Nucleon and Omega Masses
with
All HISQ Fermions

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Collaborators

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- Alexei Strelchenko (Fermilab)
- Fermilab Lattice Collaboration
- MILC Collaboration
Outline

The importance of baryon masses

Interpolators and simulation details

Preliminary data and results
  - Nucleon correlators
  - Omega baryon correlators

Conclusion
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Why Nucleon and Omega Masses?

\[
M_{\text{proton}} = 938.272081 \pm 0.000006 \text{ MeV}
\]

\[
M_{\Omega^-} = 1672.45 \pm 0.29 \text{ MeV}
\]

~10^{-7}\% uncertainty!  ~0.017\% uncertainty!

[2019 Particle Data Group]
Why Nucleon and Omega Masses?

- Nucleon mass as the first step towards nucleon matrix element (neutrino physics)
- Omega baryon mass for absolute lattice scale setting (precision physics such as muon HVP)
- Omega baryon as a test bed for staggered fermion formalism
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Simulation Details

<table>
<thead>
<tr>
<th>Set</th>
<th>( \beta )</th>
<th>( a ) (fm)</th>
<th>( a m_l )</th>
<th>( a m_s )</th>
<th>( a m_c )</th>
<th>( N_s \times N_T )</th>
<th>( n_{cfg} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.8</td>
<td>0.1529(4)</td>
<td>0.002426</td>
<td>0.06730</td>
<td>0.8447</td>
<td>32 ( \times ) 48</td>
<td>3500</td>
</tr>
<tr>
<td>2</td>
<td>6.0</td>
<td>0.1222(3)</td>
<td>0.001907</td>
<td>0.05252</td>
<td>0.6382</td>
<td>48 ( \times ) 64</td>
<td>1000</td>
</tr>
<tr>
<td>3</td>
<td>6.3</td>
<td>0.0879(3)</td>
<td>0.001200</td>
<td>0.03630</td>
<td>0.4320</td>
<td>64 ( \times ) 96</td>
<td>1047</td>
</tr>
</tbody>
</table>

[MILC collaboration, arxiv: 1212.4768]

- 2+1+1 HISQ ensembles at **physical pion masses** generated by the MILC collaboration for both valence and sea quarks
- Coulomb gauge fixed
- Wall sources
**Staggered Nucleon Operators**

Finite Lattice symmetry: $SU_I(2) \times GTS$

<table>
<thead>
<tr>
<th>GTS Irrep</th>
<th>$I = 3/2$</th>
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</tr>
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<tbody>
<tr>
<td>8</td>
<td>$3N + 2\Delta$</td>
<td>$5N + 1\Delta$</td>
</tr>
<tr>
<td>8'</td>
<td>$0N + 2\Delta$</td>
<td>$0N + 1\Delta$</td>
</tr>
<tr>
<td>16</td>
<td><strong>$1N + 3\Delta$</strong></td>
<td>$3N + 4\Delta$</td>
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[Jon A. Bailey, hep-lat/0611023]

Use 16 irrep with isospin 3/2 to extract nucleon mass without any taste complications
## Staggered Omega Baryon Operators

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[Jon A. Bailey, hep-lat/0611023]

\[\Omega: \frac{3}{2}^+ \ (J^P), \ sss, \cdots\]

\[N_s: \frac{1}{2}^+ \ (J^P), \ sss^*, \cdots\]

Use 8’ irrep with “isospin” 1/2 to extract omega baryon mass without any taste complications
### Staggered Omega Baryon Operators

For a given operator irrep and isospin, the total number of operators used is equal to the total number of states.

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[Jon A. Bailey, hep-lat/0611023]

**Question:** Can we resolve different baryon tastes?
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Nucleon Correlators

- Perform GEVP on the three by three correlator matrix for 16 irrep, isospin 3/2

- We solve the equation

\[
\frac{1}{4} \left\{ \begin{array}{l}
[C(t_0 - 1)]^{-1}C(t - 1) + 2[C(t_0)]^{-1}C(t) \\
+ [C(t_0 + 1)]^{-1}C(t + 1)
\end{array} \right\} \tilde{v}_R^1(t) = \tilde{\lambda}_1(t, t_0)\tilde{v}_R^1(t)
\]

- Perform unconstrained plateau fits (plus an excited state exponential term) to the eigenvalues

- Bayesian fits gave the same posterior masses
Nucleon Correlator: 0.15fm

\[ aM_N = 0.7556(59)_{\text{fit}}(22)_{\text{stat}} \]
Nucleon Correlator: 0.12fm

\[ aM_N = 0.5946(22)_{\text{fit}}(48)_{\text{stat}} \]
Nucleon Correlator: 0.09fm

\[ aM_N = 0.4295(8)_{\text{fit}}^{(26)}_{\text{stat}} \]
Nucleon Continuum Extrapolation

We perform Bayesian fit to \( \Lambda_{QCD} = 500 \text{ MeV} \)

\[
M_N(a) = M_{N,\text{phy}} \left\{ 1 + o_2 \left( \frac{\Lambda_{QCD} a}{500 \text{ MeV}} \right)^2 + o_4 \left( \frac{\Lambda_{QCD} a}{500 \text{ MeV}} \right)^4 \right\}
\]

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<th>Parameter</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
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<tr>
<td>(M_{N,\text{phy}} \text{ [MeV]})</td>
<td>940(50)</td>
<td>957(10)</td>
</tr>
<tr>
<td>(o_2)</td>
<td>unconstrained</td>
<td>0.07(22)</td>
</tr>
<tr>
<td>(o_4)</td>
<td>0.0(1.0)</td>
<td>0.15(96)</td>
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Nucleon Continuum Extrapolation

\[ M_N = 958(13) \text{ MeV} \]
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Omega Baryon Correlators

- Only one correlator for 8’ irrep, “isospin” 3/2 operator
- Two types of effective masses

\[ M_{\text{raw}} \equiv \frac{1}{2} \ln \left( \frac{C(t)}{C(t+2)} \right) \]

\[ M_{\text{smoothed}} \equiv \frac{1}{4} \left( M_{\text{eff}}(t-1) + 2M_{\text{eff}}(t) + M_{\text{eff}}(t+1) \right) \]

[Carleton DeTar and Song-Haeng Lee, arxiv: 1411.4676]

- We perform Bayesian fits (only set priors on the masses) to the single correlator with two even and one odd parity states
$aM_\Omega = 1.296(3)_{stat}$
$a M_\Omega = 1.038(3)_{\text{stat}}$
Omega Baryon Correlator: 0.09fm

\[ aM_\Omega = 0.748(2)_{\text{stat}} \]
We perform the same Bayesian fits as nucleon using the lattice spacings determined by the gradient flow, $w_0$.

We also illustrate the potential of using omega baryon to set the scale by plotting $w_0M_\Omega$ and compare it to the current value of

$$w_{0,phy} = 0.1715(9) \text{ fm}, \quad w_0M_\Omega = 1.454(8)$$

[HPQCD Collaboration, arxiv:1303.1670]

[MILC Collaboration, arxiv:1503.02769]
Omega Continuum Extrapolation

\[ M_\Omega = 1677(14)_{\text{stat}} \text{ MeV} \]
Scale Setting with $w_0$ and $M_\Omega$
Lastly, we are interested in the taste-breaking pattern for staggered baryons.

We will solve the same GEVP equation as nucleon for 16 irrep with “isospin” 3/2.

Recall the spectrum:

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Tastes of Omega Baryon: 0.15 fm

-\ln(\bar{\lambda}/(t - t_0))

four \( \Omega \) states

three \( N_s \) states
In Conclusion…

- We have successfully extract both nucleon and omega baryon masses and extrapolate them continuum.

- We have demonstrated the potential of using omega baryon to set the scale.

- We might be able to extract the masses of all entangled tastes if we use all operators. But this needs further studies.