

Z_b tetraquark channel and $B\bar{B}^*$ interaction

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Outline

- Discovery of Z_b in experiment

$$Z_b^+ \approx \bar{b} b \bar{d} u$$

- Why it is (to) difficult to study scattering in Z_b channel with Luscher's method
- Study of Z_b system with static b quarks
- possible indication for existence of Z_b tetraquarks under employed approximations

Z_b in experiment

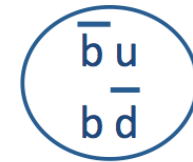
discovered by Belle in 2011 [PRL 108 (2012) 122001]

Z_b⁺(10610) , Z_b⁺(10650)

I=1, J^{PC}=1⁺⁻

$$Z_b^+ \rightarrow \Upsilon \pi^+$$

$\bar{b}b \bar{d}u$

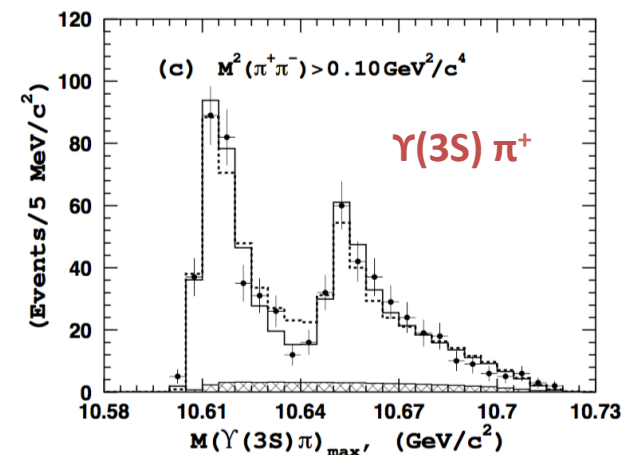
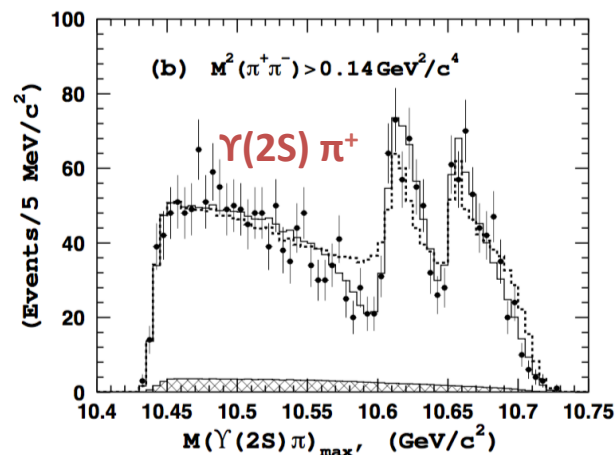
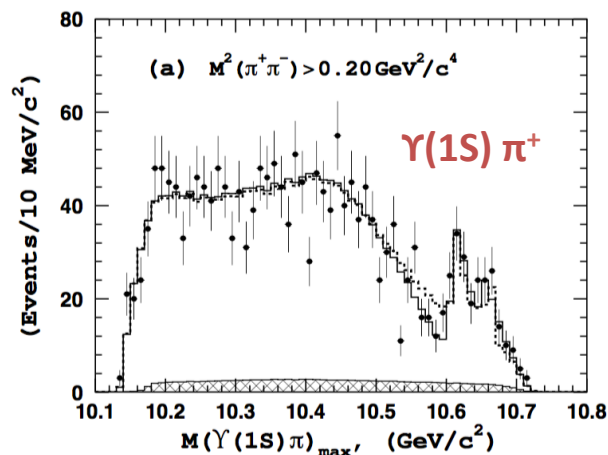


Z_b observed in decays $\Upsilon(1S,2S,3S) \pi$

$h_b(1S,2S) \pi$

$B \underline{B}^* , B^* \underline{B}^*$

Belle PRD 91 (2015) 072003



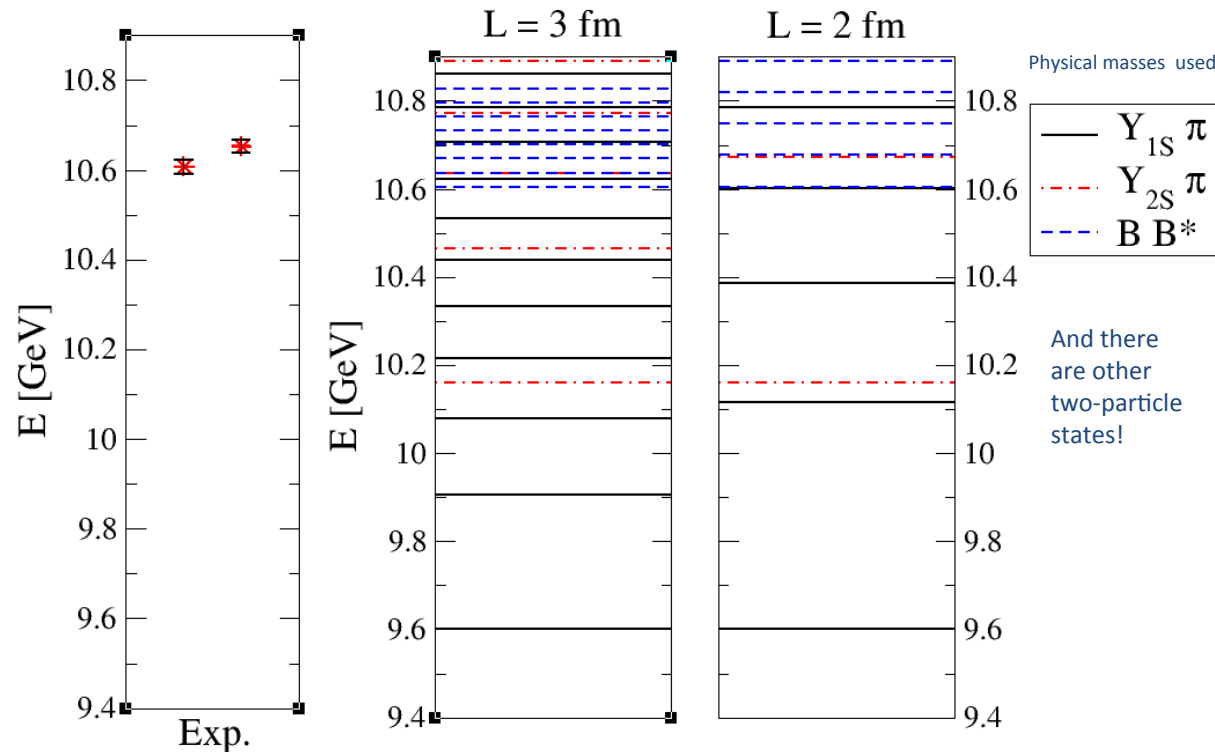
↑ $m_B + m_{B^*}$

Z_b tetraquark channel, Sasa Prelovsek

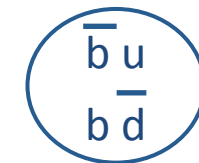
Z_b^+ with non-static b and Luscher's approach: (to) challenging

Eigen-energies in non-interacting limit

$$E^{n.i.}(L) = \sqrt{m_1^2 + \vec{p}^2} + \sqrt{m_2^2 + (-\vec{p})^2}$$



$$\vec{p} = \frac{2\pi}{L} \vec{n}$$



Rigorous treatment very challenging:

- at least 7 two-particle channels coupled
- very dense $B\bar{B}^*$ and $B^*\bar{B}$ energy levels

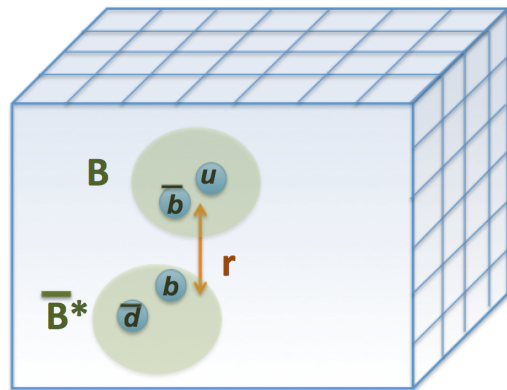
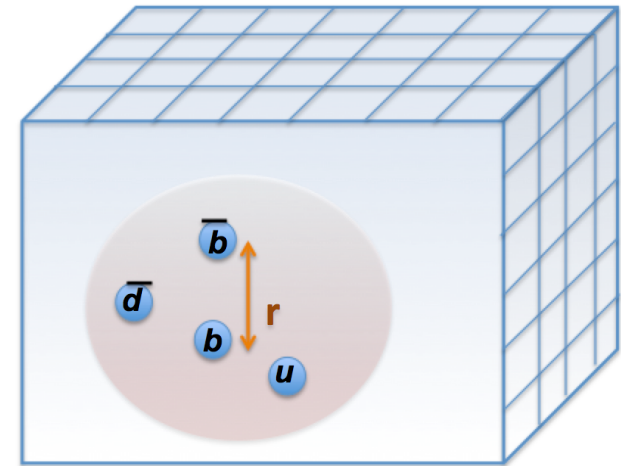
Z_b with static b and \underline{b}

Only previous lat study

Bicudo, Cichy, Peters, Wagner [proceedings Lat16: 1602.07621
proceedings Lat17: 1709.03306]

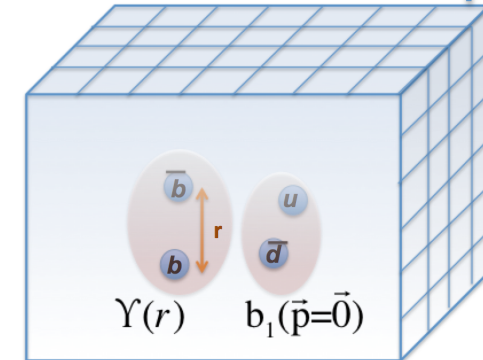
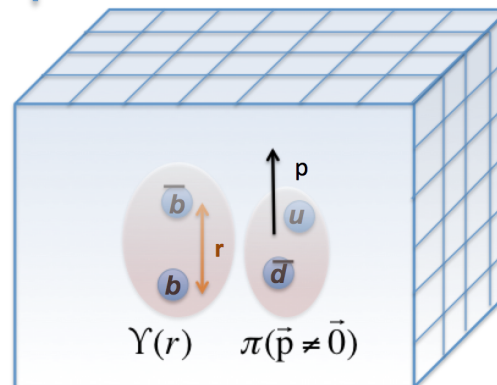
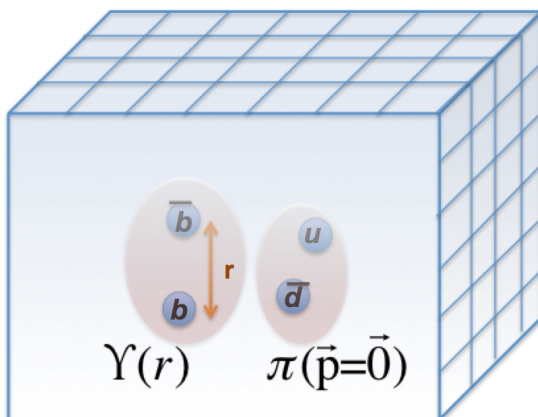
Born-Oppenheimer approach

Fock components incorporated



- main aim: extract static potential $V(r)$ between B and \underline{B}^*
- momentum of light degrees of freedom not conserved in presence of static quarks

not incorporated before

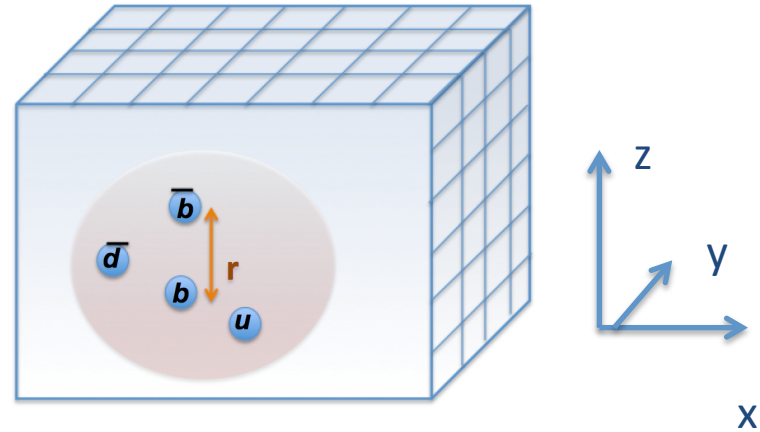


Zb with static \underline{b}

Good symmetries and quantum numbers:

$I=1$ $I_3=0$ (consider neutral Z_b)

$S_{\text{heavy}}=1$ $(S_z)_{\text{heavy}}=0$ $\bar{b}(\uparrow)b(\downarrow) - \bar{b}(\downarrow)b(\uparrow)$



heavy quark can not flip spin via gluon exchange

note: transition is not possible to final states with

$S_{\text{heavy}}=0$ (η_b, h_b) $\bar{b}(\uparrow)b(\downarrow) + \bar{b}(\downarrow)b(\uparrow)$

$(J_z)_{\text{light}}=0$ [J_x and J_y not conserved]

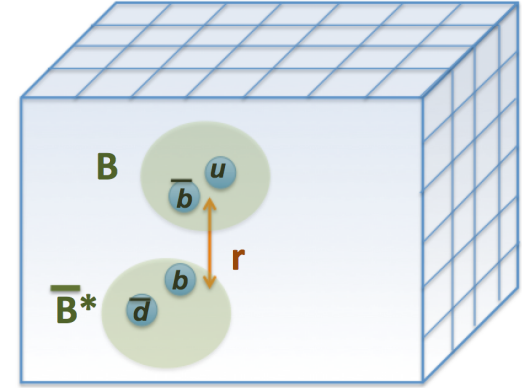
$C \cdot P = -1$ (P = inversion over midpoint between b and \underline{b})

R_{light} = reflection over yz plane = $P_{\text{light}} * R_{\text{light}}(y, \pi) : \epsilon = -1$

momentum of light degrees of freedom: not conserved

Operators O_{1-6} with given quantum numbers

in this way $(Jz)_{\text{light}}$, S_{heavy} and $(Sz)_{\text{heavy}}$
are indeed separately good quantum num.



inspired by Wagner et al.

$$\begin{aligned}
 O_1 = O^{B\bar{B}^*} &= \sum_{a,b} \sum_{A,B,C,D} \Gamma_{BA} \tilde{\Gamma}_{CD} \bar{b}_C^a(0) q_A^a(0) \bar{q}_B^b(r) b_D^b(r), \quad \Gamma = P_- \gamma_5 \quad \tilde{\Gamma} = \gamma_z P_+, \\
 &= \sum_{a,b} \sum_{A,B,C,D} \bar{q}_B^b(r) \Gamma_{BA} q_A^a(0) \bar{b}_C^a(0) \tilde{\Gamma}_{CD} b_D^b(r) \\
 &\propto [\bar{b}(0) P_- \gamma_5 q(0)] [\bar{q}(r) \gamma_z P_+ b(r)] + [\bar{b}(0) P_- \gamma_z q(0)] [\bar{q}(r) \gamma_5 P_+ b(r)] \\
 &\quad - [\bar{b}(0) P_- \gamma_x q(0)] [\bar{q}(r) \gamma_y P_+ b(r)] + [\bar{b}(0) P_- \gamma_y q(0)] [\bar{q}(r) \gamma_x P_+ b(r)]
 \end{aligned}$$

$$O_5 = O^{B\bar{B}^*} = \sum_{a,b} \sum_{A,B,C,D} \Gamma_{BA} \tilde{\Gamma}_{CD} \bar{b}_C^a(0) \nabla^2 q_A^a(0) \bar{q}_B^b(r) \nabla^2 b_D^b(r), \quad \Gamma = P_- \gamma_5 \quad \tilde{\Gamma} = \gamma_z P_+,$$

$$\mathbf{I}=1 \quad \mathbf{I}_3=0: \quad \bar{q}q \rightarrow \bar{u}u - \bar{d}d$$

Operators O_{1-6}

[..] indicate color singlets

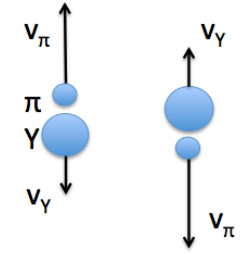
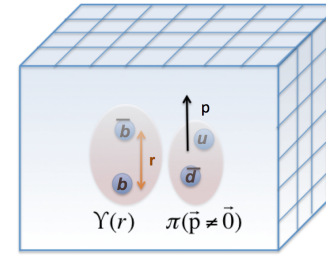
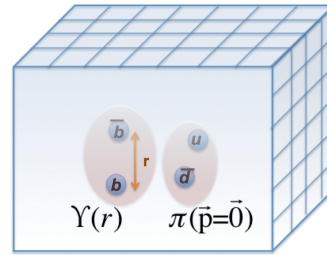


illustration of $O_{3,4}$
for non-static case

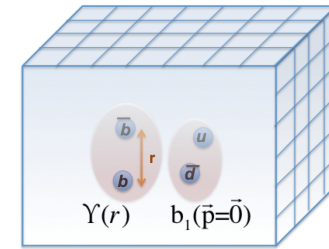
$$O_2 = O^{\Upsilon\pi(0)} = \Upsilon_z \pi_{p=000} = [\bar{b}(0)\gamma_z P_+ b(r)] [\bar{q}\gamma_5 q]_{p=000}$$

$$O_3 = O^{\Upsilon\pi(1)} = \Upsilon_z (\pi_{p=001} + \pi_{p=00-1}) = [\bar{b}(0)\gamma_z P_+ b(r)] \left([\bar{q}\gamma_5 q]_{p=001} + [\bar{q}\gamma_5 q]_{p=00-1} \right)$$

$$O_4 = O^{\Upsilon\pi(2)} = \Upsilon_z (\pi_{p=002} + \pi_{p=00-2}) = [\bar{b}(0)\gamma_z P_+ b(r)] \left([\bar{q}\gamma_5 q]_{p=002} + [\bar{q}\gamma_5 q]_{p=00-2} \right)$$

- momentum in z direction ensures that $(J_z)_{\text{light}}=0$
- the sum ensures that CP is good q.n.

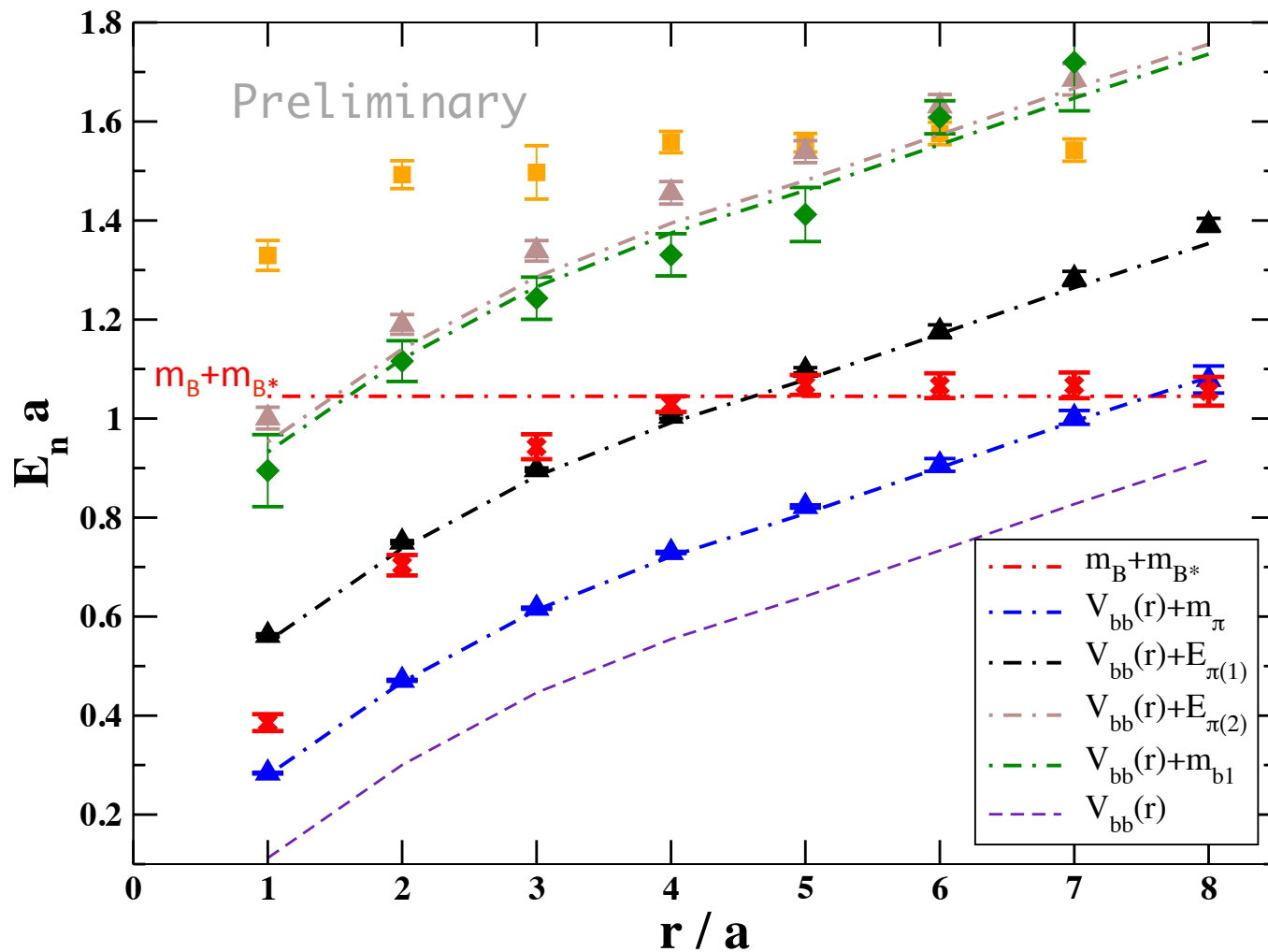
$$O_6 = O^{\Upsilon b_1(0)} = \Upsilon_z (b_1 z)_{p=000} = [\bar{b}(0)\gamma_z P_+ b(r)] [\bar{q}\gamma_x \gamma_y q]_{p=000}$$



$N_f=2$, $m_\pi \approx 266$ MeV, $a \approx 0.124$ fm, $L \approx 2$ fm [larger L would require $\Upsilon\pi(p=003)$...]

full distillation method to compute Wick contractions

Eigen-energies $E_n(r)$ of 6x6 correlation matrix from GeVP

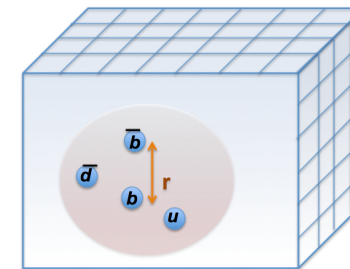


$\Delta E_n = E_n - E_n^{\text{non-int}} \neq 0$ claimed only for level $B\bar{B}^*$ at $r \leq 4$
 significance of other ΔE_n not yet systematically explored

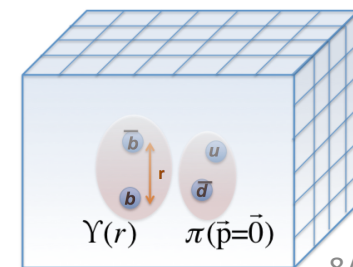
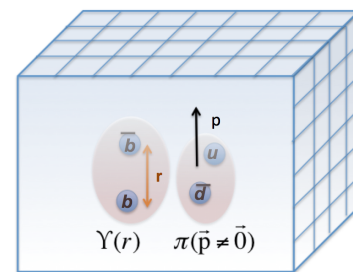
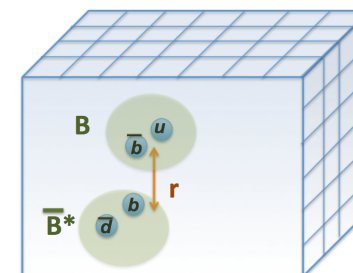
dominant operator
 in each $|n\rangle$
 according to

$$\langle O_i | n \rangle$$

- $O = B \bar{B}^*$
- ▲ $O = Y \pi(0)$
- ▲ $O = Y \pi(1)$
- ▲ $O = Y \pi(2)$
- ◆ $O = Y b_1(0)$
- ✖ $O = B \bar{B}^*$

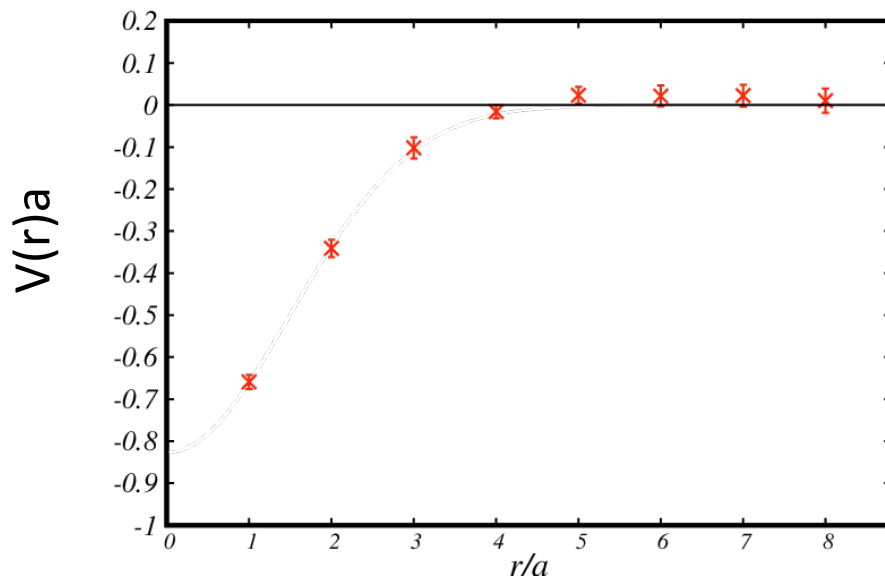


dot-dashed-lines:
 $E_n^{\text{non-int}}$

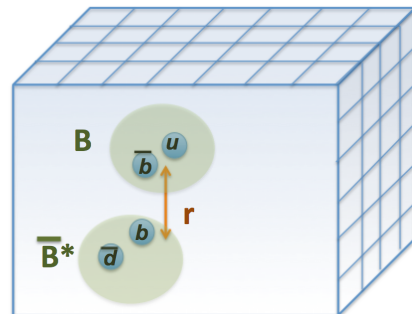


- we verified there is no other $Y M$ ($M=\rho, a_1, a_0$) same the with same quantum num.
- $\eta_b M$ and $h_b M$ have $S_{\text{heavy}}=0$, so they also have different quantum num.

Static potential $V(r)$ for interaction between B and \underline{B}^*



$V(r < 1) = ?$



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We assume that $B \underline{B}^*$ eigenstate is decoupled from $Y\pi$ and Yb_1 channels (overlaps support that).

Born-Oppenheimer approach: B and \underline{B}^* move in

$$V(r) = E_n(r) - m_B - m_{B^*} \quad (m_{B^*} = m_B)$$

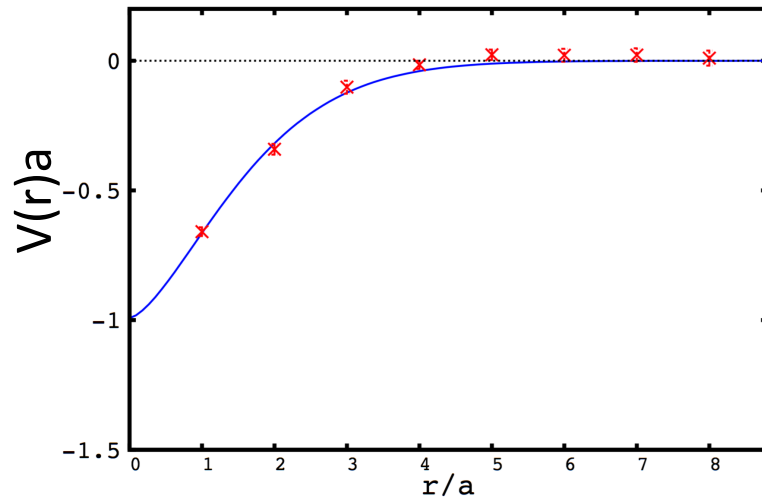
$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{\hbar^2 L(L+1)}{2\mu r^2} + V(r) \right] u(r) = E u(r)$$

$$\mu = \frac{1}{2} m_B^{\text{exp}}, \quad \psi \propto \frac{u}{r} Y_{LM}$$

We focus on most relevant : s-wave ($L=0$)

BB* (virtual) bound states in s-wave

fits with $V(r)$ with $V(0)=\text{finite}$



$$V(r) = -\alpha \exp[-(r/d)^p]$$

example: $p=1.5, r=1-4$

fit results: $\alpha = 0.99(5), d=1.8(1)$

(1) virtual bound st. : $E_B = 34 \pm 24$ MeV

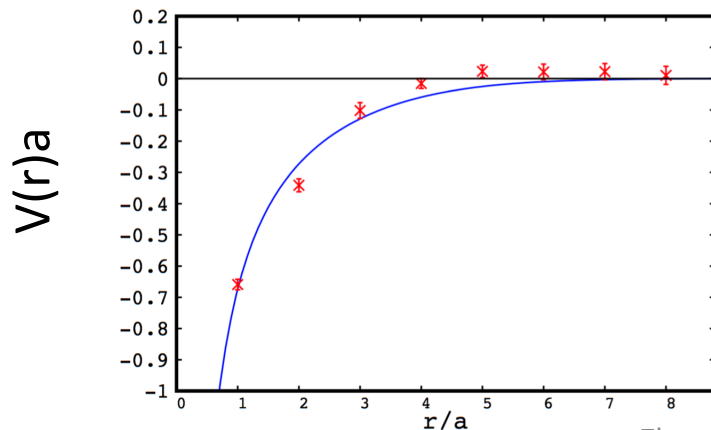
(2) deep bound st.: $E_B = 403 \pm 70$ MeV

Various fit choices for p and range in r (taking or omitting $r=1$)

(1) virtual bound st.: state with small E_B present or not

(2) deep bound st: $E_B = 250$ MeV – 500 MeV

fits with $V(r)$ with $V(0)=-\infty$



$$V(r) = -\frac{\alpha}{r} \exp[-(r/d)^p]$$

Various fit choices for p and range in r (taking or omitting $r=1$)

(1) bound state: $E_B = 2 - 10$ MeV

(2) deep bound st.: $E_B = 400$ MeV – 600 MeV

lattice

deep bound st. found on the lattice:
prediction of a new exotic state?
It could be perhaps visible only in
 $\Upsilon(1S) \pi$, since it is located below all
other thresholds.

bound st. or virtual bound st.
just below threshold is
perhaps related to Z_b

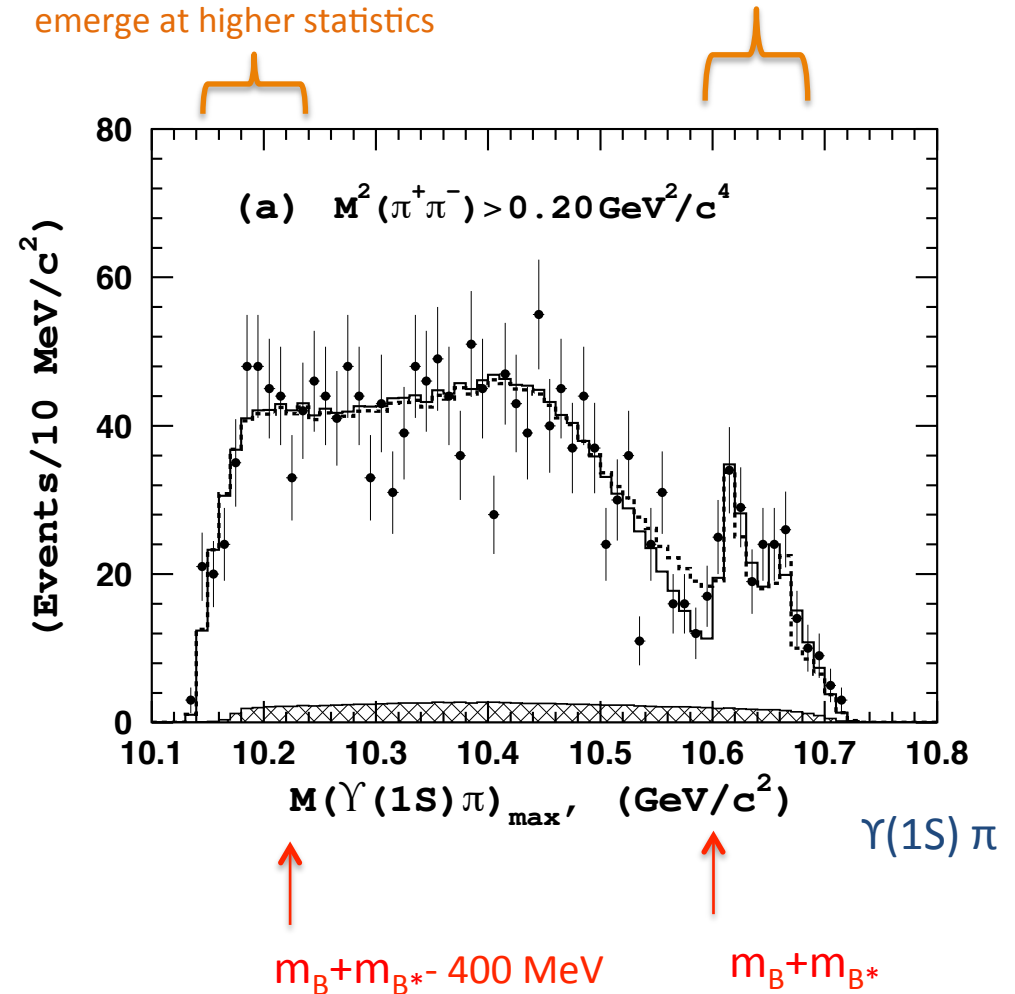
experiment

Belle PRD 91 (2015) 072003

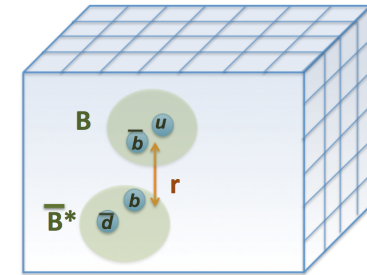
nothing claimed by Belle;
significant "bump" could perhaps
emerge at higher statistics

exp Z_b res

Relating lattice results to Belle experiment



Conclusions

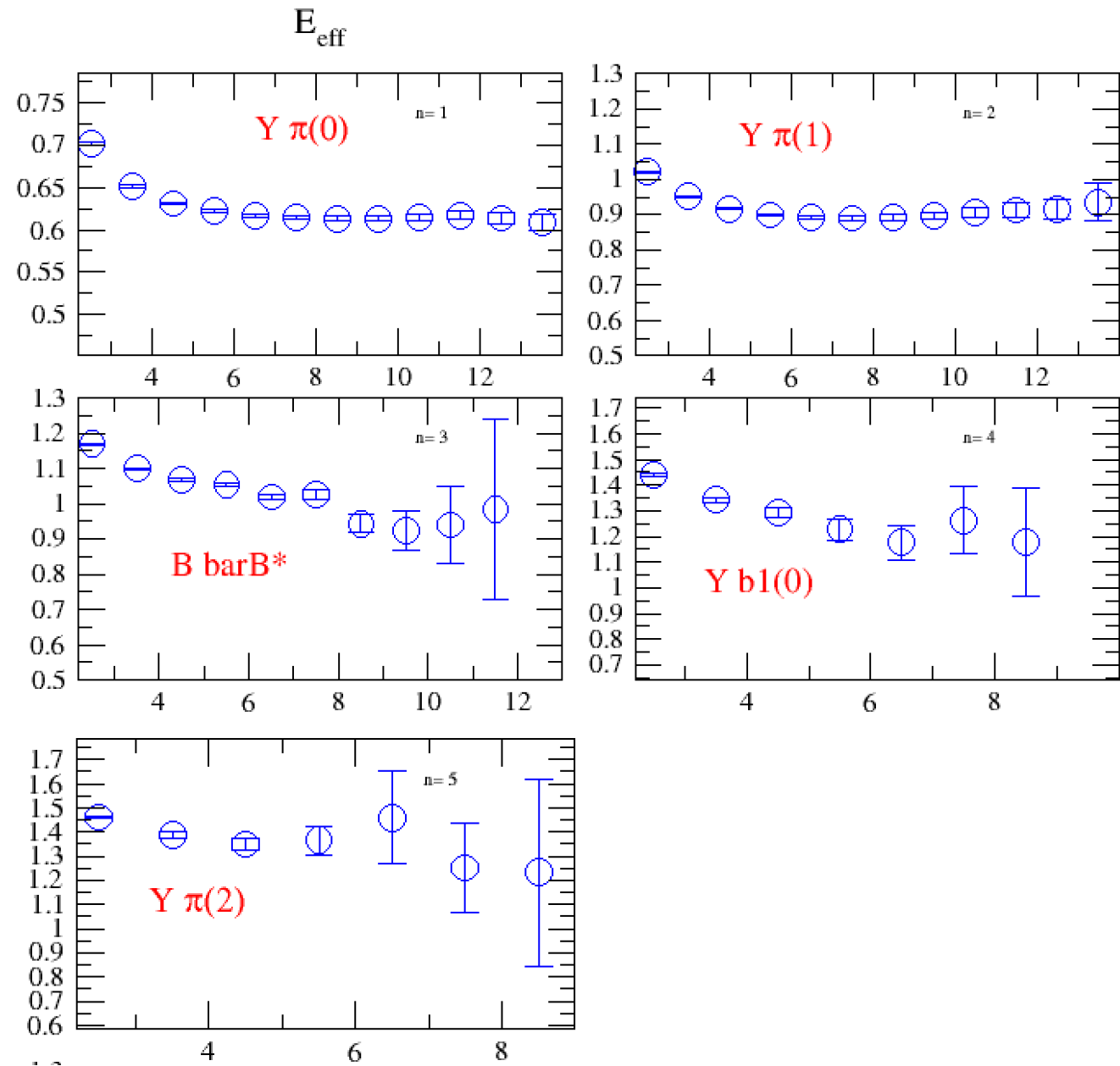


- Exploratory study of Zb channel
- Simplifications:
 - ✧ $m_b = \infty$
 - ✧ $B \underline{B}^*$ channel treated as decoupled from other channels in Schrodinger eq.
- Conclusions based on these simplifications:
 - ✧ Sizable attraction between B and \underline{B}^* at small distances
 - ✧ Bound state found far below $B\underline{B}^*$ threshold : yet undiscovered exotic state ??
 - ✧ Bound state or virtual bound state found slightly below $B\underline{B}^*$ threshold:
this state might be related to Zb from Belle experiment
- Future plans:
 - ✧ coupled-channel analogue of Schrodinger equation
 - ✧

Backup

Effective energies for $r=3$

red labels denote dominant operators in each eigenstate



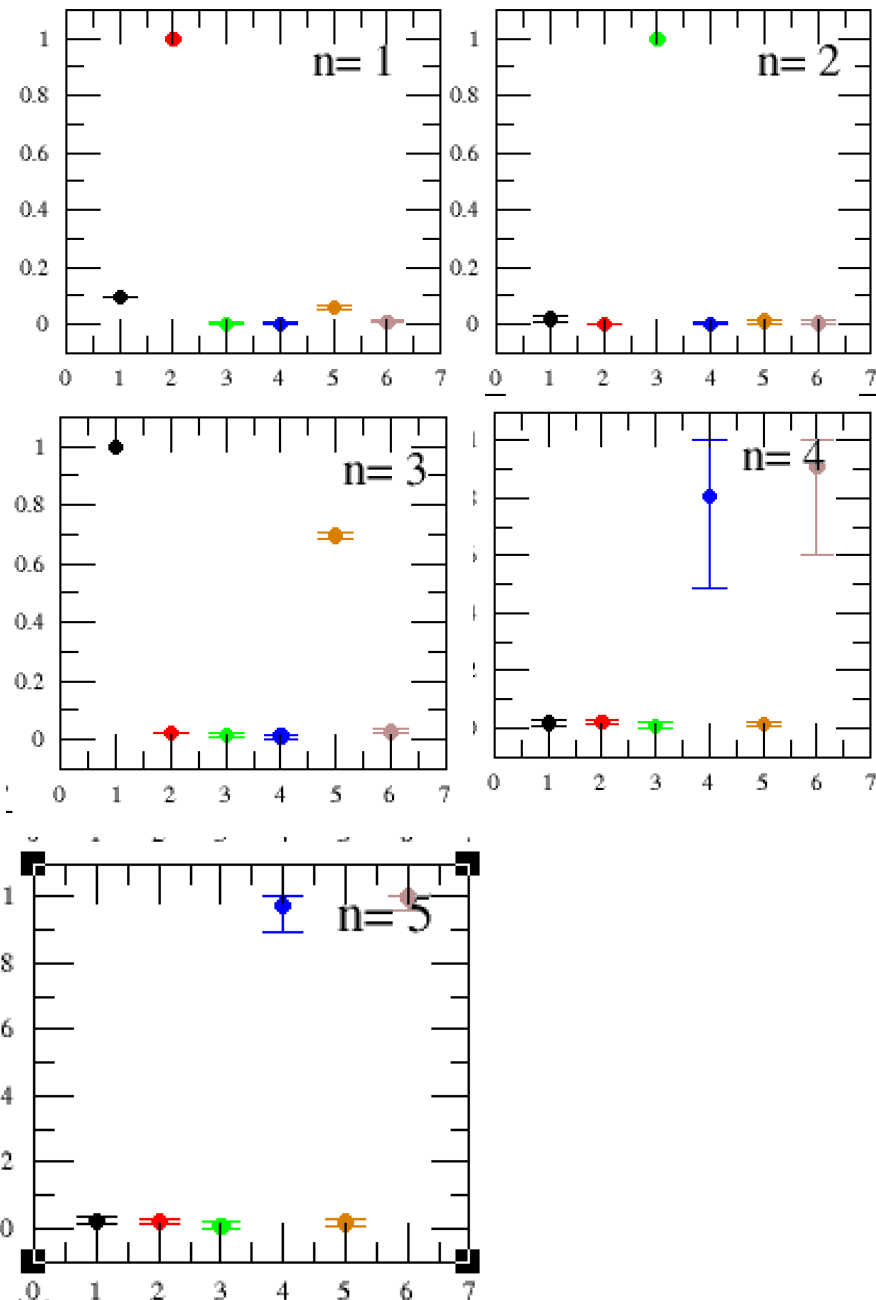
Overlaps of eigenstates $|n\rangle$ for $r=3$



$O=B \bar{b}B^*$
 $O=Y \pi(0)$
 $O=Y \pi(1)$
 $O=Y \pi(2)$
 $O=Y b_1(0)$
 $O=B \bar{b}B^*$

$$Z_i^n = \langle O_i | n \rangle$$

$$Z_i^n / \max_m (Z_i^m)$$



Consideration of symmetries in the static case

In the static limit: heavy (S_{heavy} and $S_{z_{\text{heavy}}}$) and light (J_{light}) angular mom. are separately conserved, because heavy quark can not flip direction of spin under interaction with gluon.

The operator below does not transform irreducibly separately under heavy spin and separately under light spin transformations.

$$\bar{b}P_{-}\gamma_{5}q \quad \bar{q}\gamma_{z}P_{+}b + \bar{b}P_{-}\gamma_{z}q \quad \bar{q}\gamma_{5}P_{+}b$$
$$= (S_{\text{heavy}} = 1)(S_{\text{light}} = 0) + (S_{\text{heavy}} = 0)(S_{\text{light}} = 1)$$