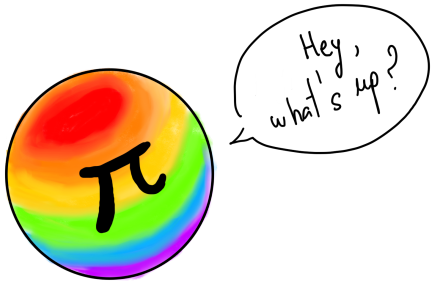
Meson interactions at Large N_c from Lattice QCD



Fernando Romero-López

In collaboration with: A. Donini, P. Hernández & C. Pena

arXiv:1607.03262, arXiv:1711.10248, arXiv:1810.06285 and on-going work

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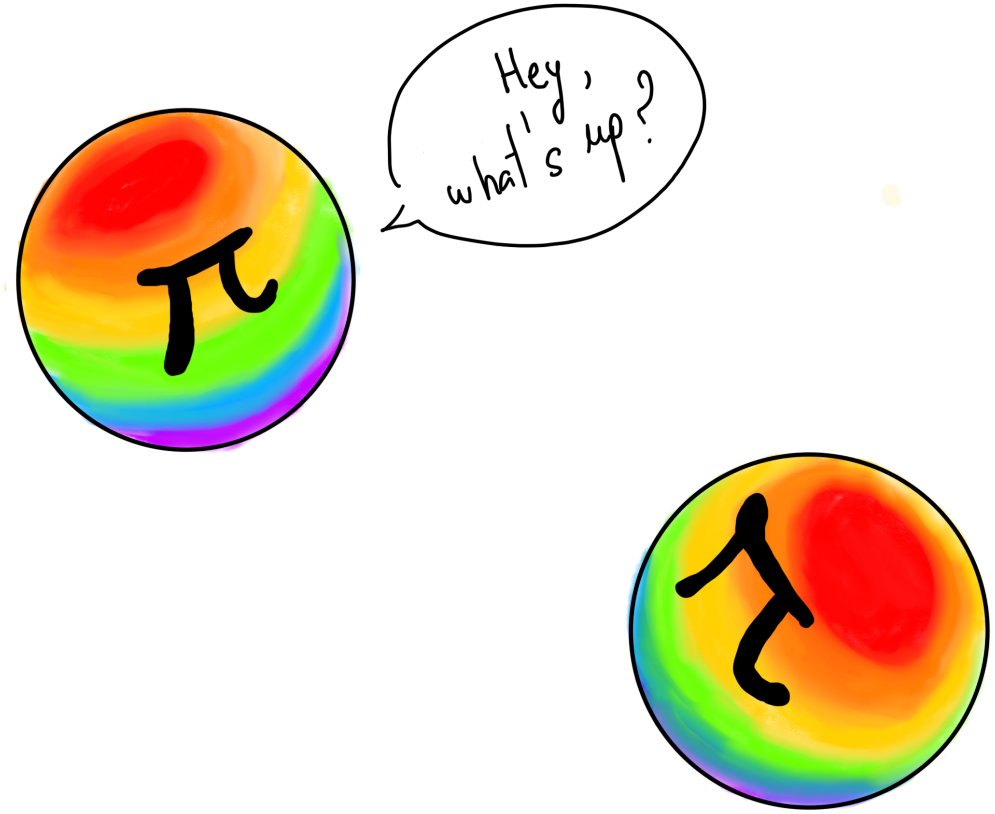
Wuhan, 18th June, 2019

Overview of meson interactions at Large N_c



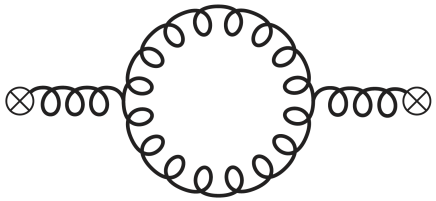
Motivation

The Large N_c limit ('t Hooft limit)

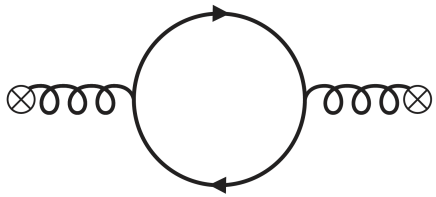
- Let QCD be a $SU(N_c)$ gauge theory
  - ★ $N_c \to \infty$
  - ★ $\alpha_s N_c =$ constant
- QCD is dominated by gluon loops and keeps relevant features (i.e. confinement, spontaneous chiral symmetry breaking...)
- In particular, ChPT is still valid at Large N_c and it is simpler!

Example of N_c counting:

1. Gluon loops$\to$ Adjoint rep. $\sim N_c^2$



2. Quark loops$\to$ Fundamental rep. $\sim N_c$



Chiral Perturbation Theory at NLO and Large N_c

1. The LO Lagrangian ($\mathcal{L}_2$) of ChPT is very predictive:

$$\mathcal{L}_2 = \frac{F_0^2}{4}\text{tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + \frac{F_0^2 B}{2}\text{tr}\left(MU + M^\dagger U^\dagger\right),$$

2. At NLO, there are more terms in the Lagrangian with additional couplings (Low Energy Constants):

$$\mathcal{L}_4 = \sum_{i=0}^{10} L_i \mathcal{O}_i.$$

- LECs encode the high energy physics information
- They have different Large N_c behaviour

L_i	Value	Order
$2L_1 - L_2$	-0.4 ± 0.2	1
L_4	-0.0 ± 0.3	1
L_6	0.0 ± 0.4	1
L_7	-0.3 ± 0.2	1
L_2	1.6 ± 0.2	N_c
L_3	-3.8 ± 0.3	N_c
L_5	1.2 ± 0.1	N_c
L_8	0.5 ± 0.2	N_c
L_9	6.9 ± 0.7	N_c
L_{10}	-5.2 ± 0.1	N_c

Bijnens & Ecker, 2014

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Bijnens & Ecker, 2014

- LECs encode the high energy physics information
- They have different Large N_c behaviour
- Large N_c allows for important simplifications!

The Large N_c limit in Phenomenology

"Large N_c-inspired" approximations are usual in phenomenology.

Updated Standard Model Prediction for ε'/ε *

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6. The SM prediction for ε'/ε

Taking into account all computed corrections in Eq. (7), our SM prediction for ε'/ε is

$$\mathrm{Re}\,(\varepsilon'/\varepsilon) = \left(15 \pm 2_{\mu} \pm 2_{m_s} \pm 2_{\Omega_{\rm eff}} \pm 6_{1/N_C}\right) \times 10^{-4}$$

- Uncertainties from Large N_c are hard to estimate.
- Can Lattice QCD improve this?

Simulations at Large N_c

Our Large N_c Ensembles with $N_f = 4$

- Iwasaki gauge action and $O(a)$ improved Wilson fermions.

Ensemble	N_c	$L \times T$	β	m_0	aM	M (MeV)
A301	3	20×36	1.778	-0.4040	0.2191(36)	570
A302		24×48		-0.4060	0.1831(17)	480
A303		24×48		-0.4070	0.1612(24)	420
A304		32×60		-0.4080	0.1384(15)	360
A401	4	20×36	3.570	-0.3725	0.2035(14)	530
A402		24×48		-0.3752	0.1804(7)	470
A403		24×48		-0.3760	0.1714(8)	440
A404		32×60		-0.3780	0.1397(8)	360
A501	5	20×36	5.969	-0.3458	0.2128(9)	560
A502		24×48		-0.3490	0.1802(6)	470
A503		24×48		-0.3500	0.1712(6)	450
A504		32×60		-0.3530	0.1328(8)	350
A601	6	20×36	8.974	-0.3260	0.2150(7)	570
A602		24×48		-0.3300	0.1801(5)	470
A603		24×48		-0.3311	0.1690(7)	450
A604		32×60		-0.3340	0.1354(7)	360

See FRL *et al.*, *arXiv:1810.06285*
Generated with HiRep, M. Hansen *arXiv:1705.11010*

Scale Setting at Large N_c

Use observables from Gradient Flow (Lüscher):

$$\langle t^2 E(t) \rangle = \frac{3}{128\pi^2} \frac{N_c^2 - 1}{N_c} \lambda_{GF}(\mu)$$

with $\mu = 1/\sqrt{8t}$ and $\lambda_{GF} = N_c g_{GF}^2$ ('t Hooft coupling).

For QCD , t_0 is defined through the implicit equation:

$$\langle t^2 E(t) \rangle \Big|_{t=t_0} = 0.3,$$

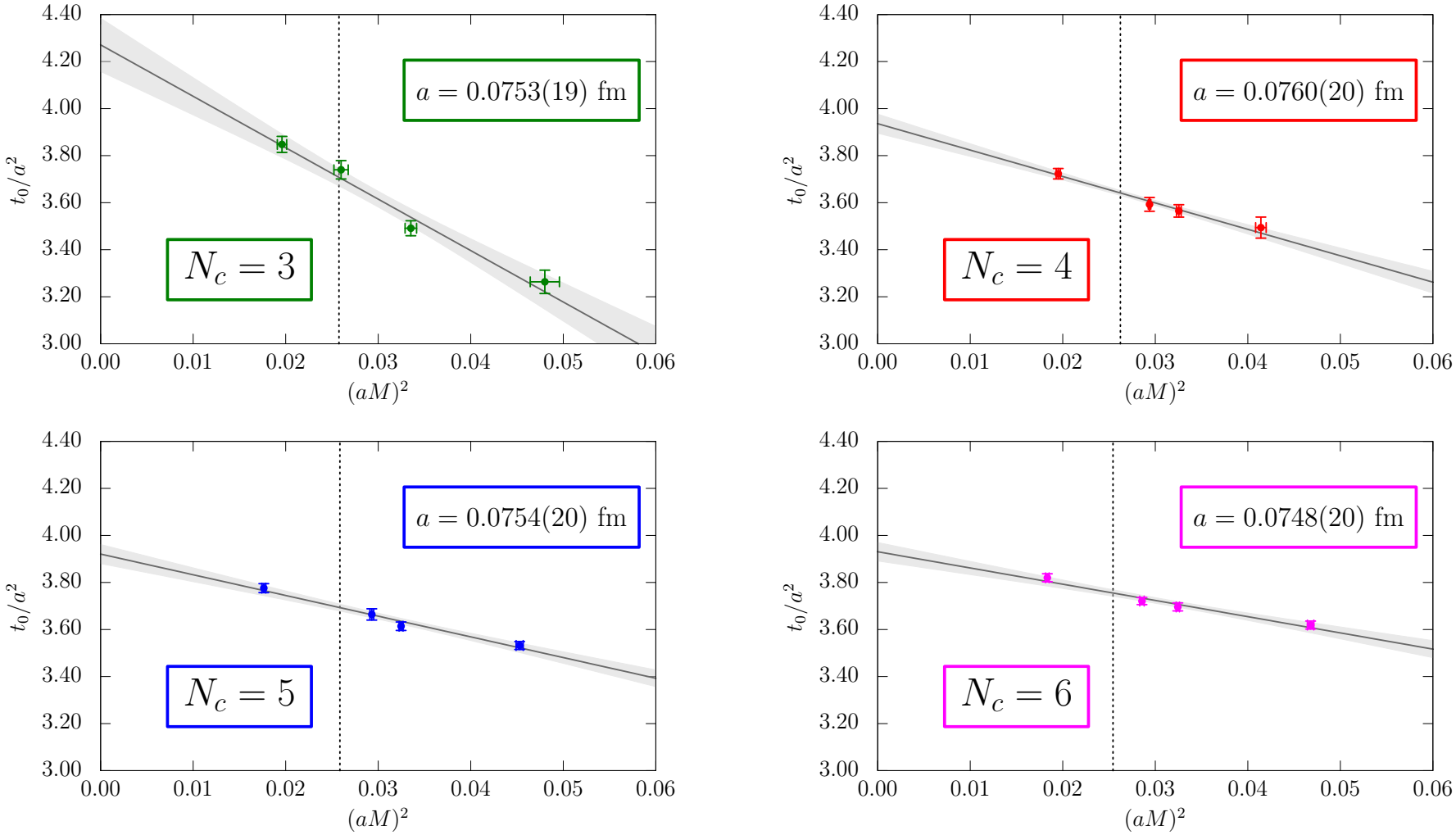
and its value in physical units is known from lattice simulations.

Generalization for arbitrary N_c:

$$\langle t^2 E(t) \rangle \Big|_{t=t_0} = 0.1125 \frac{N_c^2 - 1}{N_c}, \quad (M\sqrt{t_0})\Big|_{M=420\text{ MeV}} = 0.3090(83)$$

Mass dependence of t_0

$$t_0(M^2) = t_0^{\text{chiral}}\left(1 + kM^2\right) + O(M^4) \xrightarrow{\text{Large } N_c} t_0^{\text{chiral}}$$



Decay constant at Large N_c

Meson decay constant at Large N_c

The decay constant is defined as:

$$\langle 0|A_\mu(0)|\pi^+(q)\rangle = -iq_\mu\sqrt{2}F_\pi$$

$$C_A(t) = \langle A_0(0)A_0(t)\rangle \propto F_\pi^2 e^{-M_\pi t} \longrightarrow O(N_c).$$

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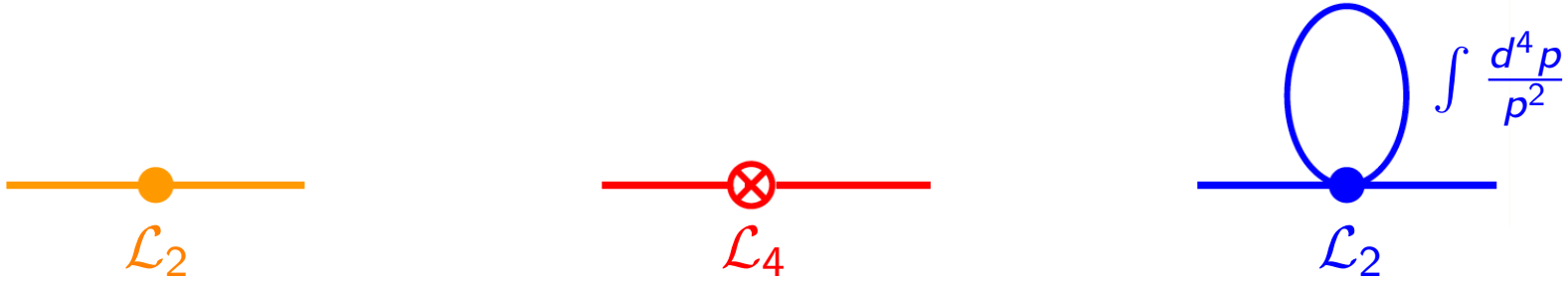
In Chiral Perturbation Theory:

$$F_\pi = F_0\left[1 + \frac{M_\pi^2}{F_\pi^2}\Big(4L_5(\mu) + 4N_f L_4(\mu)\Big) + \frac{N_f}{2}\frac{M_\pi^2}{(4\pi F_\pi)^2}\log\frac{M_\pi^2}{\mu^2}\right]$$

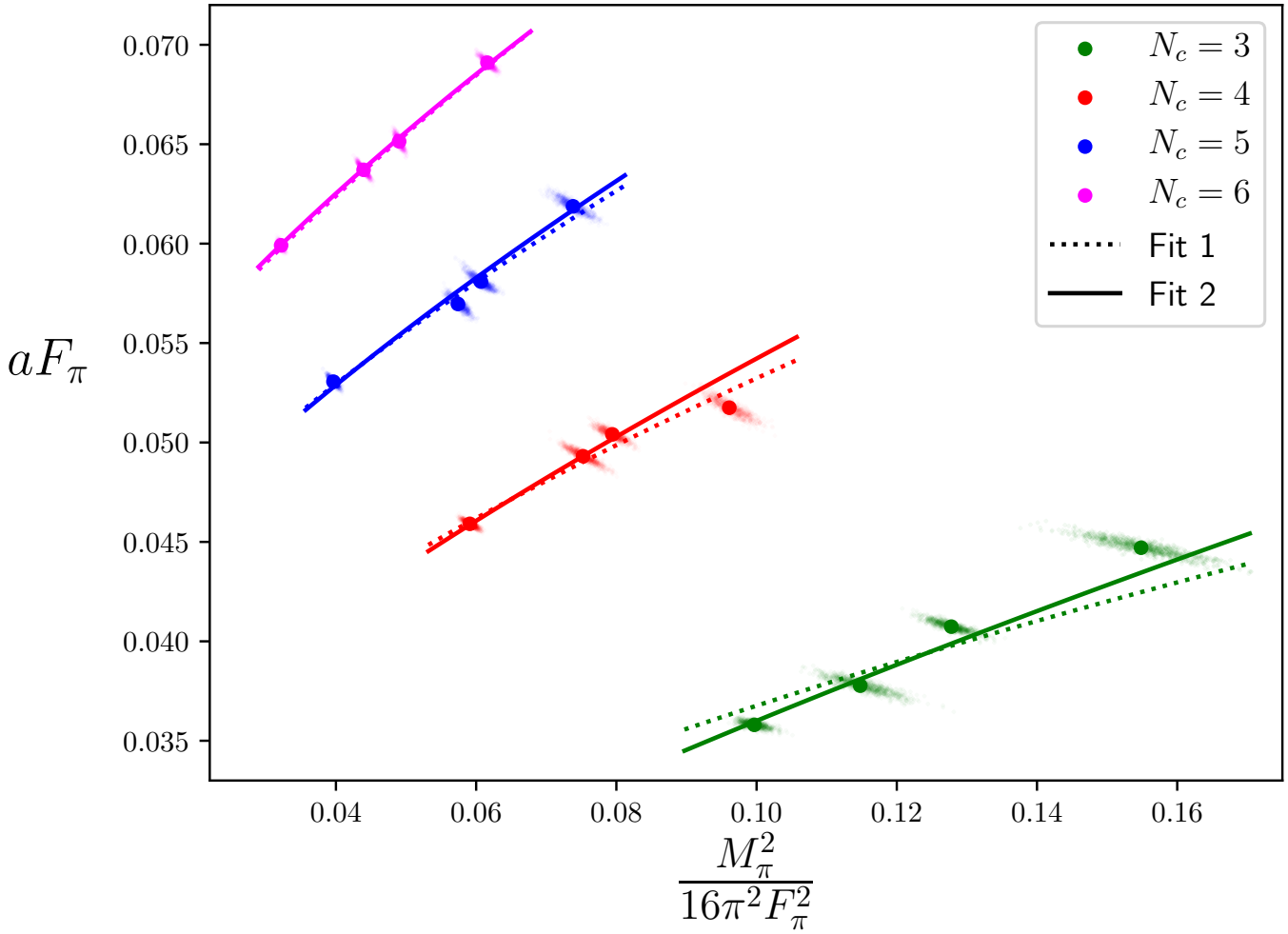


1. $F_\pi^2 = O(N_c)$
2. $L_5 = O(N_c)$
3. $L_4 = O(1)$

$$\xrightarrow{\text{Large } N_c} \quad F_\pi = F_0\left[1 + 4\frac{M_\pi^2}{F_\pi^2}L_5\right]$$

F_π at Large N_c : preliminary results

Simultaneous chiral and N_c fit.



F_π at Large N_c

- F_π is a fundamental quantity in Phenomenology.
- We expect to extract the N_c scaling of $L_F = L_5 + 4L_4$.
- We can combine our results with large N_c quenched result (Bali *et al.*) or reduced models (plenary by M. García Pérez and talk by A. González-Arroyo).
- Similar analysis for the meson mass, M_π. (arXiv:1907:xxxxx)

Scattering at Large N_c

Scattering in finite volume

Lüscher method

Make use of finite volume artefacts to study interactions

$$\det\left[\cot\delta + \mathcal{M}\right] = 0$$

S Matrix

kinematical quantity

$$k\cot\delta_0 = -\frac{1}{a_0} + \frac{1}{2}rk^2 + O(k^4)$$

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scattering length

For the ground state,a simpler formula is available:

(Lüscher, Hansen & Sharpe, See also talk by A. Rusetsky)

$$E - 2m = \frac{4\pi a_0}{mL^3}\left(1 + c_1\frac{a_0}{L} + c_2\left(\frac{a_0}{L}\right)^2 + c_3\left(\frac{a_0}{L}\right)^3 + \frac{2\pi r(a_0)^2}{L^3} - \frac{\pi a_0}{m^2L^3}\right)$$

$\Rightarrow$ At order L^{-5}, the ground state is solely explained by a_0

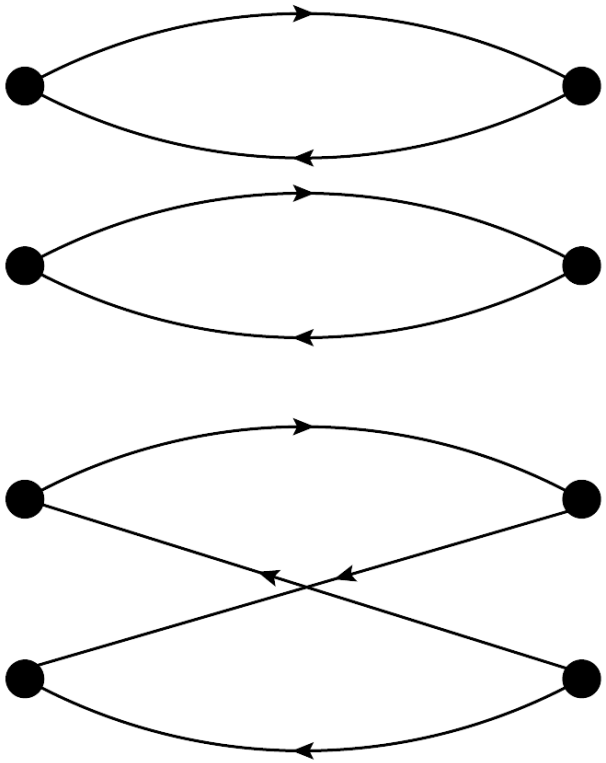
Isospin 2 $\pi\pi$ scattering

$\pi^+\pi^+ \to \pi^+\pi^+$ is the simplest application to QCD

Phenomenological value (Ynduráin, 2002)

$$M_\pi a_0^{I=2} = 0.0422(22)$$

- Weakly coupled
- Less noisy
- Extract LECs of ChPT



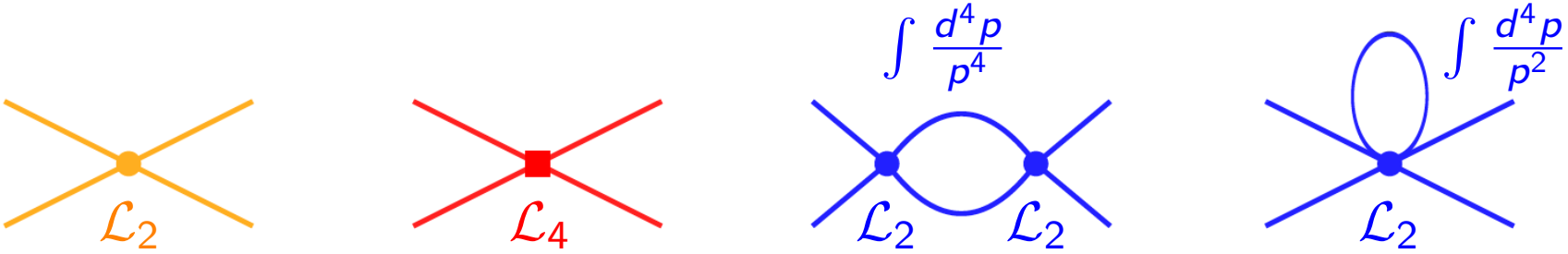
$$C_{\pi\pi} = \langle \pi\pi(0)\pi^\dagger\pi^\dagger(t)\rangle$$

I=2 $\pi\pi$ scattering at Large N_c

In Chiral Perturbation Theory with $N_f = 4$ (Bijnens *et al.*)

$$M_\pi a_0^{I=2} = -\frac{M_\pi^2}{16\pi F_\pi^2}\left[1 - \frac{16M_\pi^2}{F_\pi^2}L_{\pi\pi}(\mu) - \frac{M_\pi^2}{32\pi^2 F_\pi^2}\left(\frac{13}{4}\log\frac{M_\pi^2}{\mu^2} - \frac{3}{4}\right)\right]$$

with $L_{\pi\pi} = L_0 + 2L_1 + 2L_2 + L_3 - 2L_4 - L_5 + 2L_6 + L_8$

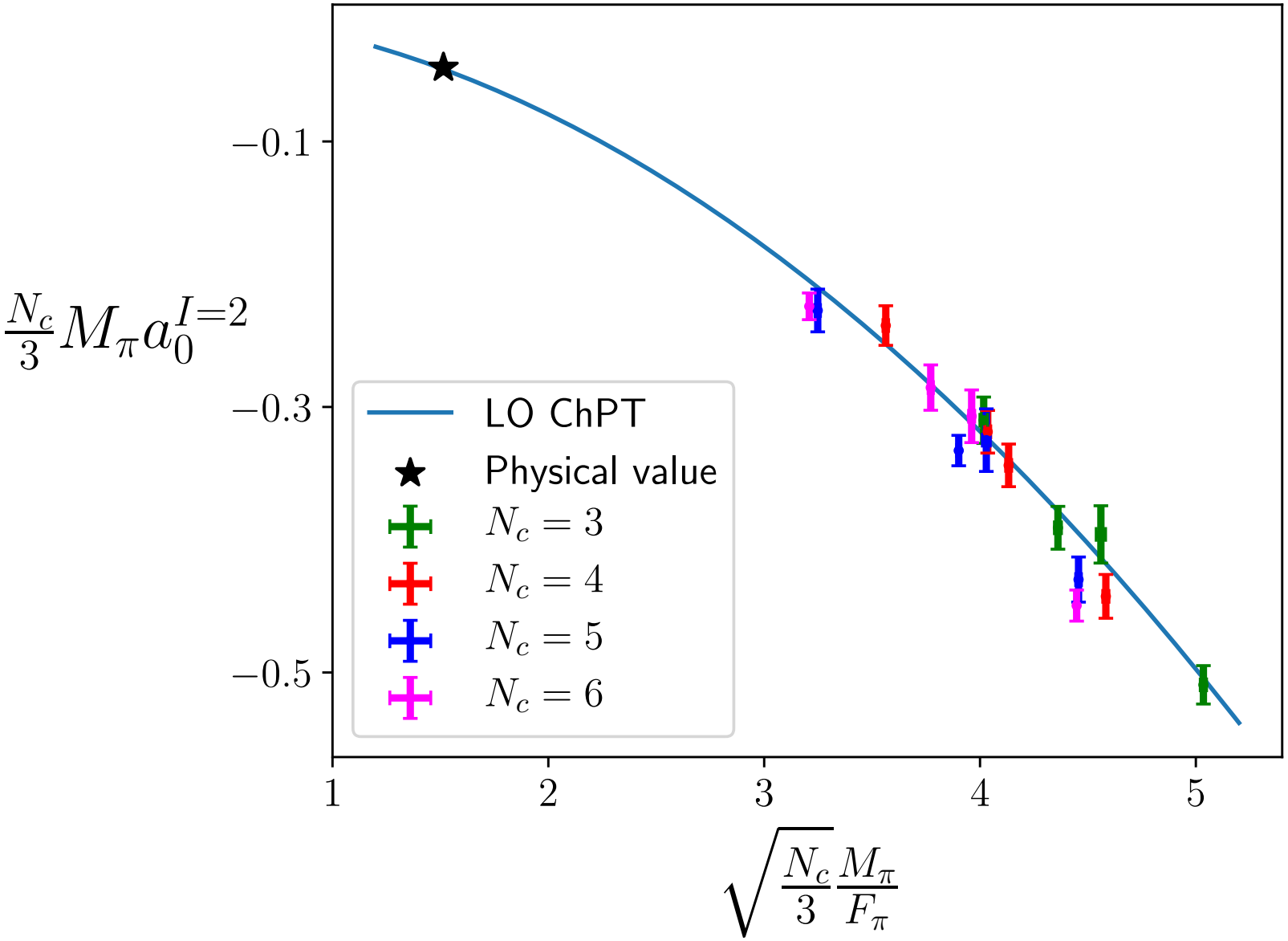


1. $F_\pi^2 = O(N_c)$
2. $L_0, L_5, L_3, L_8 = O(N_c)$
3. $L_1, L_2, L_4, L_6 = O(1)$

$$\xrightarrow{\text{Large } N_c} \quad a_0^{I=2} \propto \frac{1}{N_c}(1 + \text{LECs})$$

Preliminary results for Isospin 2 $\pi\pi$ scattering

$\Rightarrow$ All ensembles at $\sim$ 70% statistics



$K \to \pi\pi$ at Large N_c

Large N_c limit for $K \to \pi\pi$

- The Large N_c prediction for $K \to \pi\pi$ is (*Manohar, Large N QCD*):

$$\frac{\text{Re } A_0}{\text{Re } A_2} = \sqrt{2}$$

- Experimental values for $K \to \pi\pi$ are very well measured in two isospin channels, $I = 0, 2$ and Large N_c fails.

$$\frac{\text{Re } A_0}{\text{Re } A_2} \simeq 22 \gg \sqrt{2}\Big|_{\text{Large } N_c},$$

- State of the art result by *RBC-UKQCD, 2015*: (also talk by C. Kelly on Friday):

$$\frac{\text{Re } A_0}{\text{Re } A_2} = 31(11),$$

- Why does Large N_c fail? Very large $1/N_c$ corrections? Can Lattice QCD help? .

Relating $K \to \pi$ to A_2 and A_0

$$\mathcal{H}^{SM}(W^\mu, u, d...) \longrightarrow \mathcal{H}_W^{N_f=4}(u, d, c, s) \longrightarrow \mathcal{H}_W^{ChPT}(\pi, K, D, \eta)$$

$$\mathcal{H}_W^{ChPT} \propto g^+\mathcal{O}^+ + g^-\mathcal{O}^-$$

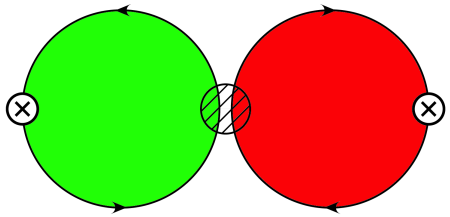
The tree level result in ChPT for the ratio is:

$$\frac{A_0}{A_2} = \frac{1}{2\sqrt{2}}\left(1 + 3\frac{g^-}{g^+}\right) \xrightarrow[g^+=g^-]{\text{Large } N_c} \sqrt{2}$$

Determine $g^\pm$ from Lattice QCD:

$$A^\pm = \langle K|\mathcal{O}^\pm|\pi\rangle \xrightarrow{M_\pi \to 0} g^\pm$$

$g^\pm \propto$



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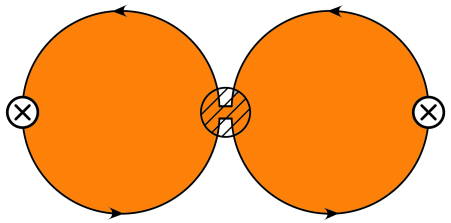
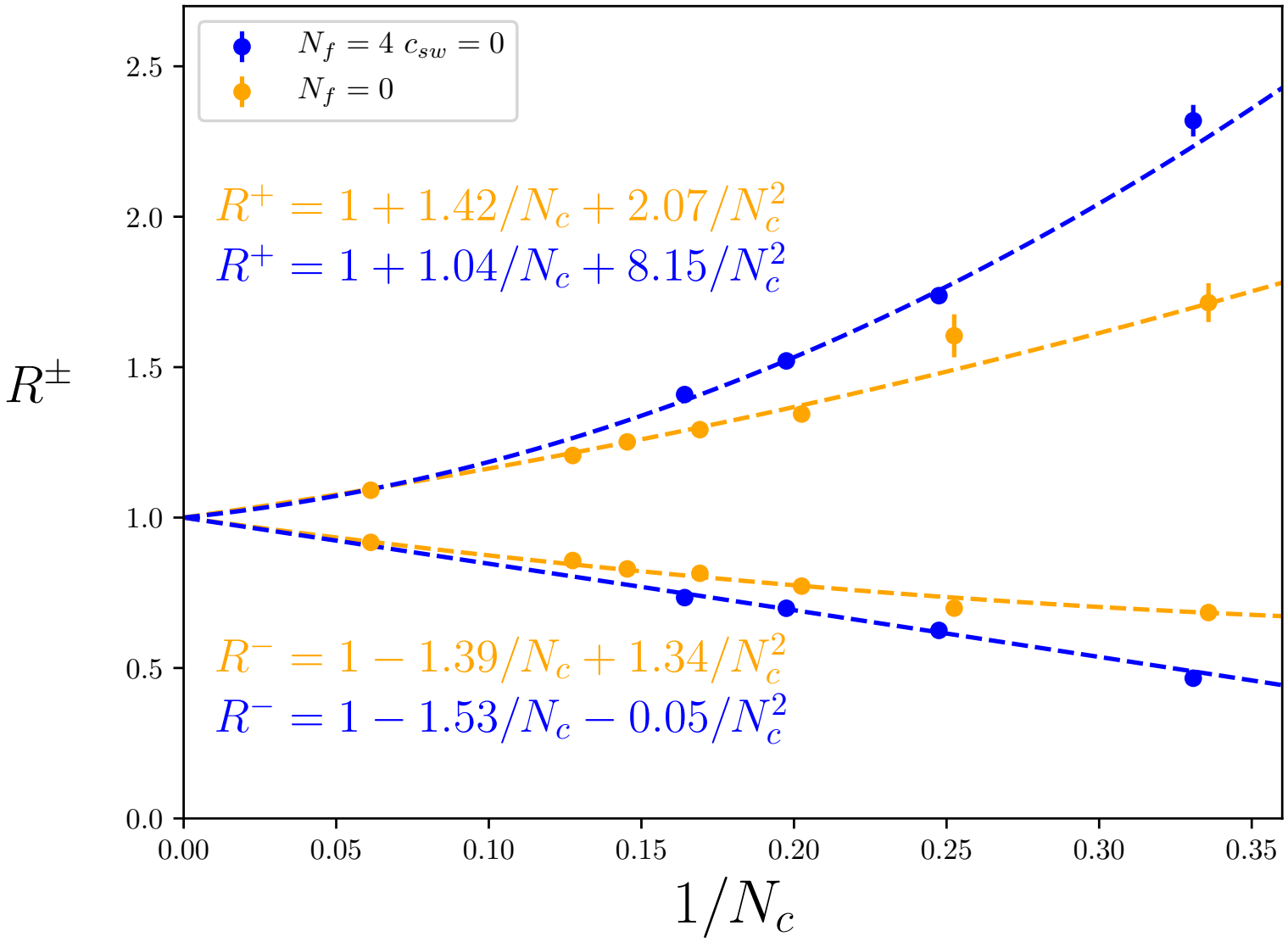


Color-disconnected $O(N_c^2)$ Color-connected $O(N_c)$

Preliminary results for $K \to \pi$ at Large N_c

Donini, Hernández, FRL, Pena, arXiv:1607.03262, arXiv:1810.06285 and on-going work



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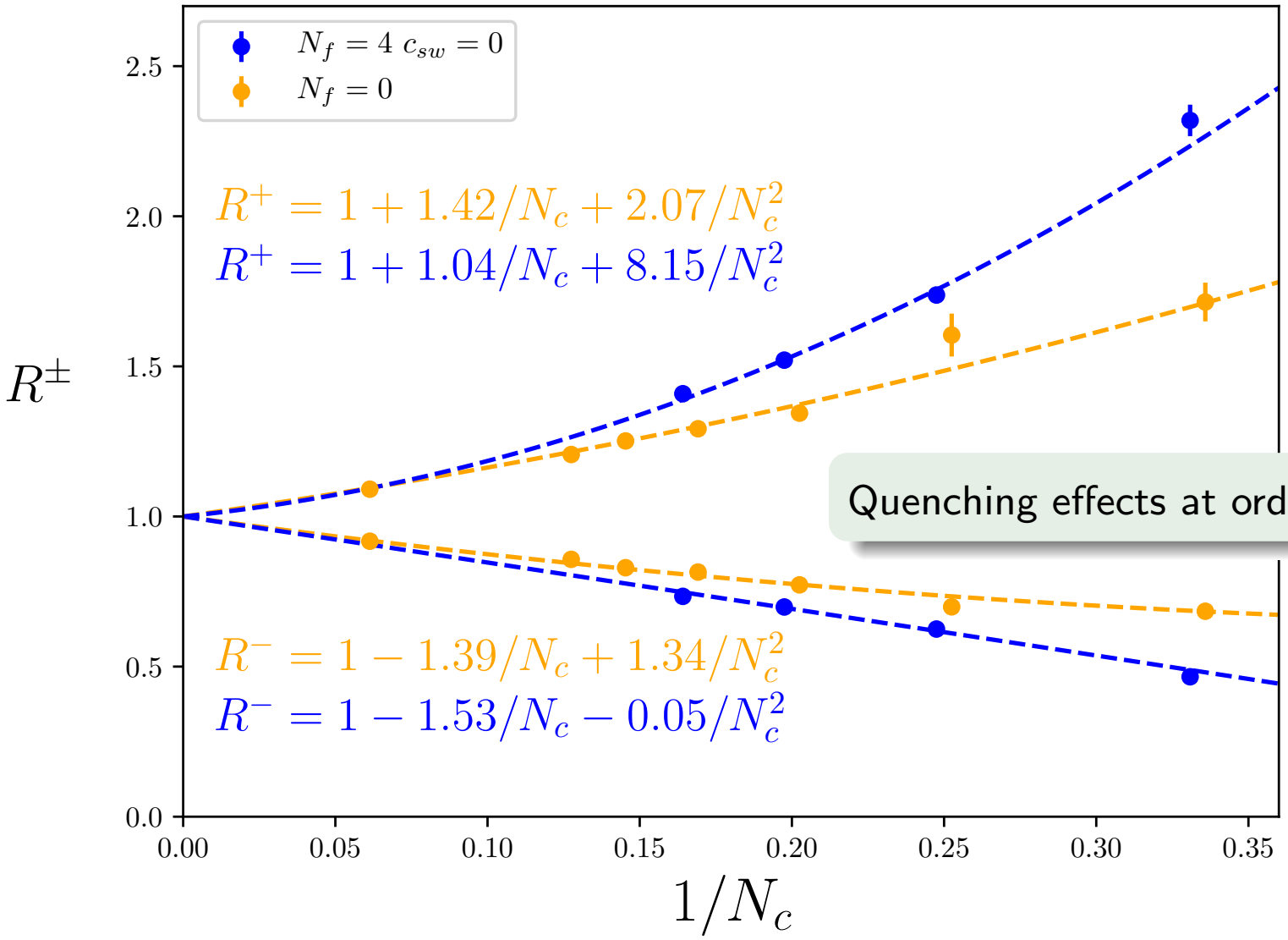

Quenching effects at order $1/N_c^2$

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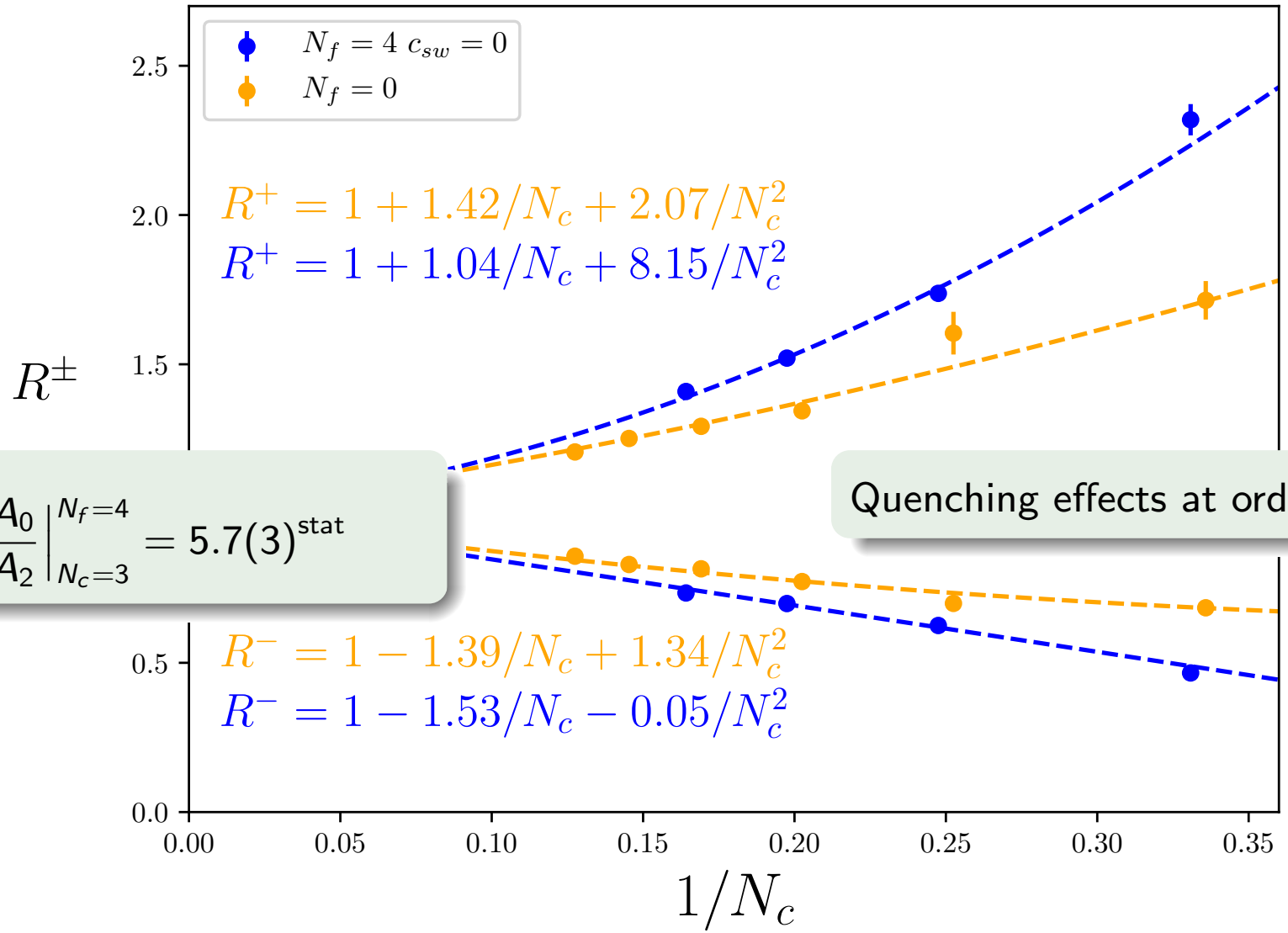

$$\frac{A_0}{A_2}\Big|_{N_c=3}^{N_f=4} = 5.7(3)^{\text{stat}}$$

Quenching effects at order $1/N_c^2$

Summary and Outlook

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1. 16 ensembles available: 4 masses $\times$ 4 colors at $a \simeq 0.075$ fm
2. We are currently testing the scaling of several observables
   - $A^{\pm}(K \to \pi) = 1 \mp O(N_c)$
   - $F_\pi = O(\sqrt{N_c})$
   - $a_0^{I=2} = O(1/N_c)$
3. Potentially useful for phenomenology $\to$ N_c scaling of LECs.
4. Future topics: $I = 0$ scattering at η' at Large N_c .

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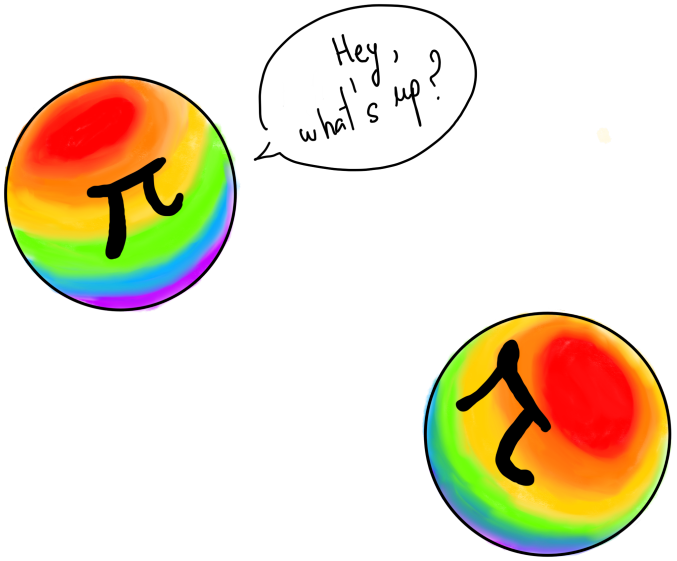
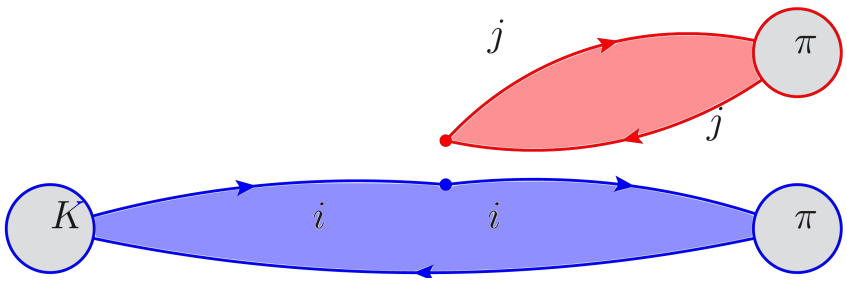
Thanks for your attention!



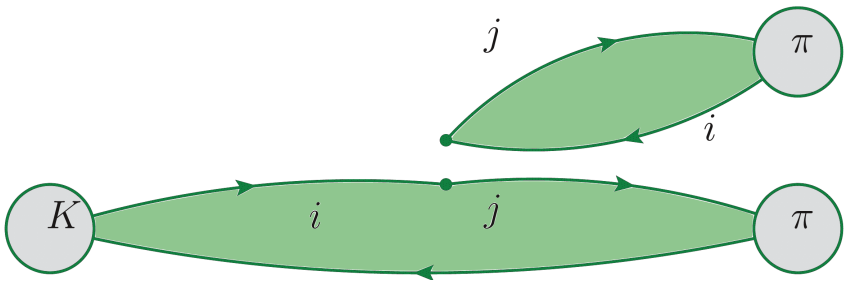
This project has received funding through the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No. 713673 and "La Caixa" Foundation

Current Lattice results for $K \to \pi\pi$

RBC-UKQCD, PRD91 (2015) and PRL115 (2015)

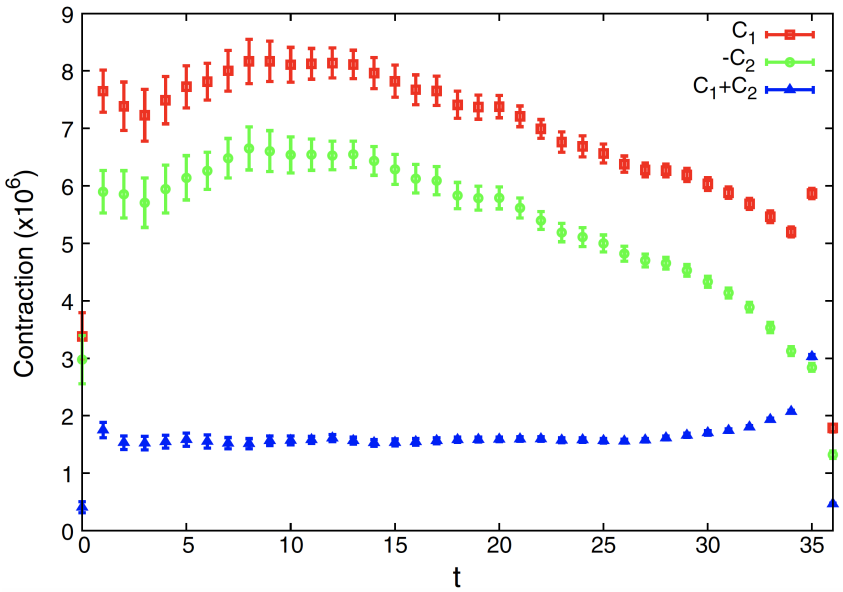


$C_1 \to$ two color traces $\to O(N_c^2)$



$C_2 \to$ one color trace $\to O(N_c)$

- $\frac{\text{Re } A_0}{\text{Re } A_2} = 31(11)$
- Source seems to be a <u>cancellation</u>, and not penguins ($m_c \gg m_u$)
- Large N_c predicts $|C_2| \sim \frac{|C_1|}{3}$
- However, $C_2 \sim -0.9 C_1$
- Very big $1/N_c$ corrections?



Framework for $K \to \pi\pi$: weak interactions in ChPT

1. Take the lowest order Lagrangian is written as:

$$\mathcal{L}_2 = \frac{F^2}{4}\text{tr}\ \left(\partial_\mu U^\dagger \partial^\mu U\right) + \frac{F^2 B}{2}\text{tr}\ \left(MU + M^\dagger U^\dagger\right),$$

2. Use covariant derivative with an external left-handed source:

$$\partial_\mu U \to D_\mu U = \partial_\mu U + T^a A^a_\mu U,$$

3. Define the left current as:

$$\mathcal{J}^a_\mu = \frac{\delta \mathcal{L}}{\delta A^a_\mu} = F^2\ \text{Tr}\ \left(T_a U \partial_\mu U^\dagger\right) \leftrightarrow \bar{q} T^a \gamma^L_\mu q$$

4. Build operators with the right irreducible representation of the $SU(4)$ flavour group $(\mathcal{O}_{\Gamma=20}, \mathcal{O}_{\Gamma=84})$. One needs 4 left indices:

$$O_\Gamma = t_{ijkl}(U\partial_\mu U^\dagger)_{ij}(U\partial_\mu U^\dagger)_{kl}$$

The electroweak Hamiltonian in ChPT is:

$$\mathcal{H}_W = g^+ \mathcal{O}^+_{\Gamma=84} + g^- \mathcal{O}^-_{\Gamma=20}$$

Framework for $K \to \pi\pi$: Effective $SU(4)_F$ Theory

M_W $\quad \mathcal{H}_{\text{SM}} \to \mathcal{H}^{N_f=4}_{\Delta S=1} = \sqrt{2} G_F V^*_{us} V_{ud} (C_\pm(M_W)\mathcal{O}^\pm(M_W))$

$$\mathcal{O}^\pm \equiv (\bar{s}\gamma^L_\mu u)(\bar{u}\gamma^L_\mu d) \pm (\bar{s}\gamma^L_\mu d)(\bar{u}\gamma^L_\mu u) - (u \leftrightarrow c)$$

Use flavour symmetries $\to$ irreps (Γ)

$SU(4)_L \times SU(4)_R$: $\mathcal{O}^+ \to (84,1) \quad \mathcal{O}^- \to (20,1)$

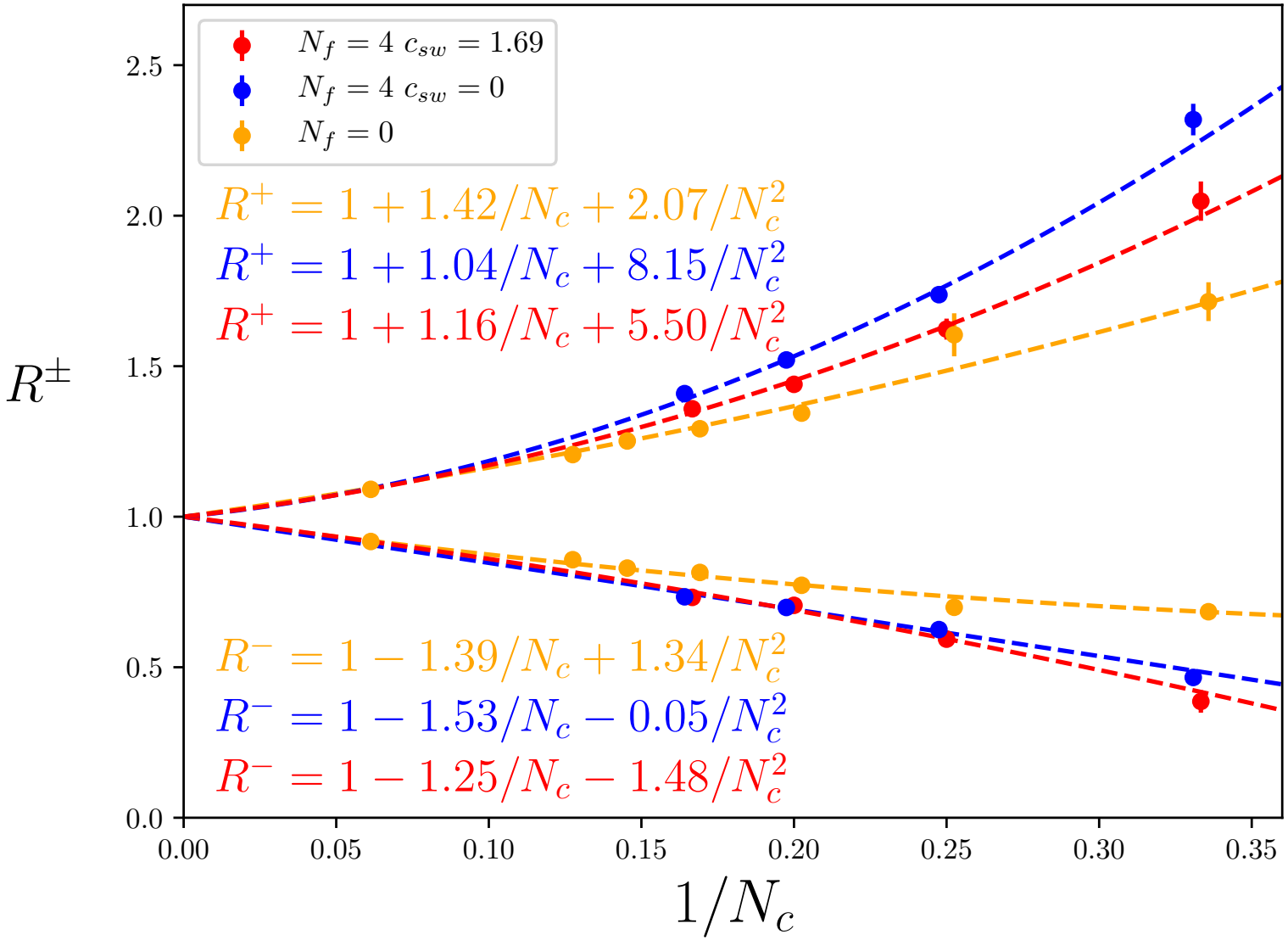
$(84,1) \to A_2, A_0$, $(20,1) \to A_0$

$$\mathcal{H}^{N_f=4}_{\Delta S=1} = \sqrt{2} G_F V^*_{us} V_{ud} (C_\pm(\mu)\mathcal{O}^\pm(\mu))$$

Λ_{ChPT} $\quad \mathcal{H}^{N_f=4}_{\Delta S=1} \xrightarrow{\text{Lattice QCD}} \mathcal{H}^{N_f=4}_{ChPT} \propto g_+\mathcal{O}^+ + g_-\mathcal{O}^-$

Preliminary results for $K \to \pi$ at Large N_c

A. Donini, P. Hernández, FRL, C. Pena, arXiv:1607.03262, arXiv:1810.06285



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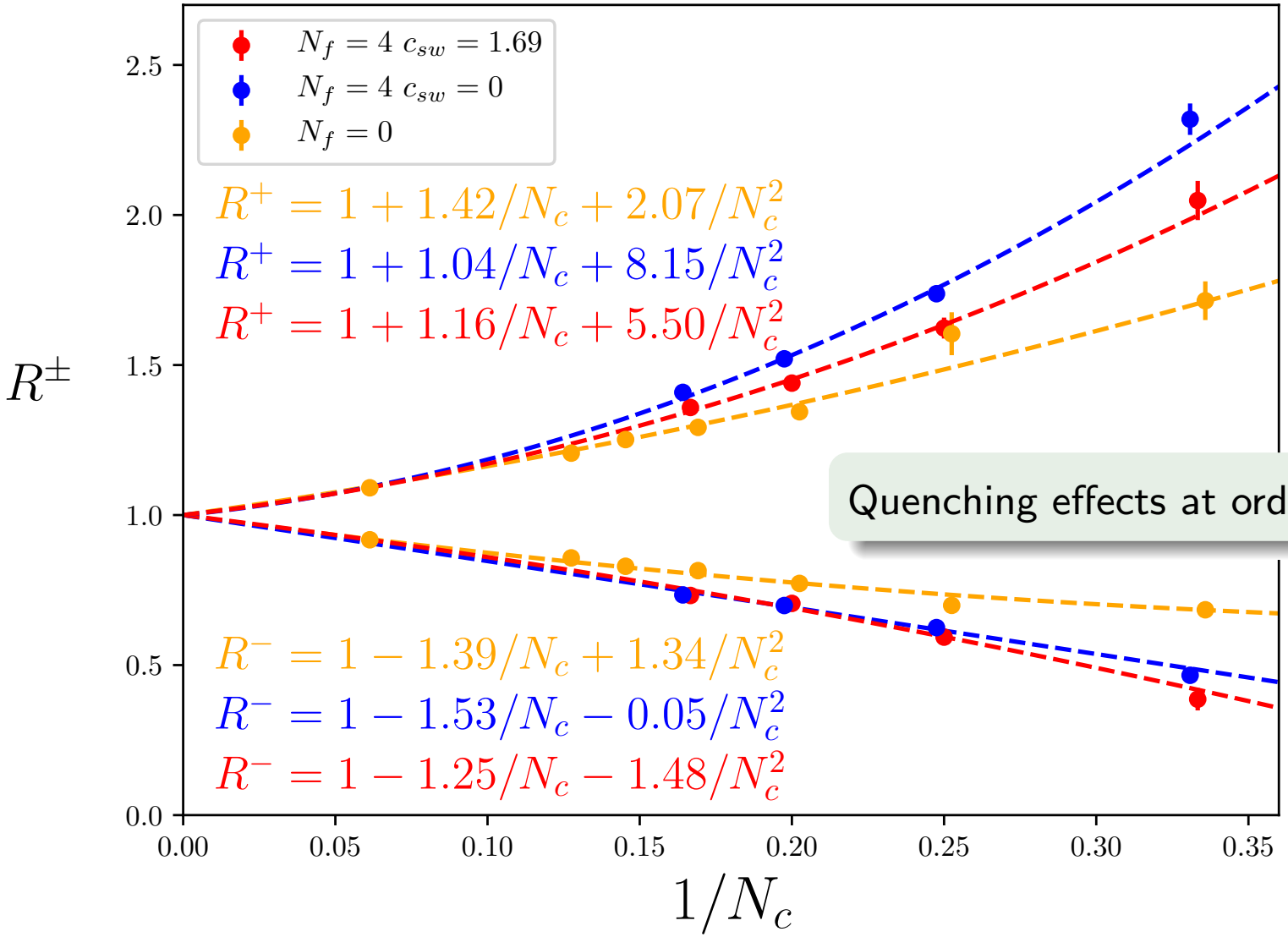

Quenching effects at order $1/N_c^2$

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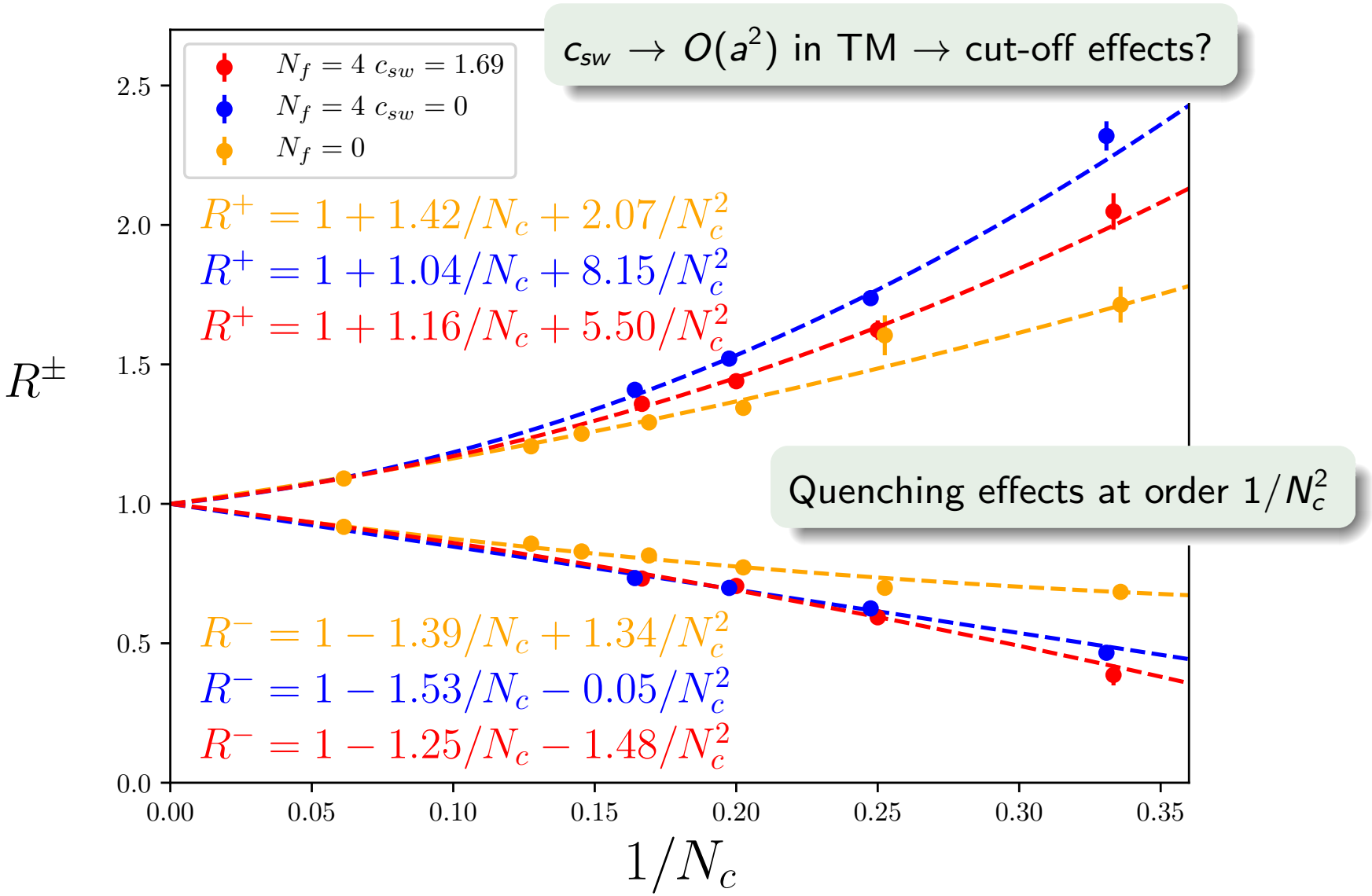

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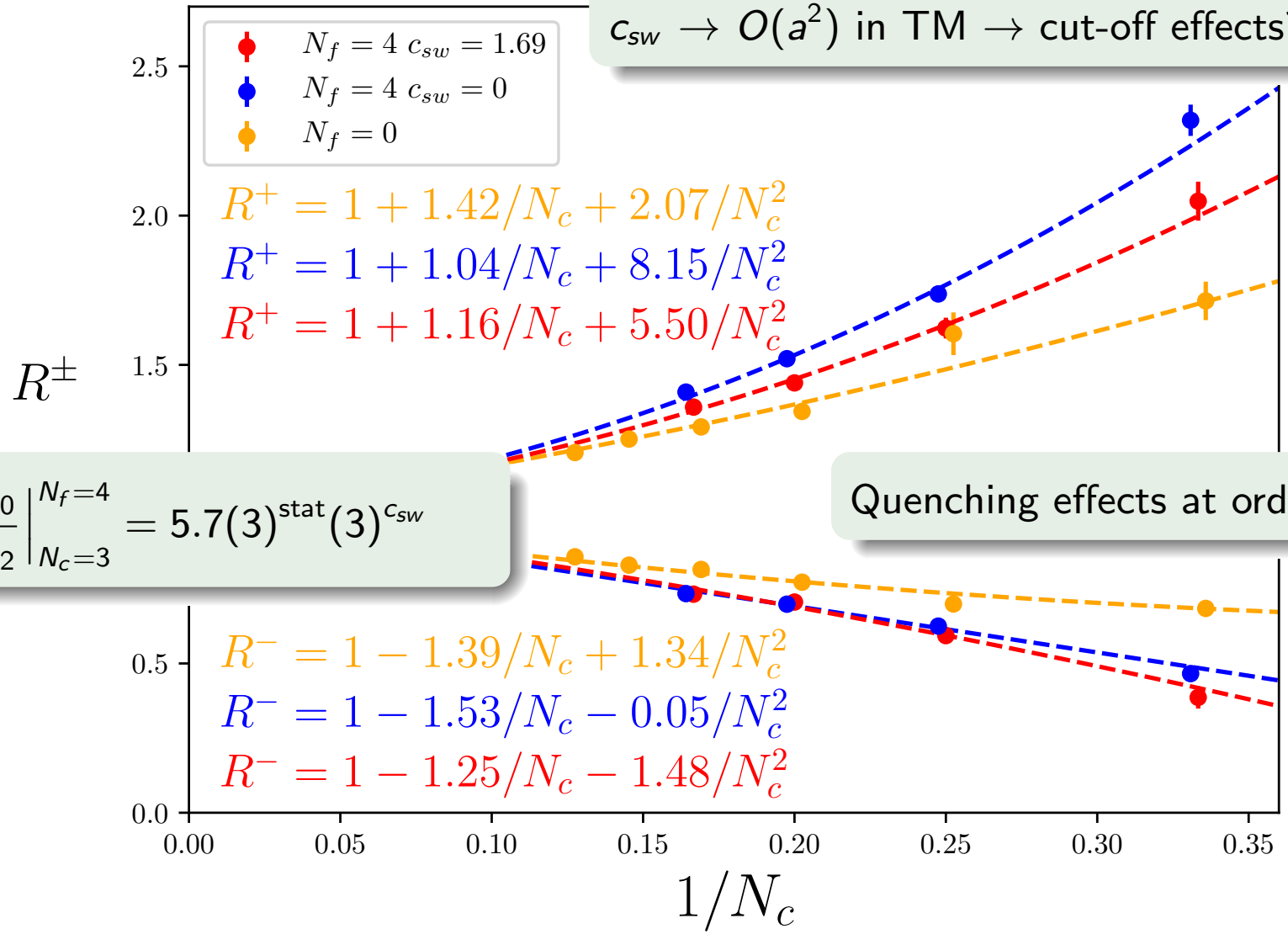
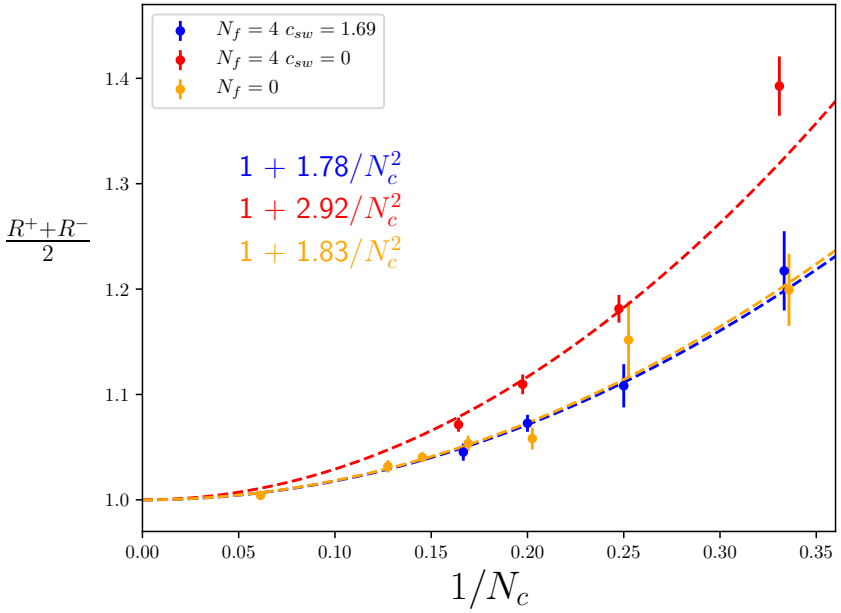
$c_{sw} \to O(a^2)$ in TM $\to$ cut-off effects?

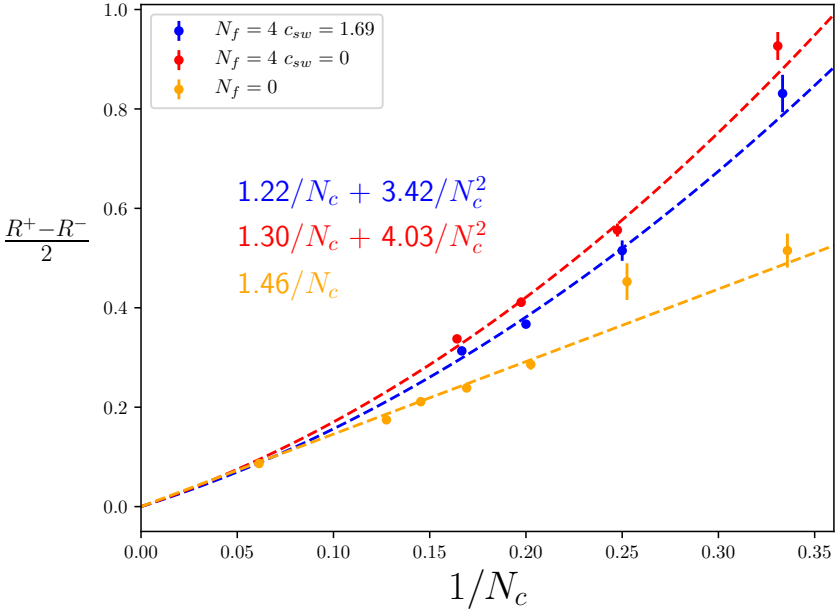


$$\frac{A_0}{A_2}\Big|_{N_c=3}^{N_f=4} = 5.7(3)^{\text{stat}}(3)^{c_{sw}}$$

Quenching effects at order $1/N_c^2$

$K \to \pi$ at Large N_c

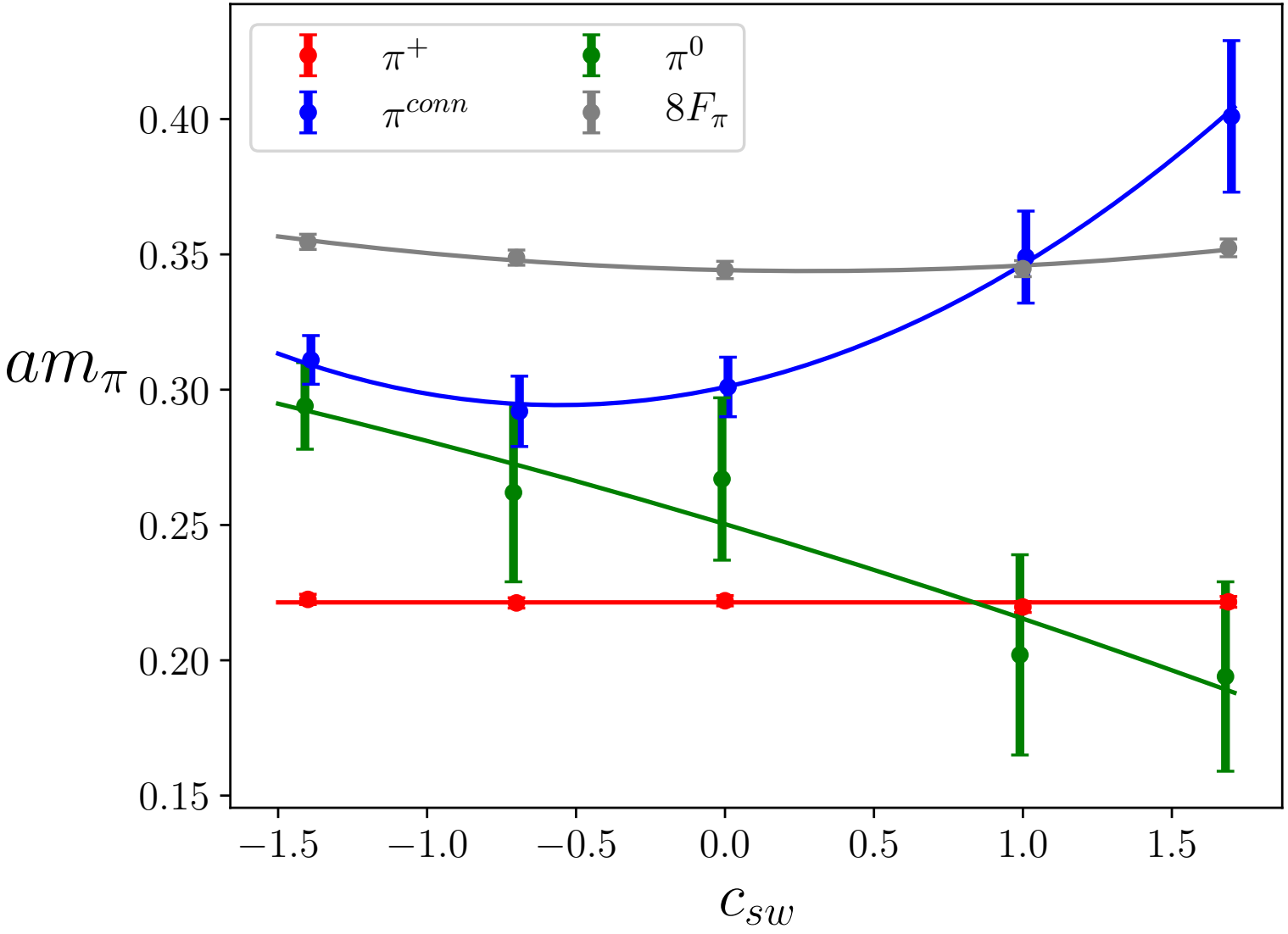




$$\frac{R^+ + R^-}{2} = 1 + \alpha \frac{1}{N_c^2} + O(N_c^{-2})$$

$$\frac{R^+ - R^-}{2} = \beta \frac{1}{N_c} + \gamma \frac{N_f}{N_c^2} + O(N_c^{-3})$$

c_{sw} effects on meson spectrum



Summary for $K \to \pi\pi$ at Large N_c

1. Our results recover the expected Large N_c limit:

$$\frac{A_0}{A_2} = \frac{1}{2\sqrt{2}}\left(1 + 3\frac{A^-}{A^+}\right) = \sqrt{2}$$

2. Current $N_c = 3$ LO result:

$$\left.\frac{A_0}{A_2}\right|_{N_f=0} = 2.9(1), \qquad \left.\frac{A_0}{A_2}\right|_{N_f=4} = 5.7(4)$$

3. Further enhancement from NLO ChPT ($\mu_{\text{eff}} = M_\rho$ or 2 GeV).

$$\frac{A_0}{A_2} = 6.8(3)_{\text{stat}}(6)_{\mu_{\text{eff}}} \ll 21,$$

4. Still missing a factor ~ 3...
5. Outlook: direct $K \to \pi\pi$ on the lattice and m_c dependence.