June 17-22, 2019 , Wuhan

CME with lattice and PV regularizations

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Outlines

Introduction to anomalous transports

CME from QFT with PV regularization

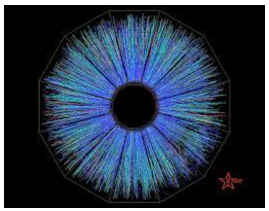
CME on lattice

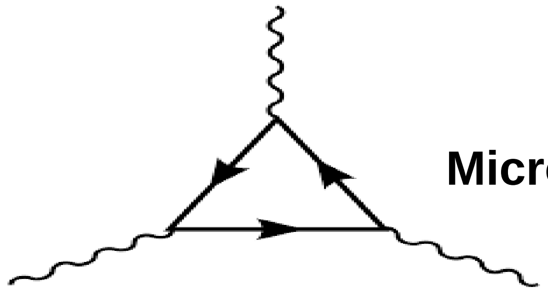
Higher order corrections to CME

Summary

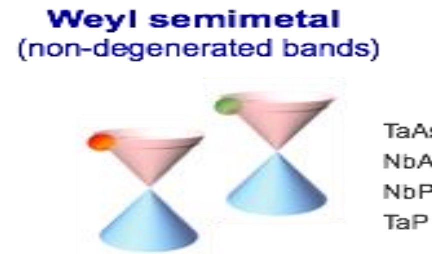
Introduction : Anomalous Transports

Micro-quantum anomaly + B/ Ω → macro-transport (CME/CVE)





Micro



Search in HIC

B Ω

$$\vec{J} = \sigma_5 \mu_5 \vec{B}$$

Macro

Nature Phys.12（2016）

Phys. Rev. X.5（2015）

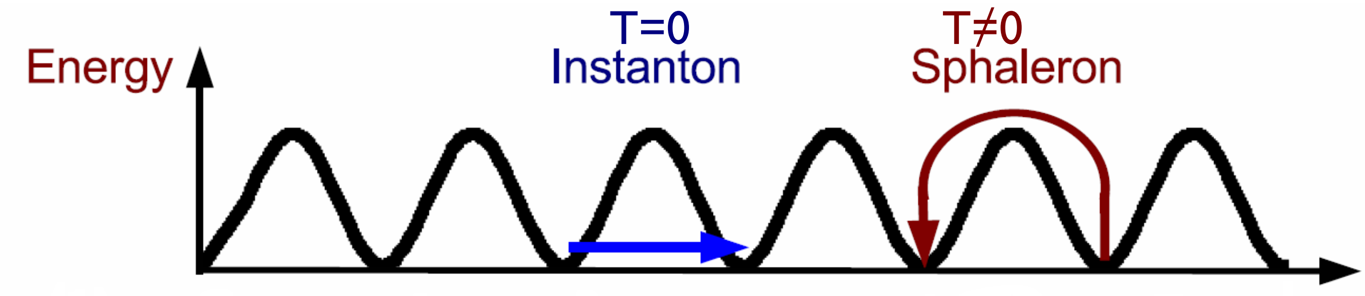
Science350（2015）413

Astrophysics, cosmology

Kharzeev,Liao,Voloshin,Wang: Prog.Part.Nucl.Phys.2016

* Net axial charge density $\mu_5 \neq 0$

Topological charge fluctuations of QCD in QGP

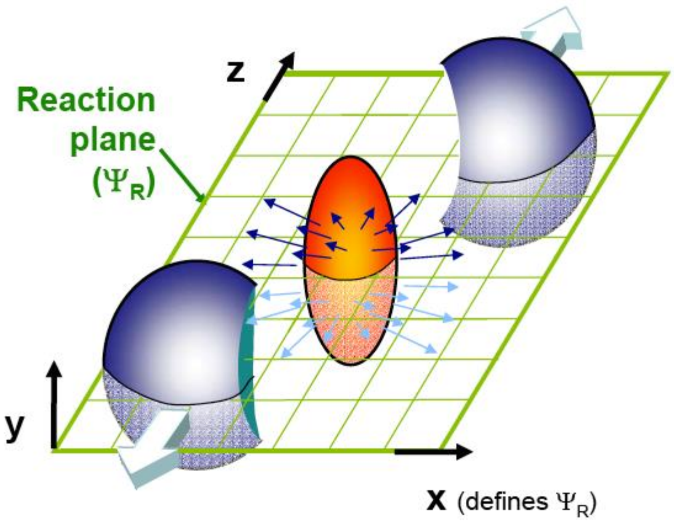


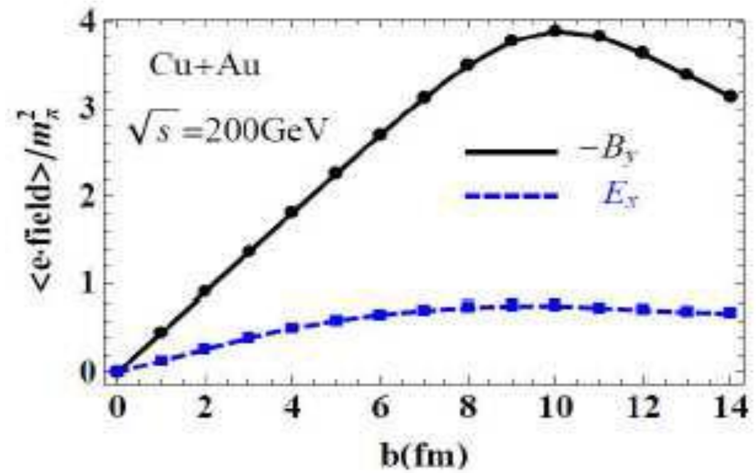
Axial anomaly

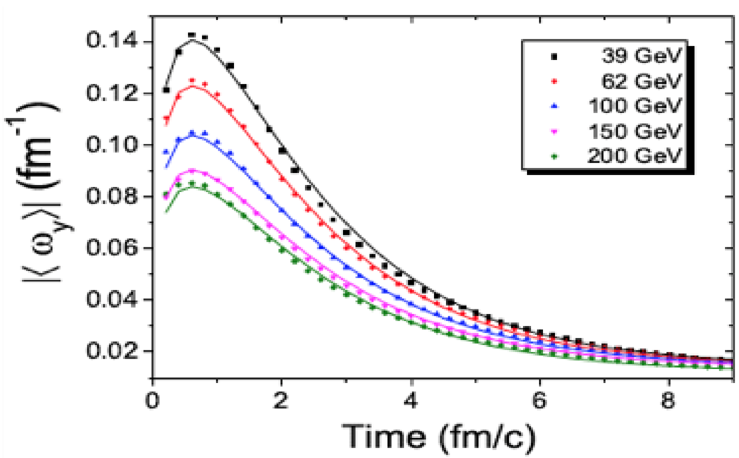
$$\Delta N_5 = -\frac{N_f g^2}{32\pi^2} \int d^4x \, \varepsilon_{\mu\nu\rho\lambda} F^l_{\mu\nu} F^l_{\rho\lambda} = n_W$$

$n_W =$ the wind number $\qquad F^l_{\mu\nu} =$ QCD field strength

Strong EM Field/Rotation/ produced in HIC







Deng, Huang, 2015

Jiang,Lin,Liao, 2016

Theoretical approaches:

--- Lattice method

--- Continuum Field theory

--- Holographic theory

--- Kinetic approach or hydrodaynamice

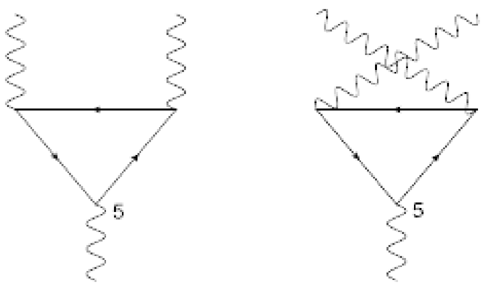
UV divergence demands regularization, IR behavior is crucuial

The relation of CME current to chiral anomaly

The CME current

$$J_i(p) = \eta\mu_5 K_{ij}(p) A_j(p) + \mathcal{O}(\mu_5^3)$$

- In terms of the AVV three point function $\Lambda_{\mu\nu\rho}(Q_1, Q_2)$



$$Q_1 = (\mathbf{q}, i(\omega + \frac{k_0}{2})),$$

$$Q_2 = (-\mathbf{q}, i(-\omega + \frac{k_0}{2}))$$

- the coefficient

$$K_{ij}(q) = \Lambda_{ij4}(q, -q) = -i \lim_{k_0 \to 0} \frac{1}{k_0} (Q_1 + Q_2)_\rho \Lambda_{ij\rho}(Q_1, Q_2)$$

- the chiral anomaly

$$(Q_1 + Q_2)_\rho \Lambda_{\mu\nu\rho}(Q_1, Q_2) = -i \frac{e^2}{2\pi^2} \epsilon_{\mu\nu\alpha\beta} Q_{1\alpha} Q_{2\beta}$$

Universal to all orders of coupling， all temperature & chemical potential .Necessary to explain $\pi^0 \to 2\gamma$

CME from continuum QFT at finite T and density

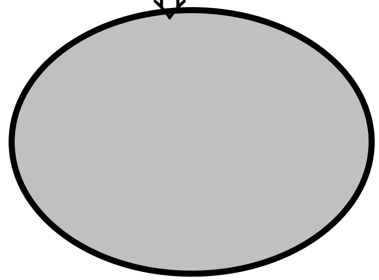
$$J_i(Q) = K_{ij}(Q) A_j(Q)$$

Hou,Liu,Ren ,JHEP 05(2011)046

$$\mu_5(\mathbf{k}, k_0)$$

$$\Downarrow$$

$$\mathbf{J}\left(\mathbf{q} + \frac{1}{2}\mathbf{k},\ \omega + \frac{1}{2}k_0\right) \Leftarrow$$



$$\Leftarrow \mathbf{B}\left(\mathbf{q} - \frac{1}{2}\mathbf{k}, \omega - \frac{k_0}{2}\right)$$

Constant μ_5*, non-constant* **B:** $\mathbf{k} = k_0 = 0$

$$\mathrm{limit}_{\mathbf{q}\to 0}\mathrm{limit}_{\omega\to 0} \Longrightarrow \quad \mathbf{J} = \eta \frac{e^2}{2\pi^2}\mu_5 \mathbf{B}$$

$$\mathrm{limit}_{\omega\to 0}\mathrm{limit}_{q\to 0} \Longrightarrow \quad \mathbf{J} = \frac{1}{3}\times\eta \frac{e^2}{2\pi^2}\mu_5 \mathbf{B}$$

Artifact of one-loop approximation. The ambiguity disappears with higher order corrections. (*Satow & Yee*)

Constant ***B****, non-constant* $\mu_5(\mathbf{k}, k_0)$

$$\Downarrow$$

$$\text{limit}_{\mathbf{k}\to 0}\text{limit}_{k_0\to 0} \Longrightarrow \qquad \mathbf{J}=0$$

$$\text{limit}_{k_0\to 0}\text{limit}_{\mathbf{k}\to 0} \Longrightarrow \qquad \mathbf{J}=\eta\frac{e^2}{2\pi^2}\mu_5\mathbf{B}$$

Follows from the EM gauge invariance and the non-renormalization of the axial anomaly.

_*with T=0 and* $\mu=0$*: relativistic invariance requires the two limit orders are equivalent*

CME from regulated Wigner function

a robust regularization scheme has to be introduced to the underlying field theory before defining the wigner function. e.g. PV scheme

$$L = -\bar{\psi}\gamma_\mu(\partial_\mu - ieA_\mu - i\gamma_5 A_{5\mu})\psi$$

$$J_\mu(x) = i\int \frac{d^4 p}{(2\pi)^4}\mathrm{tr}W(x,p)\gamma_\mu$$
$$= i\lim_{y\to 0} U(x_+, x_-) < \bar{\psi}(x_+)\gamma_\mu \psi(x_-) >$$

$$J_\mu(x) = -ie\frac{1}{2}\left[\mathrm{Tr}\gamma_\mu \mathcal{S}_0(x,x) - \sum_s C_s \mathrm{Tr}\gamma_\mu \mathcal{S}_s(x,x)\right]$$

$$J_\mu(x) = e^2 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} e^{i(q_1+q_2)\cdot x} \Lambda_{\mu\rho\lambda}(q_1, q_2) A_\rho(q_1) A_{5\lambda}(q_2)$$

Wu,Hou, Ren, PRD 2017

gives CME current：

$$\lim_{q_{20}\to 0} \lim_{\vec{q}_2\to 0} \Lambda_{ij4}(q_1, q_2) = -\frac{1}{2\pi^2}\epsilon_{ikj} q_{1k}$$

CME current canceled at thermal equilibrium.

$$\lim_{\vec{q}_2\to 0} \lim_{q_{20}\to 0} \Lambda_{ij4}(q_1, q_2) = \frac{2f(0)-1}{2\pi^2}\epsilon_{ikj} q_{1k} + O(q_1^2)$$

Karsten and Smit (1981)

using lattice QCD with Wilson term

$$I = -\sum_x \sum_\mu \frac{1}{2a} \Big[\bar{\psi}(x) \left(\frac{1}{i}\gamma_\mu - r \right) U_\mu(x) \psi(x + a_\mu)$$

$$- \bar{\psi}(x + a_\mu) \left(\frac{1}{i}\gamma_\mu + r \right) U_\mu^\dagger(x) \psi(x) \Big]$$

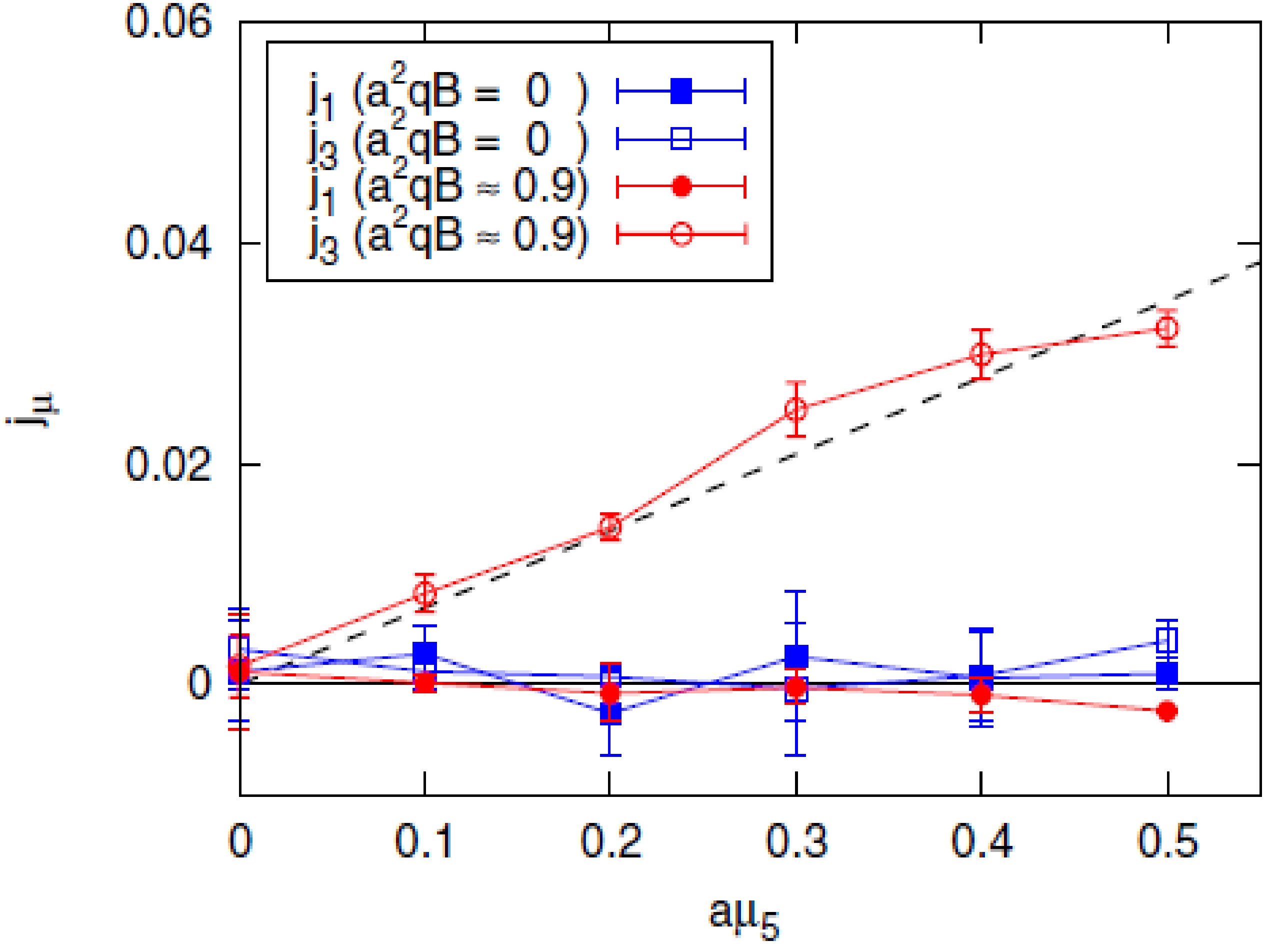
$$- \sum_x M \bar{\psi}(x) \psi(x) + \cdots$$

$$S(p) = a \left[\sum_\mu \gamma_\mu \sin a p_\mu + \mathcal{M}(ap) \right]^{-1},$$

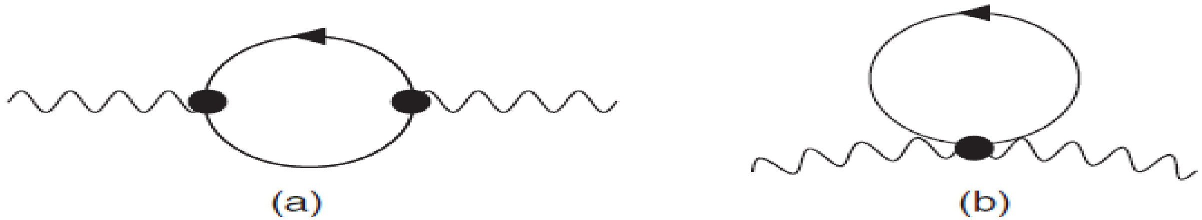
$$V_\mu(p, q) = \gamma_\mu \cos \frac{1}{2}(a p_\mu + a q_\mu) + r \sin \frac{1}{2}(a p_\mu + a q_\mu).$$

CME with Wilson Fermion with chiral chemical potential

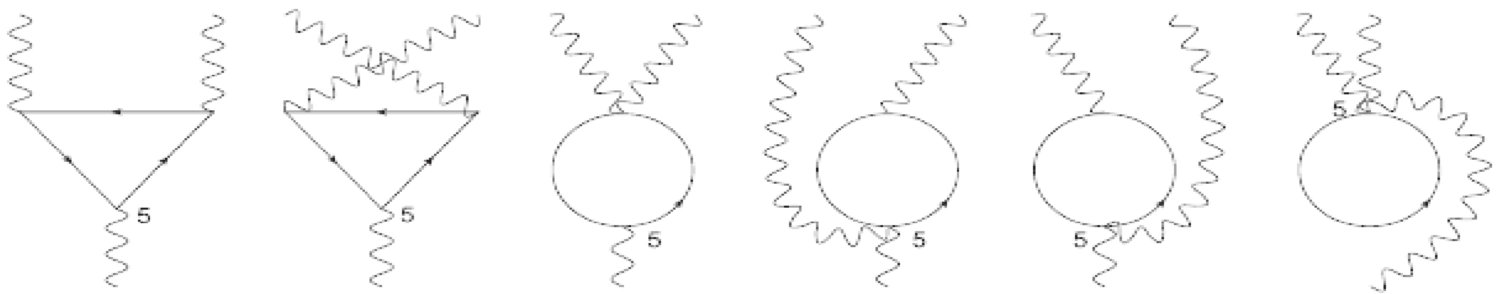
Yamamoto,PRL(2011)



One-loop contributions to $\Pi_{\mu\nu}$.



One-loop triangle diagrams corresponding to $\Pi^{(1)}_{\mu\nu}(p)$.



Feng, Hou, Liu, Ren, Wu, PRD95, (2017)

$$J_i(p) = -\Pi_{ij}(p)A_j(p)$$

One-loop self-energy on lattice of size $N_s^3 \times N_t$

$$\Pi_{ij}^{(1)}(p) = \mathcal{I}\sum_k \epsilon_{ikj}p_k + \mathcal{O}(a)$$

CME vanishes at continu. limit .

At zero temperature

$$\Pi_{ij}(q) \equiv \Lambda_{ij4}(q)$$
$$= -\lim_{q_4\to 0}\frac{1}{q_4}\sum_\rho \frac{2}{a}\sin\frac{1}{2}a(Q_1+Q_2)_\rho \Lambda_{ij\rho}(Q_1,Q_2)$$

$$\Pi_{ij}(q) = \frac{e^2}{2\pi^2}\sum_k \epsilon_{ijk}q_k$$

- numerical calculations

Lattice size	$\mathcal{I}$
$N_s = 6, N_t = 4$	1.347×10^{-2}
$N_s = 12, N_t = 4$	2.439×10^{-4}
$N_s = 20, N_t = 4$	8.886×10^{-7}
$N_s = 50, N_t = 8$	4.512×10^{-9}

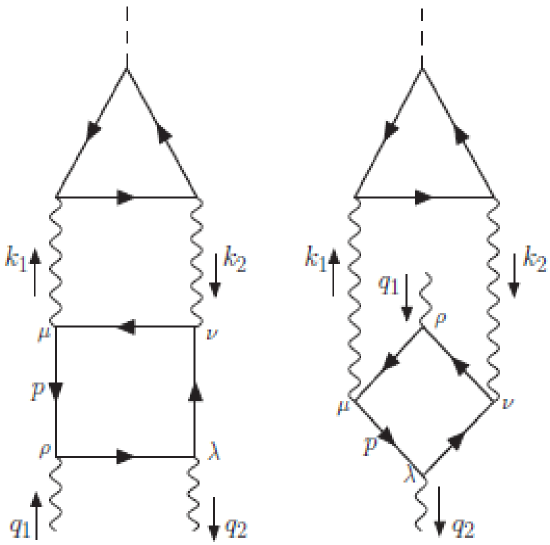
- analytical calculations(In the limit $N_s \to \infty$)

$$\mathcal{I} = 12 \frac{1}{N_t} \sum_{l_4} \int \frac{d^3\mathbf{l}}{(2\pi)^3} \frac{\mathcal{N}(l)}{\left[\sin^2 l + \mathcal{M}^2(l)\right]^3} = 0$$

QED radiative corrections to CME

Feng, Hou, Ren PRD 99 (2019)

Radiative corrections from photon-photon rescattering



Photon rescattering contribution to the AVV function

Ansel'm and Ioganson (1989')

- The anomalous Ward identity

$$(Q_1 + Q_2)_\rho \Lambda_{\mu\nu\rho}(Q_1, Q_2) = -i\frac{e^2}{2\pi^2}\epsilon_{\mu\nu\alpha\beta}Q_{1\alpha}Q_{2\beta} \times \left(1 - \frac{3e^4}{64\pi^4}\ln\frac{\Lambda^2}{k^2}\right)$$

- The kernel of CME current becomes

$$K_{ij}(q) = i\frac{e^2}{2\pi^2}\mu_5\epsilon_{ikj}q_k\left(1 - \frac{3e^4}{64\pi^4}\ln\frac{\Lambda^2}{k^2}\right)$$

- Likewisely, the same diagrams with two internal photons replaced by two gluons may also contribute to CME.

3-loop radiation correction to CME

Feng, Hou, Ren PRD99 (2019)

- the kernel of CME current

$$K_{ij}(\mathbf{q}) = i\frac{e^2}{2\pi^2}F_s\left(\frac{|\mathbf{q}|}{T}\right)\epsilon_{ikj}q_j$$

1. In low temperature limit($T << |\mathbf{q}|$): $F_s(|\mathbf{q}|/T) \to 1 - \frac{3e^4}{64\pi^4}\ln\frac{\Lambda^2}{q^2}$
2. At finite temperature(T>|**q**|): for $\lim_{Q_0\to 0}\lim_{\mathbf{Q}\to 0}$, $F_s(|\mathbf{q}|/T) \to 1$
   for $\lim_{\mathbf{Q}\to 0}\lim_{Q_0\to 0}$, $F_s(|\mathbf{q}|/T) \to 0$

If the two internal photons are replaced by gluons

$$F_s(|\mathbf{q}|/T) \to 1 - \frac{3g^4}{32\pi^4}\log\frac{\Lambda^2}{q^2}.$$

Summary

- **The zero P & zero E limits of μ_5 do not commute and the difference is robust against Higer Order correction**

- **While the CSE is expected in RHIC, its magnitude may not reach the ideal value** $\mathbf{J} = \eta \frac{e^2}{2\pi^2} \mu_5 \mathbf{B}$ **because of inhomogeneity**

. **We calculated the CME to 1-loop order with a lattice regular. With Wilson fermions and the results in continuum agree with that by PV regular. And the 1-loop results using overlapping fermions**

- **Highger-order corrections? Or finite size effect?**

Radiation corrections to CME up to 3-loop massless QED are derived at zero T and non-zero T

Thank you very much for your attention!

It follows that

$$K_{ij}(q) = i\frac{e^2}{2\pi^2}\epsilon_{ikj}q_k$$

Then the CME current

$$\mathbf{J} = \frac{e^2}{2\pi^2}\mu_5\mathbf{B} \tag{1}$$

There are, however, two shortcomings in the above establishment

1. distinction between chiral anomaly at the operator level and its matrix element

   only the former one is free from radiative corrections.

2. the constant μ_5 limit in eq.(1) becomes subtle at finite temperature

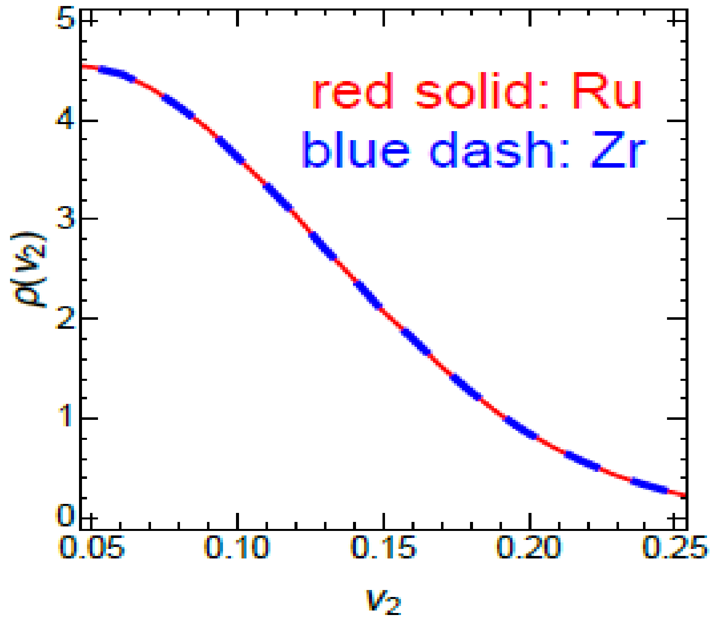
$$\lim_{k_0\to 0}\lim_{\mathbf{k}\to 0} \neq \lim_{\mathbf{k}\to 0}\lim_{k_0\to 0} \tag{2}$$

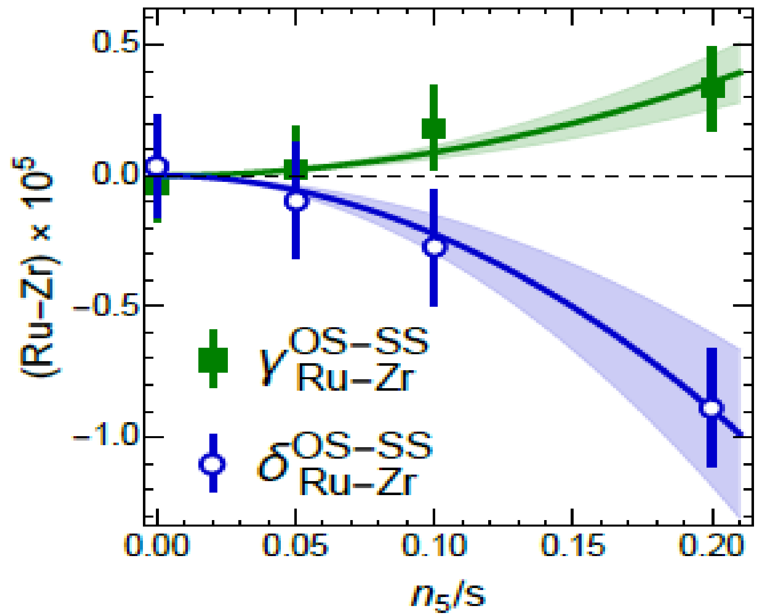
note that in the limiting process $\lim_{\mathbf{k}\to 0}\lim_{k_0\to 0}$, *the relation of CME current to chiral anomaly becomes unclear.*

Chiral Magnetic Effect in Isobaric Collisions from Anomalous-Viscous Fluid Dynamics (AVFD)

Shuzhe Shi , Hui Zhang, Defu Hou , Jinfeng Liao , QM2018 @ Venice May. 13~19, 2018

Nuclear Physics A 00 (2018) 1–4





The absolute difference between isobars, after identical multiplicity+elliptic flow cuts, will provide the most sensitive and clean probe of CME signal.

Fluct. & dissip. of axial charge from massive quark

DF Hou, S. Lin, PRD98, (2018)

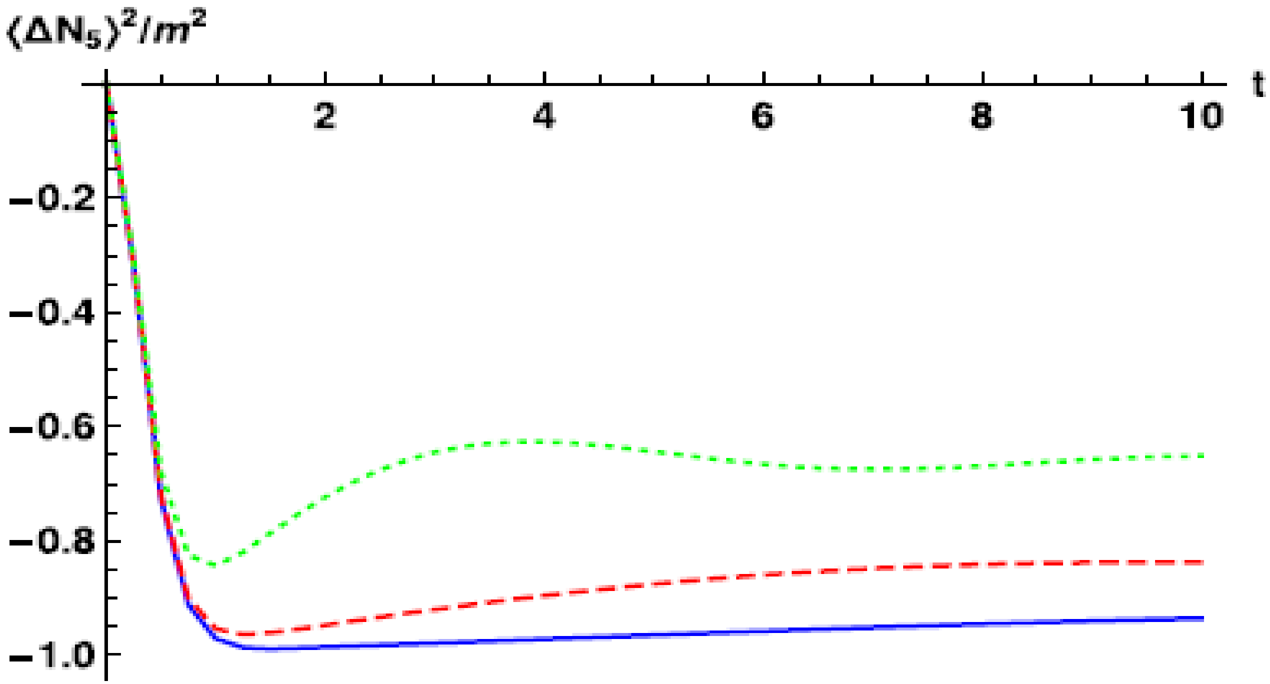


FIG. 2. Contributions from intrinsic fluctuation $\langle\Delta N_5(t)^2\rangle/m^2$ for different masses: blue solid line for $m = 1/10$, red dashed line for $m = 1/5$, and green dotted line for $m = 1/2$. The unit is set by $T = 1$. The fluctuation is characterized by an initial rise followed by oscillatory decay to asymptotic value. The case with a larger mass shows more rapid convergence to the asymptotic value.

Chiral anomaly at operator level and its matrix element

- The operator equation of the anomaly

$$\partial_\mu j^5_\mu = 2imj^5 + i\frac{\alpha_0}{4\pi}\epsilon_{\rho\sigma\lambda\nu}F_{\rho\sigma}F_{\lambda\nu}$$

  the coefficient of the anomalous term is exactly $\alpha_0/4\pi$ and *does not involve an unknown power series in the coupling constant coming from higher orders in perturbation theory.* *Adler and Bardeen (1969')*

- The matrix element between the vacuum and a state with two photons of momenta Q_1, Q_2

$$(Q_1+Q_2)_\mu\Lambda_{\mu\rho\lambda}(Q_1,Q_2) = -i\left[2mG\left(\frac{Q_1^2}{m^2},\frac{Q_2^2}{m^2},\frac{Q_1\cdot Q_2}{m^2}\right) + H\left(\frac{Q_1^2}{m^2},\frac{Q_2^2}{m^2},\frac{Q_1\cdot Q_2}{m^2}\right)\right] \times \epsilon_{\rho\lambda\alpha\beta}Q_{1\alpha}Q_{2\beta}$$

  in low energy limit

$$2mG(0,0,0)+H(0,0,0)=0, \quad H(0,0,0)=\frac{2\alpha}{\pi}$$

- For massless fermions, the low energy kinematic point cannot be attained, the matrix elements receive radiative corrections.