Hadronic light-by-light scattering contribution to the muon anomalous magnetic moment from lattice QCD

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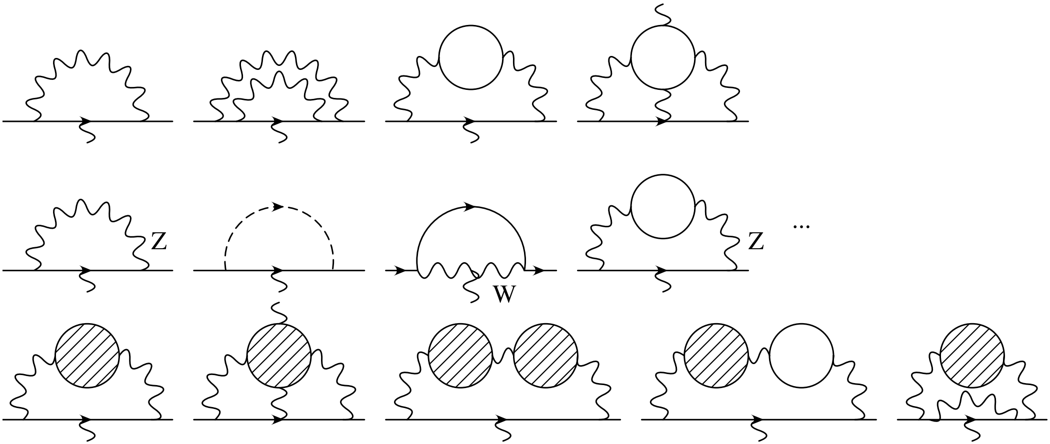
Outline I

Standard Model Theory: QED+EW+QCD

$$\langle \mu(\vec{p}')|J_\nu(0)|\mu(\vec{p})\rangle = -e\bar{u}(\vec{p}')\left(F_1(q^2)\gamma_\nu + i\frac{F_2(q^2)}{4m}[\gamma_\nu,\gamma_\rho]q_\rho\right)u(\vec{p})$$

$$a_\mu \equiv (g-2)/2 = F_2(0) \qquad (q = p' - p)$$



Experiment - Theory

SM Contribution	Value ± Error ($\times 10^{11}$)	Ref	notes
QED (5 loops)	116584718.951 ± 0.080	[Aoyama et al., 2012]	
HVP LO	6931 ± 34	[Davier et al., 2017]	$\rightarrow 3.5\sigma$
	6932.6 ± 24.6	[Keshavarzi et al., 2018]	$\rightarrow 3.7\sigma$
	6925 ± 27	[Blum et al., 2018]	lattice+R-ratio (FJ17), $\rightarrow 3.7\sigma$
HVP NLO	-98.2 ± 0.4	[Keshavarzi et al., 2018]	
		[Kurz et al., 2014]	
HVP NNLO	12.4 ± 0.1	[Kurz et al., 2014]	
HLbL	105 ± 26	[Prades et al., 2009]	
HLbL (NLO)	3 ± 2	[Colangelo et al., 2014]	
Weak (2 loops)	153.6 ± 1.0	[Gnendiger et al., 2013]	
SM Tot	116591820.5 ± 35.6	[Keshavarzi et al., 2018]	
Exp (0.54 ppm)	116592080 ± 63	[Bennett et al., 2006]	
Diff (Exp − SM)	259.5 ± 72	[Keshavarzi et al., 2018]	$\rightarrow 3.7\sigma$

main messages: QCD errors dominate, Δ HLbL $\sim$ Δ HVP, discrepancy is large

FNAL E989 running, goal to reduce BNL 821 error by 1/4; JPARC E34 approved

Outline I

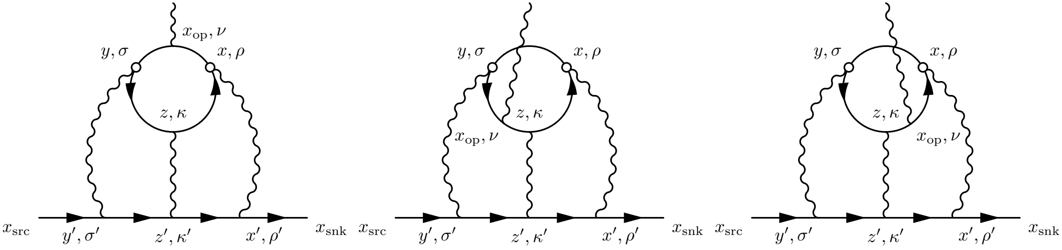
HLbL in finite and infinite volume QED (+lattice QCD)

Two different methods for treating QED

1. QED_L: both QED and QCD in discrete, finite volume
2. QED_∞: QED in infinite volume, continuum. QCD in discrete, finite volume

The hadronic parts of the amplitudes are the same for both methods

Point source method in QCD+pQED (L. Jin) [Blum et al., 2016]



- Must sum over all vertices. Muon line exact with FFT's
- Hadronic part done stochastically. Compute point source propagators at x and y
- Importance sampling is used to choose source locations:
  - Most of the contribution from $r = |x - y| \lesssim 1$ fm
  - Compute for all possible $|x - y| \leq 5$ lattice units
  - Randomly choose pairs for $|x - y| > 5$, $p(r) \propto e^{-|r|/100}/|r|^4$
- Moment in $\vec{x}_{\rm op}$ allows computation of $F_2(q^2)$ directly at $q = 0$
- Ward Identity exact for single measurement, so noise doesn't blow up when $q \to 0$

Techniques produce $O(1000)$ improvement in statistical error over original method

Lattice setup

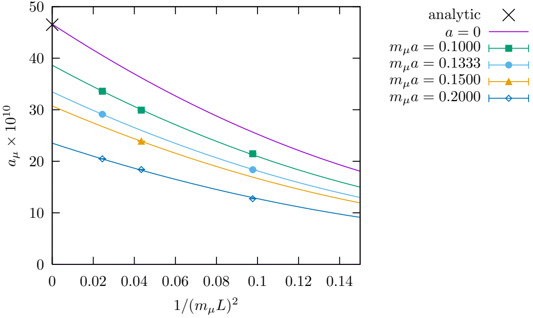
- Photons: Feynman gauge, QED_L [Hayakawa and Uno, 2008] (omit all modes with $\vec{q} = 0$)
- Gluons: Iwasaki (I) (+DSDR) gauge action (RG improved, plaquette+rectangle)
- muons: $L_s = \infty$ free domain-wall fermions (DWF)
- quarks: Möbius-DWF
- Lanczos, AMA, and zMöbius techniques used to speed up the calculation

2+1f Möbius-DWF, I and I-DSDR physical point QCD ensembles (RBC/UKQCD) [Blum et al., 2014]

	48I	64I	24D	32D	48D
a^{-1} (GeV)	1.73	2.36	1.0	1.0	1.0
a (fm)	0.114	0.084	0.2	0.2	0.2
L (fm)	5.47	5.38	4.8	6.4	9.6
L_s	48	64	24	24	24
m_π (MeV)	139	135	140	140	140
m_μ (MeV)	106	106	106	106	106
meas (con, disco)	65, 99	43, 44	158, 157	71, 70	64, 0

Continuum and ∞ volume limits in QED [Blum et al., 2016]

Test method in pure QED. QED systematics large, but under control



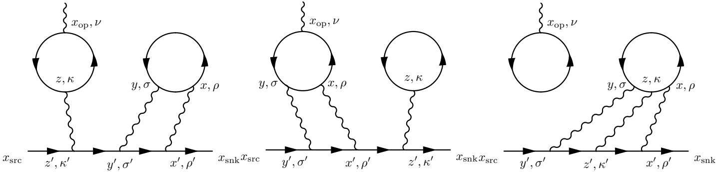
$O(1/L^2)$ finite volume (FV) error

($c.f.$ exponentially suppressed in QCD)

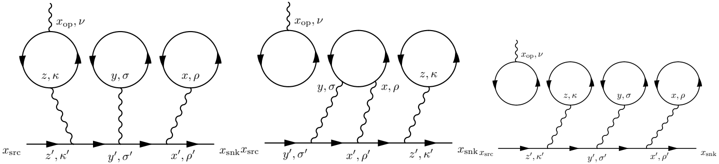
Compare to analytic result, 46.5×10^{-10}

$$F_2(a, L) = F_2 \left(1 - \frac{b_1}{(m_\mu L)^2} + \frac{b_2}{(m_\mu L)^4}\right)(1 - c_1 a^2 + c_2 a^4) \to F_2 = 46.6(2) \times 10^{-10}$$

Quark disconnected contributions



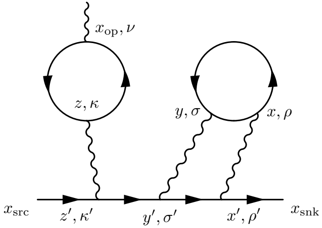
Leading $\qquad$ $(m_s - m_{u,d})$



$(m_s - m_{u,d})^2$ $\qquad$ $(m_s - m_{u,d})^4$

- only top-leftmost diagram does not vanish in SU(3) flavor limit
- Permutations of internal photons not shown
- Gluons within and connecting quark loops not shown
- To ensure loops are connected by gluons, explicit "vacuum" subtraction is required

Leading disconnected contribution

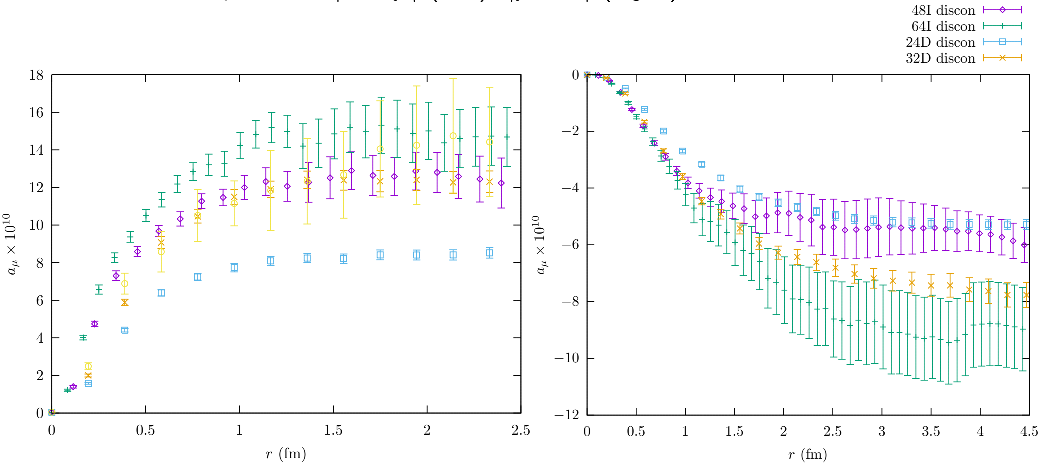


- We use two point sources at y and z, chosen randomly. The point sinks x_{op} and x are summed over exactly on lattice.
- Only point source quark propagators are needed. Compute M point source propagators (including strange) and all M^2 combinations are used to perform the stochastic sum over $r = z - y$ (M^2 trick).
- Because of parity, expectation value for (moment of) left loop averages to zero.

Outline I

HLbL contribution [Blum et al., 2017a]

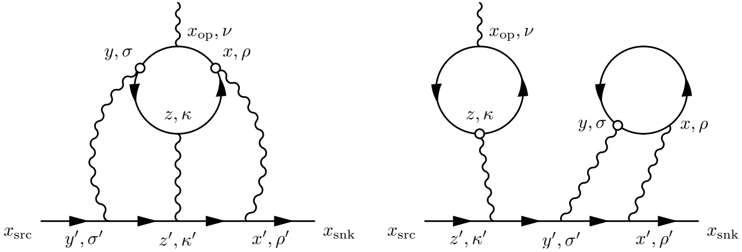
Cumulative sum up to $r = |x - y|$ (left), $|y - z|$ (right)



connected

disconnected

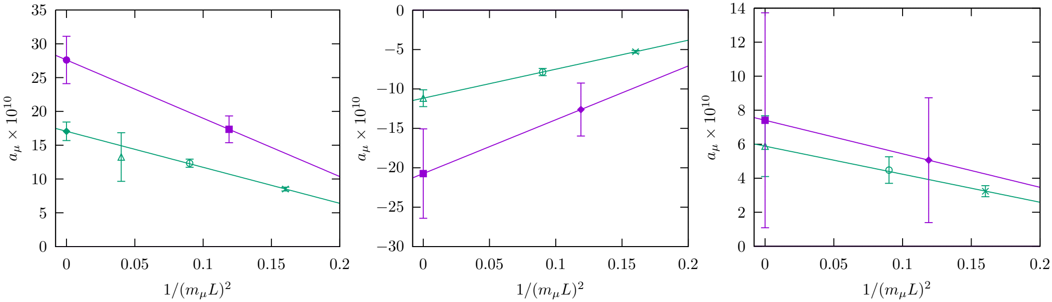
HLbL contribution, 139 MeV pion, $a = 0.114$ fm, $L = 5.5$ fm [Blum et al., 2017a]



$$a_\mu^{cHLbL} = (11.60 \pm 0.96) \times 10^{-10}$$
$$a_\mu^{dHLbL} = (-6.25 \pm 0.80) \times 10^{-10}$$
$$a_\mu^{HLbL} = (5.35 \pm 1.35) \times 10^{-10}$$

Need to extrapolate to the continuum and ∞ volume limits

QED$_L$ continuum and infinite volume extrapolation (preliminary)



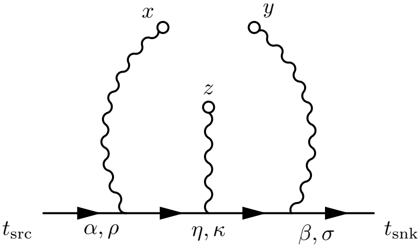
- Iwasaki ensembles: $a \to 0$ ($c_2 = 0$, conn. extrap.: up to 1 fm, 48^3 for $r > 1$ fm)
- I-DSDR ensembles: $L \to \infty$ ($b_2 = 0$)

$$
\begin{aligned}
a_\mu^{cHLbL} &= (27.61 \pm 3.51_{\text{stat}} \pm 0.32_{\text{sys},a^2}) \times 10^{-10} \\
a_\mu^{dHLbL} &= -20.20 \pm 5.65_{\text{stat}} \times 10^{-10} \\
a_\mu^{HLbL} &= 7.41 \pm 6.32_{\text{stat}} \pm 0.32_{\text{sys},a^2} \times 10^{-10}
\end{aligned}
$$

Outline I

Infinite volume QED_∞ [Green et al., 2015, Asmussen et al., 2016, Lehner and Izubuchi, 2015, Jin et al., 2015, Blum et al., 2017b]

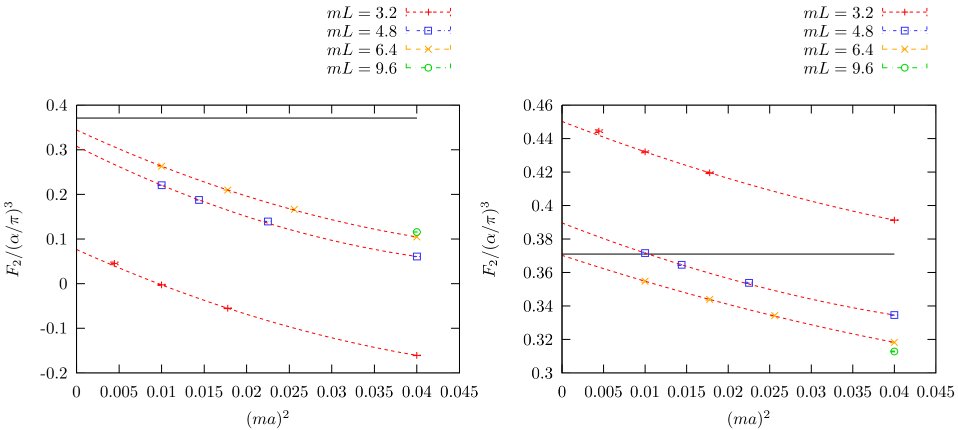
QCD in finite volume, QED in ∞ volume



$$= \quad \mathcal{Q}(x, y, z)$$

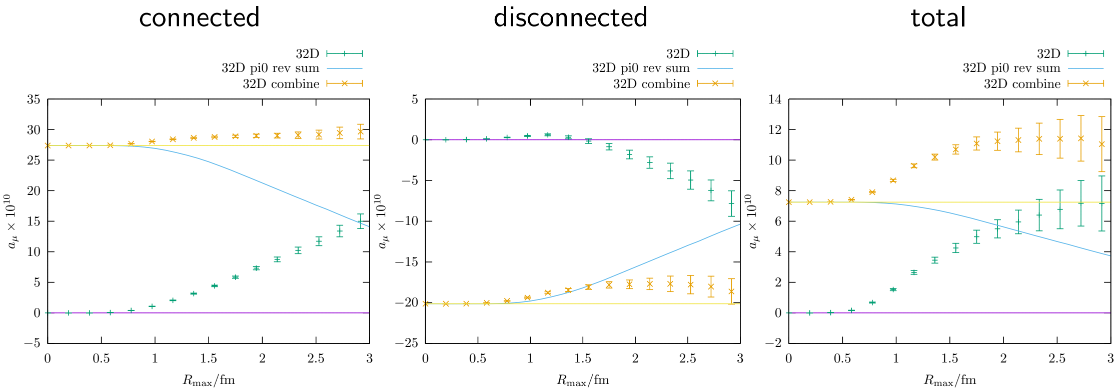
- Mainz group first proposed QED_∞ method
- QED_∞: muon, photons computed in infinite volume, continuum (*c.f.* HVP)
- QED "weight" function $\mathcal{Q}(x, y, z)$ pre-computed
- subtract terms that vanish as $a \to 0$, $L \to \infty$ to reduce $O(a^2)$ errors
- Leading FV error is exponentially suppressed (*c.f.* HVP) instead of $O(1/L^2)$
  - QCD mass gap: $\mathcal{H}(x, y, z, x_{\mathrm{op}}) \sim \exp -m_\pi \times \mathrm{dist}(x, y, z, x_{\mathrm{op}})$
  - QED weight function does not grow exponentially

QED_∞ test in pure QED [Blum et al., 2017b]



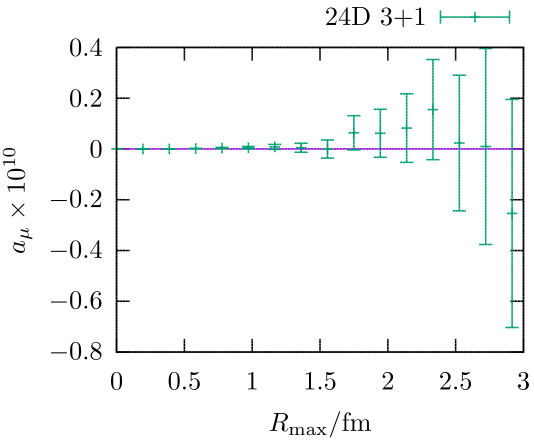
- subtraction terms (vanish when $a \to 0$) significantly reduce lattice spacing errors
- $F_2/(\alpha/\pi)^3 = 0.3686(37)(35)$ and $0.1232(30)(28)$ compared to
- QED perturbation theory results : 0.371 and 0.120

QED$_\infty$, 139 MeV pion, $a = 0.2$ fm, $L = 6.4$ fm (preliminary)



Combine full lattice result, up to R_{max}, with π^0 contribution from model or lattice from R_{max} to ∞ for most precise result (*c.f.*, QED$_L$ result)

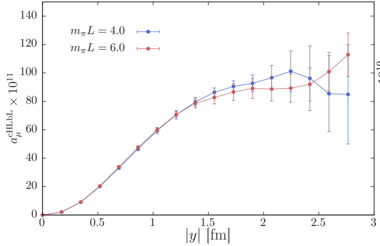
dHLbL, QED_∞ (non-leading diagram), $m_\pi = 139$ MeV, $a = 0.2$ fm (preliminary)

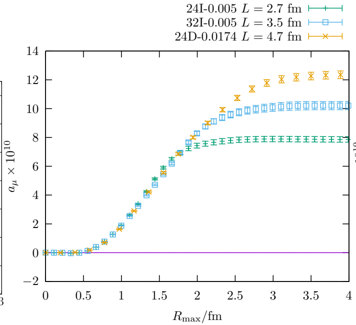


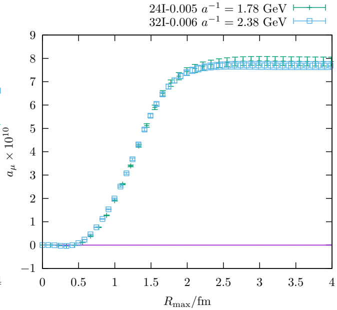
negligible contribution compared to error on leading contributions

Heavy pion mass, comparison with Mainz group

(Mainz)







$m_\pi \approx 340$ MeV, connected diagram only

a^2 errors small

Is $L = 4.7$ fm $\approx \infty$?

Need to compute disconnected diagram

Outline I

Hadronic light-by-light summary and outlook

- Lattice QCD+QED calculations done with physical masses, large boxes + improved measurement algorithms
- Physical point calculations published at $a = 0.114$ fm, 5.5 fm box [Blum et al., 2017a]
- Preliminary $a \to 0$, $L \to \infty$ limits taken in QED_L
  - connected, disconnected significant corrections
  - non-leading disconnected diagram makes small contribution
  - improving statistics
  - consistent with model, dispersive results.
- QED_∞: $a \to 0$, $L \to \infty$ (QCD) limits in progress
  - Combine with independent calculation of long distance π^0 contribution
  - a^2 and FV effects smaller than for QED_L
- Heavy pion mass comparison with Mainz
- Unlikely that HLbL contribution will rescue Standard Model

Muon g-2 Theory Initiative aims to have white paper this year, before E989 announces first results

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Outline I

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