The $\rho$-resonance from Lattice QCD
with $N_f = 2 + 1 + 1$ dynamical quark flavors

M. Werner  C. Helmes  C. Jost  B. Kostrzewa  C. Liu  L. Liu  B. Metsch
M. Petschlies  M. Ueding  C. Urbach

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Motivation

- Investigate $\pi\pi$ scattering in $l = 1, \ell = 1$
- Experimentally well-known
- Test-bed for elusive resonances
- Phase shift from Lüscher method
- Here: Chiral and continuum extrapolation

[Protopopescu et al. 1973, PRD 7, 1279]
ETMC ensembles
Baron et al. 2010 [1004.5284]; Baron et al. 2011 [1005.2042]

- 13 ensembles
- Different $M_\pi$, $L$
- Three $\beta$ values
- Iwasaki gauge
- $N_f = 2 + 1 + 1$ twisted mass fermions
  - Automatic $O(a)$ improvement [Frezzotti & Rossi 2004]
  - Isospin broken at $O(a)$ Potential $2\pi_0$ pollution, not visible
Spectrum extraction

Correlator matrix
- $\pi(d - q)\pi(q)$ with $i\gamma_5$
- $\rho(d)$ with $i\gamma_i$ and $\gamma_0\gamma_i$

Using multiple
- relative integer momenta $q$
- total momenta $p = 2\pi d/L$
- irreps $\Gamma$

$\frac{4}{\pi} \leq d^2 \leq 4$  
$q^2 \leq 4$, depending on $d^2$

Energy extraction
- Thermal state removal via *weight and shift*  
- Also without, estimates systematic error
- GEVP with up to $6 \times 6$
- Fitting principal correlators with one range by hand
Spectrum on one ensemble (A40.32)

![Graph showing spectrum with and without thermal state removal.](image)
Zeta function singularities

Use Lüscher formalism to convert energies $E_{\text{CM}}$ to phase shifts $\delta_1$

- Lüscher’s Zeta function has singularities at non-interacting energies
- Resampling distributions might cross them
- Crossing is unphysical, prohibits usage of energy level
- Jackknife distribution narrower than bootstrap distribution
- Still need to check for singularity crossing
Breit-Wigner fit to phase shifts
Brown & Goble, PRL 20, 346 (1968)

One fit per ensemble

Phase shift:

\[
\tan \delta_1 = \frac{g_{\rho \pi \pi}^2}{6\pi} \frac{p^3(E_{\text{CM}})}{E_{\text{CM}}(M_\rho^2 - E_{\text{CM}}^2)}, \quad p(E_{\text{CM}}) = \sqrt{E_{\text{CM}}^2/4 - M_\pi^2}
\]

Mass dependent width:

\[
\Gamma_\rho = \frac{2}{3} \frac{g_{\rho \pi \pi}^2}{4} \frac{p^3(M_\rho)}{M_\rho^2}
\]

Fit parameters: \( g_{\rho \pi \pi} \) and \( M_\rho \)
Phase shift on one ensemble
Chiral extrapolation

Djukanovic et al. 2009 [0902.4347]; Djukanovic et al. 2009 [1001.1772]

Combined fit to $M_\rho$ and $\Gamma_\rho$ with

$$Z = (M_\rho - i \Gamma_\rho / 2)^2$$

Fit model from EFT with vector meson dominance:

$$a^2 Z = \frac{p_{r_0/a}^{-2}}{a} \left( (p_1 + i p_2) + p_3 \left( \frac{p_{r_0/a} a M_\pi}{a M_\pi} \right)^2 - p_4 \sqrt{p_1 + i p_2} \left( \frac{p_{r_0/a} a M_\pi}{a M_\pi} \right)^3 \right.$$

$$\left. + (p_5 + i p_6) \frac{p_{r_0/a}^{-2}}{a} \right)$$

- $p_1 + i p_2$ is $\rho$ pole in chiral limit
- $p_4$ comes from $\omega \rho \pi$ coupling
- $p_{r_0/a}$ Sommer parameter, prior not shown
- Last summand is complex lattice artifact
Chiral extrapolation

Djukanovic et al. 2009 [0902.4347]; Djukanovic et al. 2009 [1001.1772]
Results & discussion

▶ Our result:

\[ M_\rho = 769(19) \text{ MeV}, \quad \Gamma_\rho = 129(7) \text{ MeV} \]

▶ Experimental result [PDG 2018]:

\[ M_\rho = 775.26(25) \text{ MeV}, \quad \Gamma_\rho = 149.1(8) \text{ MeV} \]

▶ Lattice artifact not resolvable

▶ Systematic effects:
  ▶ Long extrapolation (lowest \( M_\pi = 280 \text{ MeV} \))
  ▶ Perhaps parametrization of Breit-Wigner introduces underestimated width?
Our phase shift with experiment

Protopopescu et al. 1973, PRD 7, 1279

\[ \delta_1 \text{[rad]} \]

Protopopescu et al.

\[ M_\pi = 135 \text{ MeV} \]
\[ M_\pi = 220 \text{ MeV} \]
\[ M_\pi = 305 \text{ MeV} \]
\[ M_\pi = 390 \text{ MeV} \]
Comparison to other lattice publications

\begin{itemize}
  \item Alexandrou et al.
  \item Andersen et al.
  \item Experiment
  \item Fu et al. (MILC)
  \item HadSpec
  \item PACS-CS
  \item This work
\end{itemize}
Summary & Outlook

Summary:

- Chiral and continuum extrapolation of $\rho$ mass and width
- Lattice artifacts not resolvable
- Mass agrees with experiment, width two standard deviations too low

Outlook:

- Global fit from unitarized $\chi$ PT:
  IAM phase shift fit at NLO and NNLO
- Ensemble with physical $M_\pi$
- Including higher partial waves
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Relative momentum

\[ k = \frac{E_{CM}^2}{4} - M^2_{\pi} \]

\[ w_{lm}(q, \gamma) = \frac{1}{\pi^{3/2} \sqrt{2l + 1}\gamma^{-1}q^{-l-1} \mathcal{O}_{lm}(1, q^2)}, \quad q = k \frac{L}{2\pi} \]

Singularities of \( w_{lm}(q, \gamma) \) at non-interacting energy levels
Singularity structure in Lüscher’s Zeta function \((d^2 = 1)\)

\[
\text{acot}(\text{Re}(w_{00}))
\]
Singularity crossing with bootstrap samples

- **Black** bootstrap distribution
- **Red** central value
- **Green** singularity crossing
- **Gray** central $M_\pi$

- Bootstrap distribution is wide
- Crossing unphysical
- Use jackknife