Are dynamical charm quarks necessary?

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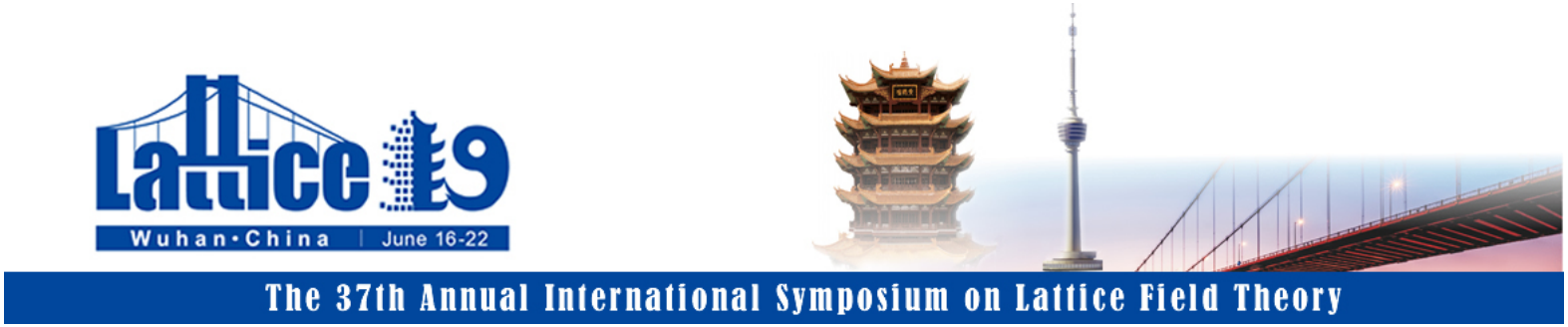
ALPHA Collaboration



Decoupling

Lattice QCD simulations often with $N_\mathrm{f} = 2 + 1$. Framework to understand the systematic errors:

Effective theory

Situation: QCD with N_f flavors out of which N_ℓ are light. Heavy quarks have RGI mass M.

- Fundamental theory: $\mathcal{L}_{\mathrm{QCD}_{N_\mathrm{f}}}$
  Parameters: coupling and masses
- Effective low energy theory: $\mathcal{L}_\mathrm{dec} = \mathcal{L}_{\mathrm{QCD}_{N_\ell}} + \frac{1}{M^2}\sum_i \omega_i \Phi_i + \ldots$
  Parameters: coupling, light masses, ω_i

  [T.Appelquist, J.Carazzone PRD11 (1975)], [S.Weinberg, Phys.Lett b91 (1980)]

Tuning of parameters: adjust parameters of $\mathcal{L}_\mathrm{dec}$ such that a set of low-energy quantities $\mathcal{S}$ of mass-dimension 1 are identical in the two theories. Then

$$\mathcal{S}^\mathrm{f} = \mathcal{S}^\ell \qquad + \qquad \underbrace{O(M^{-x})}_{x=2 \text{ for L.O. eff. theory}}$$

for all $\mathcal{S}$.

Model

Simplified setup

- Fundamental theory: QCD with two heavy quarks and no light quarks $N_\mathrm{f} = 2$, $N_\ell = 0$.
- Leading order effective theory: Yang-Mills theory

Advantages:

- No light pions $\Rightarrow$ no finite volume effects $\propto e^{-m_\pi L}$.
  $\Rightarrow$ comparatively small volumes are sufficient $L \approx 1.5$ fm
- Allows in turn to reach very fine lattice spacings
  $\Rightarrow$ small cutoff effects $\propto (aM)^2$
- Simulations in effective theory: almost for free (pure gauge)
- Two charm quarks $\Rightarrow$ $O(M^{-2})$ effects better visible

Disadvantage:

- Not the real world

Simulations

- Plaquette gauge action
- For $N_\mathrm{f} = 2$:
  - Doublet of $O(a)$ improved Wilson fermions
  - Or: Doublet of Wilson twisted-mass fermions
  - Quark masses $M \in \{M_c/8, M_c/4, M_c/2, M_c, 1.2M_c\}$
- Several lattice spacings $0.018\ \mathrm{fm} \leq a \leq 0.104\ \mathrm{fm}$
- Open boundaries in the time direction
- In pure gauge theory: Not many possibilities to choose $\mathcal{S}$ from. E.g. $1/\sqrt{t_0}$, $1/\sqrt{t_c}$, $1/w_0$, $1/r_0$, static force, glue-ball masses

  [M.Lüscher, JHEP 1008 (2010)], [S.Borsanyi et al. JHEP 09 (2012)], [R.Sommer, Nucl.Phys. B411 (1994)]
- Earlier results: Power-corrections almost impossible to resolve at $M \approx M_c$ Around 0.4% in gradient-flow quantities $\Rightarrow$ 0.2% with one charm quark

  [F.Knechtli, T.K., B.Leder, G.Moir, Phys.Lett. B774 (2017)]
- Here: study
  - perturbative vs non-perturbative decoupling
  - Partially quenched charm physics

Perturbative decoupling

$$\overline{g}^2_\ell(\mu/\Lambda_\ell) = \overline{g}^2_{\rm f}(\mu/\Lambda_{\rm f}) + c_1(\mu/\overline{m}(\mu))\,\overline{g}^4_{\rm f}(\mu/\Lambda_{\rm f}) + \ldots$$

- $\overline{m}(\mu)$ renormalized heavy quark mass $\Leftrightarrow M$
- Convenient choice of scheme and scale:
  $\overline{\rm MS}$-scheme with $\mu = m_*$ such that $\overline{m}(m_*) = m_*$
  - $c_1 = 0$
  - $log(\mu/\overline{m})$ vanishes
    $\Rightarrow c_2, \ldots, c_4$ are pure numbers
  - $c_2, \ldots, c_3$ known for $N_{\rm f} - N_\ell = 1, 2$
    c_4 known for $N_{\rm f} - N_\ell = 1$
    [K. Chetyrkin, J. Kühn, C. Sturm, Nucl. Phys. B744 (2006)]
    [Y. Schröder, M. Steinhauser, JHEP 01, 051 (2006)]

Equivalently: relation between Λ parameters

$$P_{\ell,{\rm f}}(M/\Lambda_{\rm f}) \equiv \Lambda_\ell/\Lambda_{\rm f}$$

But $\mu = \overline{m}_c$ is not a high scale. Does perturbation theory work well?

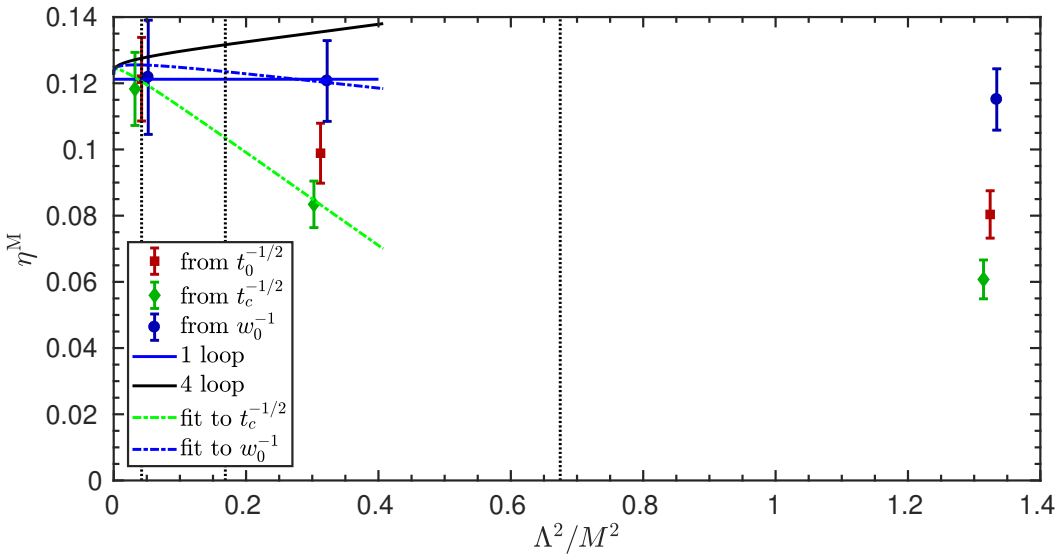
Mass scaling function

$\eta^{\mathrm{M}}(M) \equiv \frac{M}{P}\frac{\partial P}{\partial M} = \eta_0 + \eta_1 \overline{g}^2 + \ldots$ is accessible to NP study because

$$\frac{\mathcal{S}_{\mathrm{f}}(M)}{\mathcal{S}_{\mathrm{f}}(0)} = \underbrace{\frac{\mathcal{S}_\ell/\Lambda_\ell}{\mathcal{S}_{\mathrm{f}}(0)/\Lambda_{\mathrm{f}}}}_{\equiv Q^{\mathcal{S}}_{\ell,\mathrm{f}},\ \text{indep. of } M} \times \underbrace{P_{\ell,\mathrm{f}}(M/\Lambda_{\mathrm{f}})}_{\text{indep. of } \mathcal{S}} + O(M^{-2})$$

$$\Rightarrow \frac{M}{\mathcal{S}_{\mathrm{f}}}\frac{\partial \mathcal{S}_{\mathrm{f}}}{\partial M} = \eta^{\mathrm{M}} + O(M^{-2})$$

For any low energy scale $\mathcal{S}$!



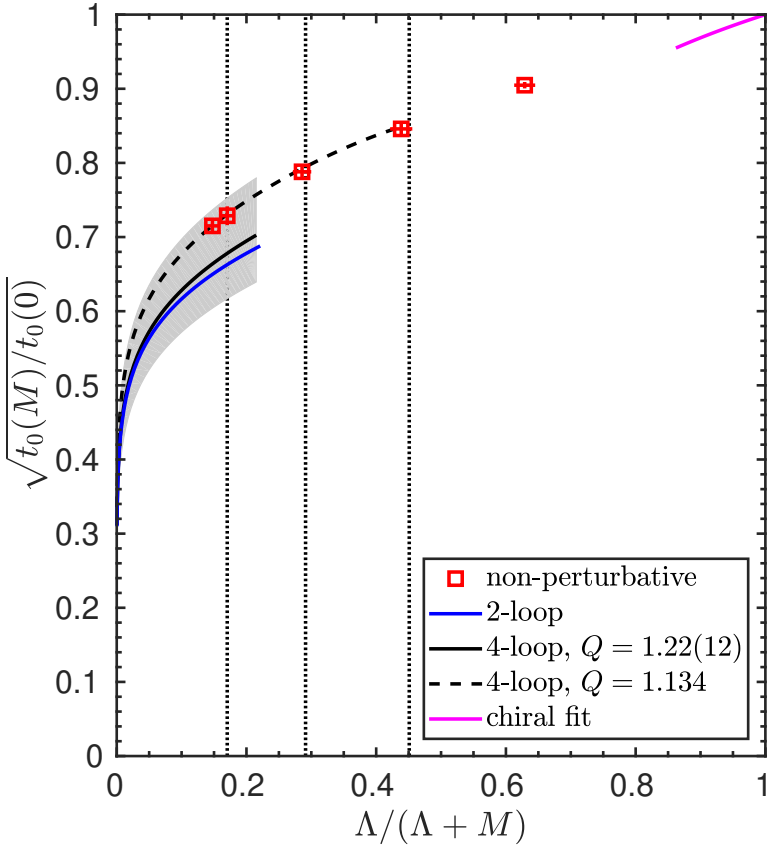
$\Rightarrow \Lambda_3/\Lambda_4$ from PT accurate to 1.5%

[A.Athenodoru et al. Nucl.Phys. B943 (2019)]

Factorization formula

$$\frac{\mathcal{S}_{\rm f}(M)}{\mathcal{S}_{\rm f}(0)} = Q^{\mathcal{S}}_{\ell,{\rm f}} \times P_{\ell,{\rm f}}(M/\Lambda_{\rm f}) \qquad + \qquad O(M^{-2})$$

- Apply to $\mathcal{S} = t_0^{-1/2}$
- P: perturbatively
- $Q = \frac{[\sqrt{t_0(0)}\Lambda]_{N_{\rm f}=2}}{[\sqrt{t_0}\Lambda]_{N_{\rm f}=0}}$
  - $[L_1\Lambda]_{N_{\rm f}=2} = 0.629(36)$
    [P.Fritzsch et al. Nucl.Phys. B865 (2012)]
  - $[r_0\Lambda]_{N_{\rm f}=0} = 0.602(48)$
    [S.Capitani, M.Lüscher, R. Sommer, H.Wittig, Nucl.Phys B544 (1999)]
  - $\left[\sqrt{t_0(0)}/L_1\right]_{N_{\rm f}=2} = 0.3881(52)$
  - $\left[\sqrt{t_0}/r_0\right]_{N_{\rm f}=0} = 0.3319(19)$
    [A.Athenodoru et al. Nucl.Phys. B943 (2019)]
- Error dominated by Λ_2
- Determine Λ from mass dependence of $\mathcal{S}$?
  $\Rightarrow$ A.Ramos Today, 15:20, Shimao 3B



[A.Athenodoru et al. Nucl.Phys. B943 (2019)]

Meson correlators

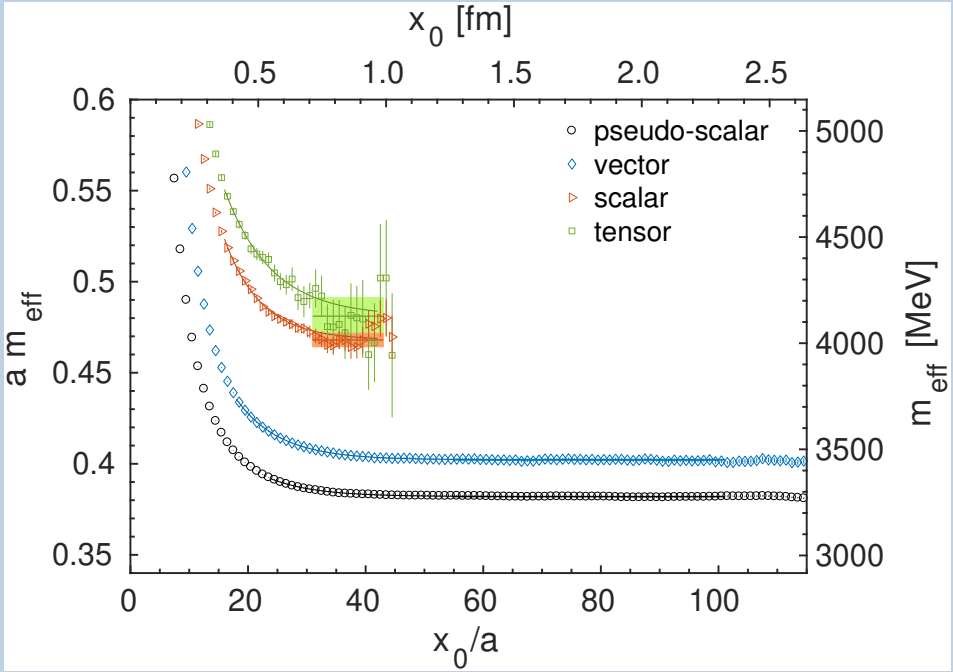
Decoupling works reasonably well at $M = M_c$. But what if we are interested in charm-physics? Observables with charm valence quarks!

Meson 2pt functions

$$f_{\mathcal{O}_1\mathcal{O}_2}(x_0, y_0) = a^6 \sum_{\mathbf{x},\mathbf{y}} \langle \mathcal{O}_1(x)\mathcal{O}_2^\dagger(y)\rangle \,,$$

- $\mathcal{O} \in \{P, S, A_\mu, V_\mu, T_{\mu\nu}\}$
- Symmetrize, e.g.: $\bar{f}_{PP}(x_0 - a) \equiv \frac{1}{2}\left(f_{PP}(x_0, a) + f_{PP}(T - x_0, T - a)\right)$
- $am^{\mathrm{eff}}(x_0 + a/2) = \ln\left(\frac{f(x_0)}{f(x_0+a)}\right)$

on finest $N_\mathrm{f} = 2$ ensemble:



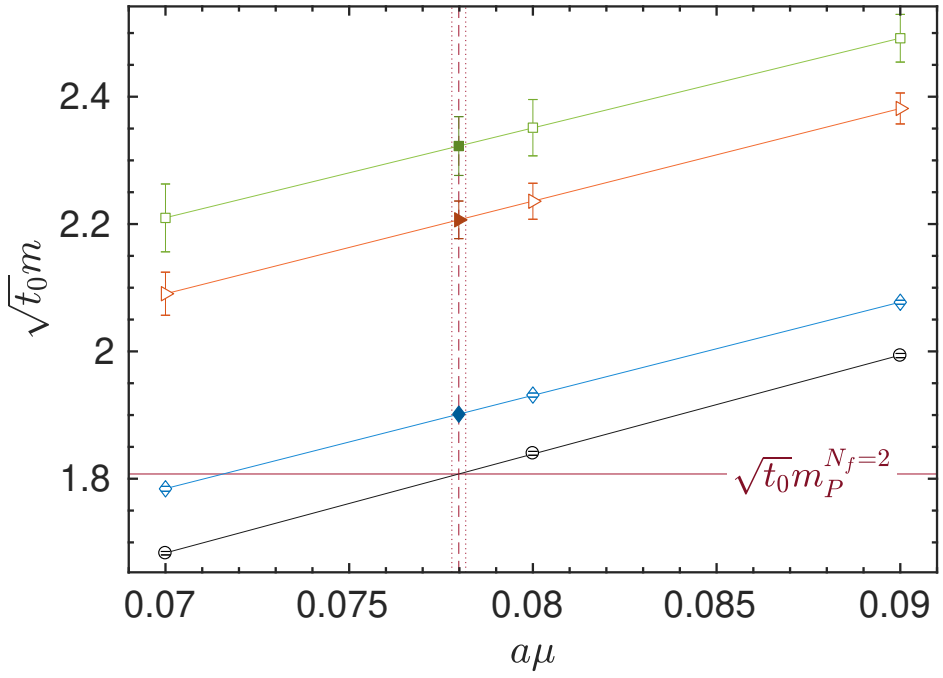
Mis-tuning corrections

In principle: twisted mass parameter μ can be computed for given M/Λ:

$$a\mu = \frac{M_c}{\Lambda_{\overline{\mathrm{MS}}}} \times Z_P \times \frac{\overline{m}_c}{M_c} \times \Lambda_{\overline{\mathrm{MS}}} L_1 \times \frac{a}{L_1} \,,$$

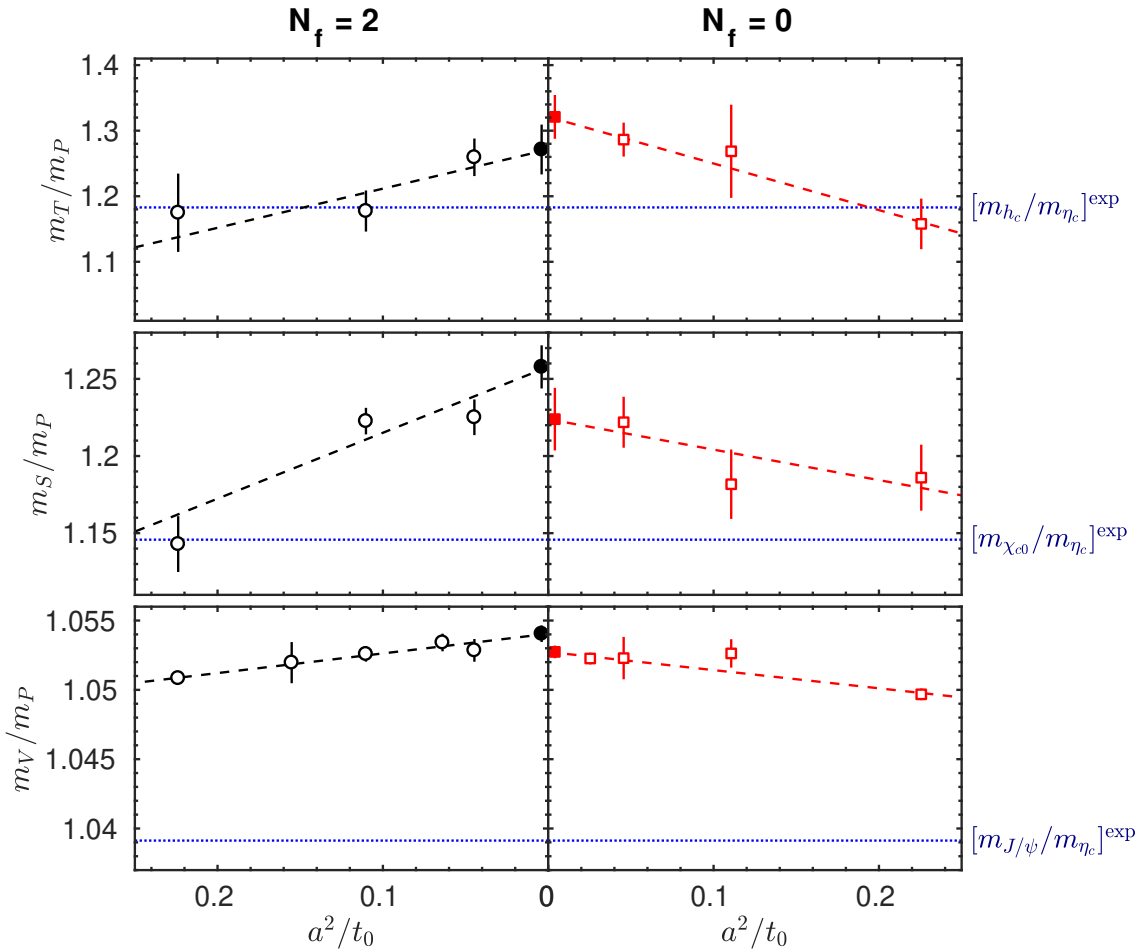
But $\approx 10\%$ errors!
Better: tune $a\mu$ to find $\sqrt{t_0} m_P = 1.807463 \Leftrightarrow M_c$ on finest ensemble

- $N_\mathrm{f} = 2$:
  Compute $\mathrm{d}/\mathrm{d}\mu$ of all observables.
  Correct:
  $\mu^\star = \mu + (\sqrt{t_0} m_P - 1.807463) \left(\frac{\mathrm{d}\sqrt{t_0} m_P}{\mathrm{d}\mu} \right)^{-1}$
  Other quantities Φ
  $\Phi(\mu^\star) = \Phi(\mu) + (\mu^\star - \mu) \frac{\mathrm{d}\Phi}{\mathrm{d}\mu}$
- $N_\mathrm{f} = 0$:
  3 valence masses + interpolation



Meson masses

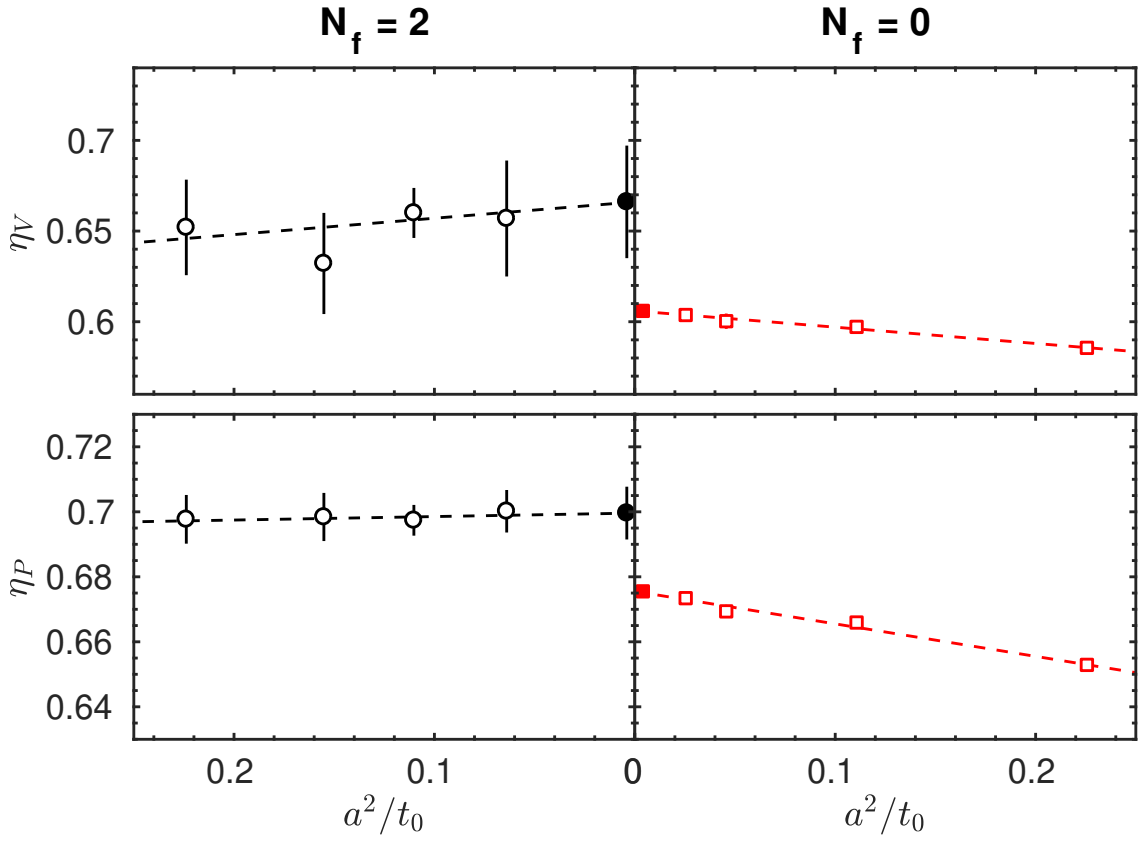
- All masses corrected to $\mu^\star$, where $\sqrt{t_0} m_P = 1.807463$
- No significant charm-loop effects on the spectrum
- $(X_{N_f=2} - X_{N_f=0})/X_{N_f=2}$
  m_V/m_P: 0.12(7) % effect
  m_S/m_P: 2.7(1.9)% effect
- Despite the simplified model:
  quite close to nature!



[S.Calì, F.Knechtli, T.K., arXiv:1905.12971]

Mass scaling functions

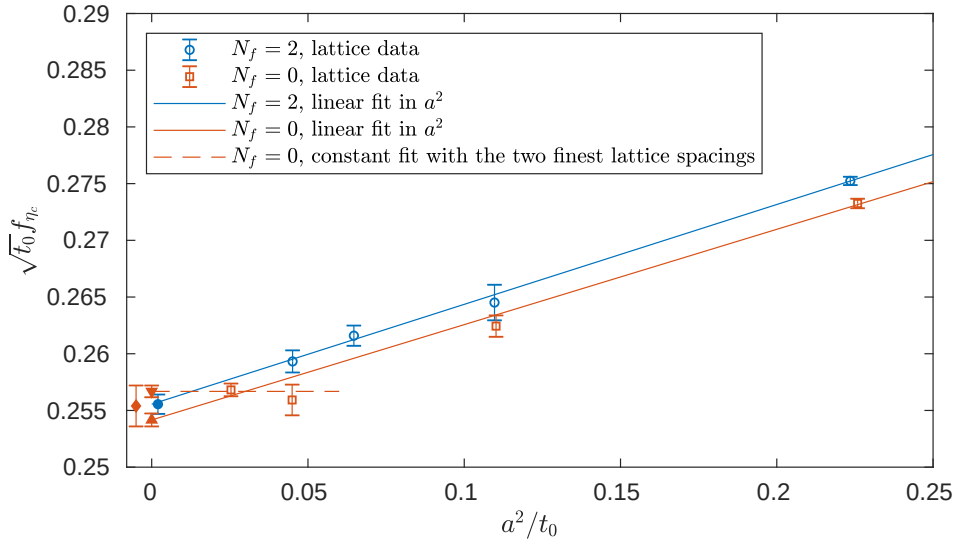
- Generalization of η^M
  $\eta_X = \frac{M}{m_X}\frac{\partial m_X}{\partial M}$
  for meson mass m_X
  $\sim$ sigma-terms
- In μ-derivatives
  $\frac{\mathrm{d}\langle A\rangle}{\mathrm{d}\mu} =$
  $-\left\langle \frac{\mathrm{d}S}{\mathrm{d}\mu} A\right\rangle + \left\langle \frac{\mathrm{d}S}{\mathrm{d}\mu}\right\rangle \langle A\rangle$
  $+\left\langle \frac{\mathrm{d}A}{\mathrm{d}\mu}\right\rangle$.
  No $\mathrm{d}S/\mathrm{d}\mu$ terms in
  $N_\mathrm{f} = 0$.
- Effect in η_P: 3.4(1.1) %
  Effect in η_V: 9.0(4.2) %

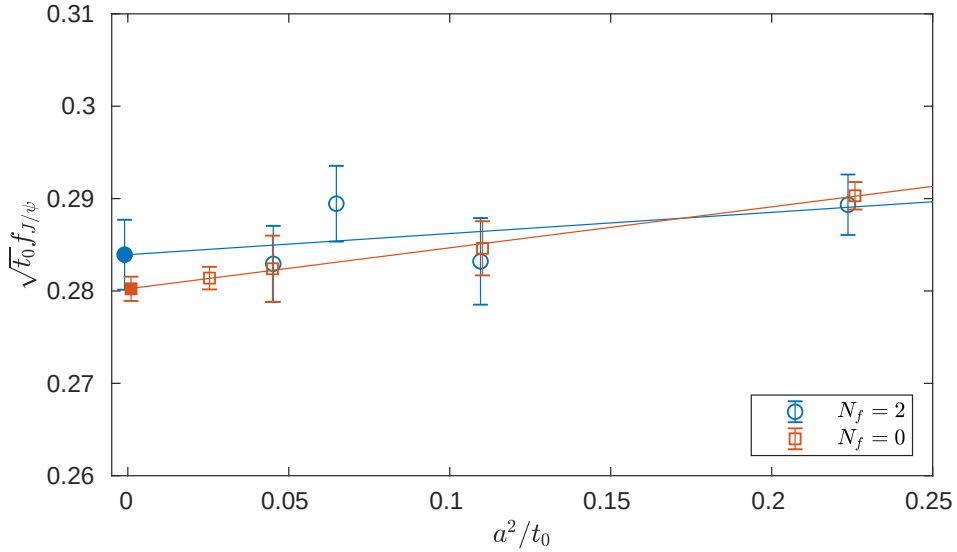


[S.Calì, F.Knechtli, T.K., arXiv:1905.12971]

Decay constants (preliminary)

- In twisted mass QCD
  - $f_{\eta_c} = \frac{2\mu\langle 0|\hat{P}^+|\eta_c\rangle}{m_{\eta_c}^2}$
  - $f_{J/\psi} = \frac{\langle 0|\hat{A}_k^+|J/\psi\rangle}{m_{J/\psi}}$
- With open b.c.
  $\langle 0|\hat{X}|\sigma\rangle \propto \sqrt{\frac{|f_X(x_0,y_0)f_X(T-x_0,y_0)|}{f_X(T-y_0,y_0)}}$
- $y_0 = a$: boundary $\rightarrow$ boundary correlator noisy and inaccurate
  - Increase y_0
  - Use distance-preconditioning
- Experimentally
  - f_{η_c} not measured
  - $f_{J/\psi} = 405(6)$ MeV
    [Donald et al. arXiv:1208.2855]



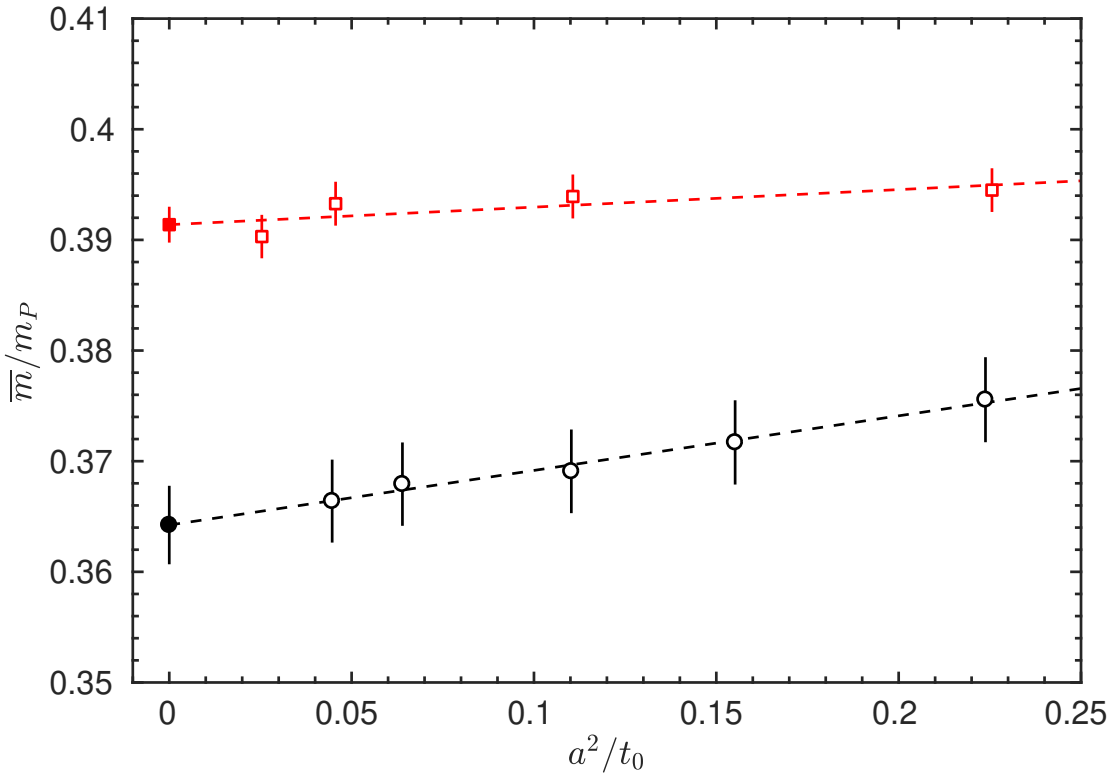


Quark mass

$\overline{m} = Z_P^{-1}\mu$

- $N_\mathrm{f} = 2$ (black)
  - Z_P and
  - $M/\overline{m} = 1.308(16)$
    [P.Fritzsch et al. Nucl.Phys.B865 (2012)]
  - $M_c/m_P = 0.4764(74)$
- $N_\mathrm{f} = 0$ (red)
  - Z_P and
  - $M/\overline{m} = 1.157(12)$
    [A.Jüttner, PhD thesis (2004)]
  - $M_c/m_P = 0.4528(51)$

$$\frac{M^{N_\mathrm{f}=2} - M^{N_\mathrm{f}=0}}{M^{N_\mathrm{f}=2}} = 5.0(1.8)\%$$



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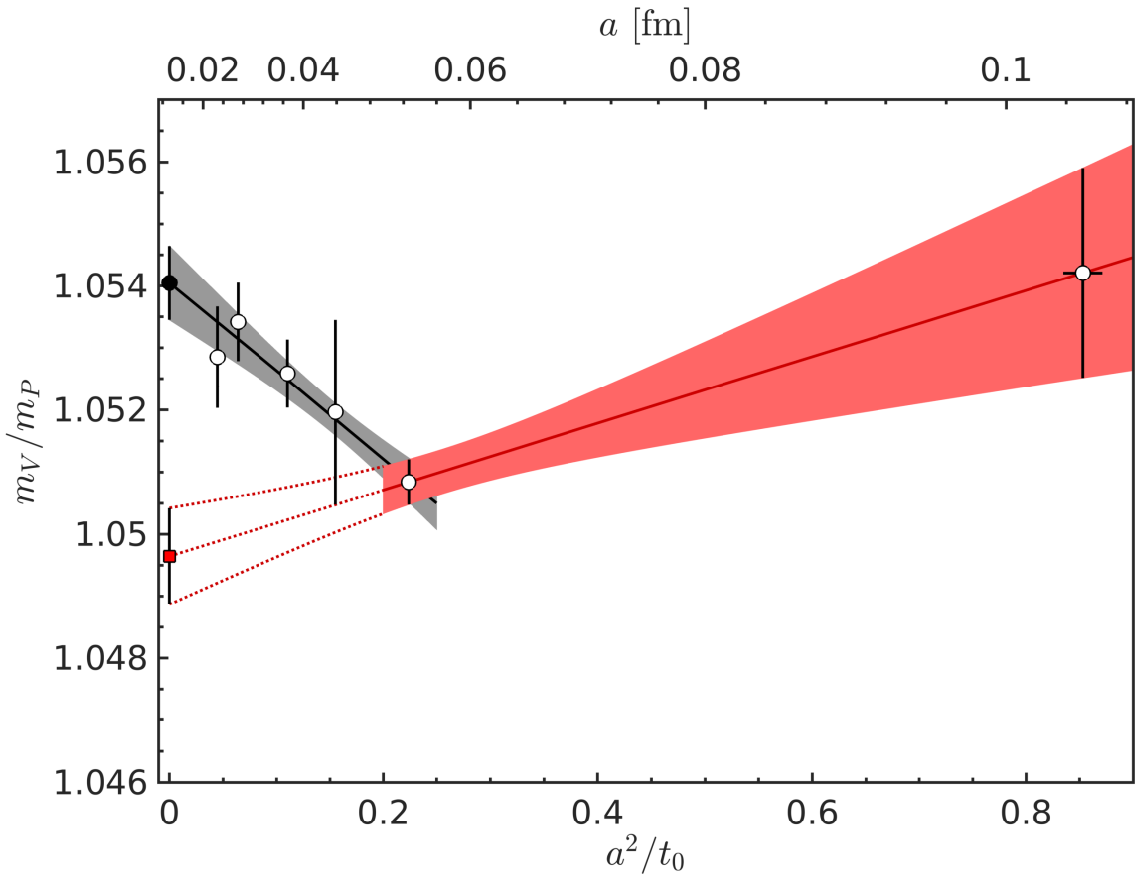
Lattice artifacts

- Without $a < 0.05$ fm: wrong result!
- Also a has lattice artifacts
  - $L_1 = 0.40(1)$ fm
    $L_1/a \Rightarrow a^{L_1}$
  - $t_0^{-1/2} = 0.113(4)$ fm
    $t_0/a^2 \Rightarrow a^{t_0}$

Ensemble	a^{L_1} [fm]	a^{t_0} [fm]
E	0.066	0.104
N	0.049	0.054
O	0.042	0.045
P	0.036	0.038
S	0.028	0.029
W	0.023	0.024
qN	-	0.054
qP	-	0.038
qW	-	0.024
qX	-	0.018



Conclusions and outlook

Conclusions

- Decoupling
  - Power corrections: tiny
  - Perturbative decoupling: OK for Λ at the 1.5% level
- Dynamical charm quark effects in quantities with valence charm quarks
  - Up to 5% effects in charm physics
  - Potentially large cutoff effects

Outlook

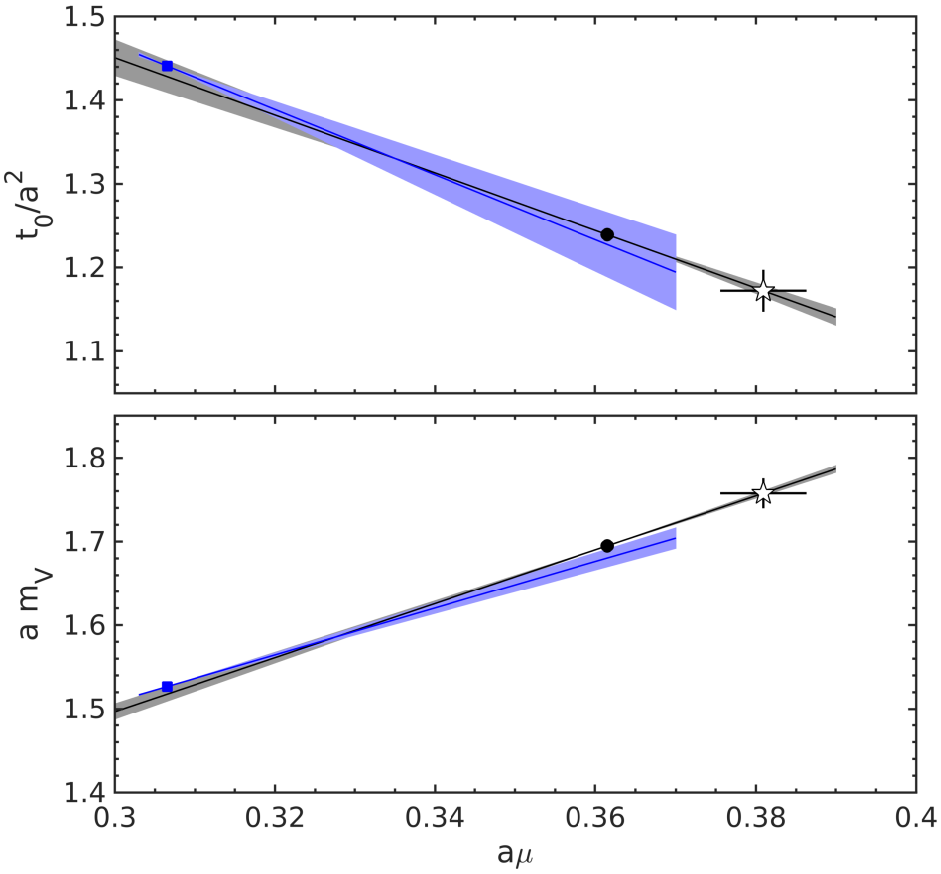
- More on decay constants
- B_c
- Iso-scalars, disconnected diagrams
- Λ from decoupling

Mass derivatives

Derivatives with respect to bare twisted mass parameter μ

$$\frac{\mathrm{d}\langle A\rangle}{\mathrm{d}\mu} = -\left\langle \frac{\mathrm{d}S}{\mathrm{d}\mu} A \right\rangle + \left\langle \frac{\mathrm{d}S}{\mathrm{d}\mu} \right\rangle \langle A\rangle + \left\langle \frac{\mathrm{d}A}{\mathrm{d}\mu} \right\rangle .$$

$$\frac{\mathrm{d}f(\langle A_1\rangle, \ldots, \langle A_N\rangle, \mu)}{\mathrm{d}\mu} = \frac{\partial f}{\partial \mu} + \sum_{i=1}^{N} \frac{\partial f}{\partial \langle A_i\rangle} \frac{\mathrm{d}\langle A_i\rangle}{\mathrm{d}\mu}.$$



Heavy quarks