

Partial Deconfinement

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In collaboration with Masanori Hanada and Goro Ishiki

[Hanada, Ishiki & HW, JHEP 1903 (2019) 145]

2019/06/21 Lattice 2019

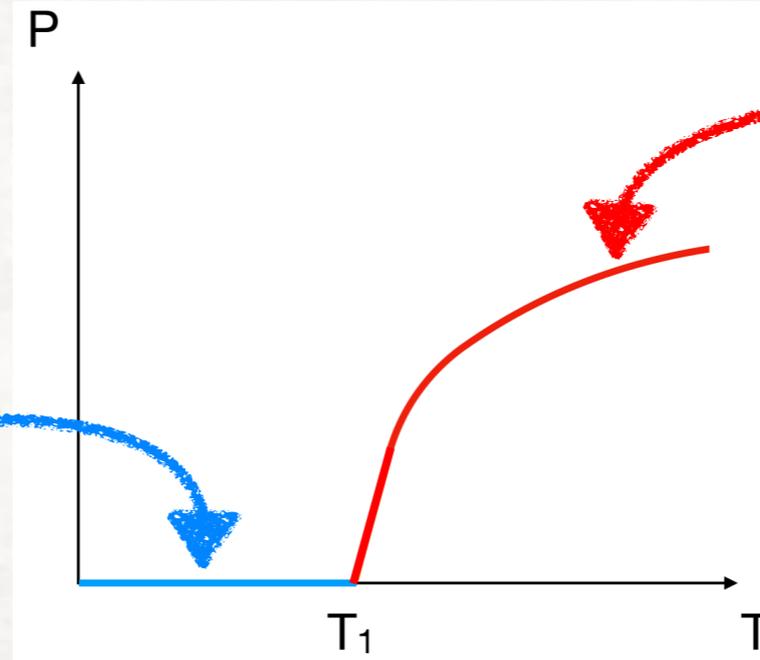
What is "Partial Deconfinement"?

P v.s. T relation

P : Polyakov loop

Confined phase

$$P = 0$$



Deconfined phase

$$P \neq 0$$

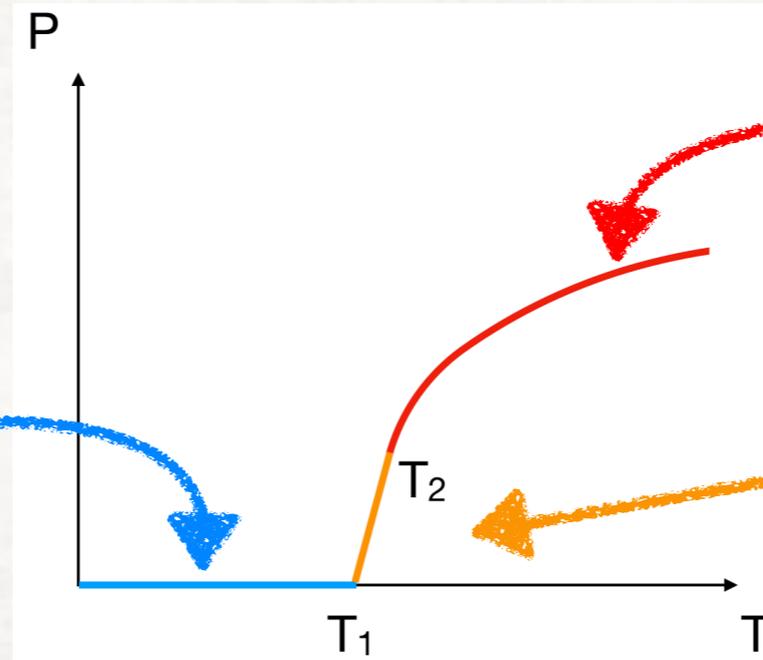
What is "Partial Deconfinement"?

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$$P = 0$$



"Fully"
deconfined phase

"Partially"
deconfined phase

- We propose the existence of "partially" deconfined phase and argued the properties of partial deconfinement.
- We study several theories which exhibit the partially deconfined phase both analytically and numerically.
- We comment on the prospects of applying to the real world QCD.

Motivation

Holographic principle or gauge/gravity correspondence



Quantum Gravity
Black Hole

"Equivalent"



A certain QFT
in lower dimension

e.g.) AdS/CFT correspondence
[Maldacena, 1997]

Gravity theory
in Anti de Sitter space



$SU(N)$ super Yang-Mills theory
(SYM)

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$SU(N)$ super Yang-Mills theory
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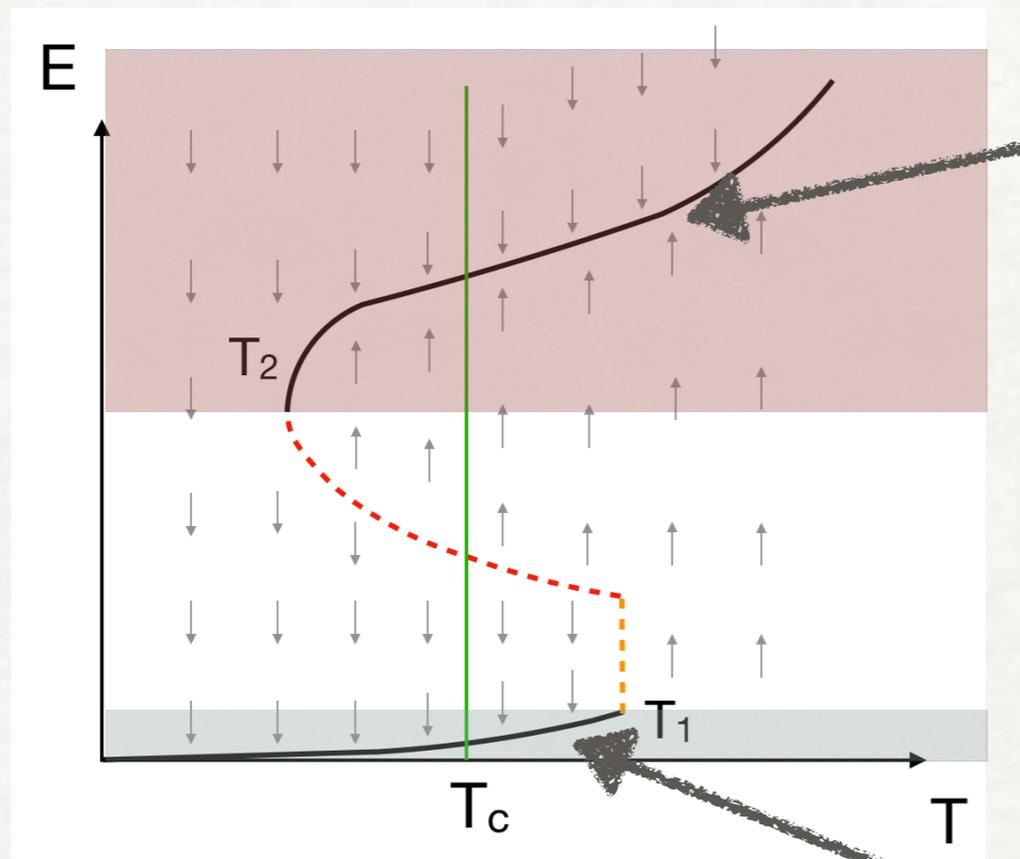
We want to study it
to learn about **quantum gravity**.



- Large N
- Adjoint representation

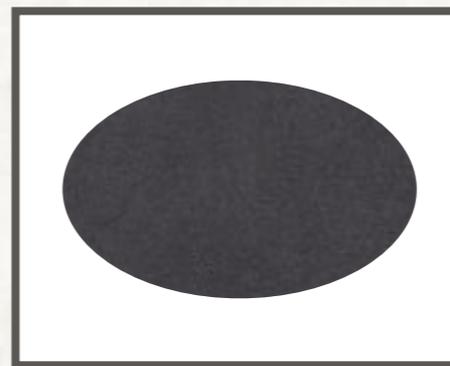
Black hole in $AdS_5 \times S^5 \iff 4d N=4 SU(N) SYM$

Strongly coupled 4d SYM / dual string theory



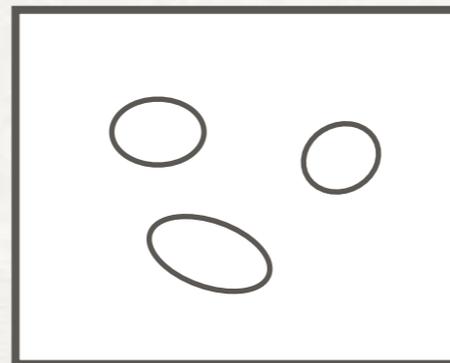
Large BH (AdS BH)

$$E \sim N^2 T^4$$



String gas

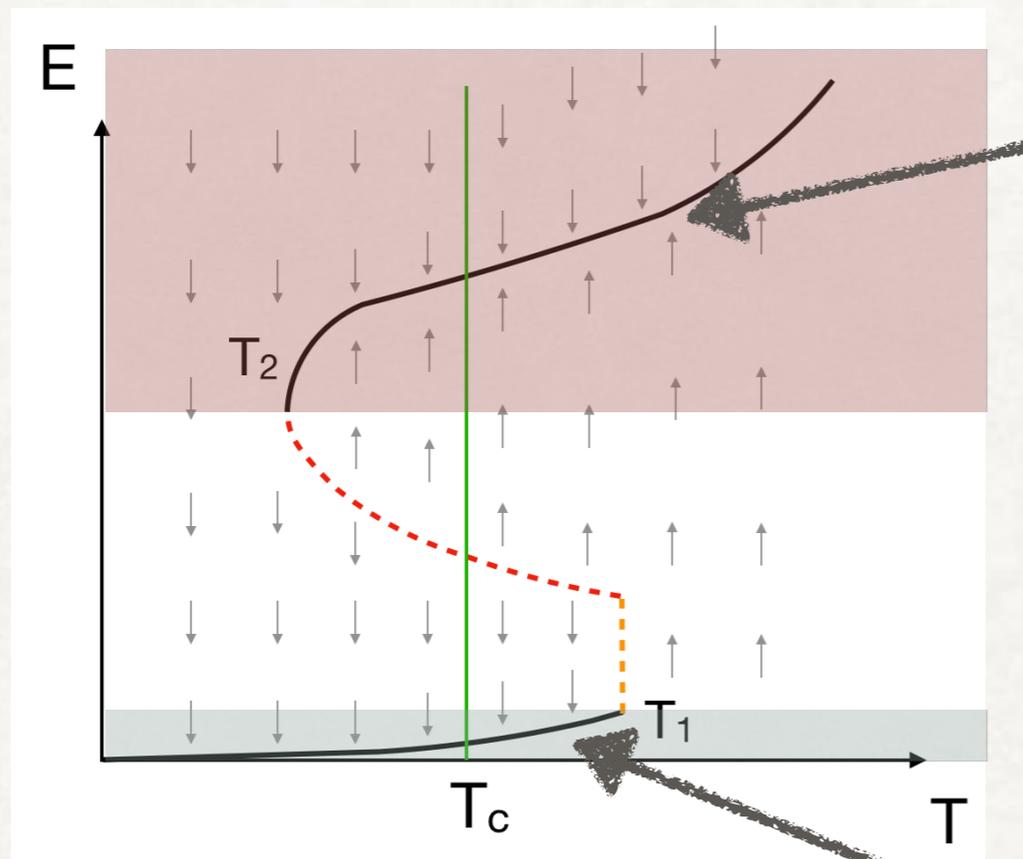
$$E \sim N^0$$



Hawking-Page transition
[Hawking & Page, (1983)]

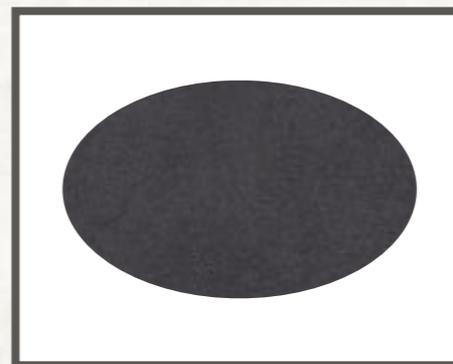
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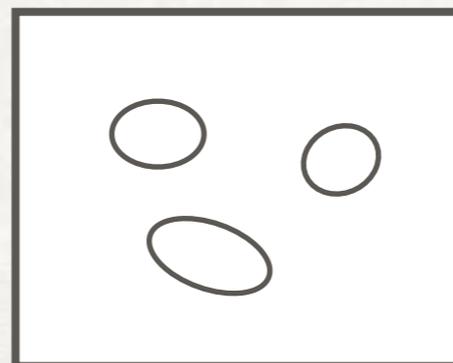
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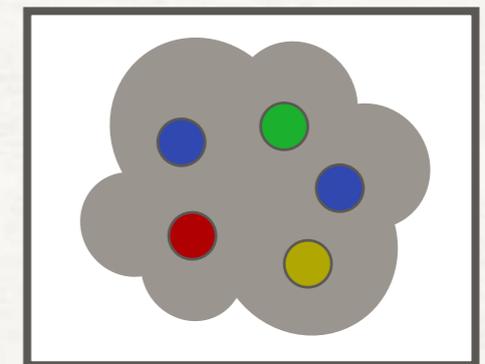
String gas

$$E \sim N^0$$



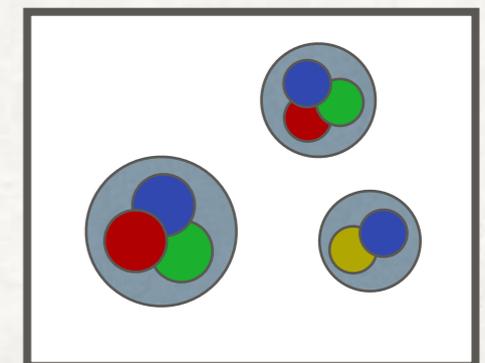
Gauge theory side;

Deconfined phase



phase transition

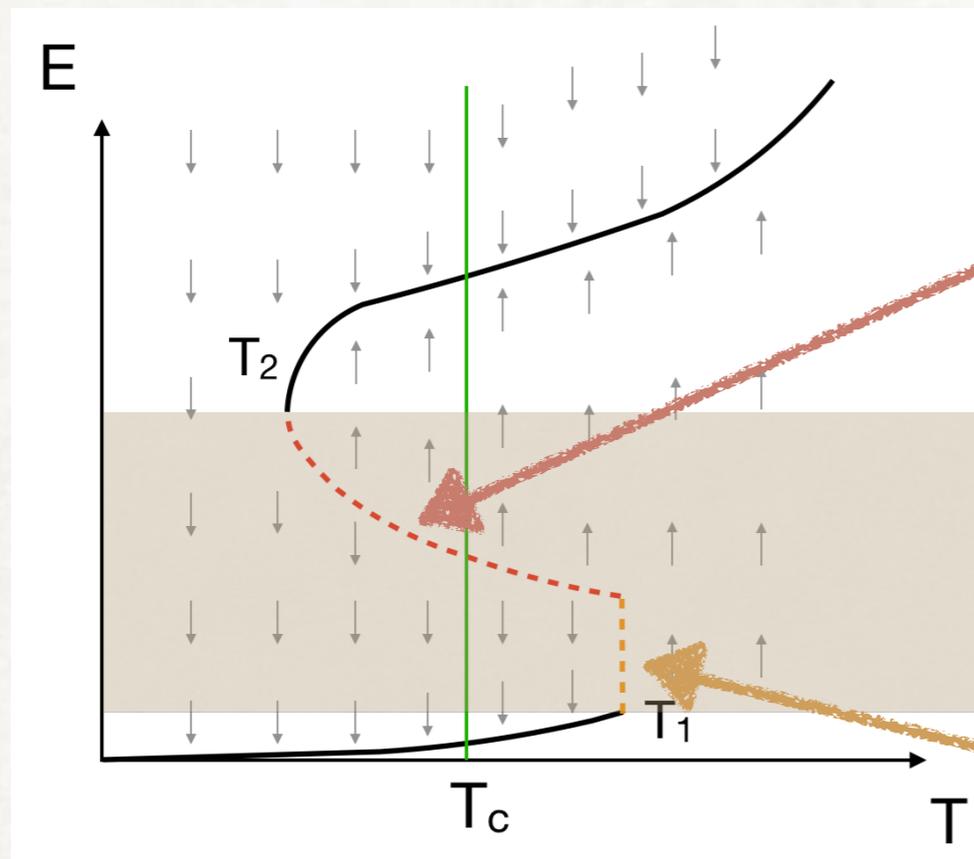
Confined phase



[Witten, (1998)]

Black hole in $AdS_5 \times S^5 \iff 4d N=4 SU(N) SYM$

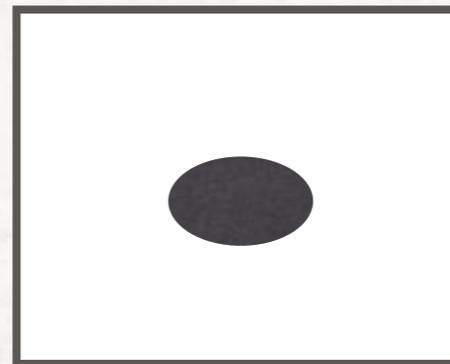
Strongly coupled 4d SYM / dual string theory



Unstable saddle

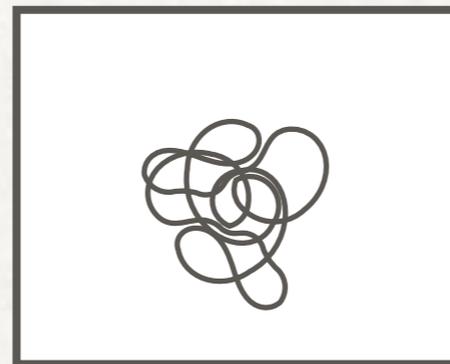
Small BH

$$E \sim N^2 T^{-7}$$



Hagedorn string

$$E \sim L_{\text{string}}$$



Gauge theory side;

??

Small BH would be understood from
"partially" deconfined phase.

Properties of the partial deconfinement

The order parameter of transition (review)

Polyakov loop : an order parameter of confine/deconfine transition

with $SU(N)$ adjoint fields and large N .

$$P = \frac{1}{N} \text{Tr} \mathcal{P} \exp \left[- \oint_{\text{temporal}} A_t \right] = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j}$$

This can be rewritten as

$$P = \int d\theta \rho(\theta) e^{i\theta}, \quad \rho(\theta) = \frac{1}{N} \sum_j \delta(\theta - \theta_j)$$

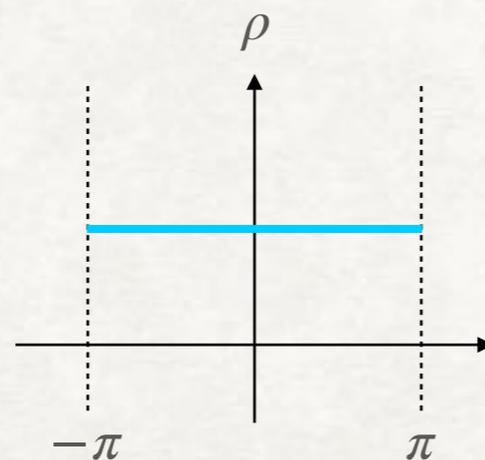
: phase distribution

$$P = 0;$$

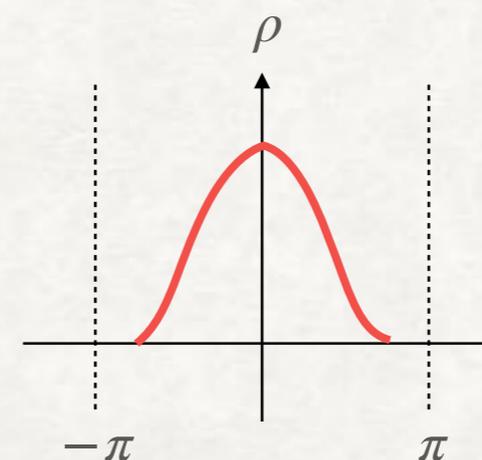
Confined phase

$$P \neq 0;$$

Deconfined phase



confined



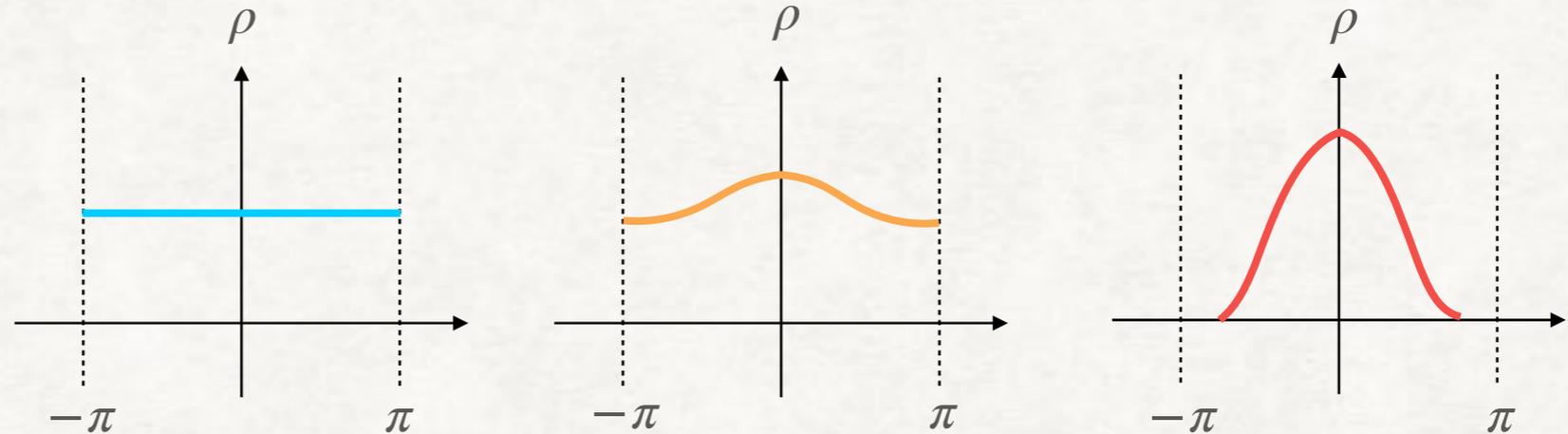
deconfined

Partial deconfinement

Polyakov loop : order parameter of confine/deconfine transition

$$P = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j}$$

$$= \int d\theta \rho(\theta) e^{i\theta}$$



$\rho(\theta)$: phase distribution

confined

"partially" deconfined

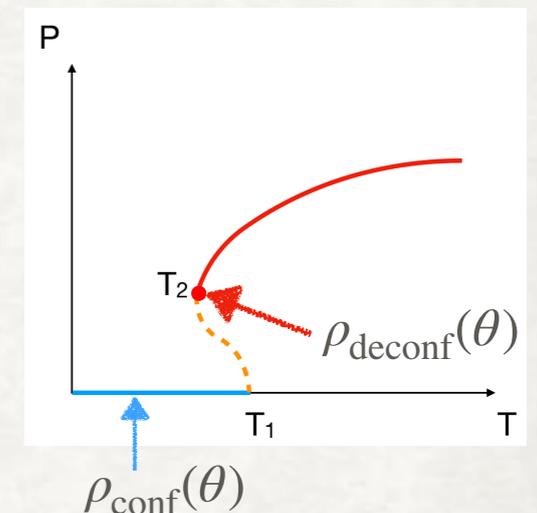
"fully" deconfined

Partial deconfinement ($M < N$)

$$\rho(\theta) = \frac{N-M}{N} \rho_{\text{conf}}(\theta) + \frac{M}{N} \rho_{\text{deconf}}(\theta) = \frac{N-M}{N} \cdot \frac{1}{2\pi} + \frac{M}{N} \rho_{\text{deconf}}(\theta)$$

Partial deconfinement is "the mixture."

- M θ_j s are in deconfined phase
- $N-M$ θ_j s are in confined phase



Examples of partial deconfinement

Partial deconfinement ($M < N$)

$$\rho(\theta) = \frac{N-M}{N} \rho_{\text{conf}}(\theta) + \frac{M}{N} \rho_{\text{deconf}}(\theta) = \frac{N-M}{N} \cdot \frac{1}{2\pi} + \frac{M}{N} \rho_{\text{deconf}}(\theta)$$

- 4d $\mathcal{N}=4$ SYM on S^3 (weak coupling) ; [Sundborg, (2000), Aharony et al, (2003)]

$$\rho(\theta) = \frac{1}{2\pi} \left(1 + \frac{2}{\kappa} \cos \theta \right)$$

$$= \left(1 - \frac{2}{\kappa} \right) \cdot \frac{1}{2\pi} + \frac{2}{\kappa} \cdot \frac{1}{2\pi} (1 + \cos \theta)$$

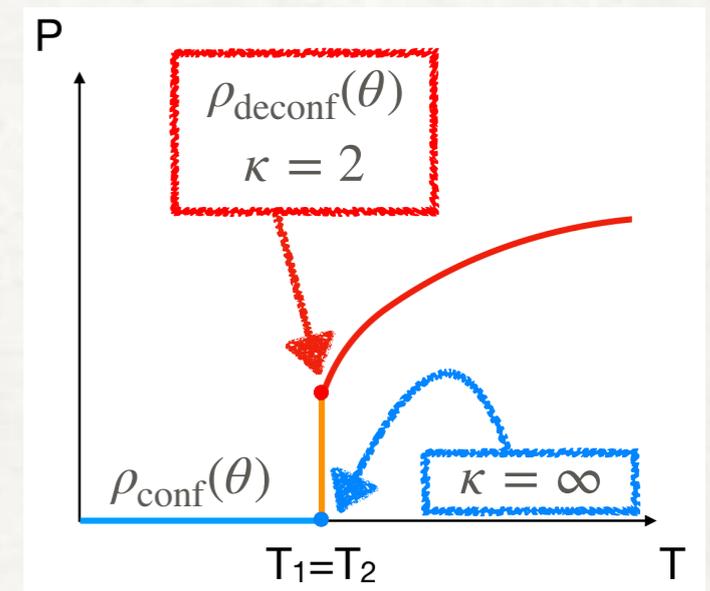
$$= \rho_{\text{conf}}(\theta)$$

$$= \rho_{\text{deconf}}(\theta)$$

Identification of $\frac{M}{N} = \frac{2}{\kappa}$

At \bullet ; "Gross-Witten-Wadia transition"

→ Satisfying the partial deconfinement!



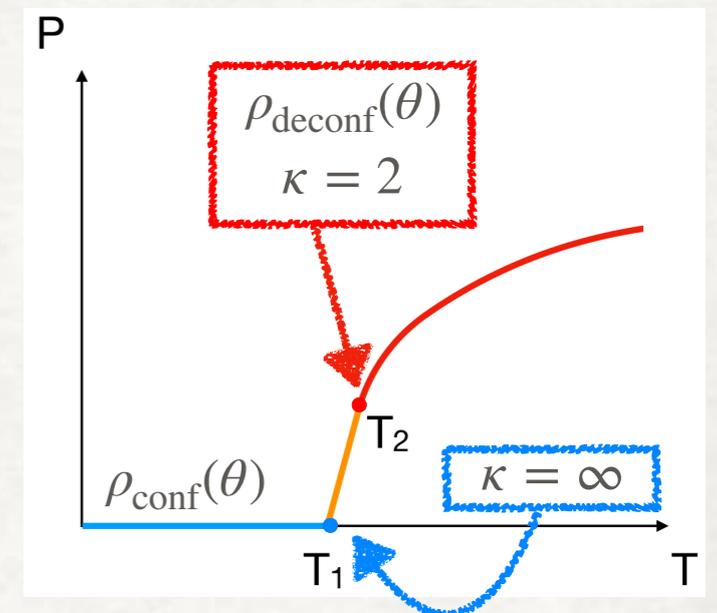
Examples of partial deconfinement

Partial deconfinement ($M < N$)

$$\rho(\theta) = \frac{N - M}{N} \rho_{\text{conf}}(\theta) + \frac{M}{N} \rho_{\text{deconf}}(\theta) = \frac{N - M}{N} \cdot \frac{1}{2\pi} + \frac{M}{N} \rho_{\text{deconf}}(\theta)$$

- 4d Yang-Mills theory with matters on S^3 (weak coupling);
[Aharony, Marsano, Minwalla, Papadodimas & Raamsdonk, (2003)/Schnitzer, (2004)]

$$\rho(\theta) = \begin{cases} \frac{1}{2\pi} & (T \leq T_1) \\ \frac{1}{2\pi} \left(1 + \frac{2}{\kappa} \cos \theta \right) & (T_1 < T < T_2) \\ \frac{2}{\pi\kappa} \cos \frac{\theta}{2} \sqrt{\frac{\kappa}{2} - \sin^2 \frac{\theta}{2}} & (T \geq T_2, |\theta| < 2 \arcsin \sqrt{\kappa/2}) \end{cases} \quad \frac{M}{N} = \frac{2}{\kappa}$$



→ Satisfying the partial deconfinement!

At ●; "Gross-Witten-Wadia transition"

- Free vector model, etc...

Partial-to-Full deconfinement transitions are GWW more broadly?

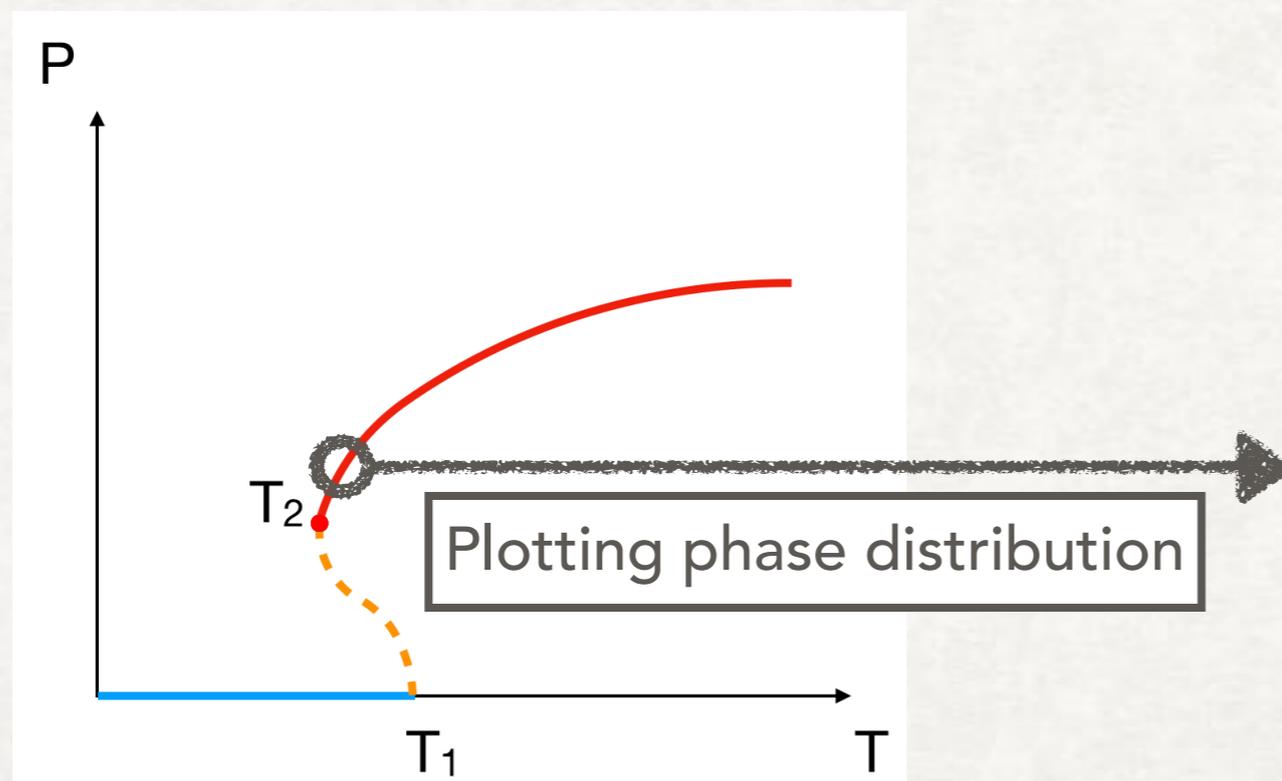
Examples of partial deconfinement

- The bosonic part of plane wave matrix model (PWMM or BMN matrix model) = the mass deform. of (0+1)d SYM / Matrix quantum mechanics.

$$L = N \text{Tr} \left(\frac{1}{2} \sum_{I=1}^9 (D_t X_I)^2 + \frac{1}{4} \sum_{I,J=1}^9 [X_I, X_J]^2 - \frac{\mu^2}{2} \sum_{i=1}^3 X_i^2 - \frac{\mu^2}{8} \sum_{a=4}^9 X_a^2 - i \sum_{i,j,k=1}^3 \mu \epsilon^{ijk} X_i X_j X_k \right)$$

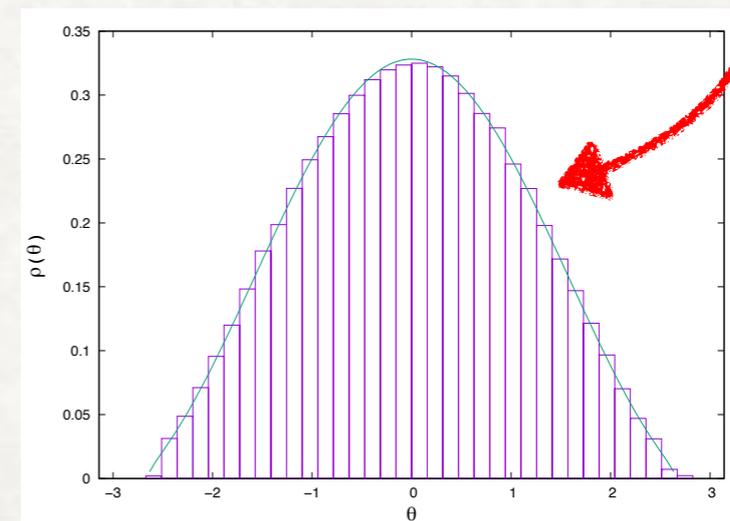
Check by Monte Carlo simulation;

- Hysteresis ($T_2 \leq T_1$)
- Phase distribution



Assumed GWW transition;

$$\rho(\theta) = \begin{cases} \frac{1}{2\pi} \\ \frac{1}{2\pi} \left(1 + \frac{2}{\kappa} \cos \theta \right) \\ \frac{2}{\pi\kappa} \cos \frac{\theta}{2} \sqrt{\frac{\kappa}{2} - \sin^2 \frac{\theta}{2}} \end{cases}$$



Fitting

$$\begin{pmatrix} N = 128 \\ \mu = 5 \end{pmatrix}$$

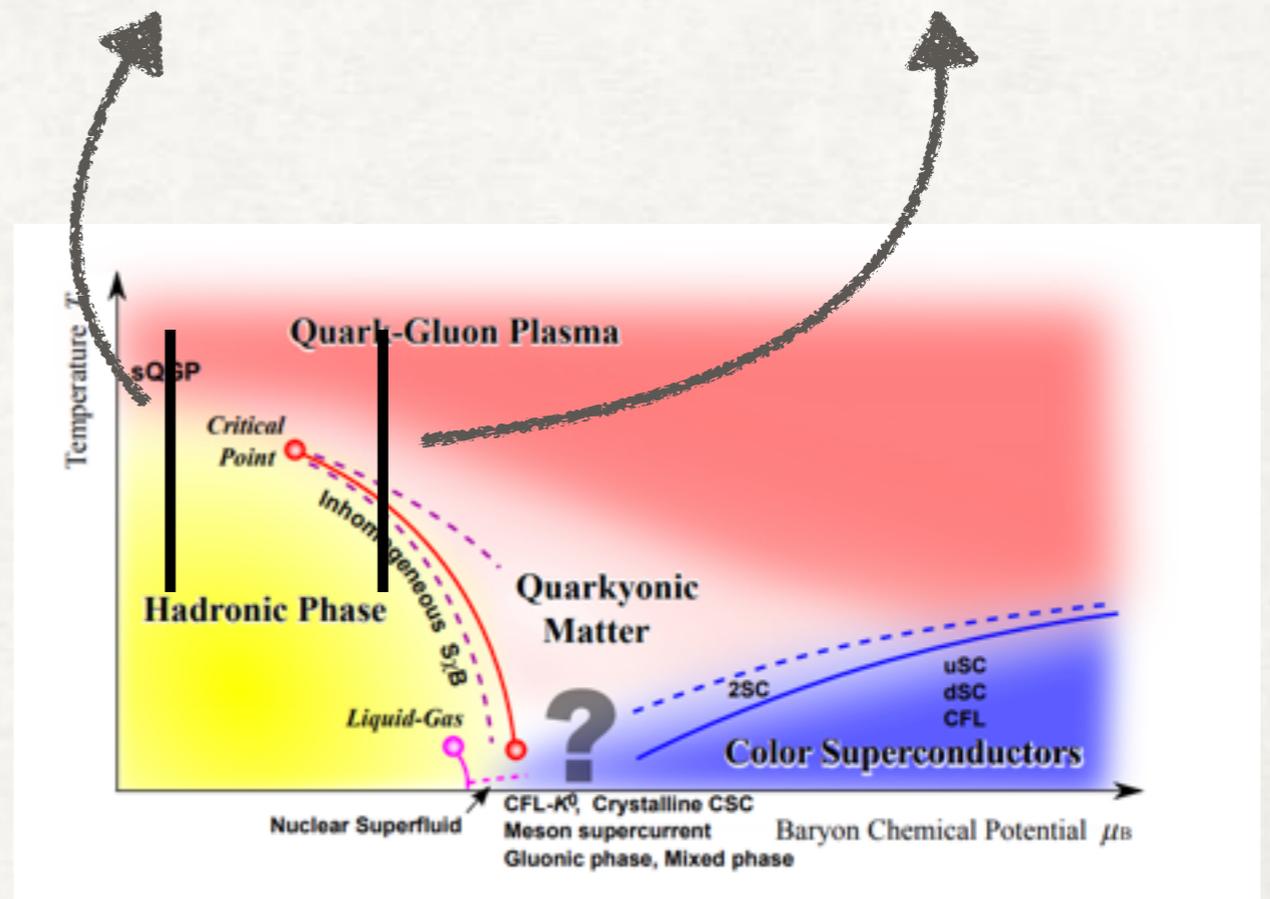
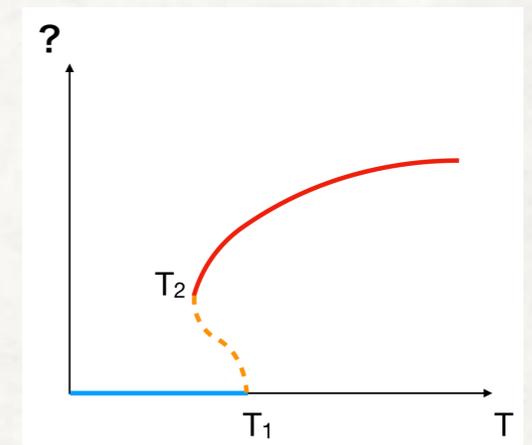
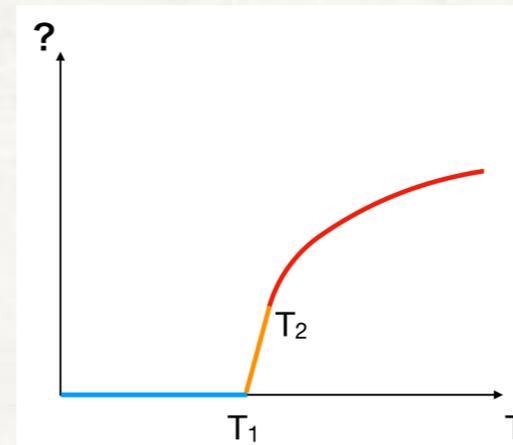
The prospects to apply

The properties of QCD phase diagram;

- The difference of the order of transition in QCD at zero and finite chemical potential
- The mechanism that the order changes when the baryon density has been changed

Note; there is the subtlety between chiral transition and deconfinement

might be explained by "partial deconfinement"



From [Fukushima & Hatsuda (2011)]

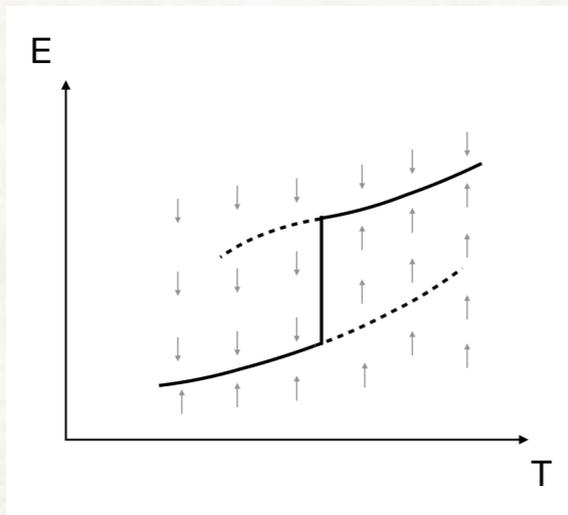
Summary & Discussion

- We proposed the existence of partially deconfined phase and its generic form of phase distribution.
- It's important to study gauge theory, in order to understand *quantum gravity*.
 - Small black hole is corresponding to the partially deconfined phase.
 - Can we attack the properties of unstable saddle by analytically or numerically (e.g. Monte Carlo simulation)?
- The basic idea that the partial deconfinement is the mixture could be applied to the real world QCD.

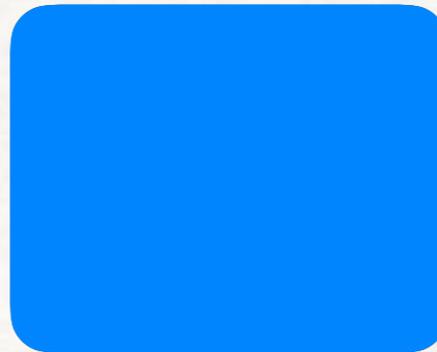
Backup Slides

How occurs the phase transition

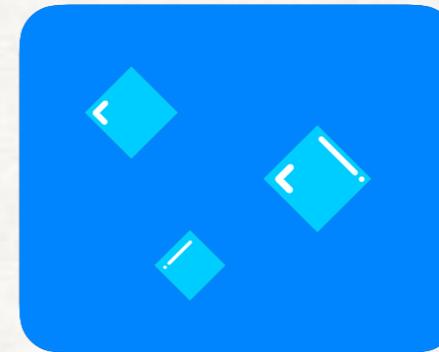
water & ice



water



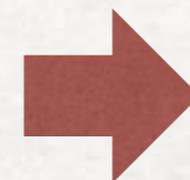
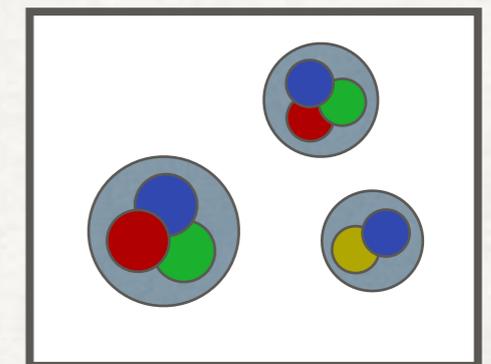
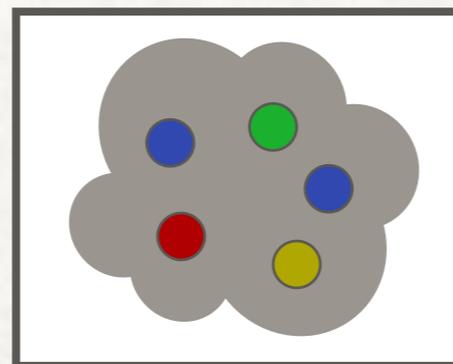
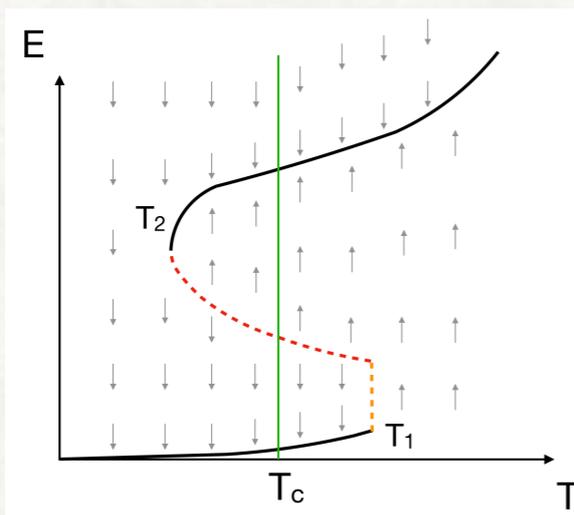
$\sim 0^\circ\text{C}$



ice



Strongly coupled 4d SYM

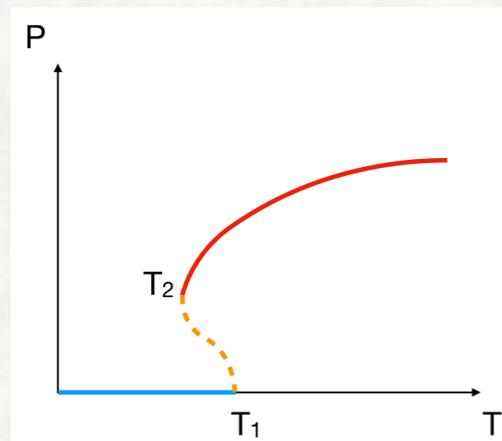


"Partially" deconfined phase

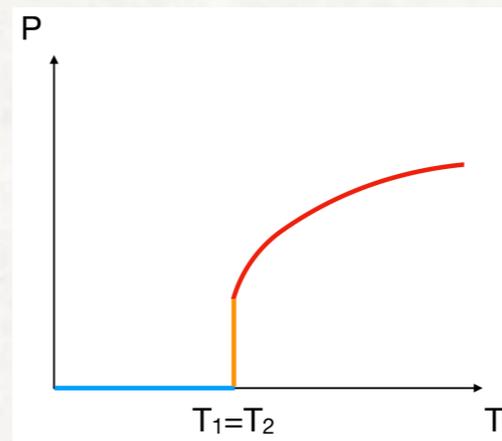
Phase structure

Partial deconfinement ($M < N$)

$$\rho(\theta) = \frac{N - M}{N} \rho_{\text{conf}}(\theta) + \frac{M}{N} \rho_{\text{deconf}}(\theta) = \frac{N - M}{N} \cdot \frac{1}{2\pi} + \frac{M}{N} \rho_{\text{deconf}}(\theta)$$

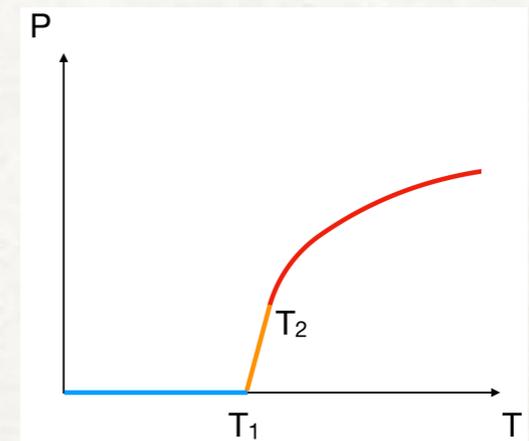


Strongly coupled 4d SYM



Weakly coupled 4d SYM

[Sundborg, (2000)]



No counterpart in SYM

Weakly coupled
4d free YM

[Aharony et al. (2004)]

Quantitative tests for partial deconfinement

Partial deconfinement ($M < N$)

$$\rho(\theta) = \frac{N-M}{N} \rho_{\text{conf}}(\theta) + \frac{M}{N} \rho_{\text{deconf}}(\theta) = \frac{N-M}{N} \cdot \frac{1}{2\pi} + \frac{M}{N} \rho_{\text{deconf}}(\theta)$$

- Free vector model ;

$$\rho(\theta) = \frac{1}{2\pi} + \frac{2b^2}{\pi} f(\theta) \quad b = \frac{T}{\sqrt{N}} \leq b_{\text{GWW}} = \frac{\sqrt{3}}{\pi}, \quad f(\theta) = -\frac{\pi^2}{12} + \frac{\theta^2}{4}$$

[Shenker & Yin, (2011)]

$$\rho(\theta) = \frac{1}{2\pi} - \frac{\pi b^2}{6} + \frac{b^2 \theta^2}{2\pi} = \frac{1}{2\pi} \left(1 - \frac{b^2}{b_{\text{GWW}}^2} \right) + \frac{b^2}{b_{\text{GWW}}^2} \cdot \frac{b_{\text{GWW}}^2 \theta^2}{2\pi} = \rho_{\text{deconf}}(\theta)$$

$$\frac{M}{N} = \frac{b^2}{b_{\text{GWW}}^2} \quad T = b\sqrt{N} = b_{\text{GWW}}\sqrt{M}$$

"Gross-Witten-Wadia transition"

Small black hole & partial deconfinement

Strongly coupled 4d SYM

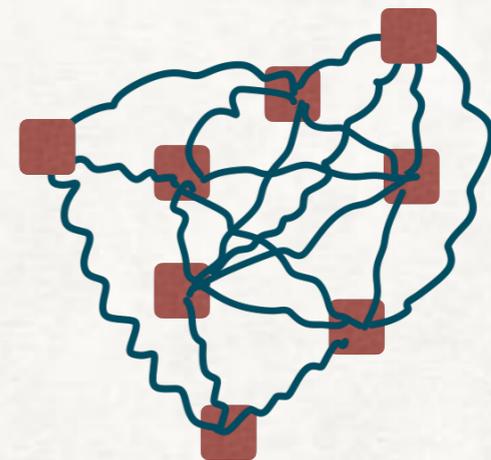
$N \times N$ matrices

$$X^\mu =$$



Open strings between D-branes

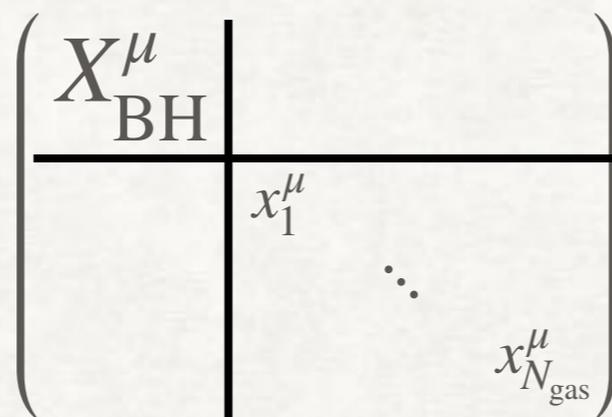
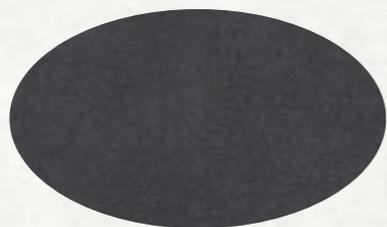
Position of D-branes



Bound state (bunch)

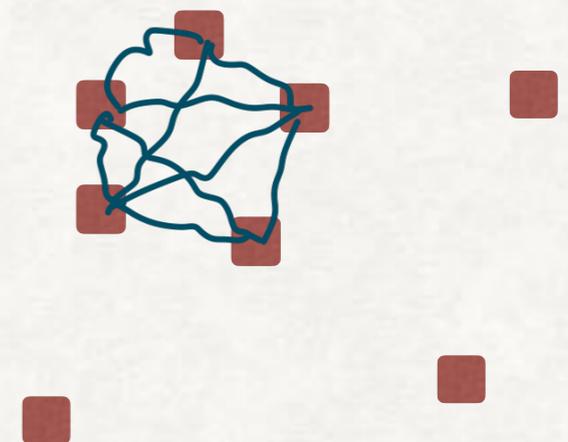
Gravity dual

black hole



$$N_{\text{BH}} = M$$

$$N_{\text{gas}}$$

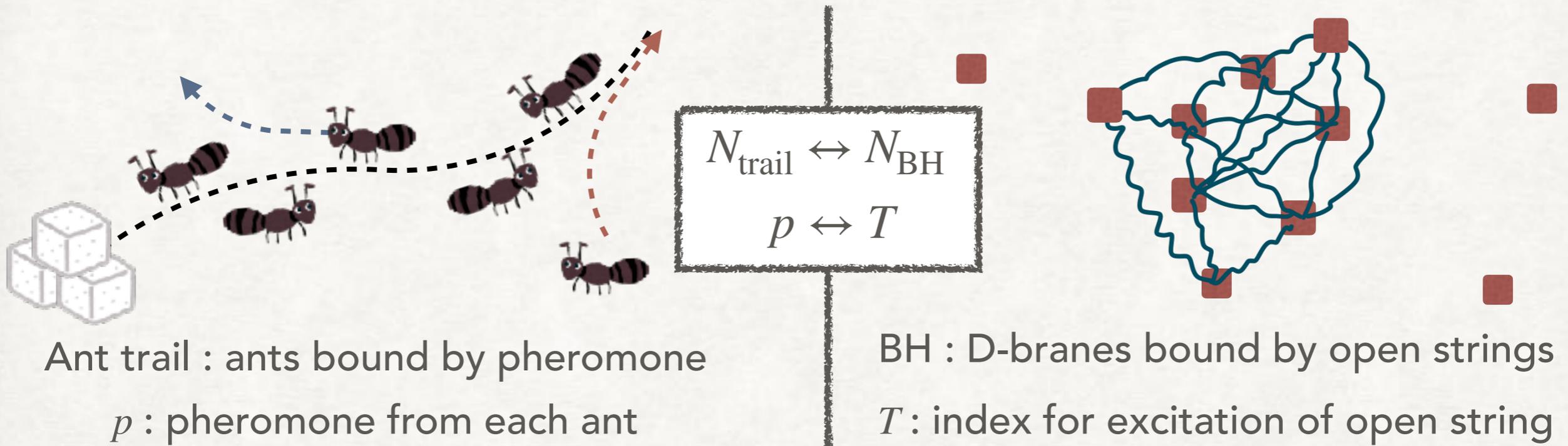


Bunch & gas

[Hanada & Maltz, (2016)]

"partially deconfined"

Ant model and partial deconfinement



$$\frac{dN_{\text{trail}}}{dt} = (\alpha + pN_{\text{trail}})(N - N_{\text{trail}}) - \frac{sN_{\text{trail}}}{s + N_{\text{trail}}}$$

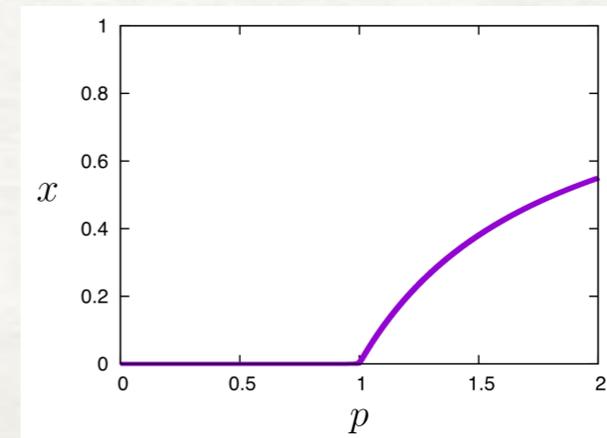
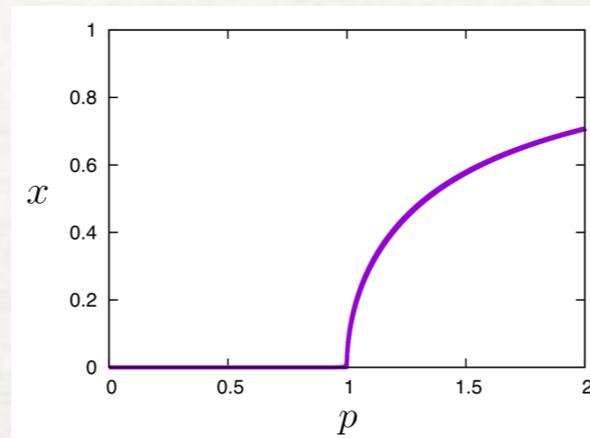
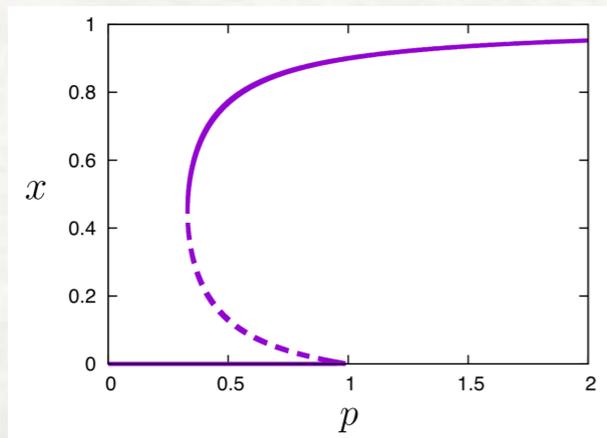
Inflow effect

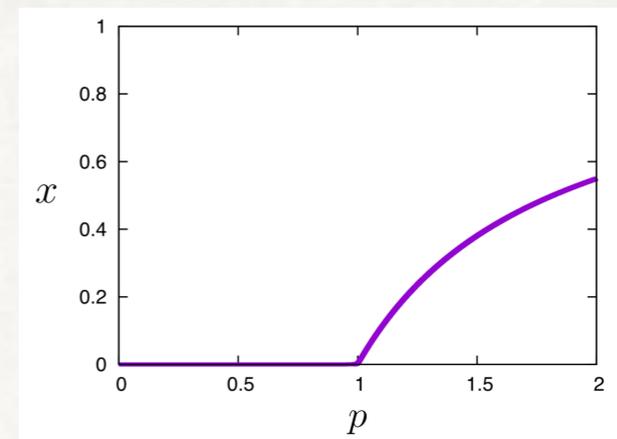
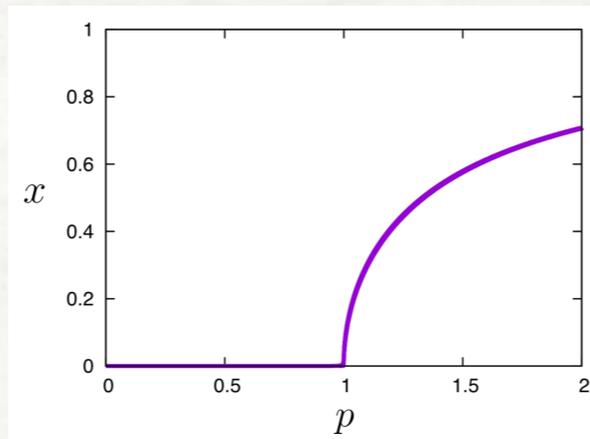
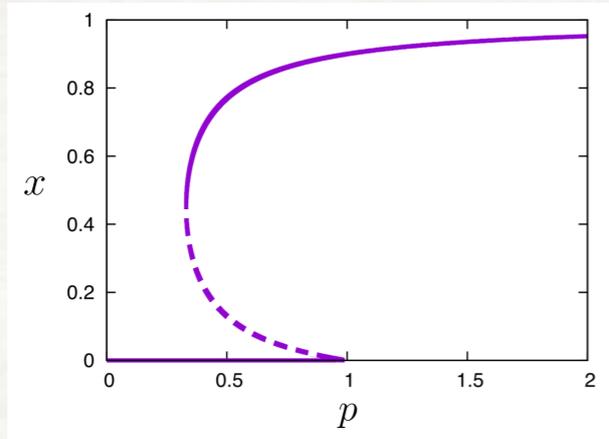
Outflow effect

saddle point

$$\frac{dN_{\text{trail}}}{dt} = 0, \quad x \equiv \frac{N_{\text{trail}}}{N}$$

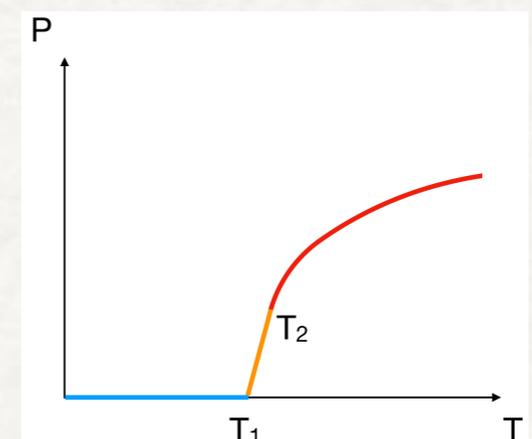
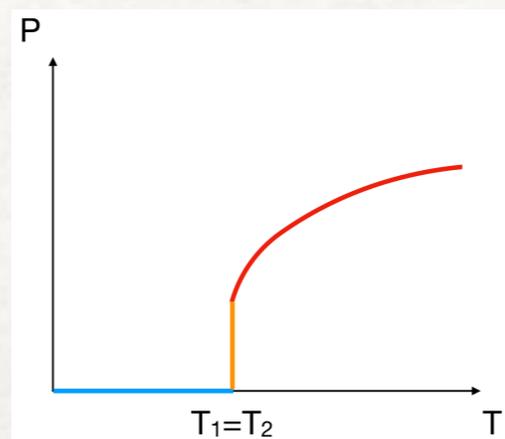
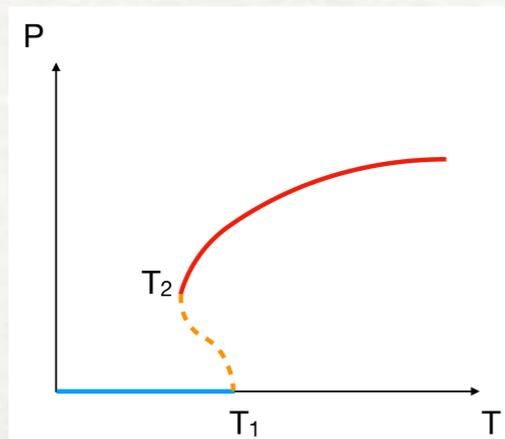
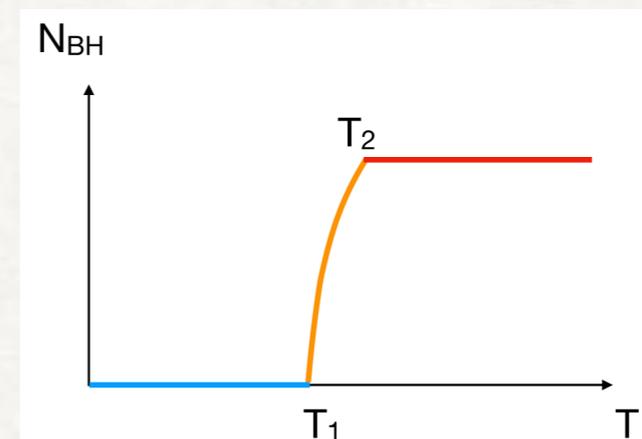
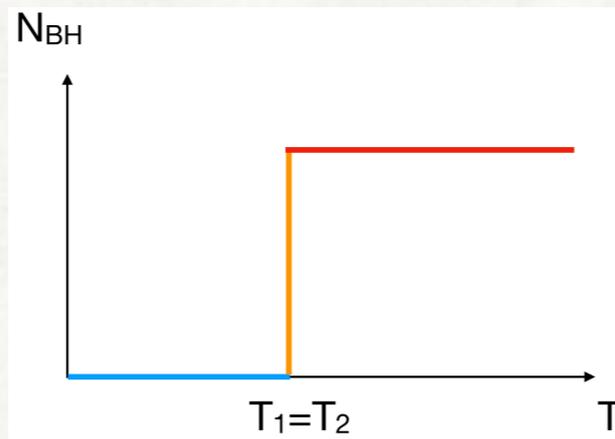
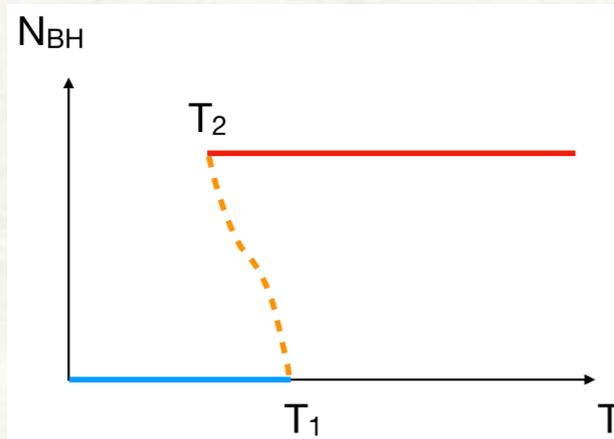
[Beekman, Sumpter & Ratnieks, (2001)]





**Strongly coupled
4d SYM**

**Weakly coupled
4d SYM**



Modified ant model (Ant-Man model)

$$\frac{dx}{dt} = (\tilde{\alpha} + px)(1 - x) - \frac{\tilde{s}x}{\tilde{s} + x} \cdot (1 - x^2) \quad x \equiv \frac{N_{\text{trail}}}{N}, \quad \tilde{s} = \frac{s}{N}, \quad \tilde{\alpha} = \frac{\alpha}{N}$$

saddle point

