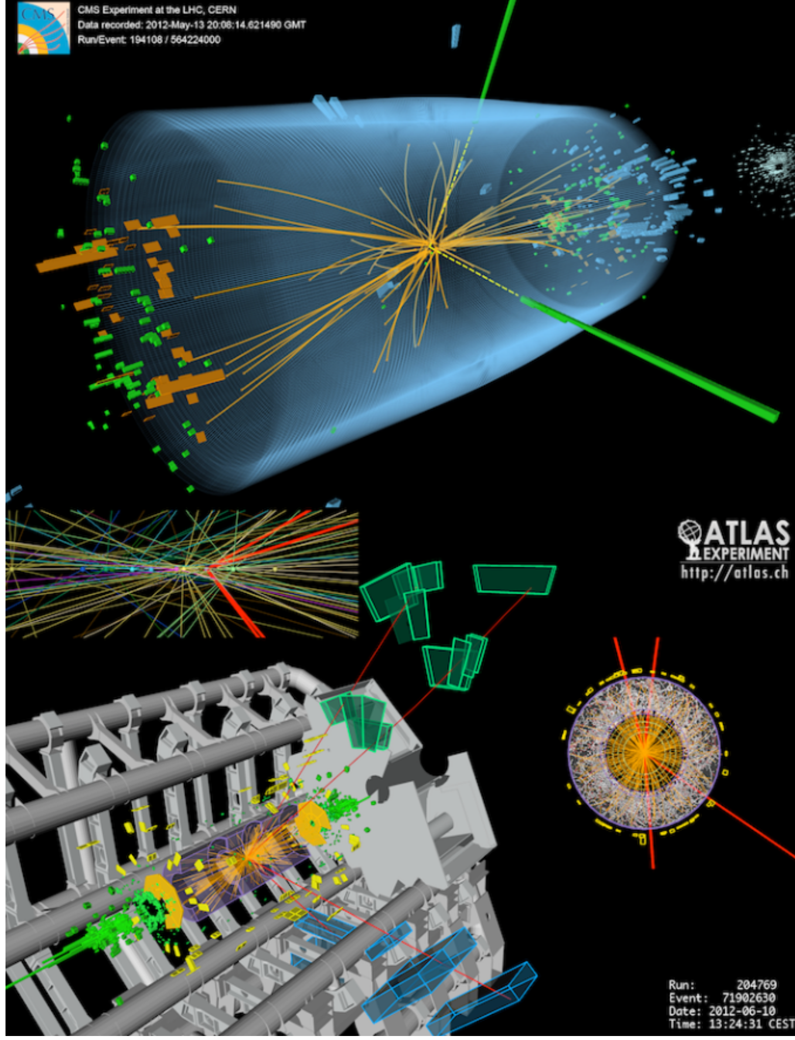
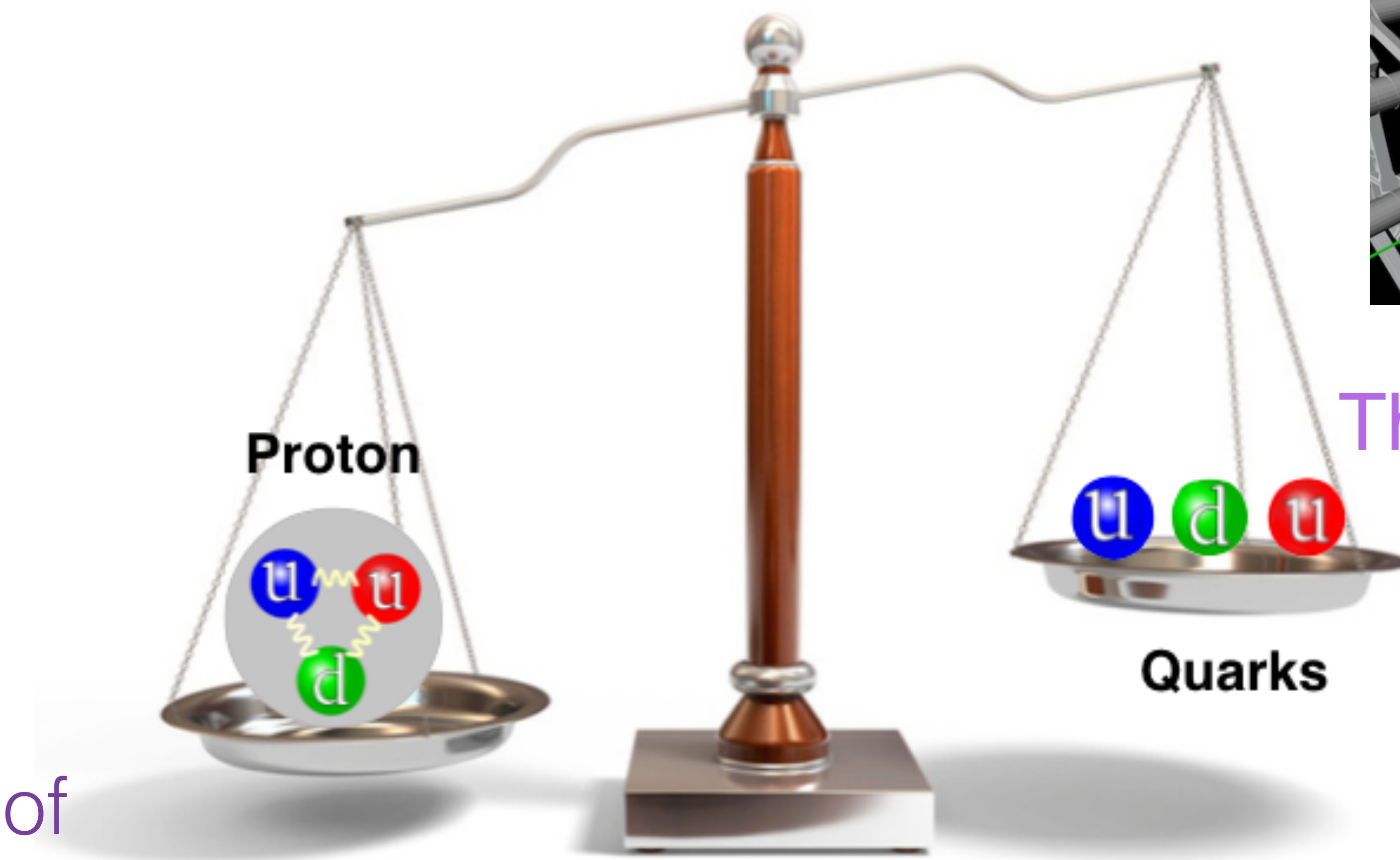
Trace anomaly under lattice regularization

Yi-Bo Yang

Jun. 20th, 2019

How does the mass of nucleon arise?





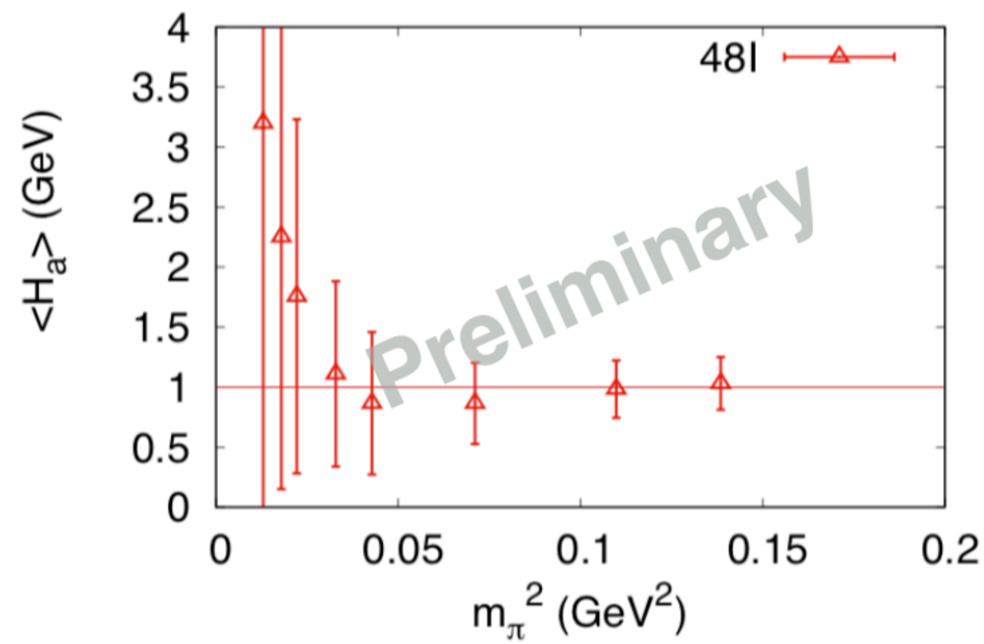
The Higgs boson make the u/d quark having masses (2GeV MS-bar):

$m_u = 2.08(09)$ *MeV*
$m_d = 4.73(12)$ *MeV*

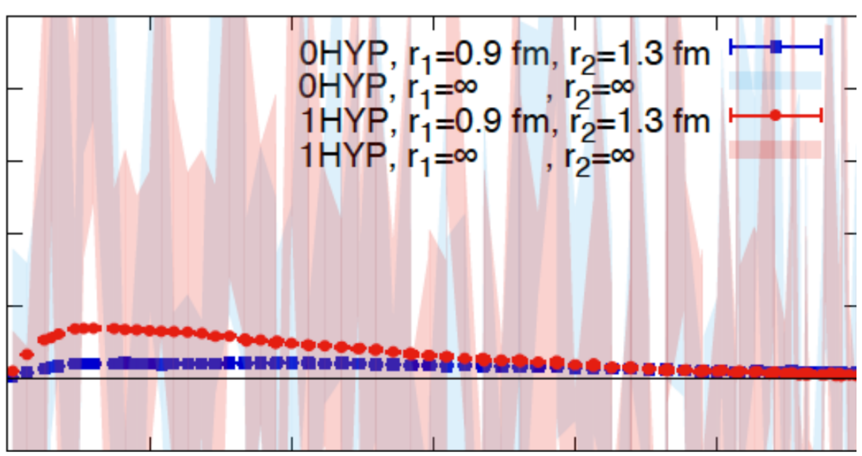
http://flag.unibe.ch/2019/Quark%20masses

But the mass of the proton is
938.272046(21) MeV.
~100 times of the sum of the quark masses!

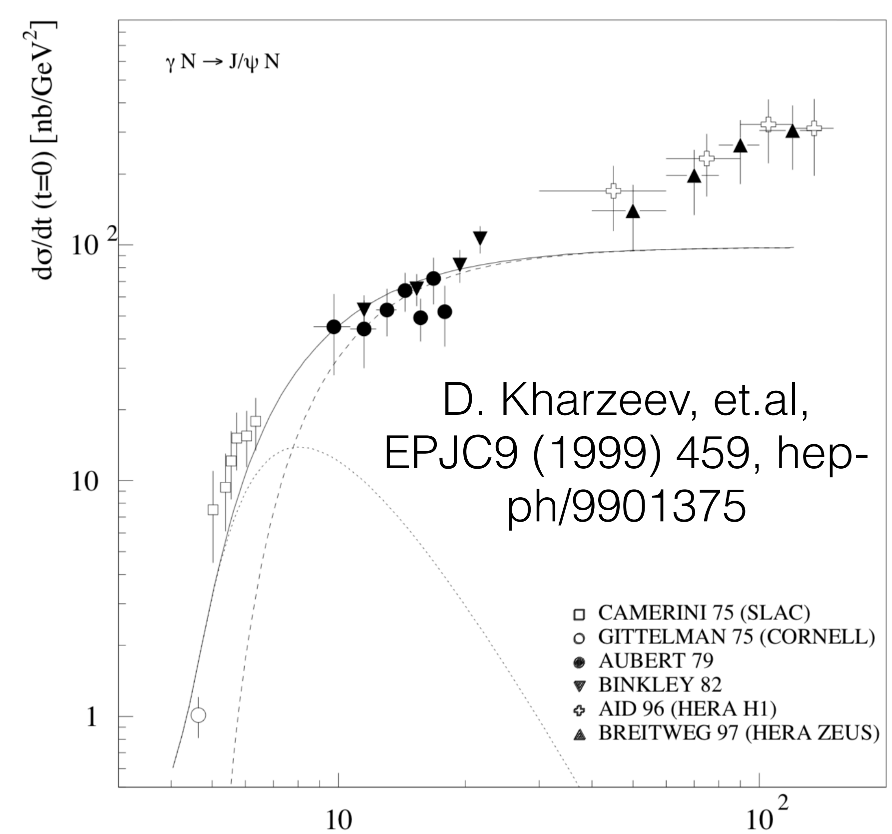
Outline



Hadron mass and trace anomaly



Non-perturbative renormalization



D. Kharzeev, et.al, EPJC9 (1999) 459, hep-ph/9901375

Towards the trace anomaly on EIC

Trace anomaly in Dim. Reg.

Under the dimensional regularization, the QCD EMT can be decomposed into the trace part and the traceless part:

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + F_{\mu\rho}F_{\nu}{}^{\rho} - \frac{1}{4}g_{\mu\nu}F^2$$

$$= \left(T_{\mu\nu} - \frac{g_{\mu\nu}}{d}T^{\alpha}_{\alpha}\right) + \frac{g_{\mu\nu}}{d}T^{\alpha}_{\alpha} \equiv \bar{T}_{\mu\nu} + \hat{T}_{\mu\nu},$$

where $T^{\alpha}_{\alpha} = m\bar{\psi}\psi - 2\epsilon\frac{F^2}{4} + \mathcal{O}(\epsilon^2)\,.$

See Y. Hatta, et.al., JHEP12(2018)008 as an example

After the renormalization,

$$m\bar{\psi}\psi = \left(m\bar{\psi}\psi\right)_R,\ F^2 = -\frac{1}{\epsilon}\left(\frac{\beta_R}{g_R}F_R^2 + 2\gamma_m^R\left(m\bar{\psi}\psi\right)_R\right) + \mathcal{O}(\epsilon^0),$$

And then,

$$T^{\alpha}_{\alpha} = (1+\gamma_m^R)\left(m\bar{\psi}\psi\right)_R + \frac{\beta_R}{2g_R}F_R^2\,.$$

M. Shifman, et.al., PLB 78 (1978) 443

Trace anomaly in other Regularizations

For the Pauli-Villars regularization, EMT trace anomaly comes from the massive regulators, likes the relation between the heavy quark mass term and F^2:

$$m_Q\bar{\psi}_Q\psi_Q = -\frac{1}{12\pi}\alpha_s F^2 + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\frac{1}{m_Q}),$$

— argument from M. Polyakov as mentioned in Y. Hatta, et.al., JHEP12(2018)008; or hinted by the 1-loop QED calculation in J. Cui, et.al., PRD84(2011)025020

For the Lattice regularization?

- The regularization itself should break the traceless of EMT at O(a^2);
- The quantum effect introduces some power divergence and make the total loop correction to be finite.
- The same form up to the 2-loop level, regardless regularization.

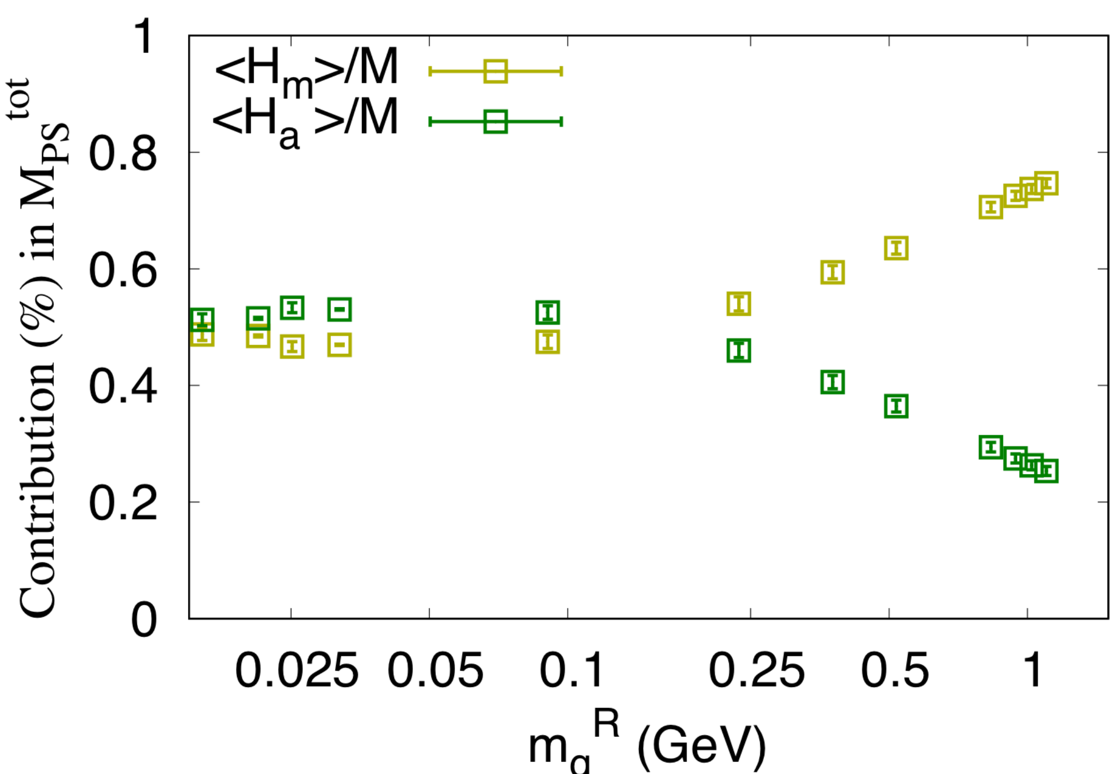
Trace anomaly in the hadron state

Thus we have the sum rule for the hadron mass

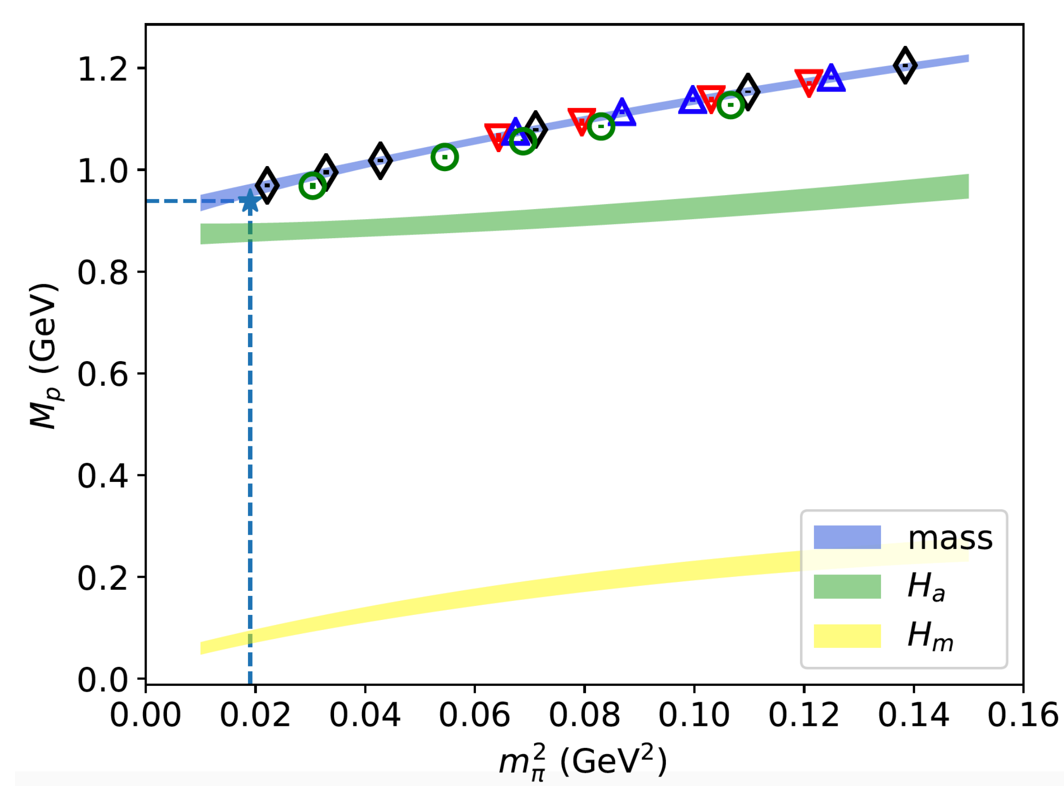
$$M = \langle M | T^{\alpha}_{\alpha} | M \rangle = \underbrace{\langle M | \sum_q m_q \bar{\psi}_q \psi_q | M \rangle}_{H_m} + \underbrace{\langle M | \gamma_m \sum_q m_q \bar{\psi}_q \psi_q + \frac{\beta}{2g} F^2 | M \rangle}_{H_a}$$

which is renormalization invariant while n_f depends.

n_f=3: quark mass ~ 90 MeV, anomaly 850 MeV
n_f=6: quark mass ~270 MeV, anomaly 670 MeV



YBY, et. al., χQCD collaboration, PRD91 (2015) 074516



YBY, J. liang, et. al., χQCD collaboration, PRD94 (2016) 054503

Trace anomaly in the nucleon state

Direct calculation

If we ignore the γ_m term which is ~3% at MS-bar 2GeV for the nucleon, and use the relation between the pion mass and quark mass:

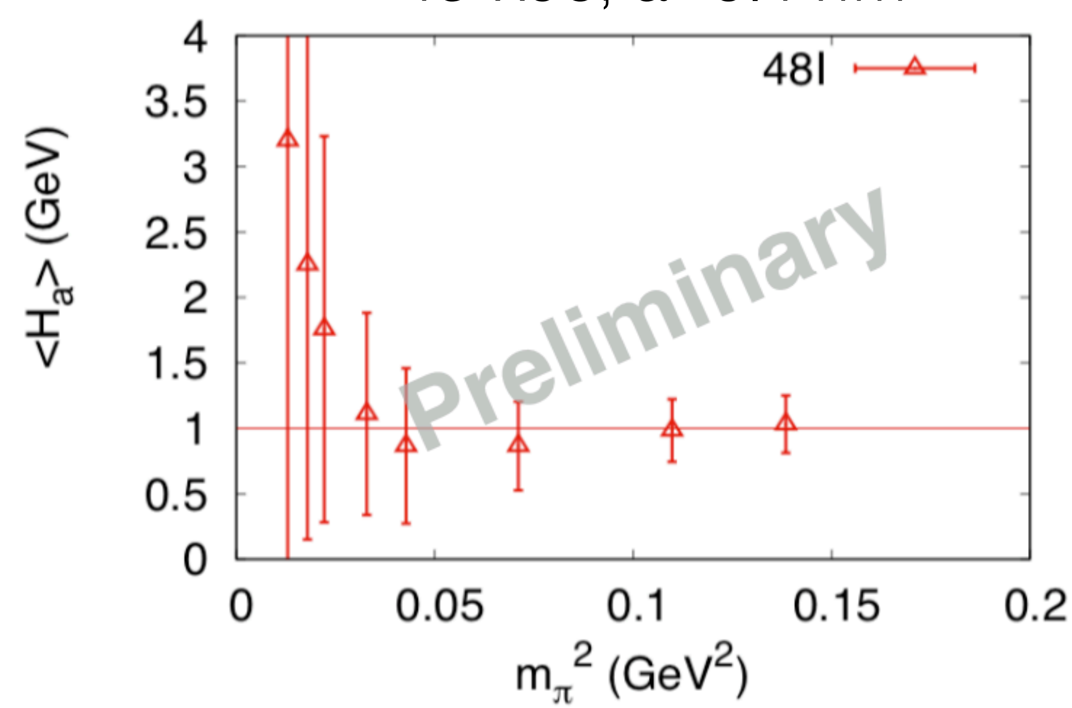
$$\langle \pi | H_a | \pi \rangle = m_\pi - \langle \pi | H_m | \pi \rangle = m_\pi - m_q \frac{\partial m_\pi}{\partial m_q} \sim \frac{1}{2} m_\pi$$

We can have the following expression for the nucleon:

$$\langle N | H_a | N \rangle \simeq \langle N | \frac{\beta}{2g} F^2 | N \rangle$$

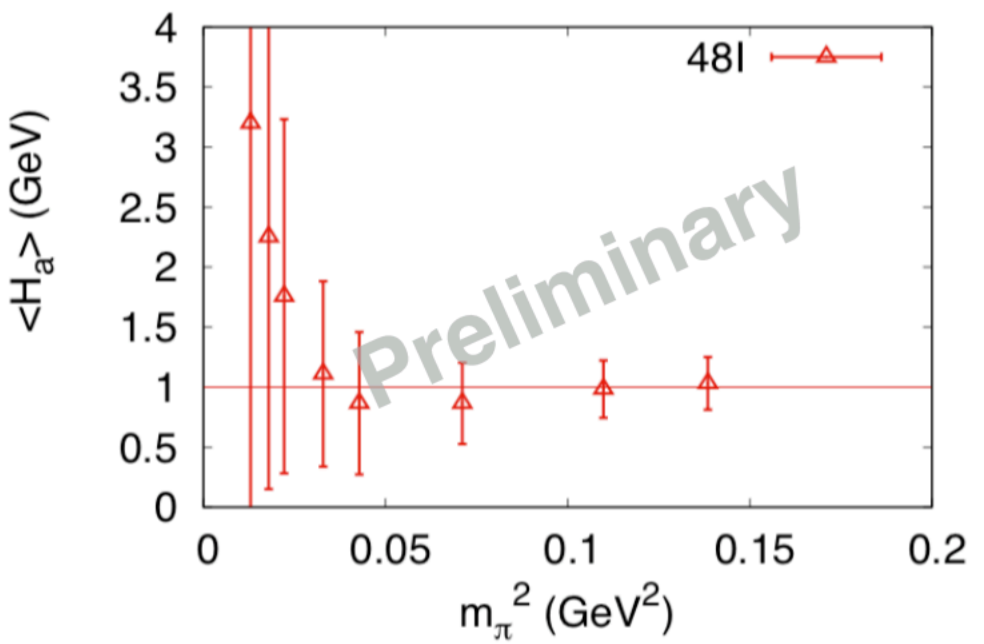
$$\simeq \frac{m_\pi}{2} \frac{\langle N | \frac{\beta}{2g} F^2 | N \rangle}{\langle \pi | \frac{\beta}{2g} F^2 | \pi \rangle} = \frac{m_\pi}{2} \frac{\langle N | F^2 | N \rangle}{\langle \pi | F^2 | \pi \rangle}$$

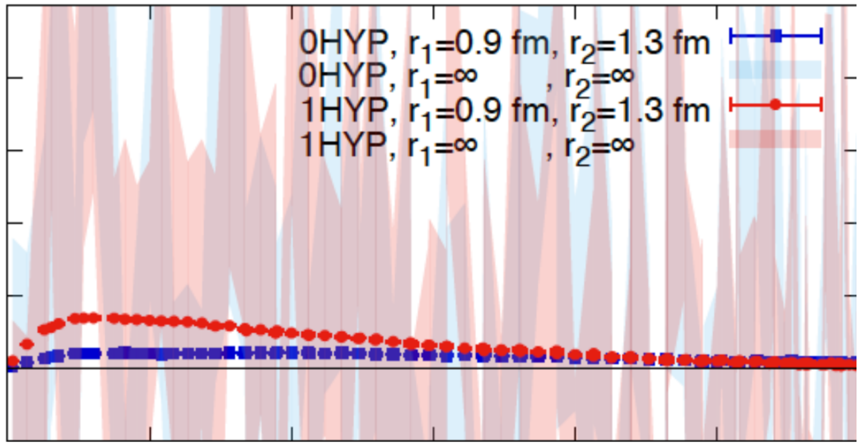
48^3x96, a=0.11fm



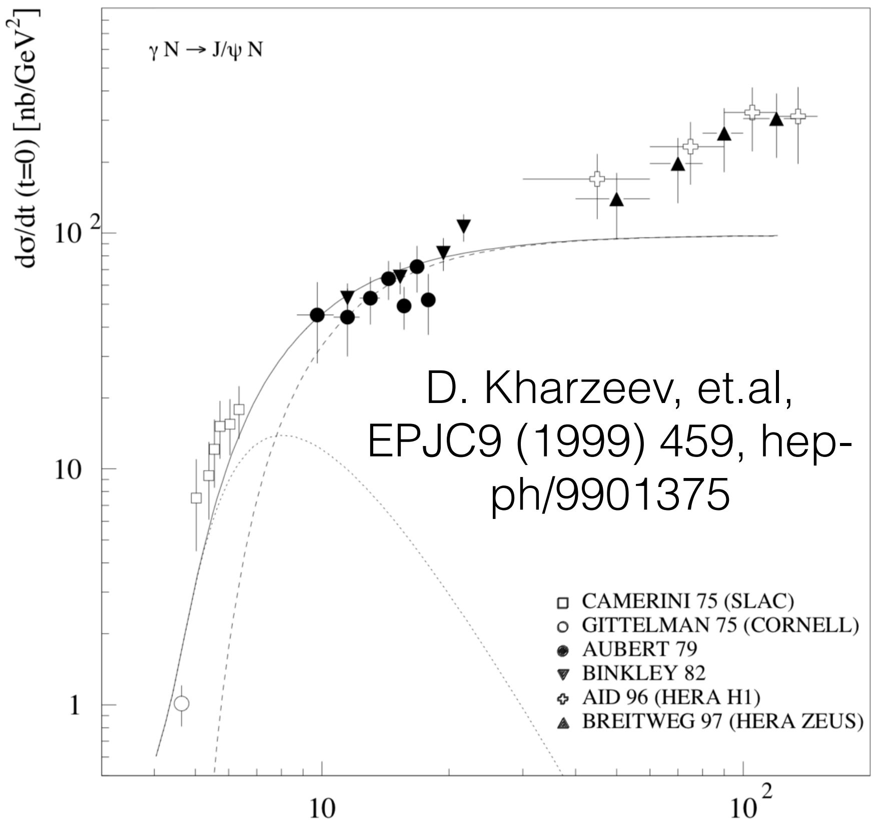
Outline

Hadron mass and trace anomaly





Non-perturbative renormalization



D. Kharzeev, et.al, EPJC9 (1999) 459, hep-ph/9901375

Towards the trace anomaly on EIC

Renormalize the F² term through RI/MOM

Another possibility, is to renormalize the F^2 term to the MS-bar scheme, and multiply the MS-bar β function:

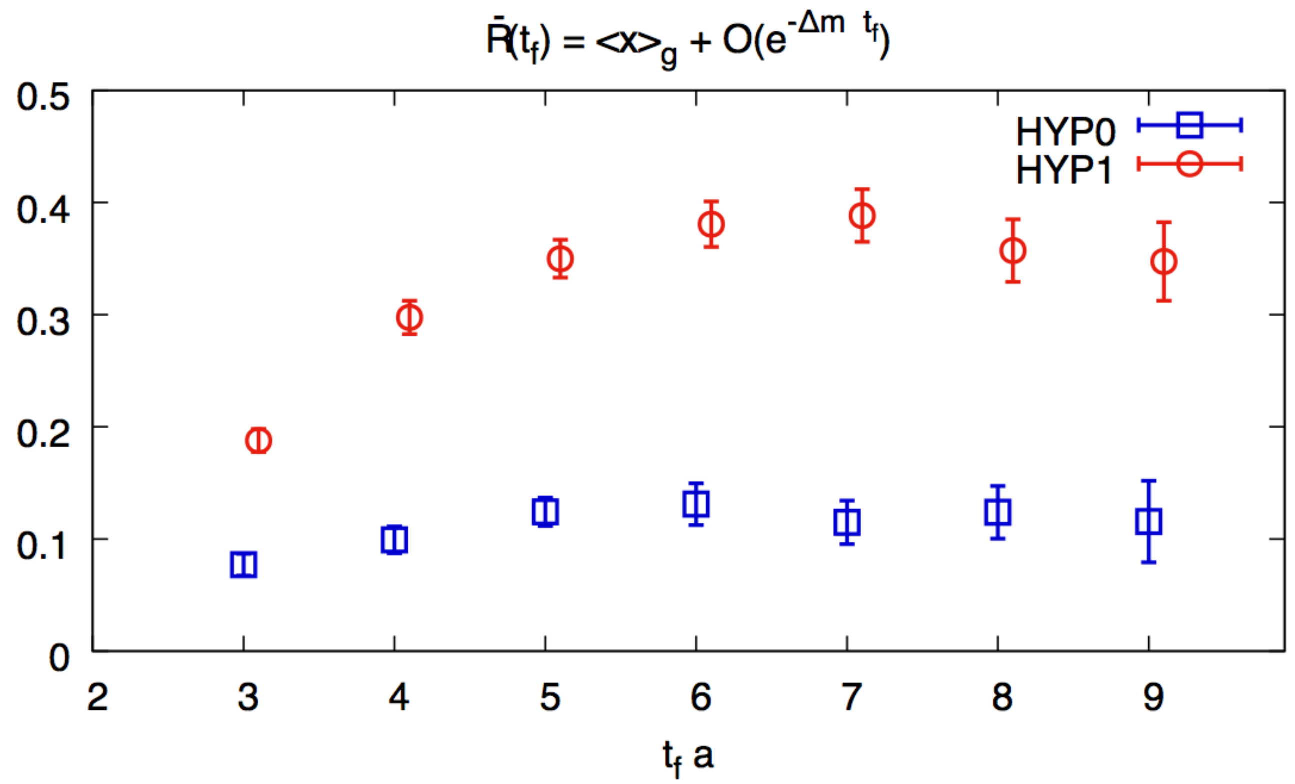
$$\langle N | H_a | N \rangle \simeq \frac{\beta_{MS}}{2g_{MS}} \langle N | F^2_{MS} | N \rangle = \frac{\beta_{MS}}{2g_{MS}} \frac{Z^{MS}}{Z^{RI}} Z^{RI} \langle N | F^2_{bare} | N \rangle$$

The matching factor can be calculated perturbatively in the continuum,

$$\frac{Z^{MS}}{Z^{RI}} = 1 + \frac{\beta}{2g} \left[\log(\mu^2/p^2) + ?? \right] + \mathcal{O}(\alpha_s^2)$$

while the Z^{RI} should be calculated non-perturbatively on the lattice.

Renormalized **glue** momentum fraction?



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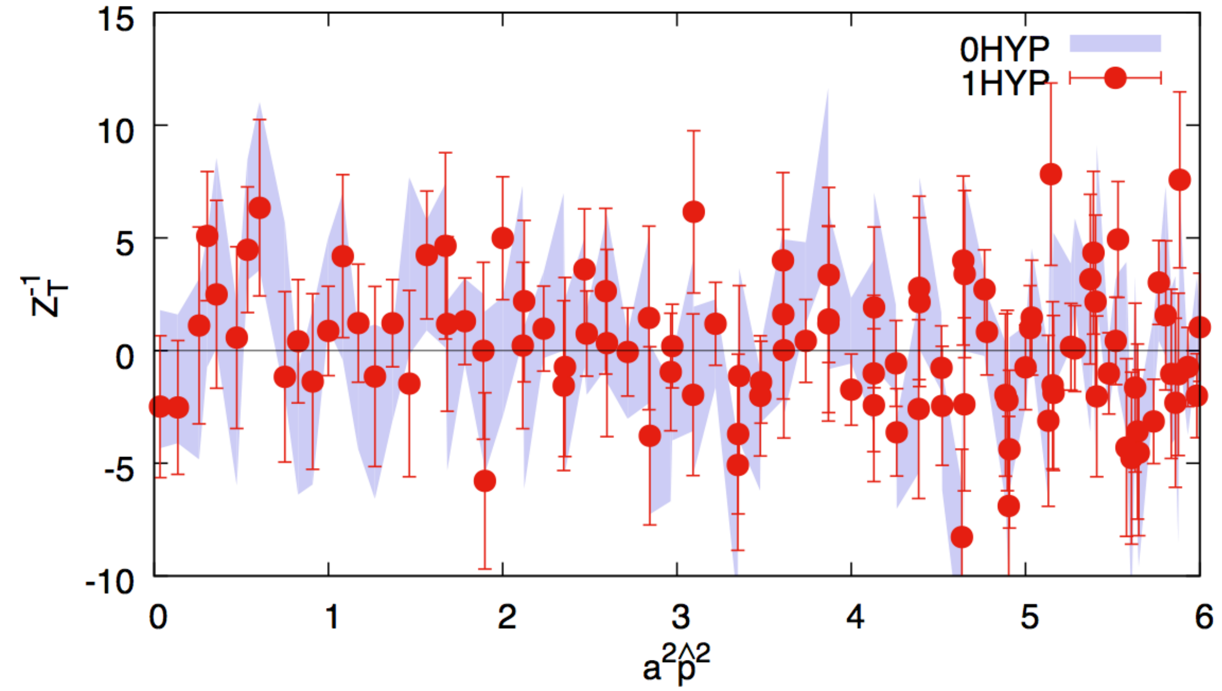

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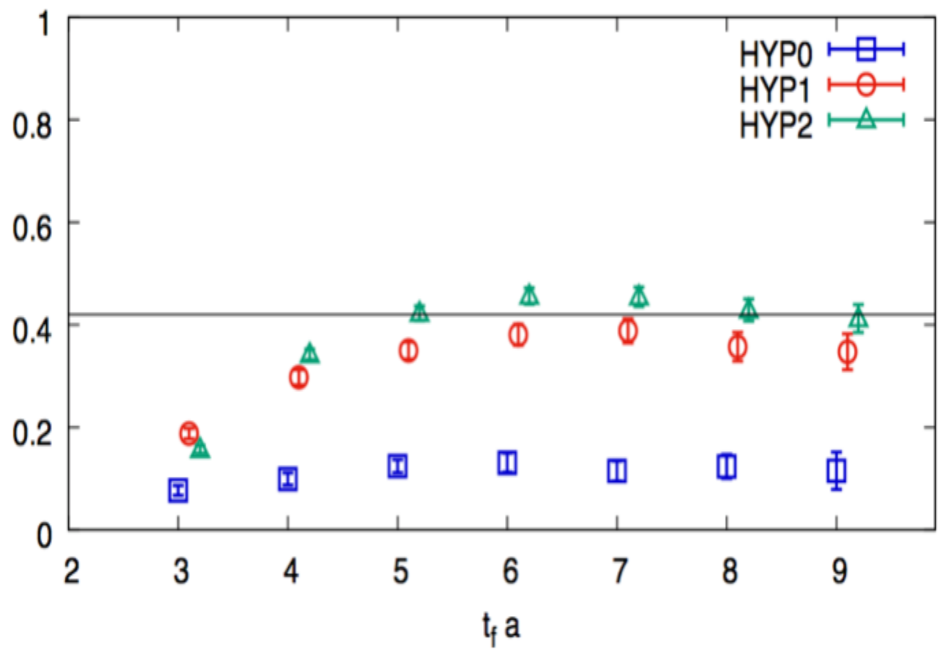
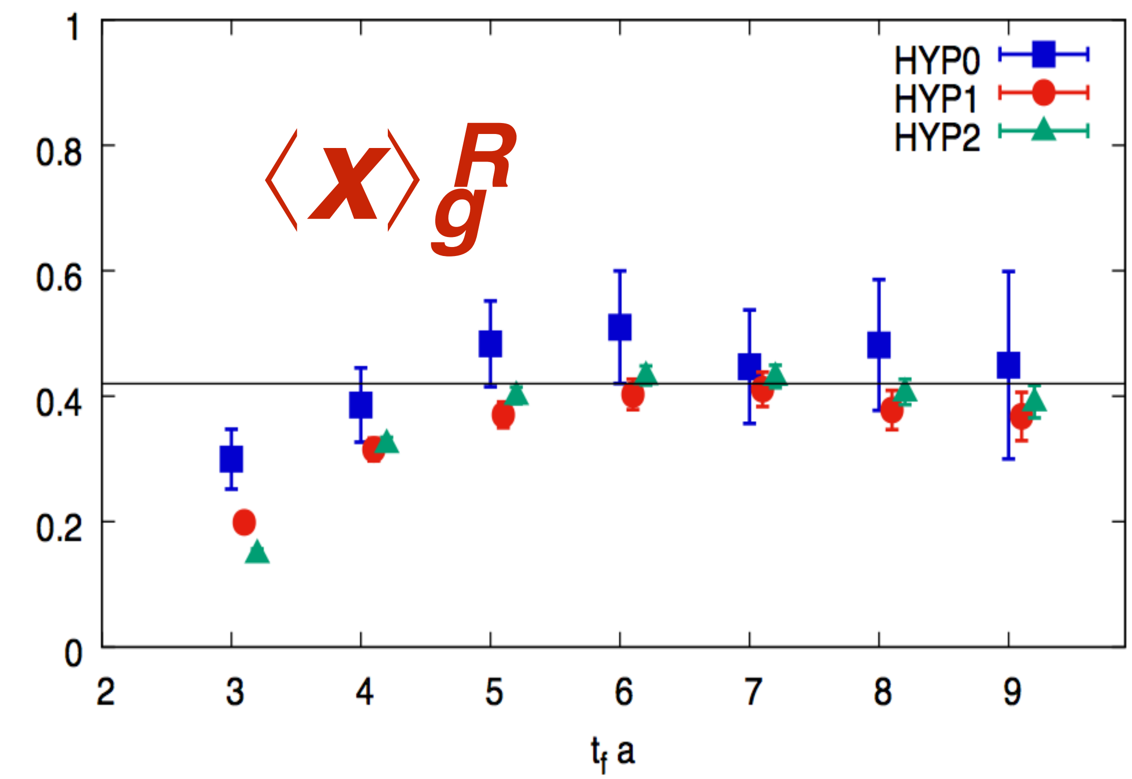
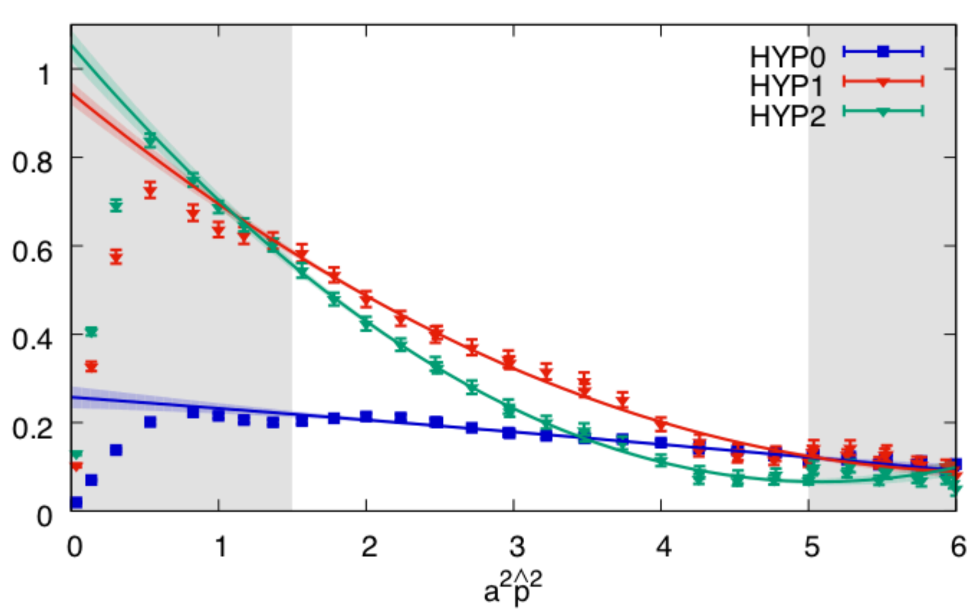
Seem to be impossible to obtain the renormalization of the glue operators ***non-perturbatively?***

Non-perturbative renormalized glue momentum fraction



$$\frac{\langle x\rangle_g^{bare}}{Z^{-1}} = \langle x\rangle_g^{R}$$



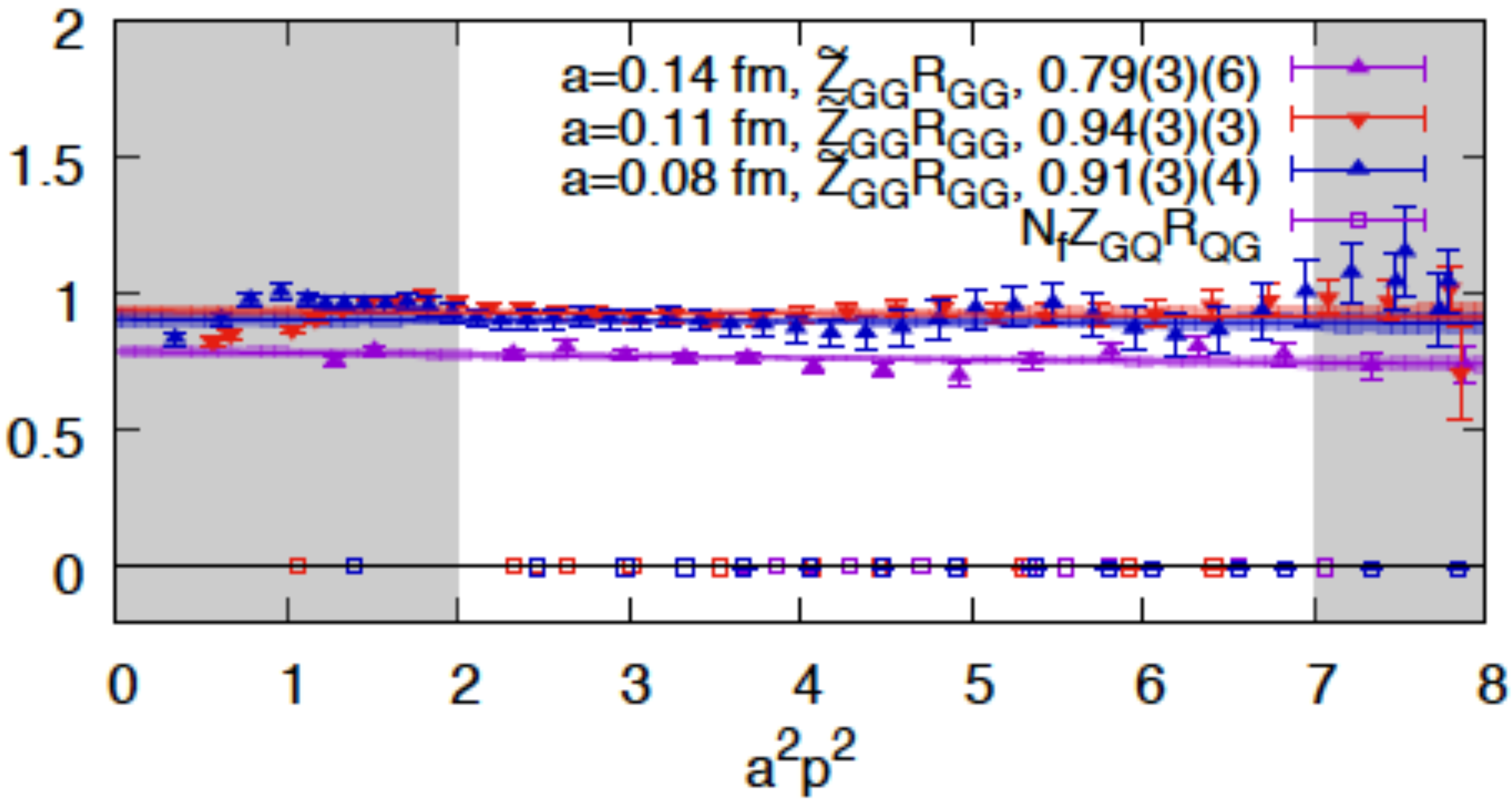


- The lattice regularization effects are **fully cancelled** within the statistical uncertainties;
- The renormalization factor can be majorly understood by the tadpole improvement factor.

YBY, et. al., χQCD collaboration, PRD98(2018) 074506

Renormalization

of the **glue** momentum fractions



It becomes possible after kinds of the improvements are applied!

YBY, J. Liang, et. al., χQCD Collaboration, PRL121(2018)212001, **ViewPoint and Editor's suggestion**

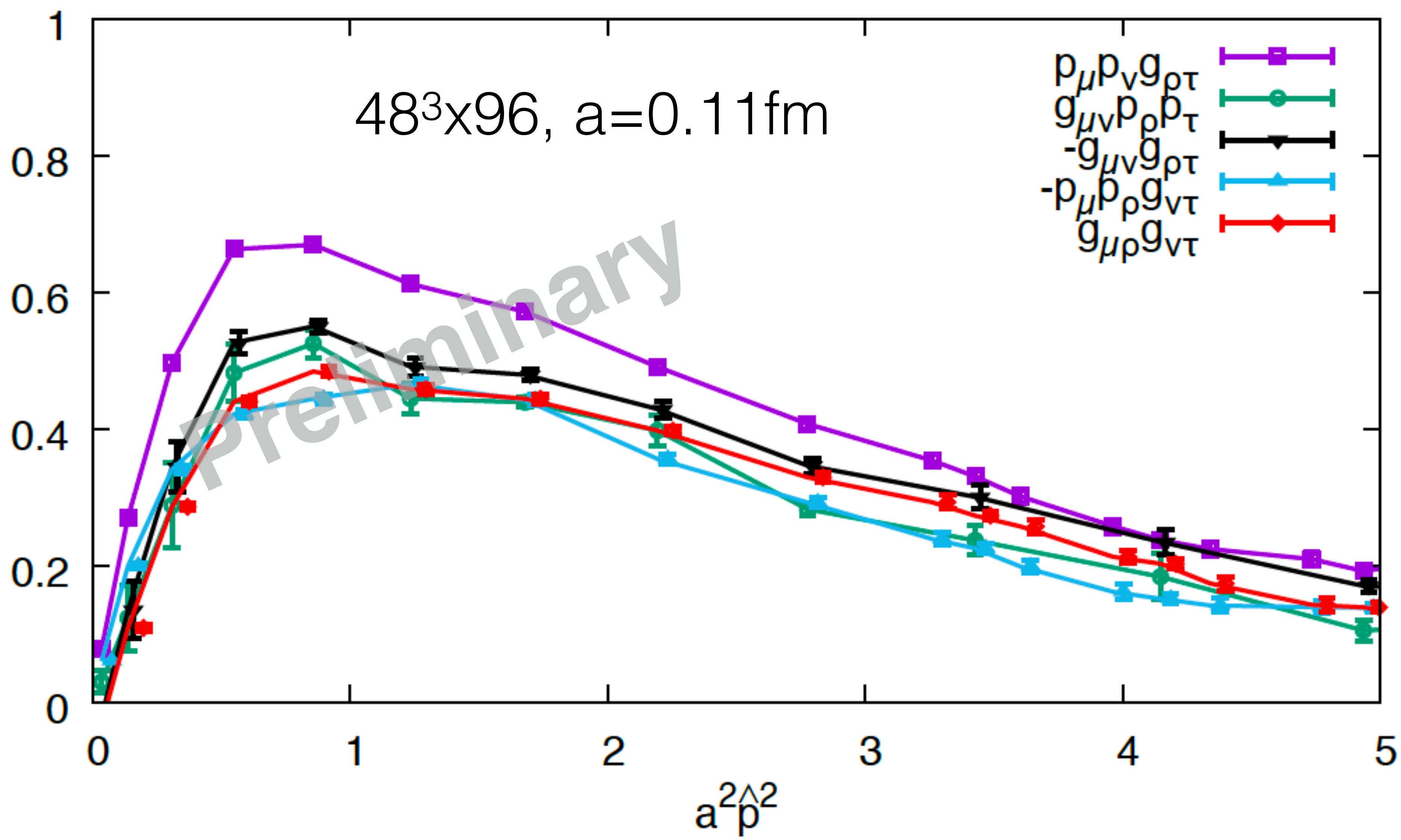
Z^{RI} of the F^2 term

The gluon EMT includes kinds of the structures and can have different renormalization constants:

Term for the traceless EMT

$$\overline{T}^{(0)}_{g,\mu\nu} = \Big(\boxed{2p_\mu p_\nu g_{\rho\tau}} - p_\mu p_\rho g_{\nu\tau} + p^2 g_{\rho\mu} g_{\nu\tau} - p_\tau p_\nu g_{\rho\mu} - p_\nu p_\rho g_{\mu\tau} + p^2 g_{\rho\nu} g_{\mu\tau} - p_\tau p_\mu g_{\rho\nu} + g_{\mu\nu}\left(p_\tau p_\rho - \boxed{p^2 g_{\tau\rho}}\right)\Big) A_\rho(p) A_\tau(-p),$$

F^2 term



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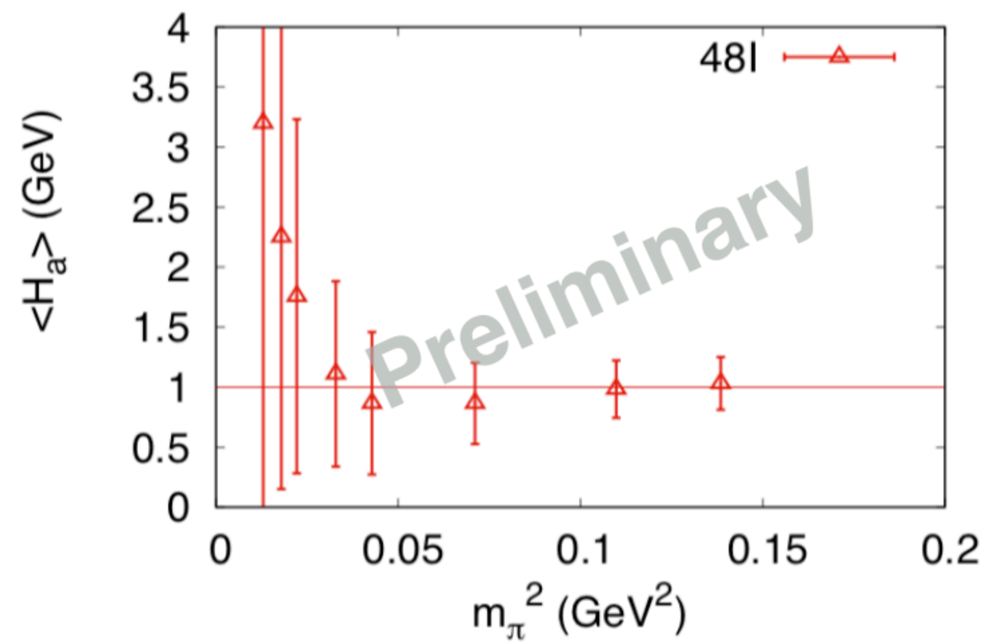
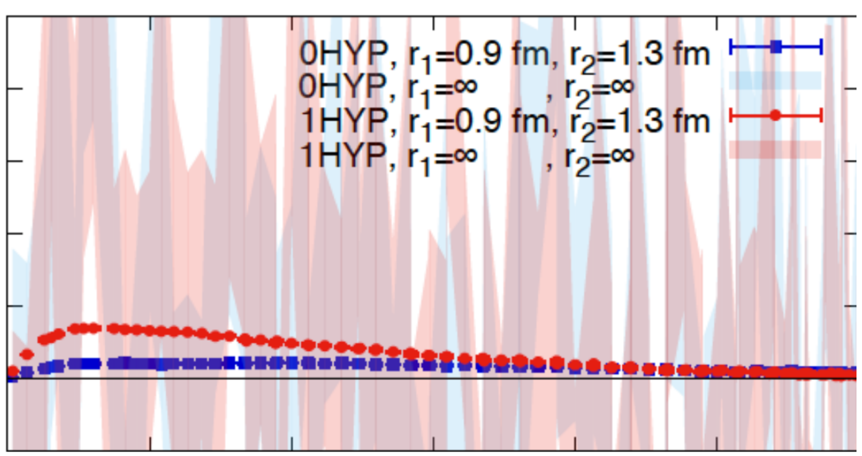
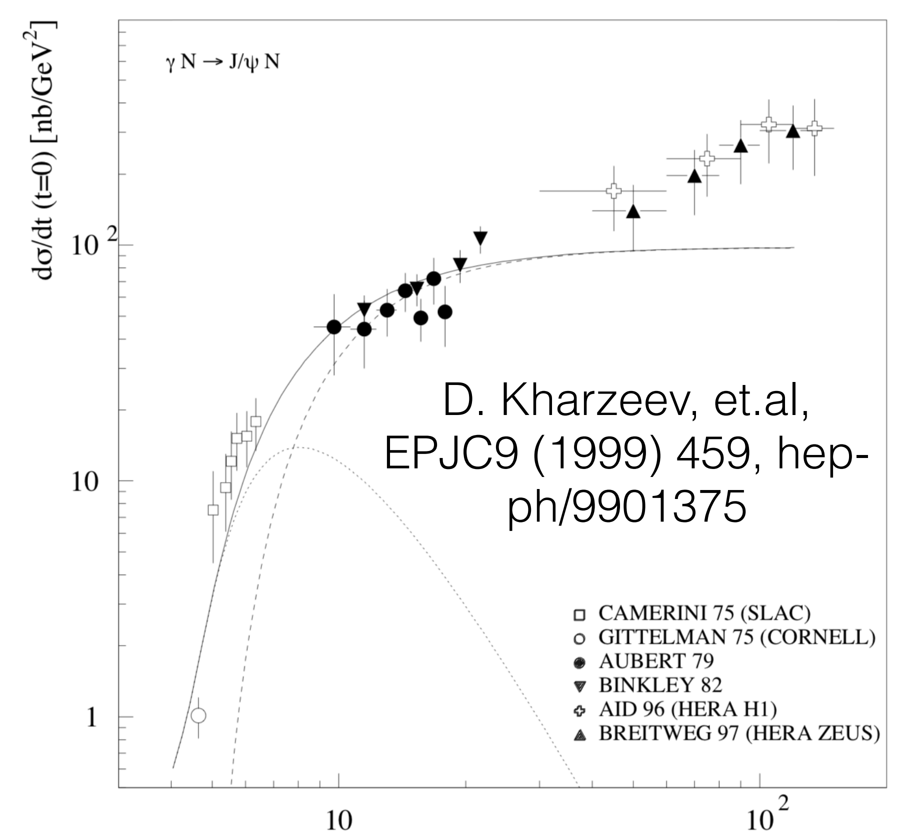




Non-perturbative renormalization

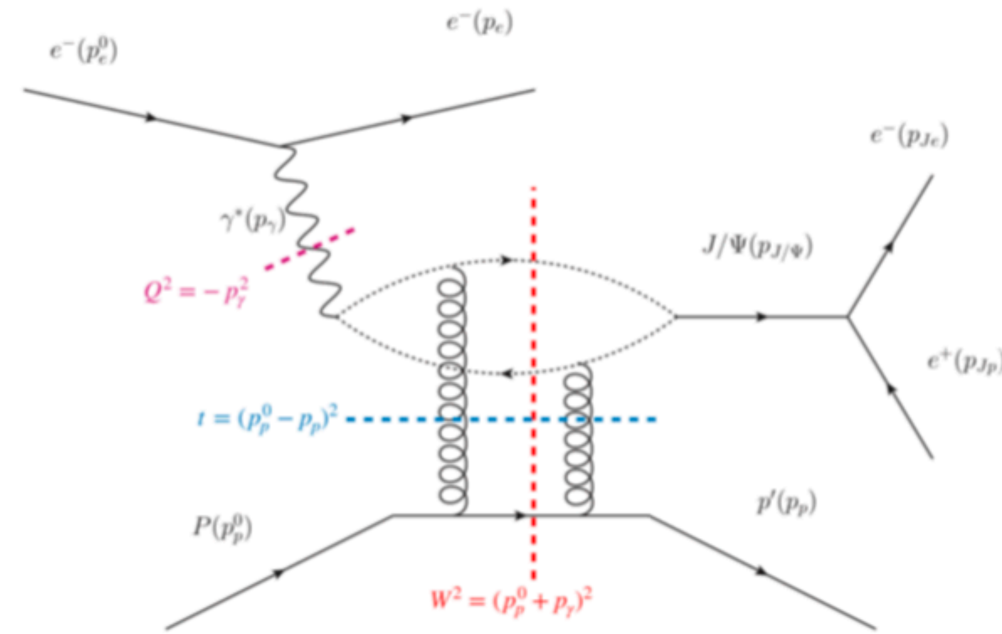
Towards the trace anomaly on EIC



D. Kharzeev, et.al, EPJC9 (1999) 459, hep-ph/9901375

Possible experiments related to the trace anomaly

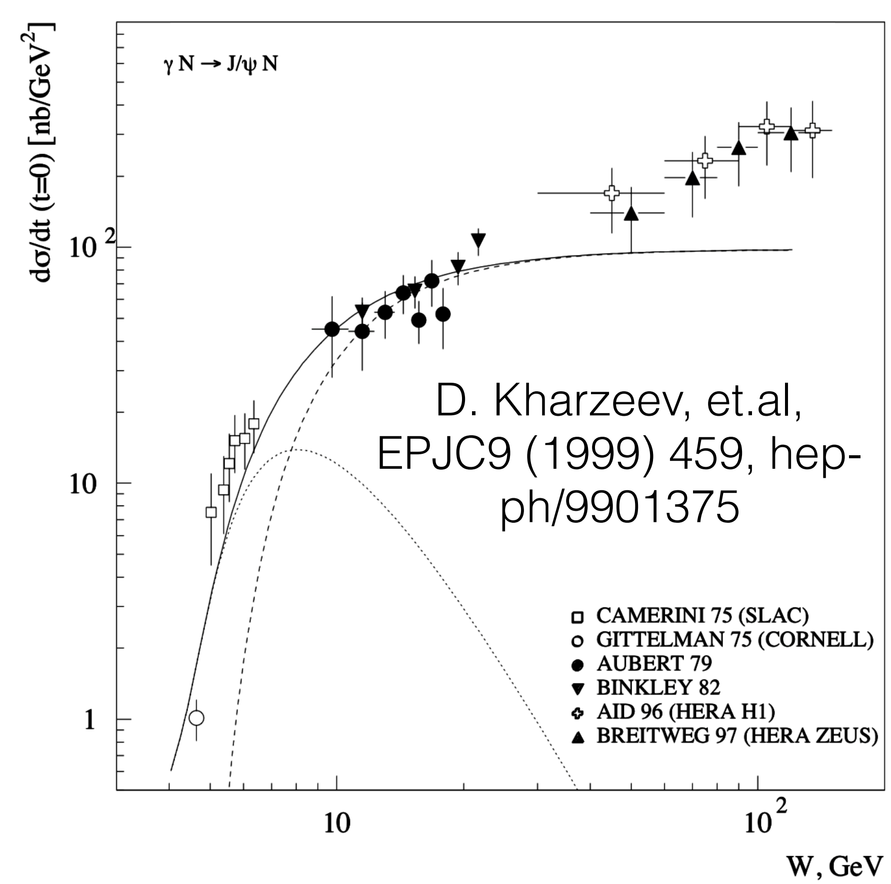
- The near-threshold γ+N->J/psi+N photo production cross section would be sensitive to the form factor of the trace anomaly…



D. Kharzeev, Proc. Int. Sch. Phys. Fermi 130 (1996), 105

- …if is Q2 dependence is similar to that of the traceless part of EMT

Y. Hatta and D. L. Yang, PRD98(2018)074003

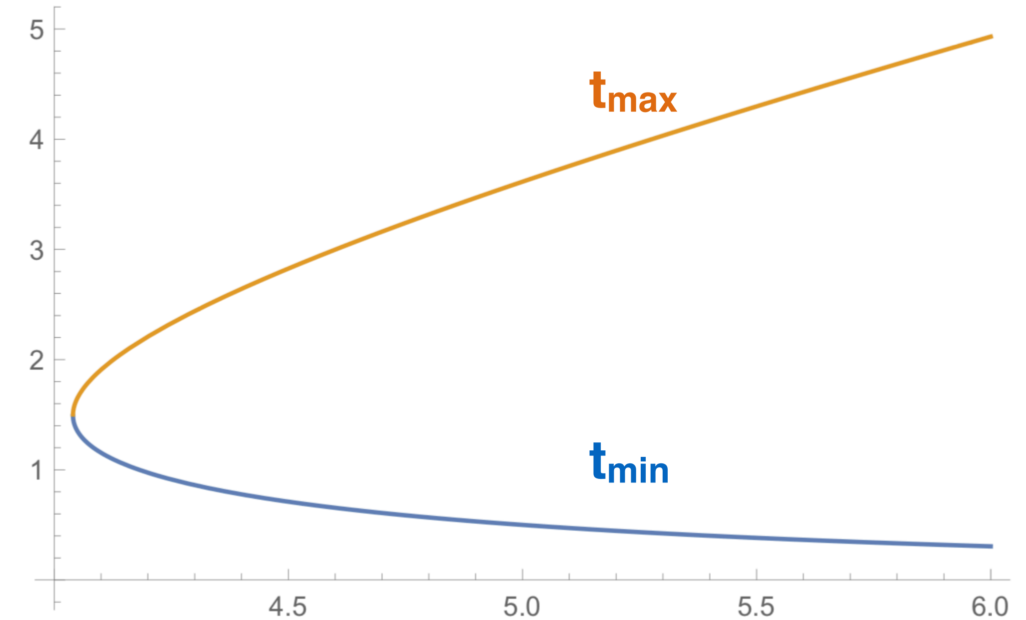


What LQCD can do

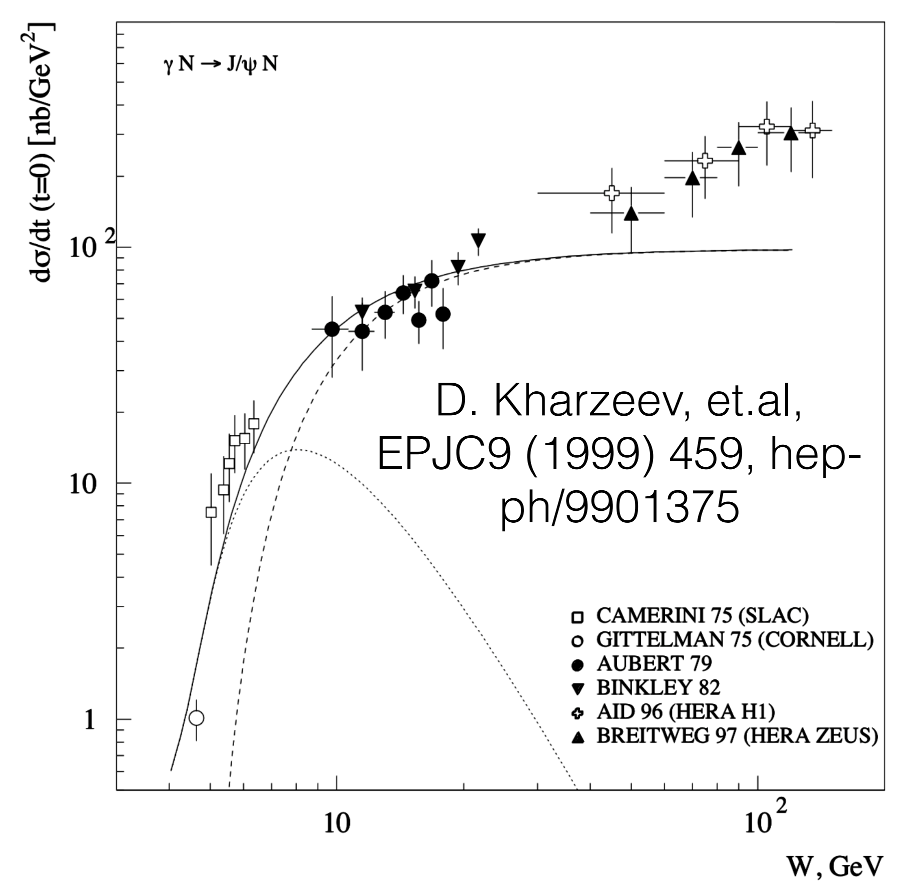
$$\langle P | E_E^2 | P' \rangle = \frac{1}{2} \langle P | E_E^2 - B_E^2 | P' \rangle + \frac{1}{2} \langle P | F_E^2 | P' \rangle$$

$$t = -(P - P')^2$$

- The second term (proportional to the trace anomaly) dominants in the forward case.
- But the off-forward case (which is needed by the experiment) requires further investigation.
- LQCD calculation on the Q^2 dependence is free of the renormalization issue, up to the mixing with the quark mass term.



center-of-mass energy of the J/Ψ+N system, W



Summary

- Lattice QCD can conform the large anomaly contribution to the nucleon mass, and help EIC to measure it.

- An accurate LQCD calculation is on-going.

- If we combine the flavor-depend anomaly contribution with the quark mass part:

Just an approximation.
See K. Tanaka,
JHEP1901 (2019) 120
for the details

$$M = \langle M | (1+\gamma_m) \sum_q m_q \bar{\psi}_q \psi_q + \frac{\beta(n_f) - \beta(0)}{2g} F^2 | M \rangle + \langle M | \frac{\beta(0)}{2g} F^2 | M \rangle$$

Then the second term contributes ~110% of the nucleon mass, regardless n_f=2,3,…,6.