The ρ-resonance from Lattice QCD with $N_f = 2 + 1 + 1$ dynamical quark flavors

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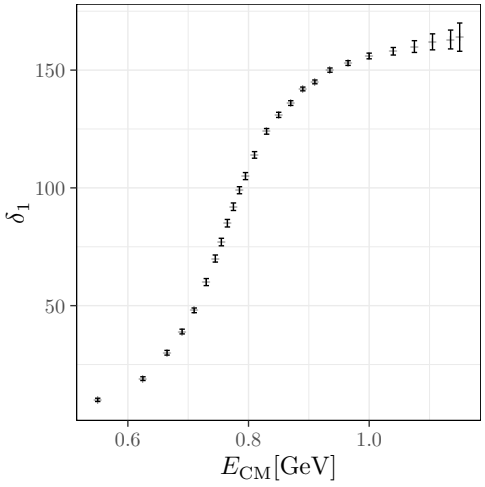
Motivation

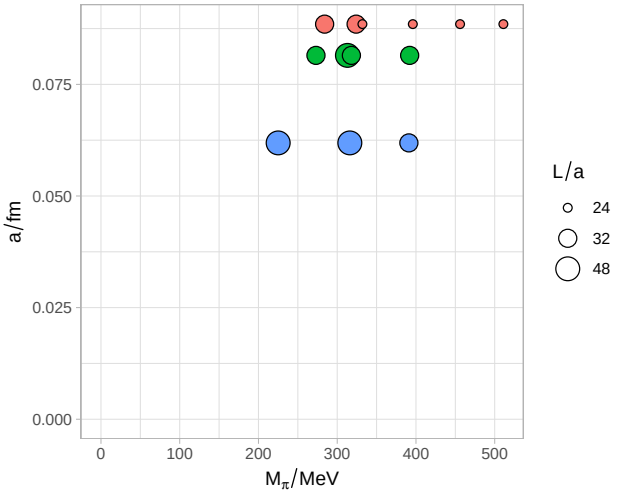


[Protopopescu et al. 1973, PRD 7, 1279]

- Investigate $\pi\pi$ scattering in $I = 1$, $\ell = 1$
- Experimentally well-known
- Test-bed for elusive resonances
- Phase shift from Lüscher method
- Extensively studied in LQCD:
  Feng et al. 2011, Lang et al. 2011, Dudek et al. 2013, RQCD 2015, Fu & Wang 2016, Guo et al. 2013, Alexandru et al. 2017, Alexandru et al. 2019, Andersen et al. 2019
- Here: Chiral and continuum extrapolation

ETMC ensembles

Baron et al. 2010 [1004.5284]; Baron et al. 2011 [1005.2042]



- 13 ensembles
- Different M_π, L
- Three β values
- Iwasaki gauge
- $N_f = 2 + 1 + 1$ twisted mass fermions
  - Automatic $O(a)$ improvement [Frezzotti & Rossi 2004]
  - Isospin broken at $O(a)$ Potential $2\pi_0$ pollution, not visible

Spectrum extraction

Correlator matrix

- $\pi(\boldsymbol{d}-\boldsymbol{q})\,\pi(\boldsymbol{q})$ with $\mathrm{i}\gamma_5$
- $\rho(\boldsymbol{d})$ with $\mathrm{i}\gamma_i$ and $\gamma_0\gamma_i$

Using multiple

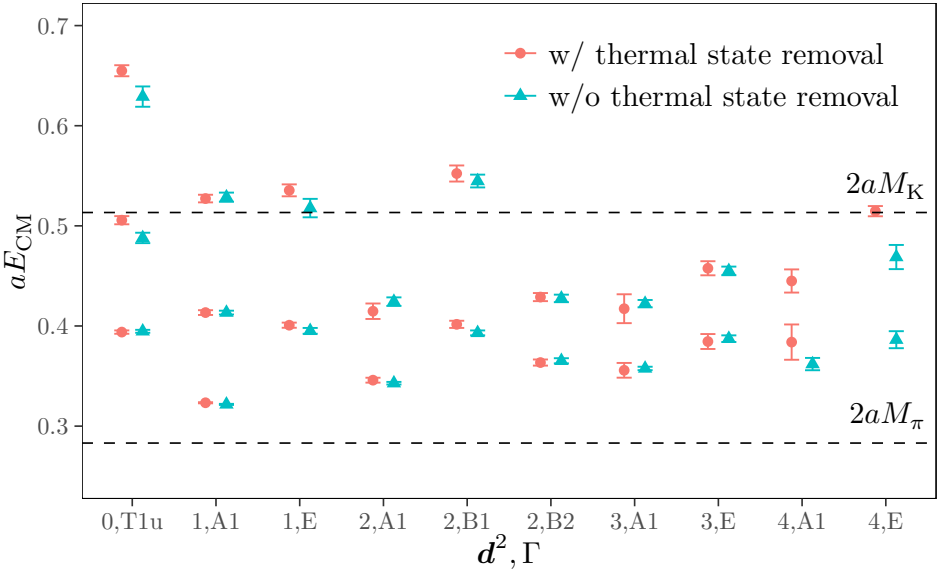
- relative integer momenta $\boldsymbol{q}$
- total momenta $\boldsymbol{p} = 2\pi\boldsymbol{d}/L$
- irreps Γ

$\boldsymbol{d}^2 \leq 4$
$\boldsymbol{q}^2 \leq 4$, depending on $\boldsymbol{d}^2$

Energy extraction

- Thermal state removal via *weight and shift* [Dudek et al. 2012, Phys. Rev. D 86, 034031]
- Also without, estimates systematic error
- GEVP with up to 6×6
- Fitting principal correlators with one range by hand

Spectrum on one ensemble (A40.32)



Zeta function singularities

Use Lüscher formalism to convert energies E_{CM} to phase shifts δ_1

- Lüscher's Zeta function has singularities at non-interacting energies
- Resampling distributions might cross them
- Crossing is unphysical, prohibits usage of energy level
- Jackknife distribution narrower than bootstrap distribution
- Still need to check for singularity crossing

Breit-Wigner fit to phase shifts

Brown & Goble, PRL 20, 346 (1968)

One fit per ensemble

Phase shift:

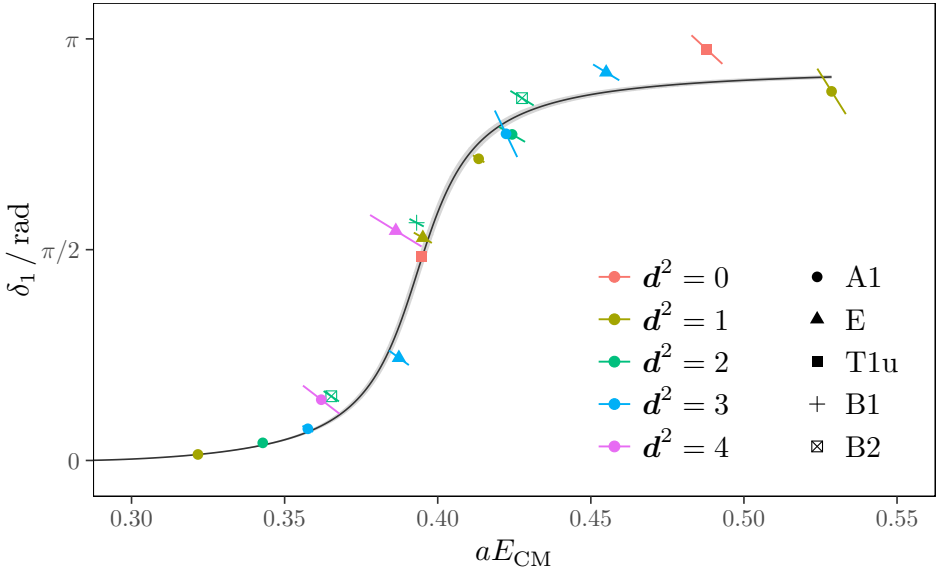
$$\tan\delta_1 = \frac{g_{\rho\pi\pi}{}^2}{6\pi}\frac{p^3(E_{\mathrm{CM}})}{E_{\mathrm{CM}}(M_\rho{}^2 - E_{\mathrm{CM}}^2)}, \qquad p(E_{\mathrm{CM}}) = \sqrt{E_{\mathrm{CM}}^2/4 - M_\pi^2}$$

Mass dependent width:

$$\Gamma_\rho = \frac{2}{3}\frac{g_{\rho\pi\pi}{}^2}{4}\frac{p^3(M_\rho)}{M_\rho{}^2}$$

Fit parameters: $g_{\rho\pi\pi}$ and M_ρ

Phase shift on one ensemble



Chiral extrapolation

Djukanovic et al. 2009 [0902.4347]; Djukanovic et al. 2009 [1001.1772]

Combined fit to M_ρ and Γ_ρ with $Z = (M_\rho - \mathrm{i}\Gamma_\rho/2)^2$

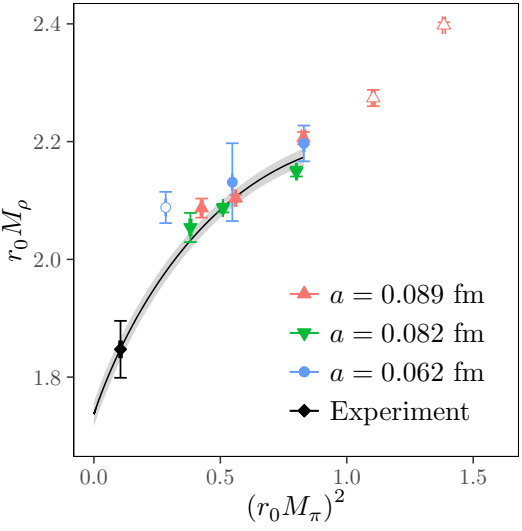
Fit model from EFT with vector meson dominance:

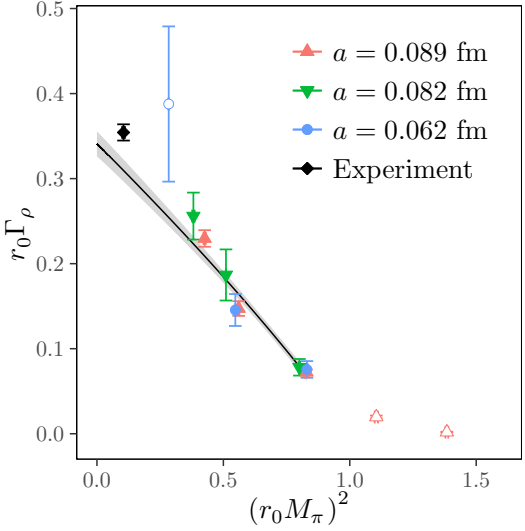
$$a^2 Z = p_{r_0/a}^{-2}\Big((p_1 + ip_2) + p_3 \left(p_{r_0/a} a M_\pi\right)^2 - p_4 \sqrt{p_1 + ip_2}\left(p_{r_0/a} a M_\pi\right)^3 + (p_5 + ip_6)\, p_{r_0/a}^{-2} \Big)$$

- $p_1 + \mathrm{i}p_2$ is ρ pole in chiral limit
- p_4 comes from $\omega\rho\pi$ coupling
- $p_{r_0/a}$ Sommer parameter, prior not shown
- Last summand is complex lattice artifact

Chiral extrapolation

Djukanovic et al. 2009 [0902.4347]; Djukanovic et al. 2009 [1001.1772]





Results & discussion

- Our result:

$$M_\rho = 769(19)\,\text{MeV}, \quad \Gamma_\rho = 129(7)\,\text{MeV}$$

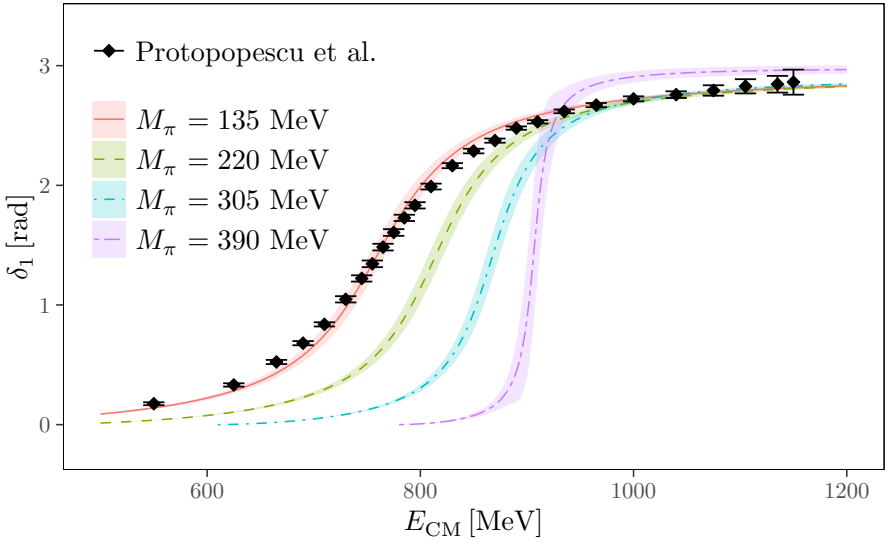
- Experimental result [PDG 2018]:

$$M_\rho = 775.26(25)\,\text{MeV}, \quad \Gamma_\rho = 149.1(8)\,\text{MeV}$$

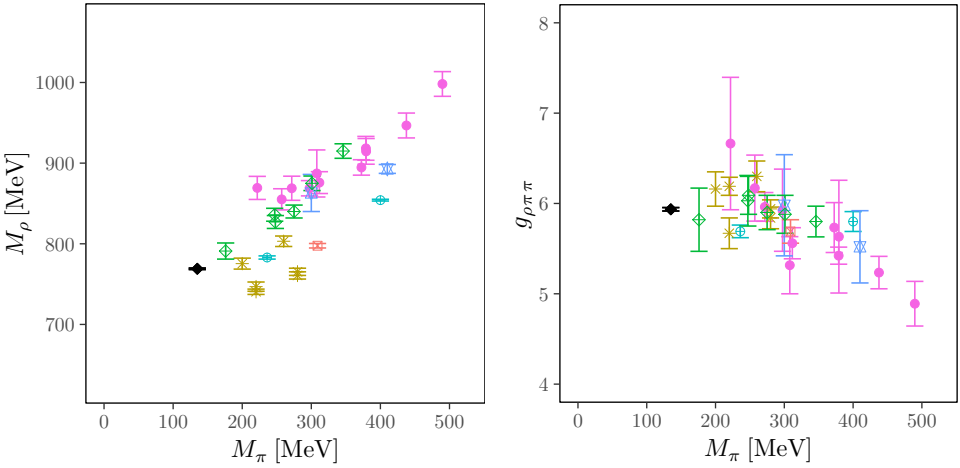
- Lattice artifact not resolvable
- Systematic effects:
  - Long extrapolation (lowest $M_\pi = 280\,\text{MeV}$)
  - Perhaps parametrization of Breit-Wigner introduces underestimated width?

Our phase shift with experiment

Protopopescu et al. 1973, PRD 7, 1279



Comparison to other lattice publications



Summary & Outlook

Summary:

- Chiral and continuum extrapolation of ρ mass and width
- Lattice artifacts not resolvable
- Mass agrees with experiment, width two standard deviations too low

Outlook:

- Global fit from unitarized χPT:
  IAM phase shift fit at NLO and NNLO
- Ensemble with physical M_π
- Including higher partial waves

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Energies to phase shifts

Göckeler et al. 2012 [1206.4141]

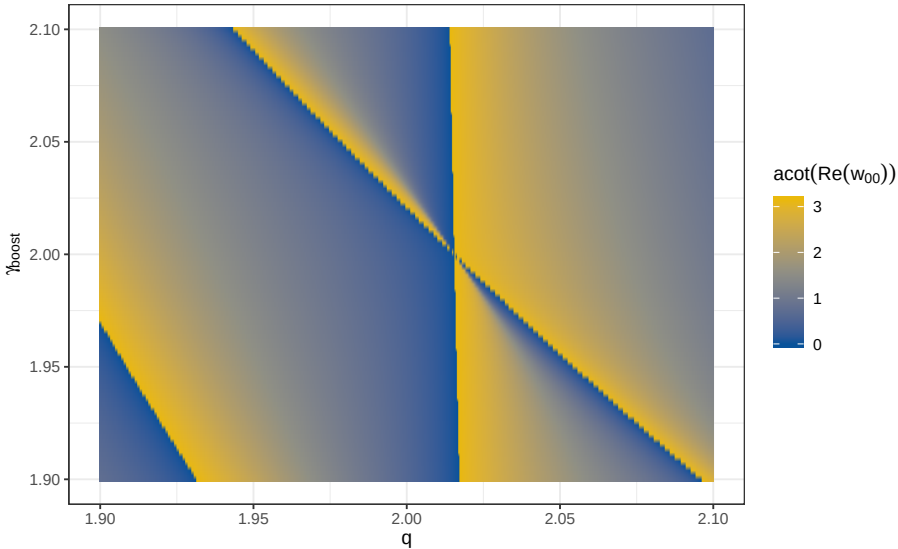
Relative momentum

$$k = \frac{E_{\mathrm{CM}}^2}{4} - M_\pi^2$$

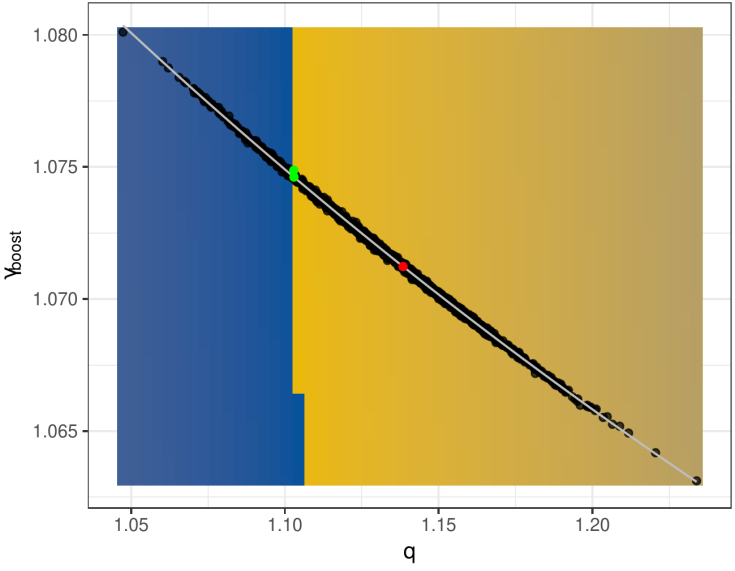
$$w_{lm}(q,\gamma) = \frac{1}{\pi^{3/2}\sqrt{2l+1}\gamma^{-1}q^{-l-1}\mathcal{Z}_{lm}(1,q^2)}, \quad q = k\frac{L}{2\pi}$$

Singularities of $w_{lm}(q,\gamma)$ at non-interacting energy levels

Singularity structure in Lüscher's Zeta function ($d^2 = 1$)



Singularity crossing with bootstrap samples



black	bootstrap distribution
red	central value
green	singularity crossing
gray	central M_π

- Bootstrap distribution is wide
- Crossing unphysical
- Use jackknife