Trace anomaly under lattice regularization

Yi-Bo Yang

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The Higgs boson makes the u/d quark having masses (2GeV MS-bar):

\[ m_u = 2.08(09) \text{ MeV} \]
\[ m_d = 4.73(12) \text{ MeV} \]

But the mass of the proton is 938.272046(21) MeV.

~100 times of the sum of the quark masses!

http://flag.unibe.ch/2019/Quark%20masses
Outline

Hadron mass and trace anomaly

Non-perturbative renormalization

Towards the trace anomaly on EIC

Trace anomaly in Dim. Reg.

Under the dimensional regularization, the QCD EMT can be decomposed into the trace part and the traceless part:

\[
T_{\mu\nu} = \frac{1}{4} \bar{\psi} \gamma_{(\mu} \overleftrightarrow{D}_{\nu)} \psi + F_{\mu\rho} F^{\rho}_{\nu} - \frac{1}{4} g_{\mu\nu} F^2
\]

\[
= \left(T_{\mu\nu} - \frac{g_{\mu\nu}}{d} T^\alpha_{\alpha}\right) + \frac{g_{\mu\nu}}{d} T^\alpha_{\alpha} \equiv \bar{T}_{\mu\nu} + \hat{T}_{\mu\nu},
\]

where

\[
T^\alpha_{\alpha} = m\bar{\psi}\psi - 2\epsilon \frac{F^2}{4} + \mathcal{O}(\epsilon^2) .
\]

See Y. Hatta, et.al., JHEP12(2018)008 as an example

After the renormalization,

\[
m\bar{\psi}\psi = (m\bar{\psi}\psi)_R, \quad F^2 = -\frac{1}{\epsilon} \left(\frac{\beta_R}{g_R} F^2_R + 2\gamma^R_m (m\bar{\psi}\psi)_R\right) + \mathcal{O}(\epsilon^0),
\]

And then,

\[
T^\alpha_{\alpha} = (1 + \gamma^R_m) (m\bar{\psi}\psi)_R + \frac{\beta_R}{2g_R} F^2_R .
\]

M. Shifman, et.al., PLB 78 (1978) 443
Trace anomaly in other Regularizations

For the Pauli-Villars regularization, EMT trace anomaly comes from the massive regulators, likes the relation between the heavy quark mass term and $F^2$:

$$m_Q\bar{\psi}_Q\psi_Q = -\frac{1}{12\pi}\alpha_s F^2 + O(\alpha_s^2) + O(\frac{1}{m_Q})$$

— argument from M. Polyakov as mentioned in Y. Hatta, et.al., JHEP12(2018)008; or hinted by the 1-loop QED calculation in J. Cui, et.al., PRD84(2011)025020

For the Lattice regularization?

• The regularization itself should break the traceless of EMT at $O(a^2)$;

• The quantum effect introduces some power divergence and make the total loop correction to be finite.

• The same form up to the 2-loop level, regardless regularization.
Trace anomaly in the hadron state

Thus we have the sum rule for the hadron mass

\[ M = \langle M | T^\alpha_\alpha | M \rangle = \langle M | \sum_q m_q \bar{\psi}_q \psi_q | M \rangle + \langle M | \gamma_5 \sum_q m_q \bar{\psi}_q \psi_q + \frac{\beta}{2g} F^2 | M \rangle \]

which is renormalization invariant while \( n_f \) depends.

\[ n_f=3: \text{quark mass} \sim 90 \text{ MeV}, \text{anomaly} 850 \text{ MeV} \]

\[ n_f=6: \text{quark mass} \sim 270 \text{ MeV}, \text{anomaly} 670 \text{ MeV} \]
Trace anomaly in the nucleon state

Direct calculation

If we ignore the $\gamma_m$ term which is $\sim 3\%$ at MS-bar 2GeV for the nucleon, and use the relation between the pion mass and quark mass:

$$
\langle \pi \mid H_a \mid \pi \rangle = m_\pi - \langle \pi \mid H_m \mid \pi \rangle = m_\pi - m_q \frac{\partial m_\pi}{\partial m_q} \sim \frac{1}{2} m_\pi
$$

We can have the following expression for the nucleon:

$$
\langle N \mid H_a \mid N \rangle \simeq \langle N \mid \frac{\beta}{2g} F^2 \mid N \rangle
$$

$$
\simeq \frac{m_\pi}{2} \frac{\langle N \mid \frac{\beta}{2g} F^2 \mid N \rangle}{\langle \pi \mid \frac{\beta}{2g} F^2 \mid \pi \rangle} = \frac{m_\pi}{2} \frac{\langle N \mid F^2 \mid N \rangle}{\langle \pi \mid F^2 \mid \pi \rangle}
$$
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Renormalize the $F^2$ term through RI/MOM

Another possibility, is to renormalize the $F^2$ term to the MS-bar scheme, and multiply the MS-bar $\beta$ function:

$$\langle N | H_a | N \rangle \simeq \frac{\beta_{MS}}{2g_{MS}} \langle N | F_{MS}^2 | N \rangle = \frac{\beta_{MS}}{2g_{MS}} \frac{Z^{MS}}{Z^{RI}} Z^{RI} \langle N | F_{bare}^2 | N \rangle$$

The matching factor can be calculated perturbatively in the continuum,

$$\frac{Z^{MS}}{Z^{RI}} = 1 + \frac{\beta}{2g} \left[ \log(\mu^2/p^2) + ?? \right] + \mathcal{O}(\alpha_s^2)$$

while the $Z^{RI}$ should be calculated non-perturbatively on the lattice.
Renormalized glue momentum fraction?

\[ \hat{R}(t) = \langle \chi \rangle_g + O(e^{-\Delta m t}) \]

Seem to be impossible to obtain the renormalization of the glue operators non-perturbatively?
Non-perturbative renormalized glue momentum fraction

\[ \langle x \rangle_{g}^{bare} = Z^{-1} \]

- The lattice regularization effects are **fully cancelled** within the statistical uncertainties;
- The renormalization factor can be majorly understood by the tadpole improvement factor.

YBY, et. al., $\chi$QCD collaboration, PRD98(2018) 074506
Renormalization of the glue momentum fractions

It becomes possible after kinds of the improvements are applied!
Z^{RI} of the $F^2$ term

The gluon EMT includes kinds of the structures and can have different renormalization constants:

$$\bar{T}^{(0)}_{g,\mu\nu} = (2p_\mu p_\nu g_{\rho\tau} - p_\mu p_\rho g_{\nu\tau} + p_\rho^2 g_{\mu\nu} - p_\tau p_\nu g_{\rho\mu}$$

$$- p_\nu p_\rho g_{\mu\tau} + p_\rho^2 g_{\nu\mu} - p_\tau p_\mu g_{\rho\nu}$$

$$+ g_{\mu\nu} \left( p_\tau p_\rho - p_\rho^2 g_{\tau\rho} \right) A_\rho(p) A_\tau(-p),$$

$F^2$ term

48$^3 \times 96$, $a=0.11\text{fm}$

Diagram: Graph showing various contributions with different colors and markers, illustrating the $a^2$ dependence in a 2D plot.
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The near-threshold $\gamma+N\rightarrow J/\psi+N$ photo production cross section would be sensitive to the form factor of the trace anomaly...

...if is Q2 dependence is similar to that of the traceless part of EMT


Y. Hatta and D. L. Yang, PRD98(2018)074003

...Possible experiments related to the trace anomaly

What LQCD can do

\[ \langle P | E_E^2 | P' \rangle = \frac{1}{2} \langle P | E_E^2 - B_E^2 | P' \rangle + \frac{1}{2} \langle P | F_E^2 | P' \rangle \]

\[ t = - (P - P')^2 \]

- The second term (proportional to the trace anomaly) dominants in the forward case.

- But the off-forward case (which is needed by the experiment) requires further investigation.

- LQCD calculation on the \( Q^2 \) dependence is free of the renormalization issue, up to the mixing with the quark mass term.
Summary

• Lattice QCD can conform the large anomaly contribution to the nucleon mass, and help EIC to measure it.

• An accurate LQCD calculation is on-going.

• If we combine the flavor-depend anomaly contribution with the quark mass part:

\[
M = \langle M | (1 + \gamma_m) \sum_q m_q \bar{\psi}_q \psi_q + \frac{\beta(n_f) - \beta(0)}{2g} F^2 | M \rangle + \langle M | \frac{\beta(0)}{2g} F^2 | M \rangle
\]

Then the second term contributes ~110% of the nucleon mass, regardless \( n_f=2,3,\ldots,6 \).