Interglueball potential in SU(N) lattice gauge theory

Nodoka Yamanaka
(YITP, Kyoto U.)

In Collaboration with

H. Iida (Tokyo U.), A. Nakamura (Far Eastern Federal U.),
M. Wakayama (Pukyong National U.)

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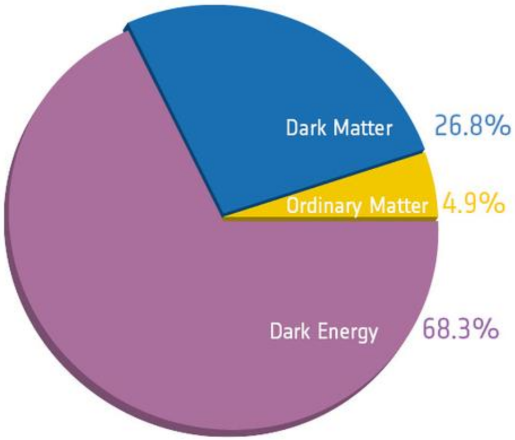
Motivation

Dark matter is representing a significant fraction of the energy content of the Universe

But, we do not know what it is…



Many candidate theories are under discussion

(WIMPs, axions, primordial blackholes, entropic gravity …)

Let us consider the SU(N) Yang-Mills theory :

Generation of mass scale, logarithmic dependence, no important fine-tuning

⇒ Theory with very high naturalness

Lightest particles are glueballs ! ⇒ SU(N) glueballs are candidate of DM

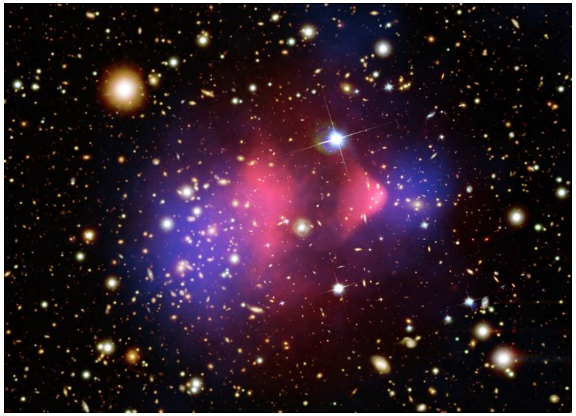
Important feature of DM : Self-interaction

(= DM-DM scattering)

DM self-interaction is constrained by observations (collisions of galaxies, structure formation)

⇒ We need to evaluate the

interglueball scattering cross section!



Bullet cluster : collision of galaxies

Object of study

Lattice gauge theory calculation is the only way to quantify nonperturbative physics of nonabelian SU(N) gauge theory.

Object:

In this work, we study the interglueball interaction of SU(2) Yang-Mills theory (YMT) on lattice, derive the glueball scattering cross section, and constrain the SU(2) YMT scale parameter from observational data.

Setup

We consider the SU(2) pure Yang-Mills theory

- Standard SU(2) plaquette action :

  Lattice spacings : $\beta = 2.5$ (N_c=2)

  Volume : 16^3x24

  Confs. generated with pseudo-heat-bath method (1 M confs.)

- Use SX-ACE (@RCNP, Osaka U.), vector machine

- Improvement of glueball operator : APE smearing

  We use all space-time translational and cubic rotational symmetries to effectively increase the statistics (like the all-mode average for meson and baryon observables)

  Reduction of the statistical error w/ cluster decomposition principle

Scale determination

We do not know the scale of the YM theory, so we leave it as a free parameter Λ

Nevertheless, all quantities calculated on lattice depend on Λ

$\Rightarrow$ We express all quantities in unit of Λ (and finally constrain Λ from other data).

Relation between Λ and string tension:

$$\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.503(2)(40) + \frac{0.33(3)(3)}{N^2}$$

$$= 0.586(41) \qquad \text{(for SU(2))}$$

Fitted from the analysis of the running coupling

C. Allton et al., JHEP **0807** (2008) 021
M. Teper, Acta Phys. Polon. B **40** (2009) 3249

String tension for $\beta = 2.5$ in SU(2) YM :

β	$a\sqrt{\sigma}$
2.5	0.186(3)

M. Teper, Phys. Lett. B **397** (1997) 223; hep-th/9812187

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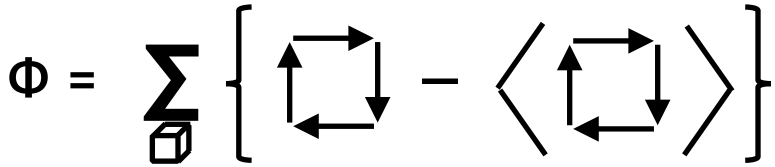
String tension for β = 2.5 in SU(2) YM :

β	$a\sqrt{\sigma}$	a (in unit of Λ^{-1})
2.5	0.186(3)	0.317(23)

⇒ Lattice spacing is now expressed in unit of Λ

Glueball operator and operator improvement

0⁺⁺ glueball operator:



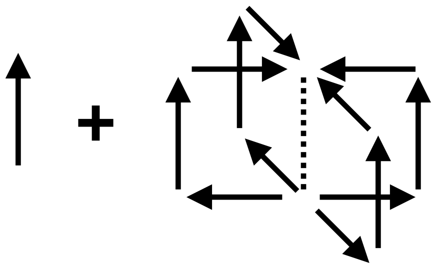
Glueball has expectation value → subtract

Sum over cubic rotational invariance

APE smearing :

$U^{(n+1)}$ so as to maximize **Re Tr [$U^{(n+1)}$ $V^{(n)\dagger}$]**

where $V^{(n)} = \alpha \times$



⇒ Gaussian spread: $2\sqrt{\frac{n}{4+\alpha}}$

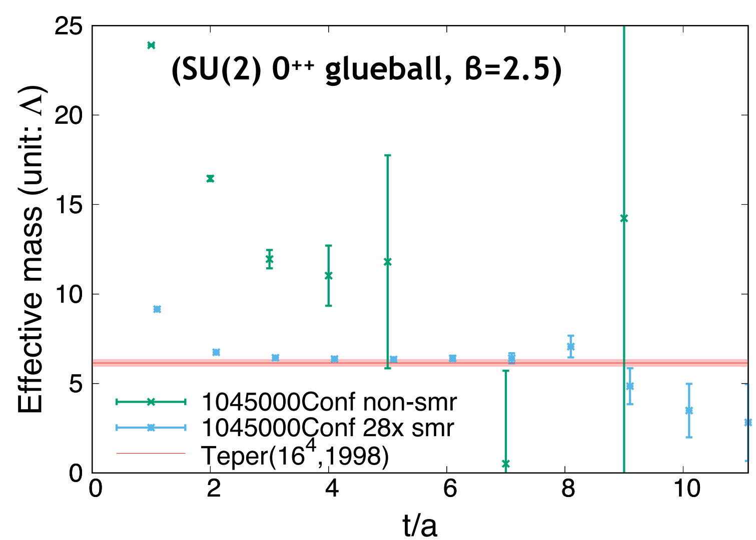
(in lattice unit)

Ape Collaboration, PLB **192** (1987) 163
N. Ishii et al., PRD **66**, 094506 (2002)

Optimal parameters
for SU(2), β=2.5:

n = 28

α = 2.0



Nambu-Bethe-Salpeter amplitude

$$C_{\phi\phi}(t, \mathbf{x}-\mathbf{y}) \equiv \frac{1}{V}\sum_{\mathbf{r}} \langle 0 \,| T[\phi(\mathbf{x}+\mathbf{r},t)\phi(\mathbf{y}+\mathbf{r},t)\cdot \mathcal{J}(0)] |\, 0\rangle$$

$\mathcal{J}(0)$: source op.

The source is smeared, but the sinks are not

For the glueball, caution is needed :

- 2-glueball (0^{++}) state mixes with all other multi-glueball states:

  ⇒ The source may be chosen as 1-body, 2-body, etc, on convenience

- Multi-glueball operators also have expectation value!

  (often called "VEV", but it corresponds to the divergence caused by the mixing with the identity operator)

  ⇒ We then have to subtract the "VEV" of both source and sink

  (removing the source "VEV" will automatically remove sink "VEV":

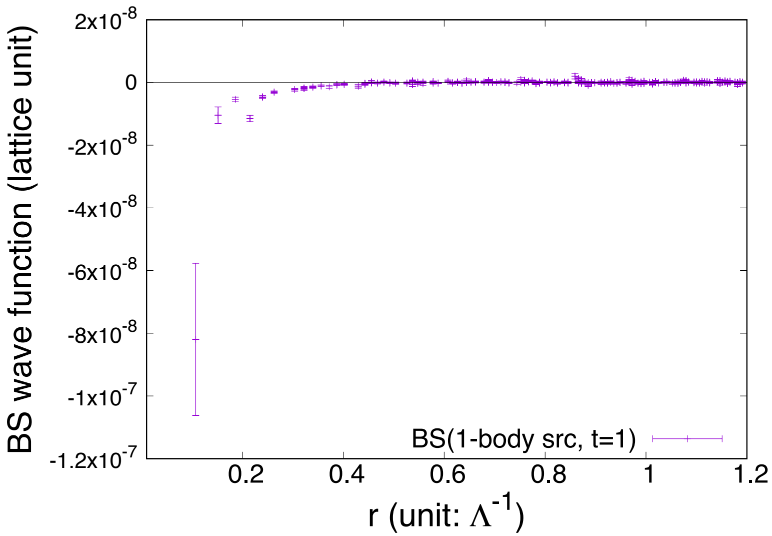
  $\langle(\varphi_{src}\varphi_{src}-\langle\varphi_{src}\varphi_{src}\rangle)(\varphi_{snk}\varphi_{snk}-\langle\varphi_{snk}\varphi_{snk}\rangle)\rangle=\langle(\varphi_{src}\varphi_{src}-\langle\varphi_{src}\varphi_{src}\rangle)\varphi_{snk}\varphi_{snk}\rangle$)

  ⇒ Important consequence : fulfills the cluster decomposition principle!

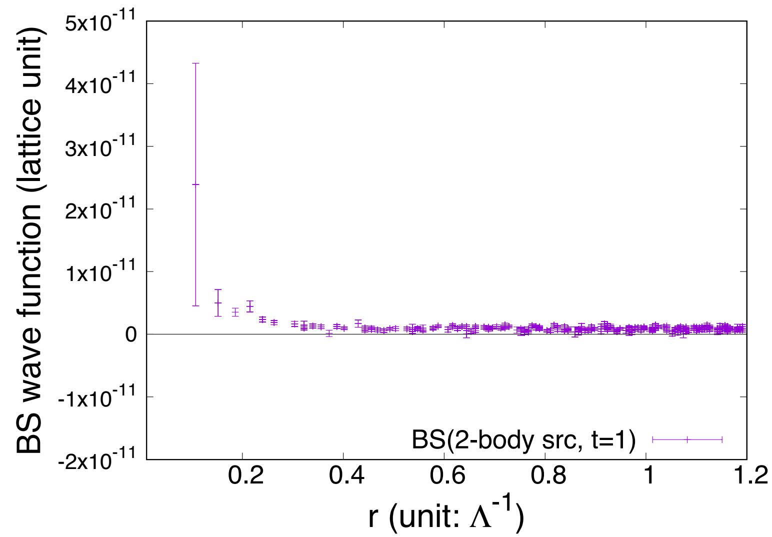
Glueball NBS wave function (with wall source)

(case of SU(2), β=2.5)

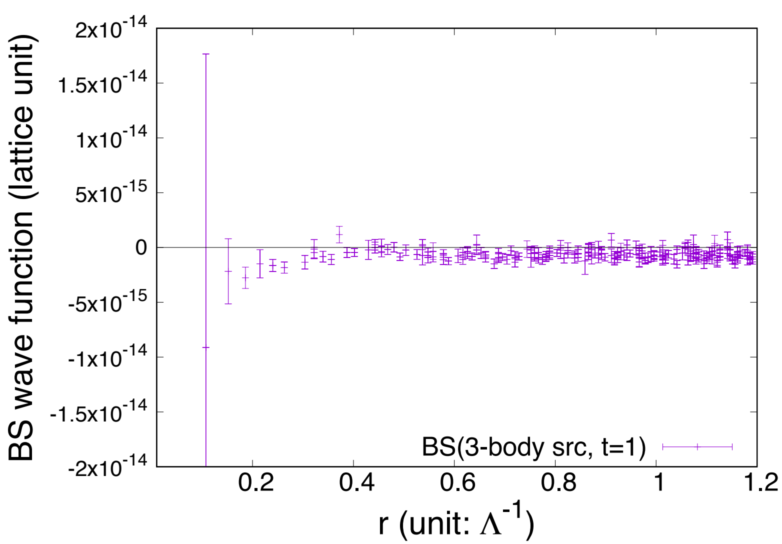
1-body source:



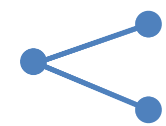
2-body source:



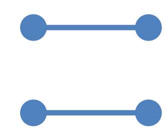
3-body source:



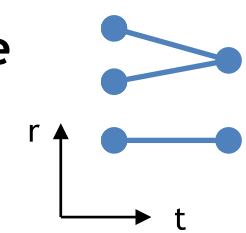
- **1-body src BS is 0 at large r due to cluster decomposition**



- **2-body src BS is finite at large r ⇒ Two free glueballs**



- **3-body src BS should be finite at large r, but large error**



Time-dependent HALQCD method

Extract the potential from the NBS wave function

$$\left[\frac{1}{4m_\phi}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{1}{m_\phi}\nabla^2\right] R(t,\mathbf{r}) = \int d^3r' U(\mathbf{r},\mathbf{r}') R(t,\mathbf{r}')$$

$$R(t,\mathbf{r}) \equiv \frac{C_{\phi\phi}(t,\mathbf{r})}{e^{-2m_\phi t}}$$

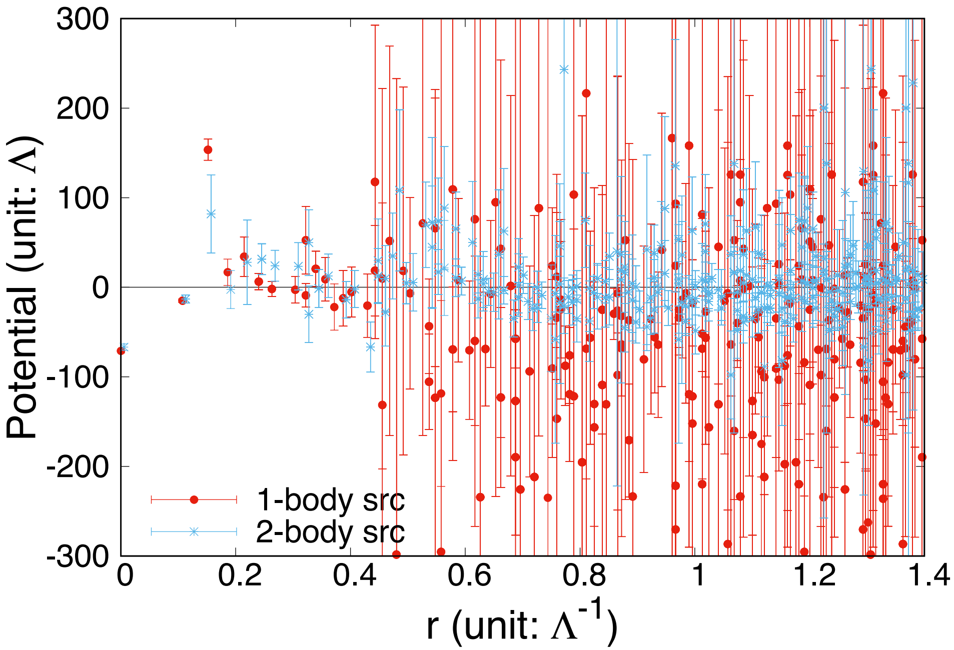
N. Ishii et al., PLB **712** (2012) 437.

- Crucial advantage : do not need ground state saturation

➡ Mandatory to use time-dependent HAL method for the glueball analysis, since the glueball correlator becomes very noisy before ground state saturation

- Inelastic threshold for glueball = $3m_\varphi$: high enough so that we may consider t=2,3 data

SU(2) result : potential plot (local central only, wall source)



(β = 2.5,
1045000 confs)

3 regions :

- **Very short range (lattice unit 0 and 1) : artifact due to ?**

  (also appeared in the SU(3,4) case, maybe related w/ the failure of Luescher's method)

- **Short range ($r < 0.4\ \Lambda^{-1}$) : looks repulsive (determined from 1-body src)**

- **Long range ($r > 0.4\ \Lambda^{-1}$) : flat, vanish (determined from 2-body src)**

Reduction of statistical error with cluster decomposition pr.

Cluster decomposition principle:

"If you are far you are uncorrelated"

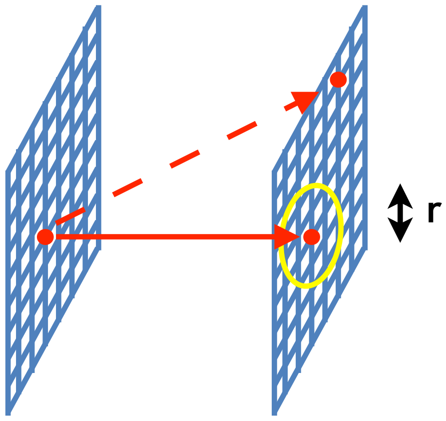
⇒ Almost zero contribution for r > cutoff

For disconnected diagram, noise remains constant

⇒ Integration over r > cutoff accumulates noise

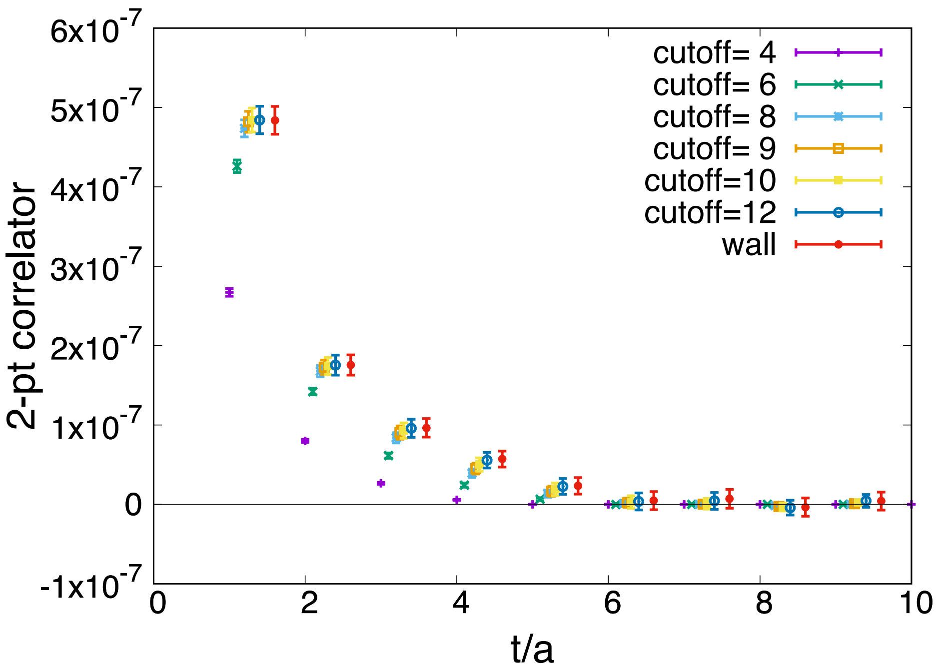
⇒ Remove r > cutoff will reduce the noise?

chiQCD Collaboration, PRD**97**, 034507 (2018)

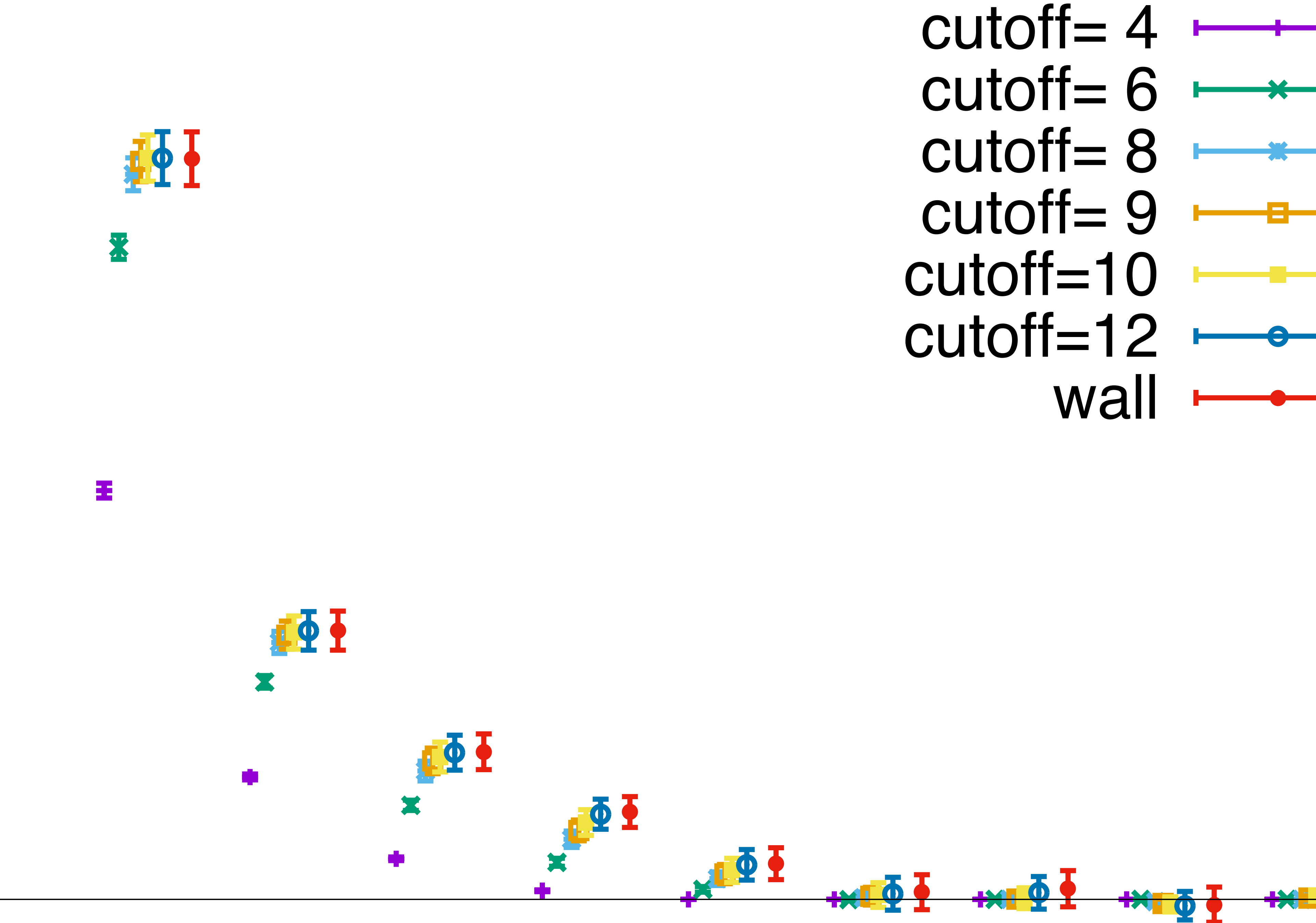


wall x wall correlation

Example of glueball 2pt-correlator:



Reduction of statistical error with cluster decomposition pr.



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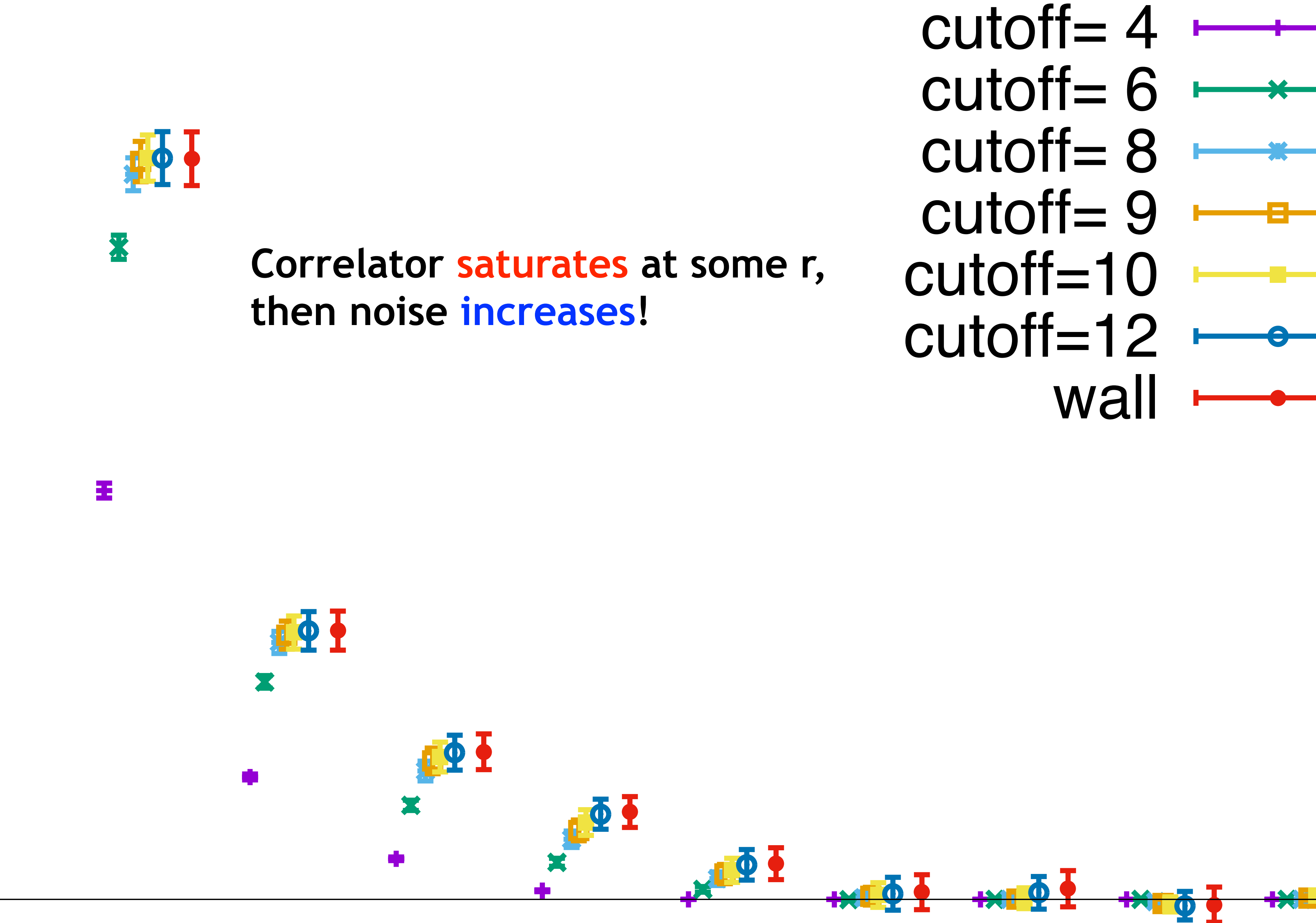

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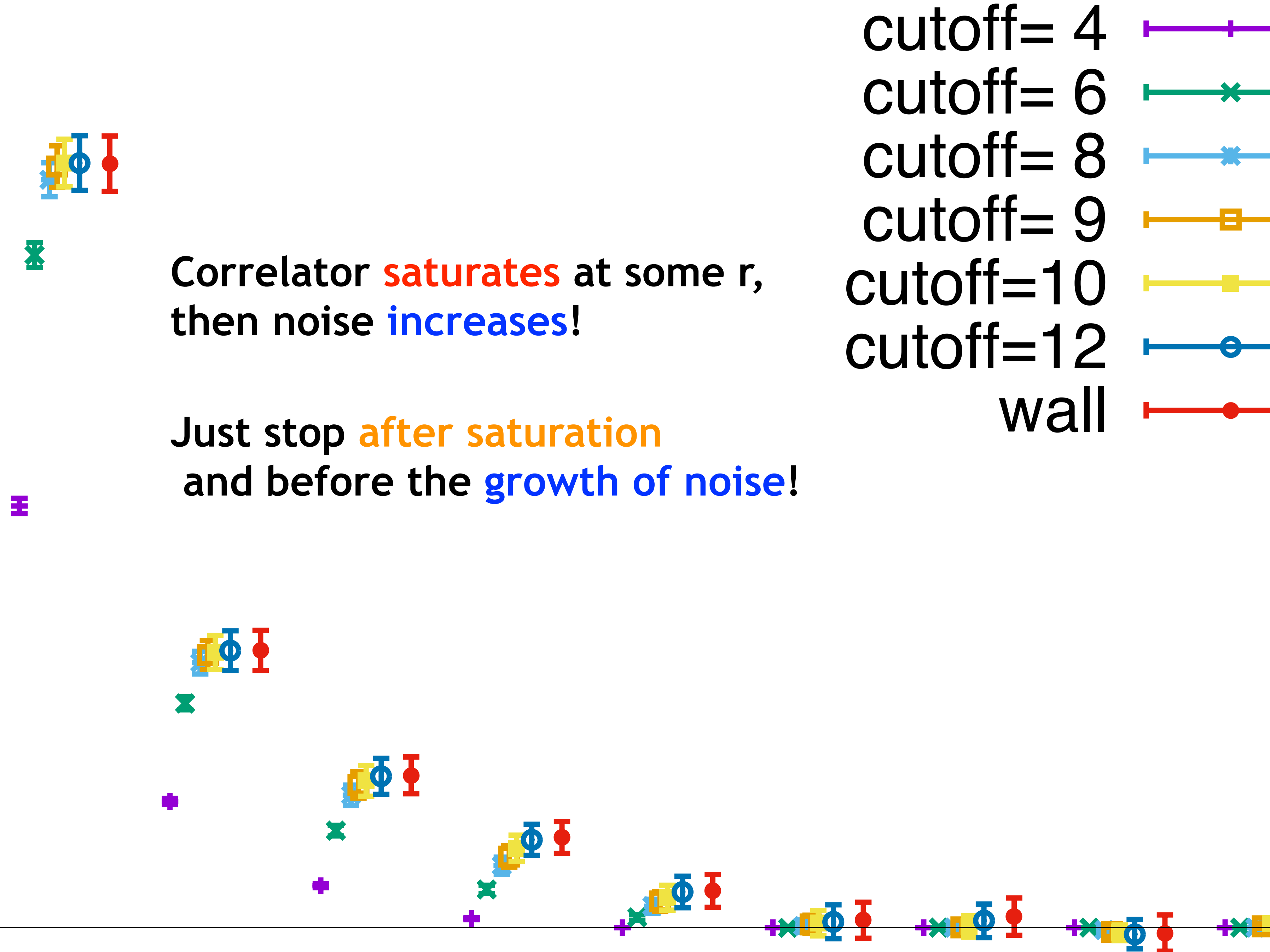
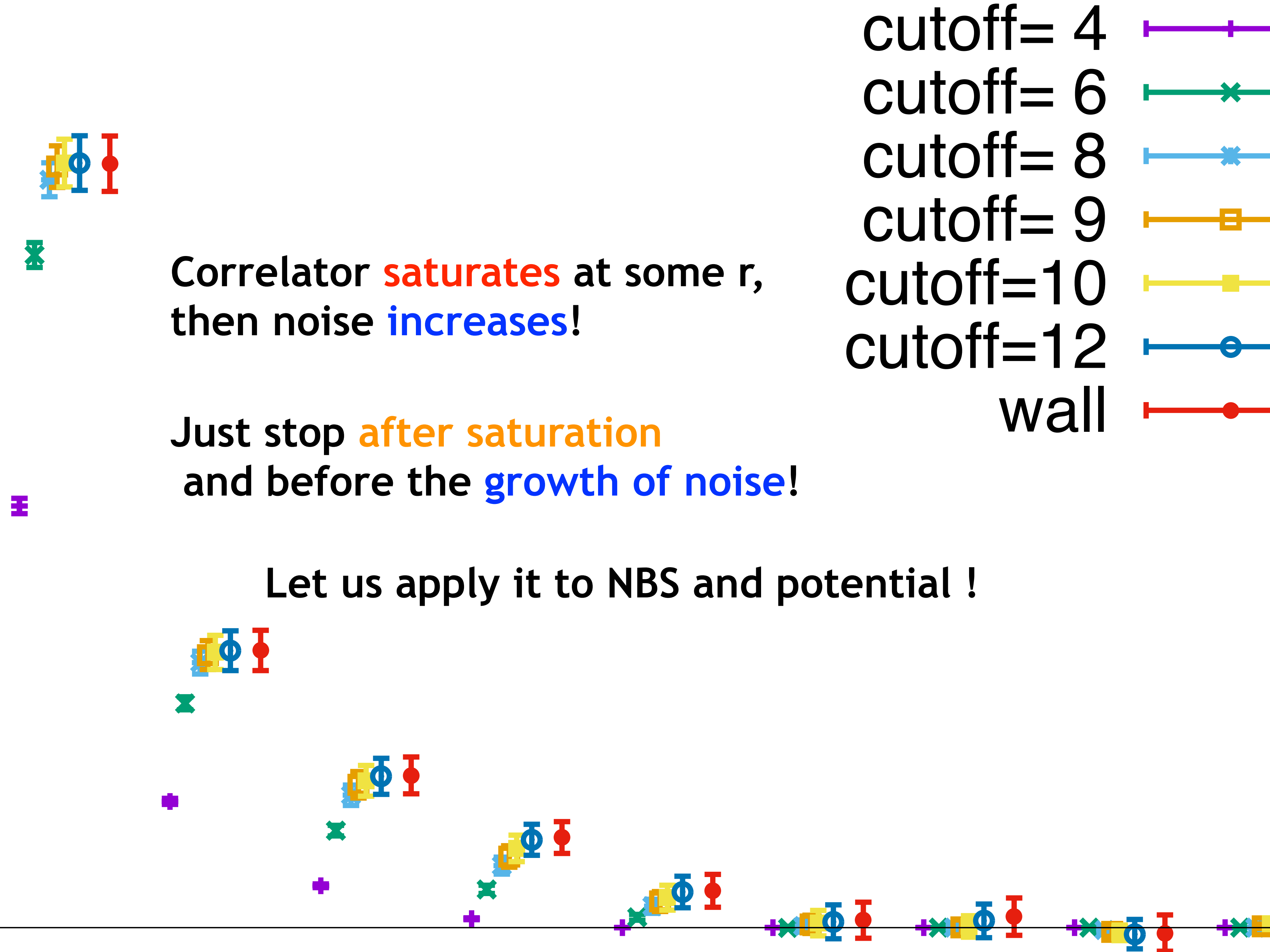


Reduction of statistical error with cluster decomposition pr.



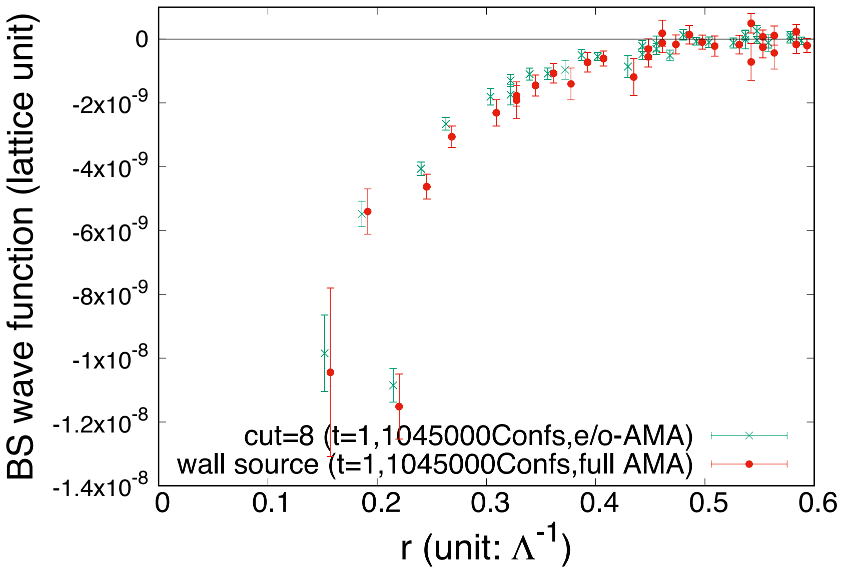
Correlator saturates at some r,
then noise increases!

Just stop after saturation
and before the growth of noise!

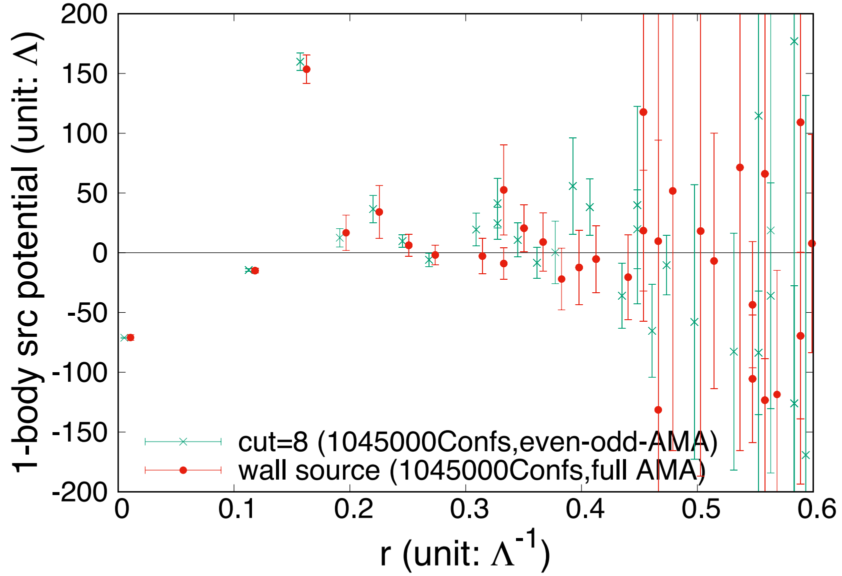
Let us apply it to NBS and potential !

Improvement of NBS and potential with CD principle

NBS amplitude:



Potential:



Statistical error ∼ 1/2, with 1/2 statistics (8 times improvement)!

Fit of the potential

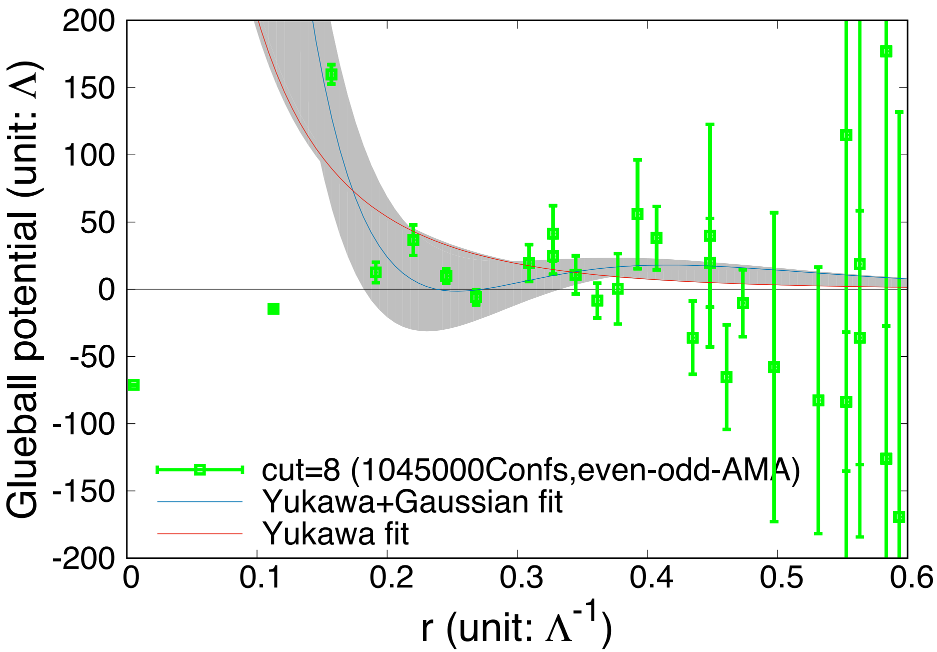
We test two fitting forms:

- **Yukawa:** $V(r) = V_1 \frac{e^{-m_\phi r}}{r}$

  V_1 = 38.2 ± 2.1 (latt. unit) χ^2 d.o.f. = 12.6

- **Yukawa + Gaussian:** $V(r) = V_1 \frac{e^{-m_\phi r}}{r} + V_2 e^{-\frac{(m_\phi r)^2}{2}}$

  V_1 = 219.1 ± 15.1 , V_2 = -68.2 ± 5.6 (latt. unit) χ^2 d.o.f. = 3.1



Note 1:
We do not take into account the 0 and 1 lattice unit points.
⇒ Artifact due to double plaquettes?

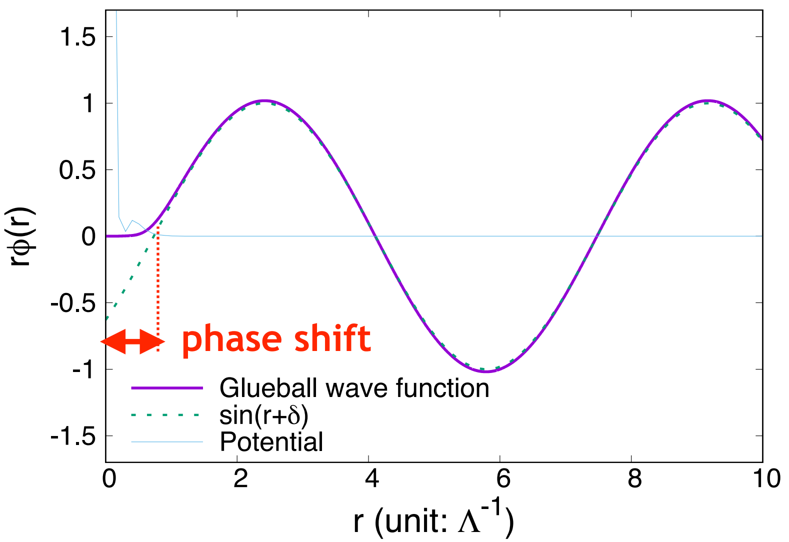
Note 2:
The zigzag behavior is due to the lattice artifact of angular momentum (see HALQCD works).

From potential to scattering cross section

Potential ⇒ Scattering phase shift:

Solve $$\left(\frac{\partial^2}{\partial r^2} + k^2 + U(r)\right)\phi(r) = 0$$

➡ $$\phi(r) \propto \sin[r + \delta(k)] \qquad (r \to \infty)$$



Scattering phase shift ⇒ Cross section:

We are interested in low energy DM cross section, s-wave dominant :

➡ $$\sigma_{\mathrm{tot}} = \frac{4\pi}{k^2}\sin^2[\delta(k \to 0)]$$

Yukawa: σ_{tot} = (3.2 - 3.4)Λ^{-2} (stat.)

Yukawa+Gaussian: σ_{tot} = (6.7 - 7.1)Λ^{-2} (stat.)

➡ **σ_{tot} = (3.2 - 7.1) Λ^{-2} (stat.and sys.)**

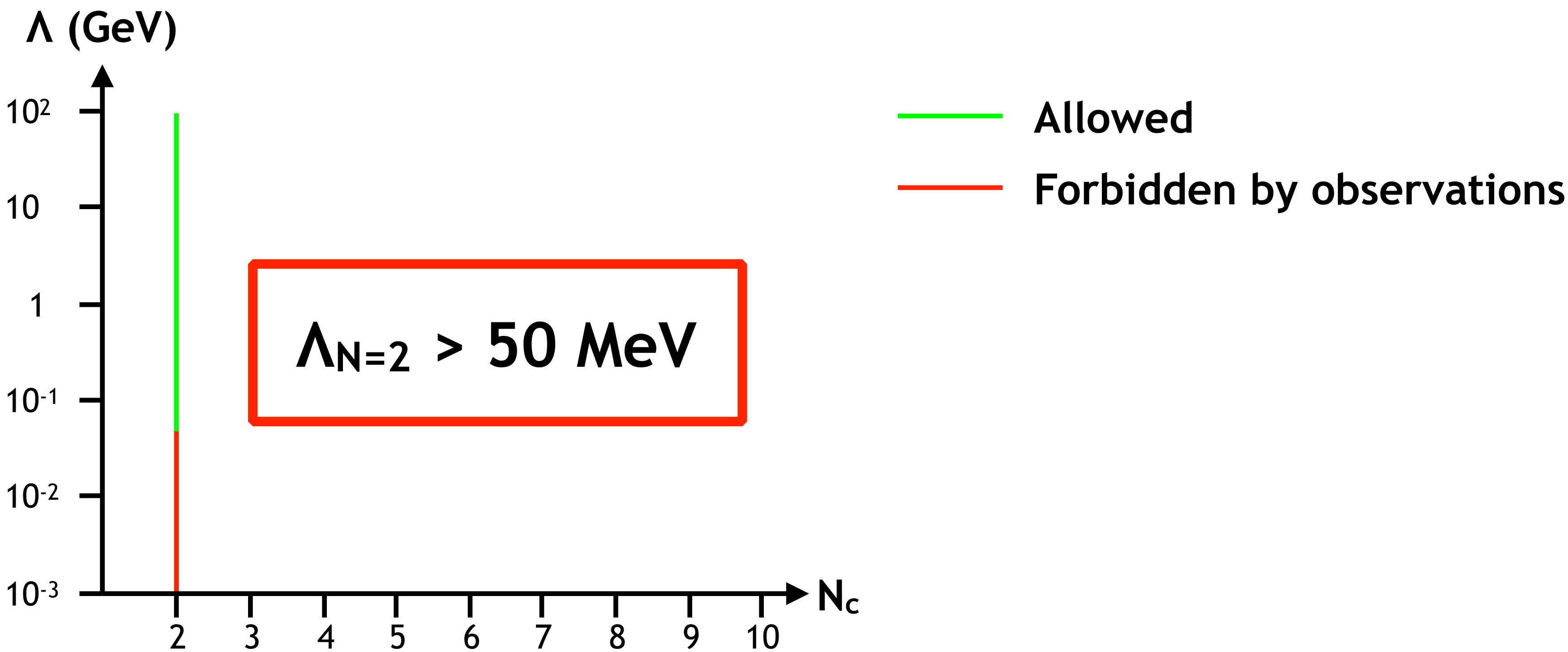
(sys. due to fitting forms)

Constraint on SU(N) YM scale parameter from DM X section

Observational constraints: $\frac{\sigma_{tot}}{m_\Phi} < 1.0\ \mathrm{cm}^2/\mathrm{g}$

Robust constraint from galactic cluster shape, collisions (upper limit)

A. H. Peter et al., MNRAS **430**, 81 (2013), **430**, 105 (2013); S. W. Randall et al., APJ **679**, 1173 (2008).



N_c vs. scale parameter (Λ) diagram

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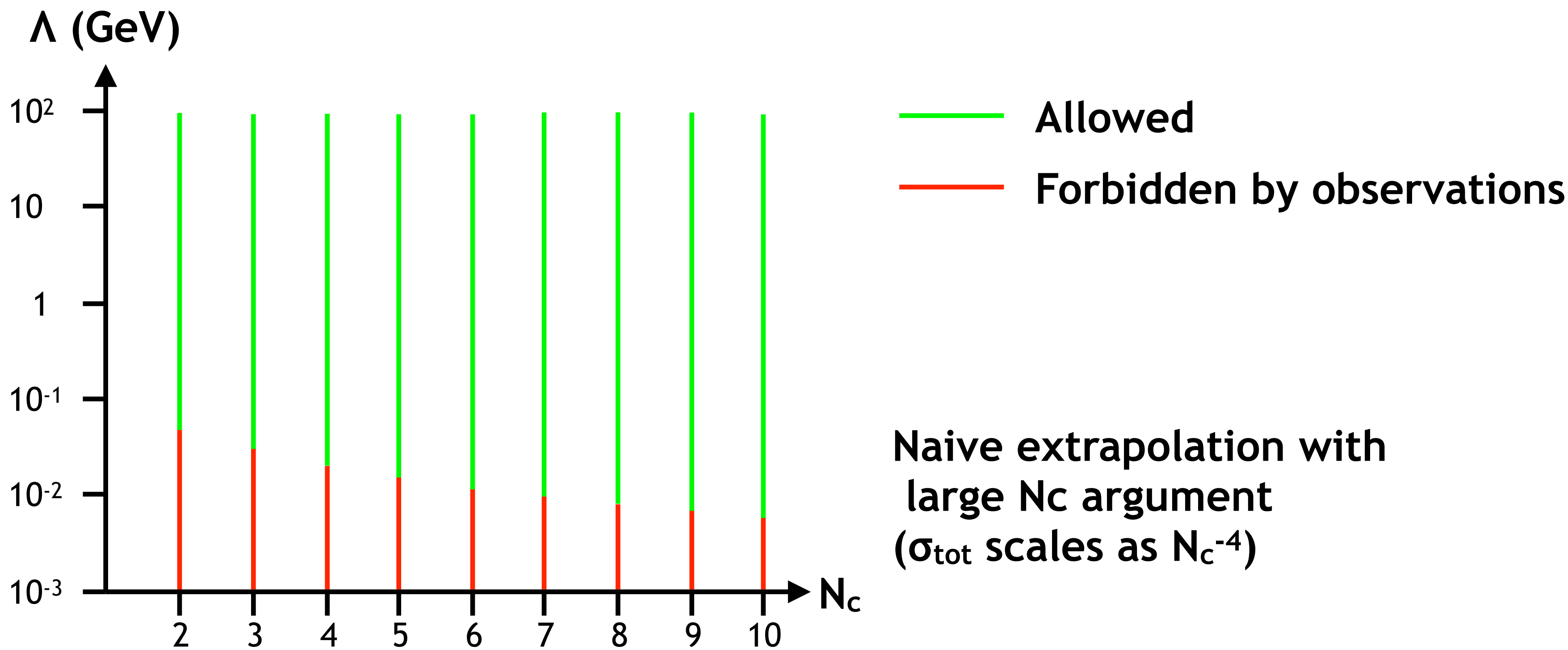


Naive extrapolation with large Nc argument (σ_{tot} scales as N_c^{-4})

N_c vs. scale parameter (Λ) diagram

Constraint on SU(N) YM scale parameter from DM X section

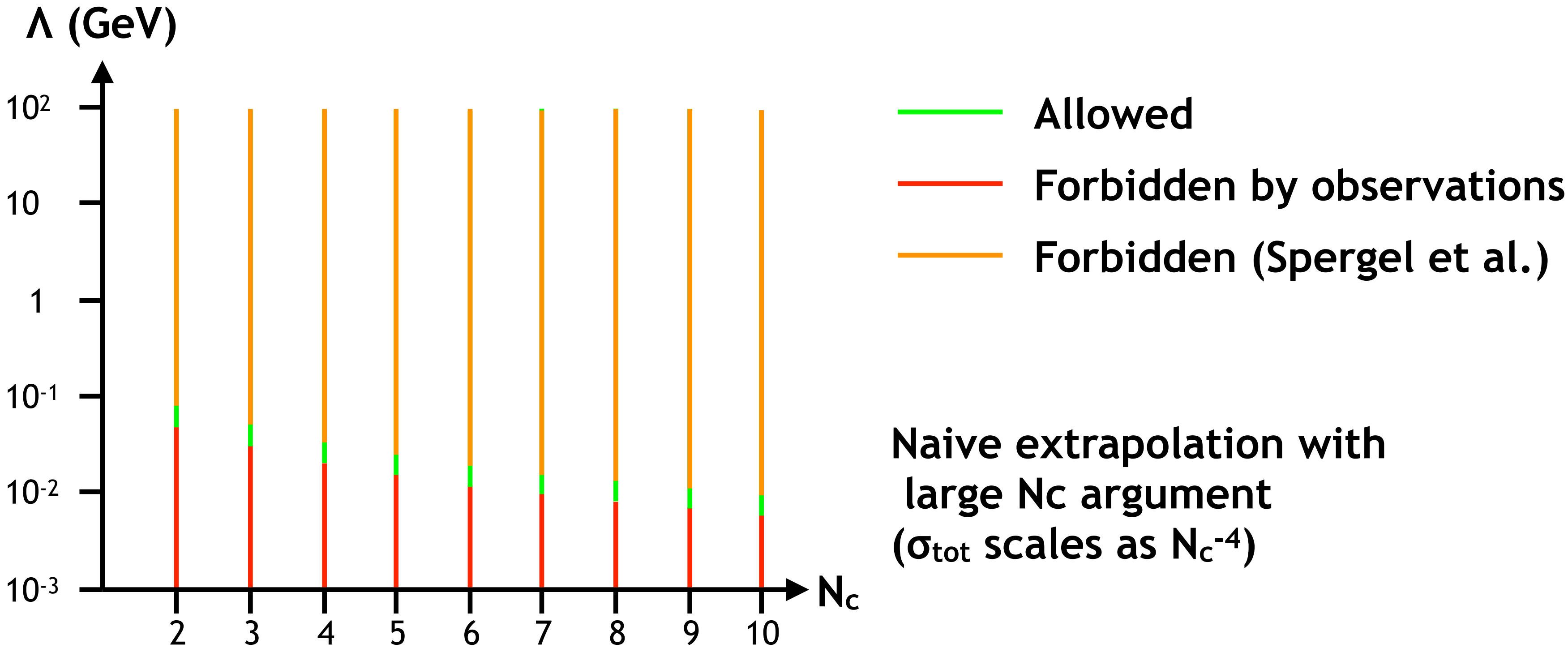
Observational constraints: $0.45\ \mathrm{cm^2/g} < \frac{\sigma_{tot}}{m_\Phi} < 1.0\ \mathrm{cm^2/g}$

Robust constraint from galactic cluster shape, collisions (upper limit)

A. H. Peter et al., MNRAS **430**, 81 (2013), **430**, 105 (2013); S. W. Randall et al., APJ **679**, 1173 (2008).

Constraint from Spergel et al. (lower limit), might disappear?

D. N. Spergel et al., PRL **84**, 3760 (2000).



Naive extrapolation with large Nc argument (σ_{tot} scales as N_c^{-4})

N_c vs. scale parameter (Λ) diagram

Summary

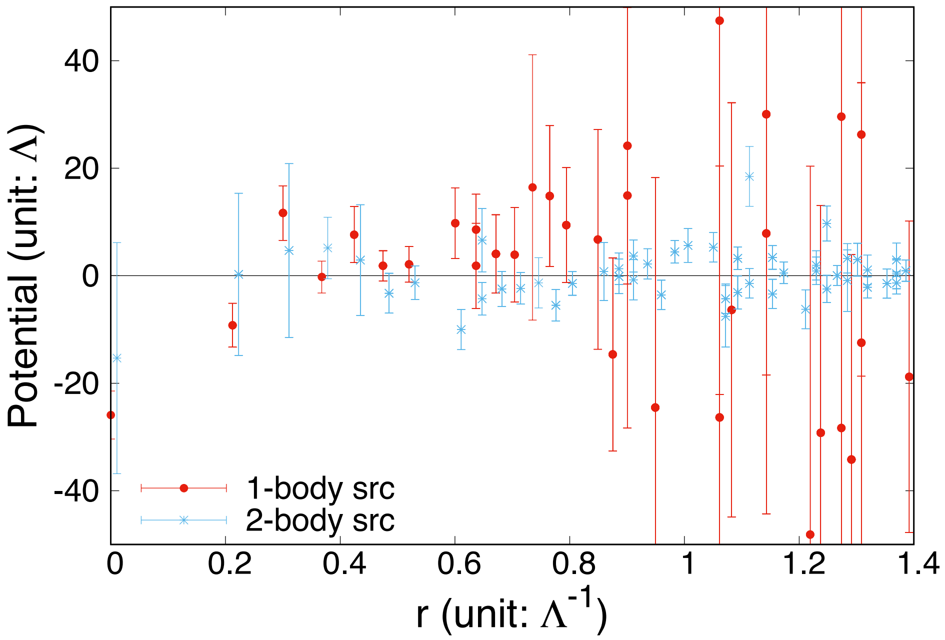
- Glueballs of the SU(N) Yang-Mills theory are good candidates of dark matter : study of self-interaction is important.
- We calculated the interglueball potential in the SU(2) Yang-Mills theory: Time-dependent HALQCD method is important for the interglueball potential because the signal becomes noisy before the ground state saturation.
- We used the cluster decomposition principle to reduce the statistical noise.
- Interglueball potential repulsive for $r < 0.4\Lambda^{-1}$, flat at $r > 0.4\Lambda^{-1}$.
- We calculated the scattering phase shift and derived the interglueball cross section.
- We could constrain Λ of SU(2) YMT for the 1st time from observational data : $\Lambda > 50$ MeV.

Homeworks:

- Complete analyses of other lattice spacings.
- Complete analyses of other SU(N) Yang-Mills theories.
- Lattice artifacts to be discussed.

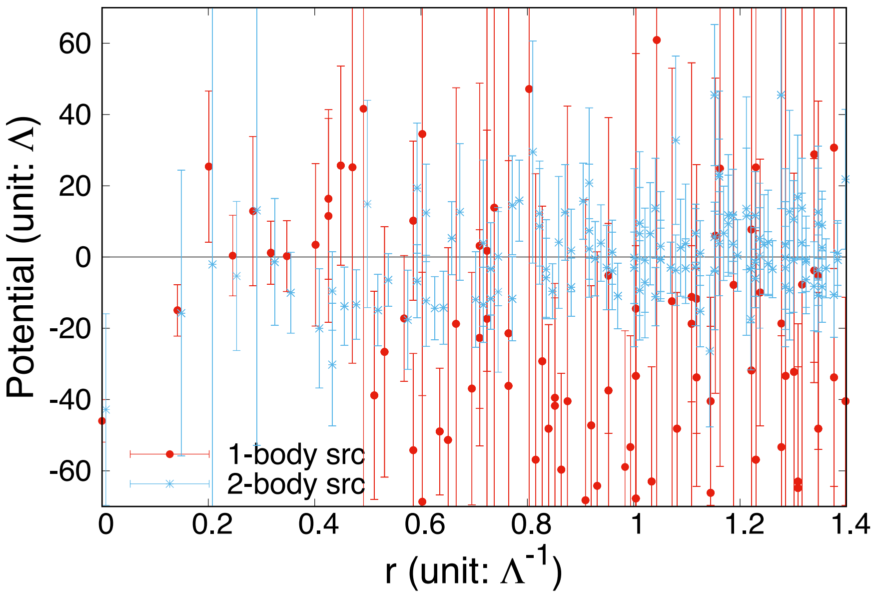
Backup

SU(3) result (wall source)

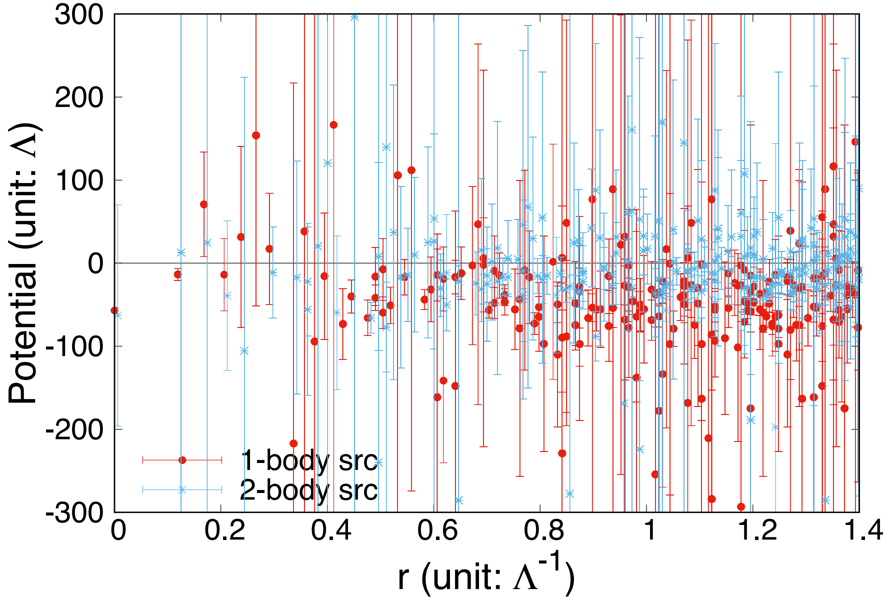


(β = 5.7,
158641 confs)

SU(4) result (wall source)



(β = 10.789,
176000 confs)



(β = 10.9,
210000 confs)

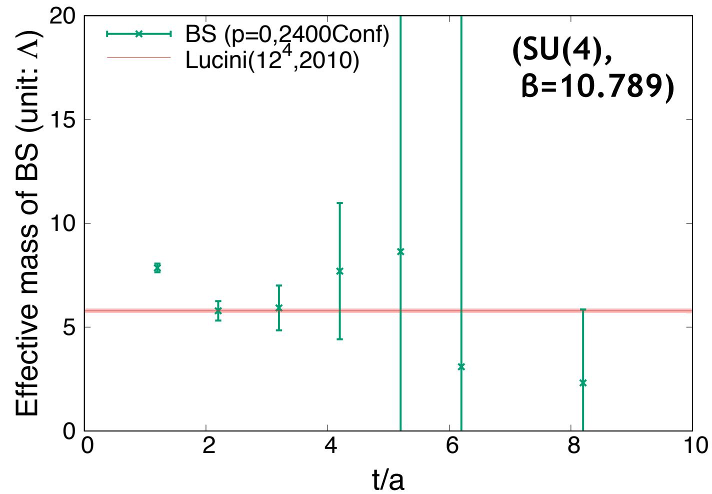
Luescher's method

Calculate the scattering phase shift : need the modulation of the energy of NBS wavefunction in momentum

Problem for the interglueball scattering :

⇒ The glueball 2-body state mixes with 1-body state (at least for 0^{++})

⇒ GS saturation of 2-body scattering dominated by 1-glueball state !

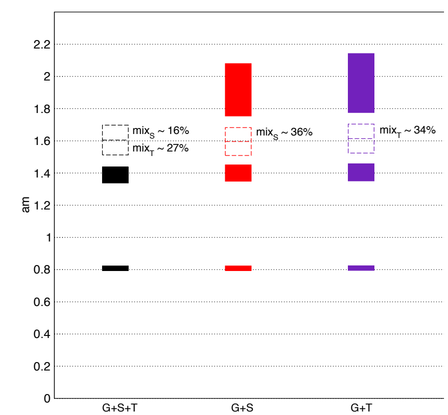


What about diagonalization? (remove 1-body state)

⇒ Many glueball states with energy close to $2m_{GB}$...?

⇒ Maybe difficult to distinguish the $2m_{GB}+\Delta E$ level from other glueball states

(momentum modulation may be visible, but challenging)



B. Lucini et al., JHEP **1008** (2010) 119

Difficult to calculate interglueball scattering with Luescher's method

Self-interacting dark matter (Spergel et al., PRL 84, 3760)

The DM distribution can be predicted in N-body simulation with gravity only

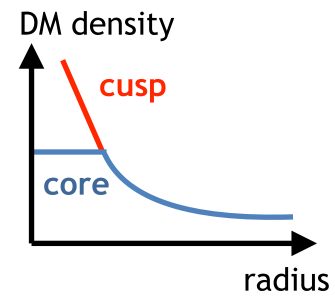
⇒ Successful in describing the large scale structure (scale > Mpc)

Introducing DM self-interaction changes its distribution smaller than Mpc

There are (were?) several problems in the galactic DM distribution:

- **Core vs Cusp problem:**

  N-body simulation predicts cuspy DM distribution near the galactic center, whereas observations suggest flat ones.



- **Too-big-to-fail problem:**

  Satellite galaxies are less dense than those predicted by the N-body simulation.

- **Missing satellite problem:**

  More satellite galaxies than those predicted by the N-body simulation are observed (resolved?).

DM-DM self-interaction ↔ DM-DM scattering ↔ DM-DM potential must be studied