80 configurations

(**The choice of beta values is based on**
- M. Okamoto et al.[CP-PACS Collaboration], Phys. Rev. D60, 094510 (1999)
- A.A. Khan et al.[CP-PACS Collaboration], Nucl. Phys. B (Proc. Suppl.), 83, 176 (2000))



Step 2. Calculation of the screening masses of mesons

- Lattice size : N_t=4, N_s=16
- quark masses for each β are decided by setp1 calculations.
- 1,600-2,000 Configurations

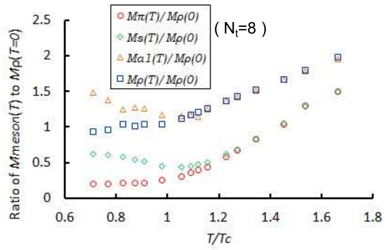


$T \le T_C$
All screening masses don't change.

$T > T_C$
All screening masses increase monotonically with ***T***.
Chiral partners degenerates , $M_\pi \sim M_s$ ***and*** $M_\rho \sim M_{a1}$, in high temperature.

Previous work by other research groups

- Tree-level improved gauge action
- Highly improved Staggered Fermion action
- Dynamical 2+1 flavor quarks
- Lattice size : N_t=8,10,12, N_s/N_t=4
- m_π ~ 160MeV (, m_K ~ 504MeV)



[1] Y. Maezawa, F. Karsch, S. Mukherjee & P. Petreczky, Pos (Lattice2015) 199

Note : This graph is created based on [1].

Summary

- **We calculate the screening masses at finite temperature with quenched truncated overlap fermion action.**
- **At low temperature below Tc, the screening masses of pseudo scalar, vector, scalar and axial vector are almost independent of temperature.**
- **At High temperature, the screening masses increase with temperature. And, the screening masses of pseudo scalar and scalar (vector and axial vector) tend to degenerate.**
- **Previous work's results show that the graph curves of the screening mass of scalar and a1 mesons are convex downward in low temperature. But, such tendency is not seen in our simulation. Is it due to dynamical quark effect ? Is our quark mass too heavy ?**

Future work

Calculations with lighter quark masses and larger lattice size are needed. For that purpose, it is necessary to speed up the calculation of the CG method.

(COST (Truncated Overlap Fermion) ~ COST (Wilson Fermion) × 10-100)