Real-Time-Evolution of Heavy-Quarkonium Bound States

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Heavy Quarkonia in Heavy Ion Collisions

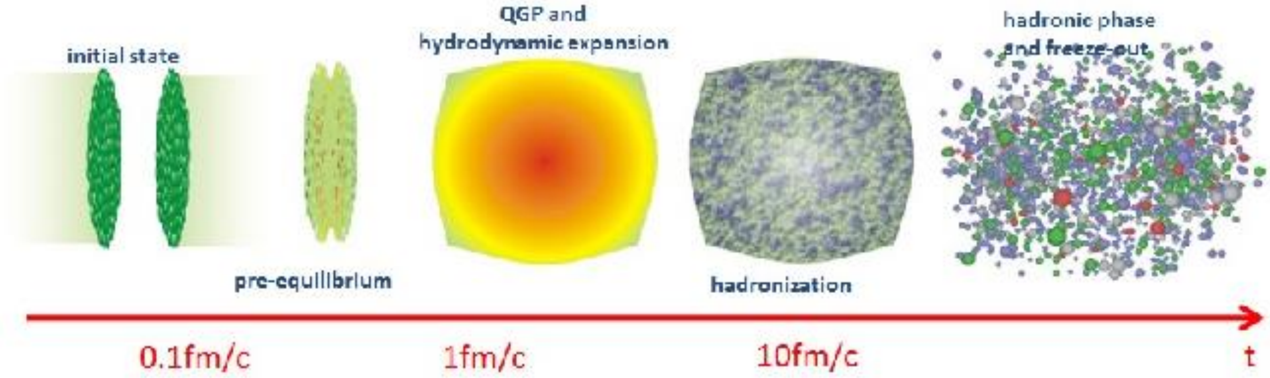

illustration by
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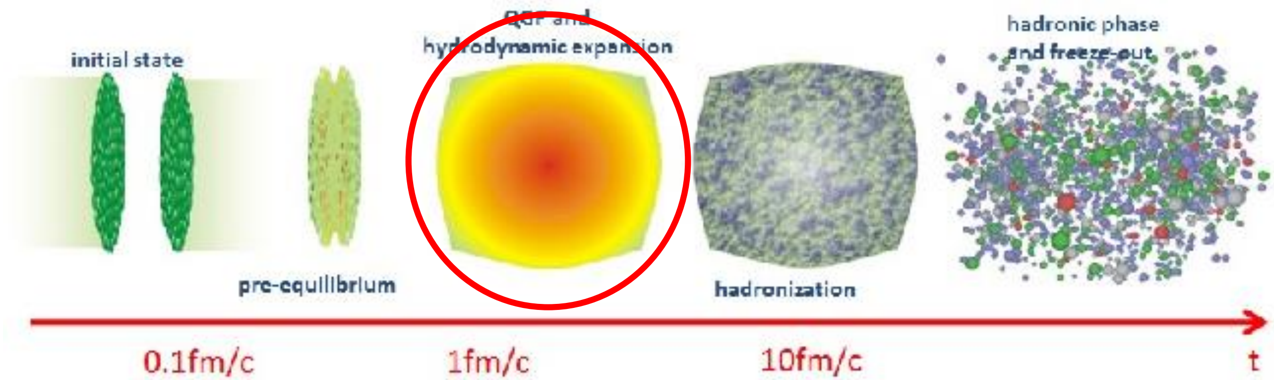

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- Heavy Quarkonia (Charmonium, Bottomonium) are **well controlled** experimental and theoretical **probes** for the **quark-gluon-plasma**
- Phenomenological models describe quarkonium supression via a Schrödinger equation + **assumption** of **early formation** of bound states

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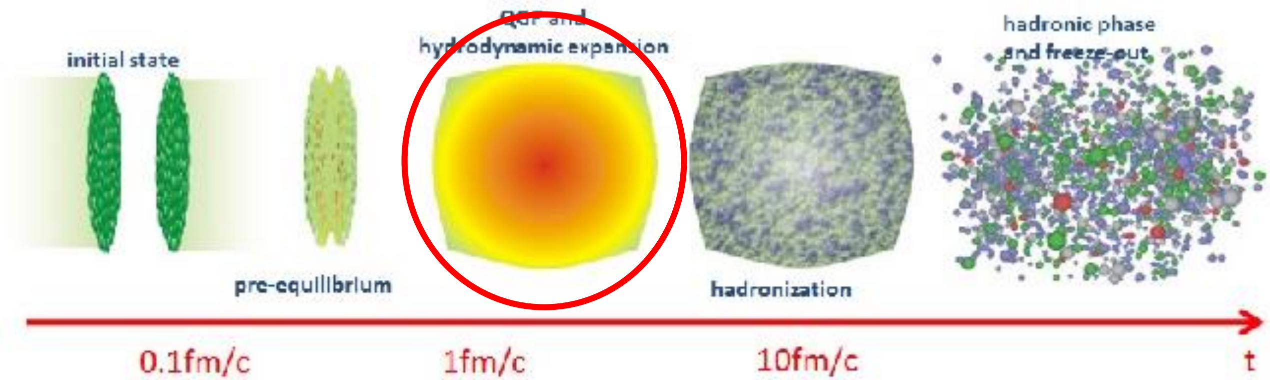

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- Phenomenological models describe quarkonium supression via a Schrödinger equation + **assumption** of **early formation** of bound states
- Characterized by a **separation of scale**:

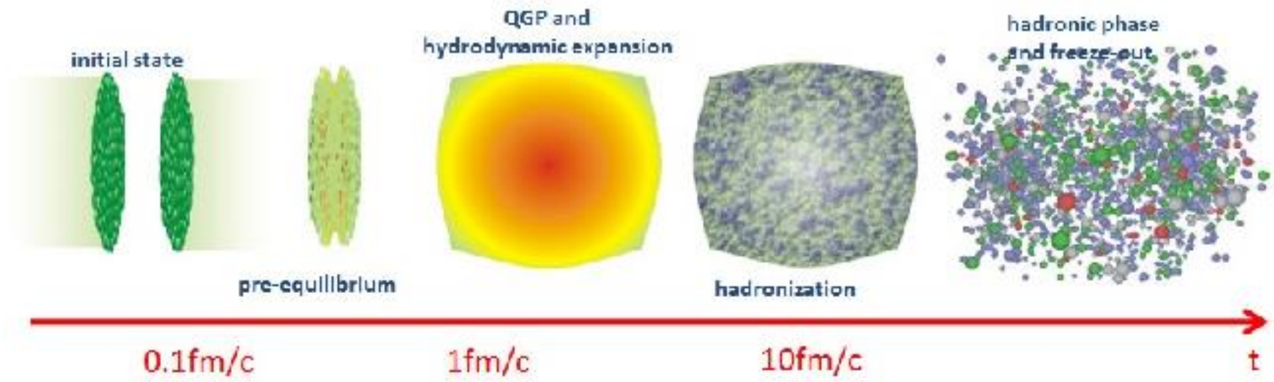
$$M_Q \gg M_Q v \gg M_Q v^2 \gg \Lambda_{\mathrm{QCD}}$$

- Very heavy states, e.g. $\Upsilon(1S)$, already bound **Coulombically**

M_Q … heavy quark mass ($m_{Bottom} = 4.18(3)$GeV [PDG 2017])
v … relative velocity in centre of mass frame ($v^2_{Bottom} \approx 0.1$)
Λ_{QCD} … momentum scale below which gluons strongly interacting

Mv … typical momentum
Mv^2 … typical kinetic or potential energy

Heavy Quarkonia in Early Stages in HICs



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Early dynamics of heavy quarkonium in HIC largely **unexplored**

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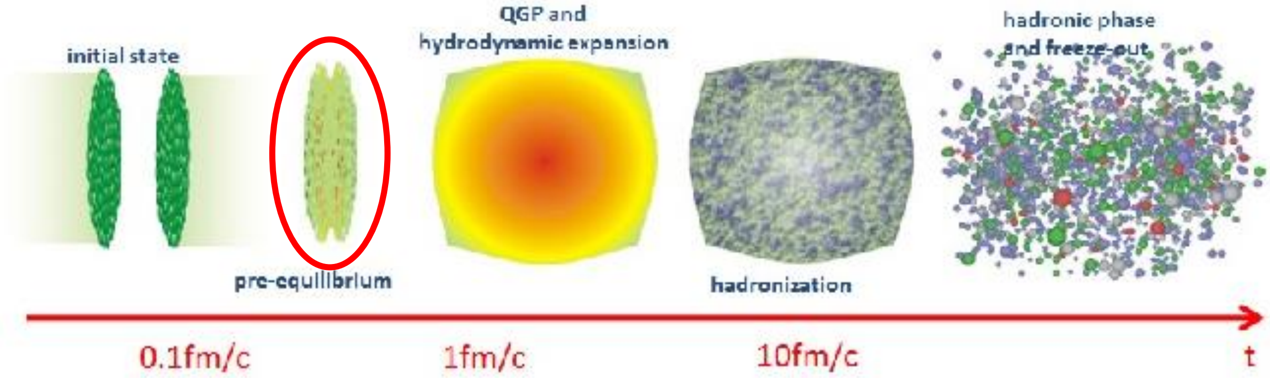

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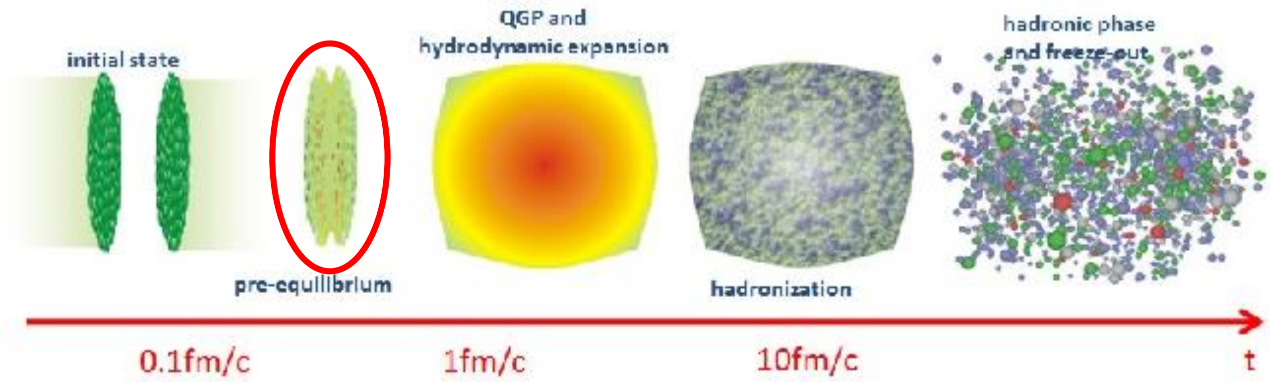

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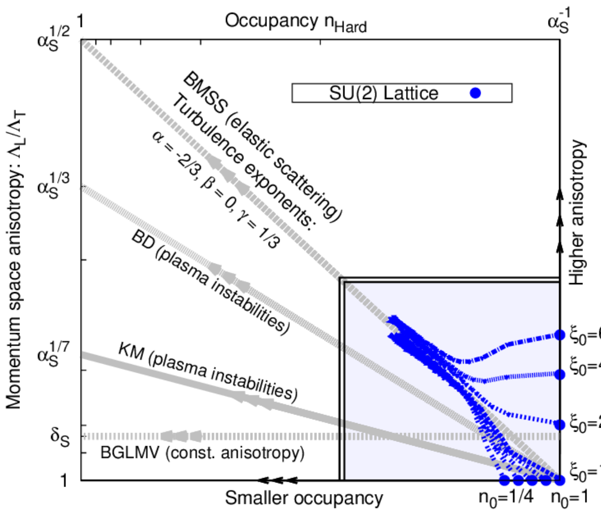
Can we find hints for heavy-quarkonium formation in the glasma?

Real-Time Evolution of the Gauge Fields

- Vital insight into glasma dynamics via **classical statistical simulations** of gauge fields in expanding geometry



J. Berges, K. Boguslavski, S. Schlichting, R. Venugopalan, PRL. 114 (2015) 061601

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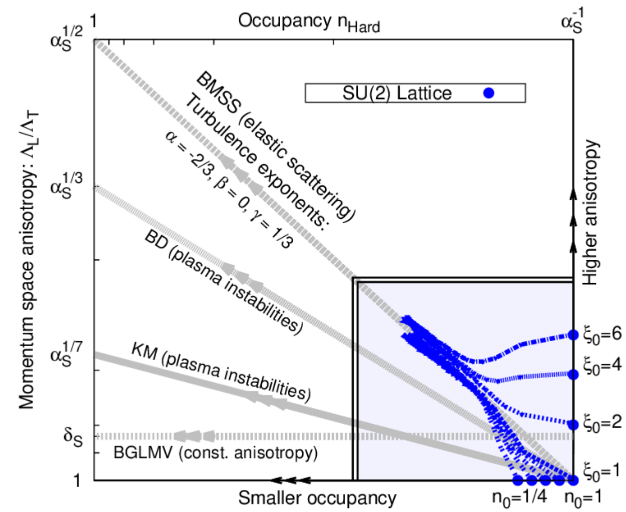

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$$\partial_t U_j(x,t) = iE_j^a U_j(x,t) \qquad \partial_t E_j^a(x,t) = -2\mathrm{ImTr}\left\{T^a \sum_{j\neq k}\left[U_{ij}(t,x) + U_{i(-j)}(t,x)\right]\right\}$$

$$U_j(x) = \exp\left(ia_j A_j^a T^a\right) \quad E_j^a = F_{0j}^a = a_0 a_j 2ImTr[T^a U_{0j}]$$

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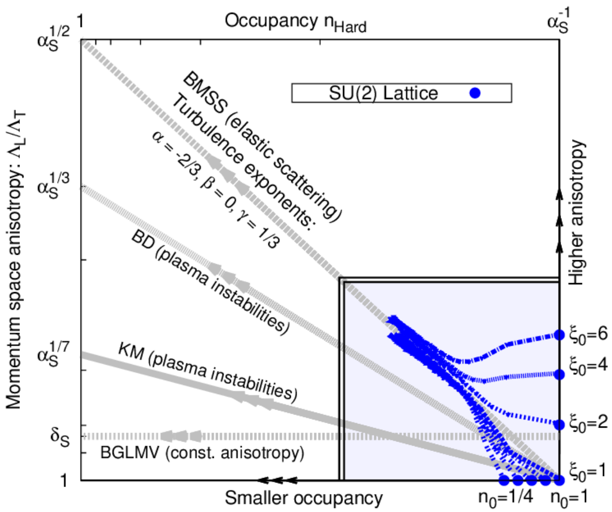

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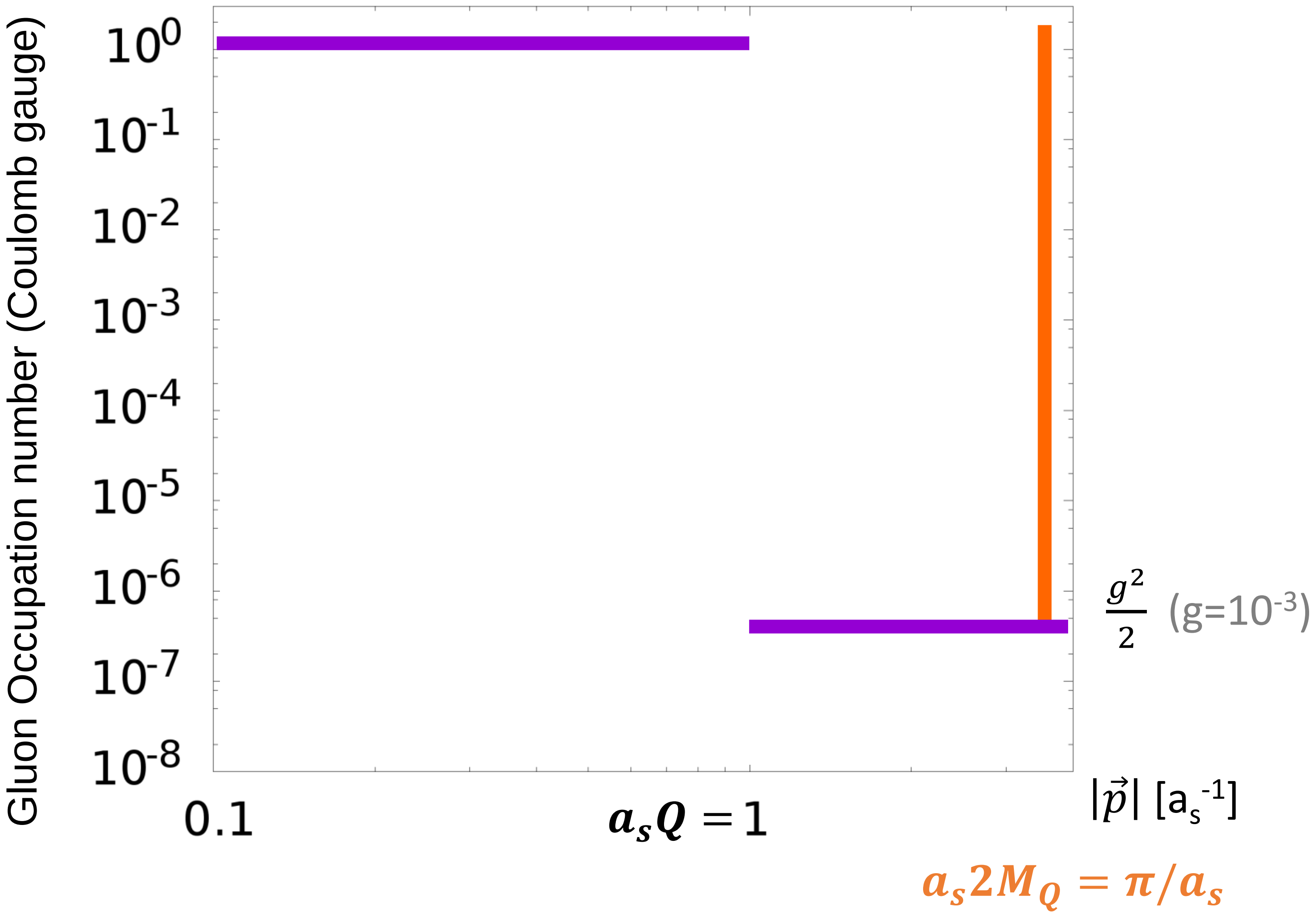
- Initial conditions drawn from a statistical ensemble

The real-time NRQCD setup



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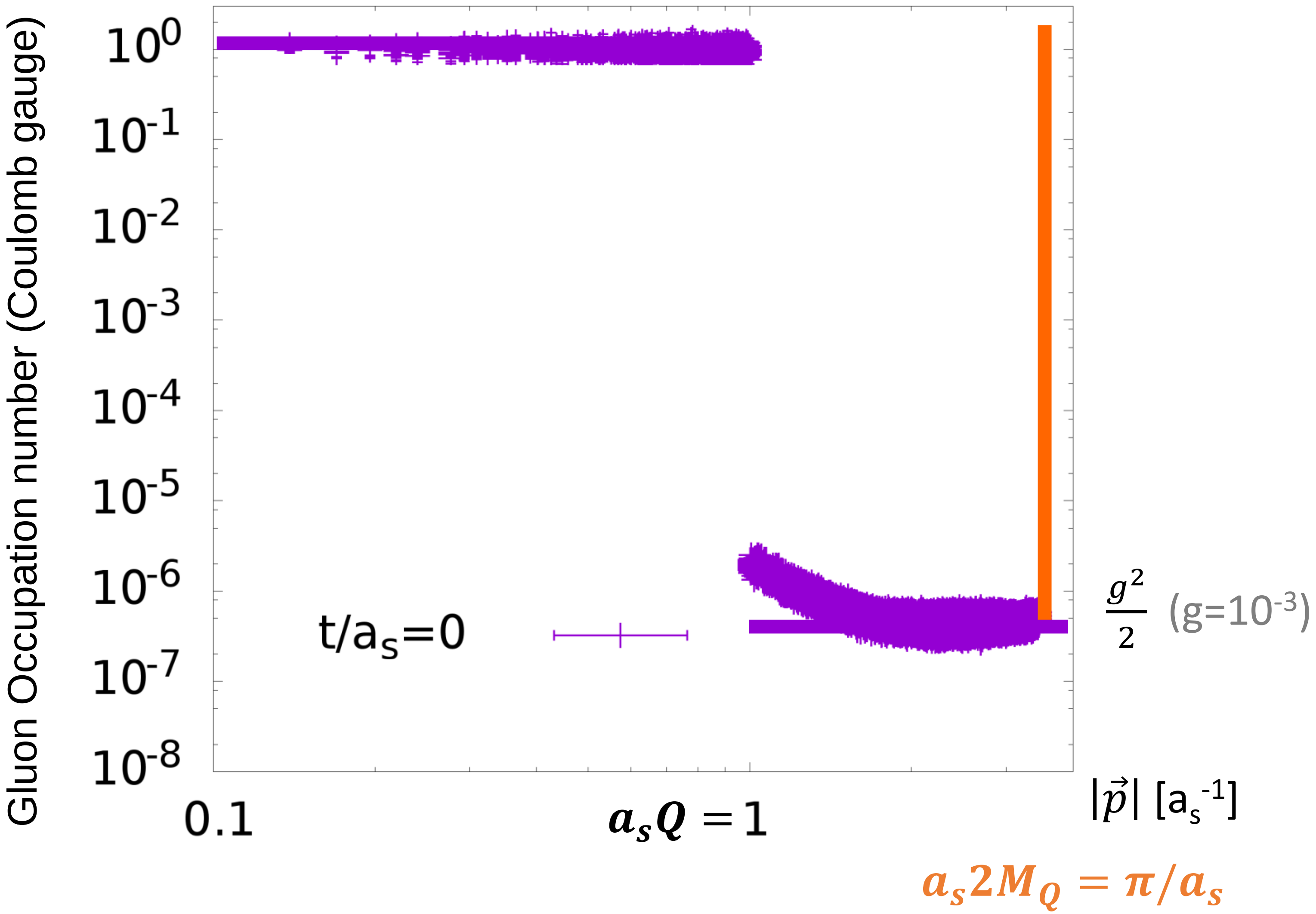

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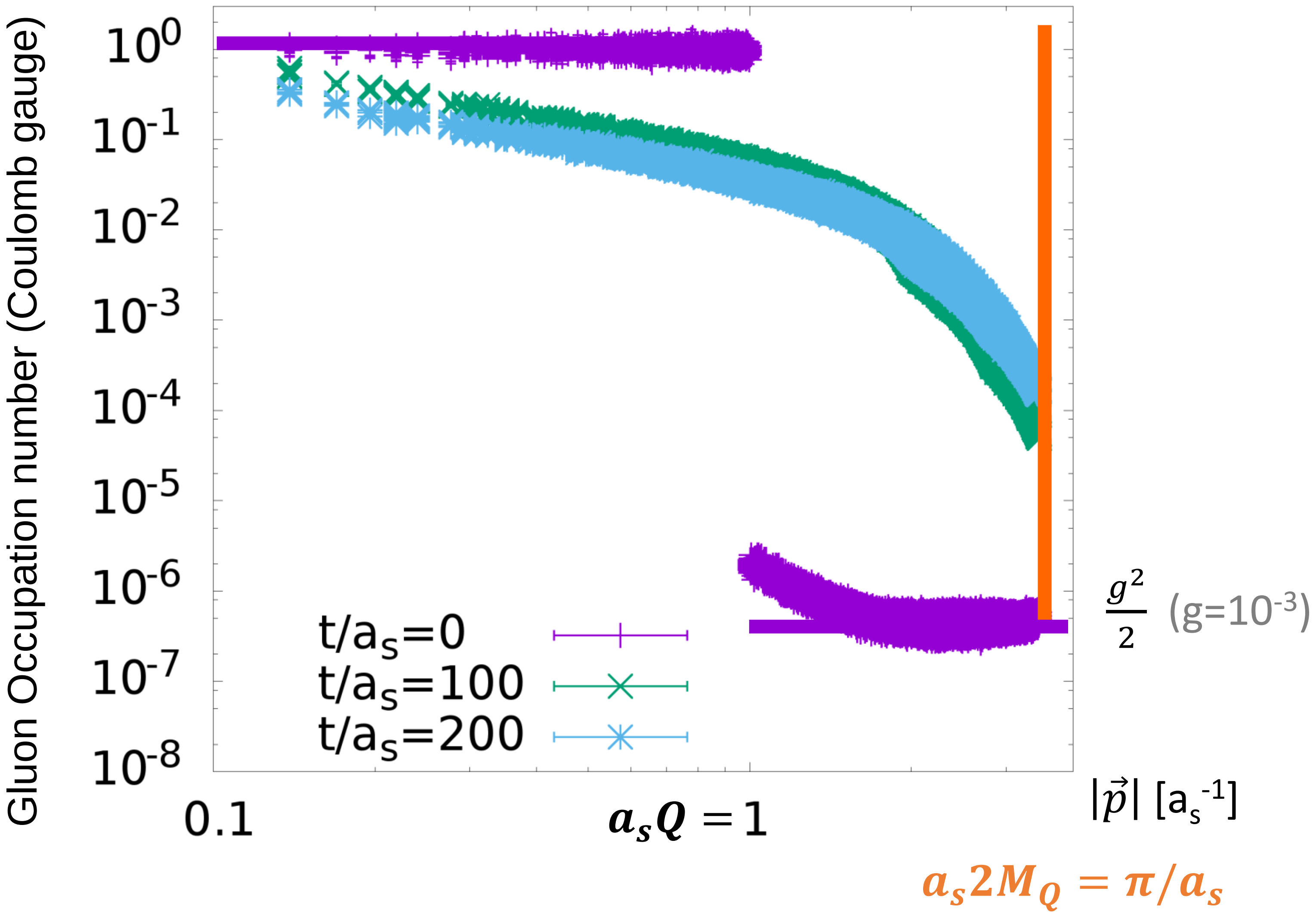

Real-time Lattice NRQCD

- Effective **non-relativistic formulation** of heavy quarks from systematic expansion of QCD action in quark velocity v for 2-component **pauli spinors** $\boldsymbol{\psi, \chi}$
- Hamiltonian to order O(v^3) with leading order Wilson coefficients c_i=1

$$H^{\psi} = -\frac{\vec{D}^2}{2M} - c_1 \frac{g}{2M}\vec{\sigma}\cdot\vec{B} - c_2 \frac{g}{8M^2}\vec{D}\cdot\vec{E} - c_3 \frac{ig}{8M^2}\vec{\sigma}\cdot\left(\vec{D}\times\vec{E} - \vec{E}\times\vec{D}\right)$$

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$$D_V^>(t_2,t_1) = < J^i_{NRQCD}(t_2)\, J^+_{i,NRQCD}(t_1) > \qquad D_V^>(t) = \int DU\, G^{\psi}\sigma^i G^{\chi +}\sigma_i\, e^{iS[U]}$$

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$$G(t + a_t) = (1 - ia_t H[U(t)])\cdot G(t)$$

Often via forward Euler: cheap but 1st order in dt, inherently unstable (Courant), range of validity of NRQCD mixed with breakdown of discretization

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Optimal rational approximation of exp (Crank-Nicholson, O(dt²)): **unconditionally stable**, no mixing of range of validity. (No operator splitting via MPI PETSC)

Wigner Coordinates for Non-Equilibrium

- **No time translational invariance**: need to correctly account for relative and central time coordinate in 2pt functions:

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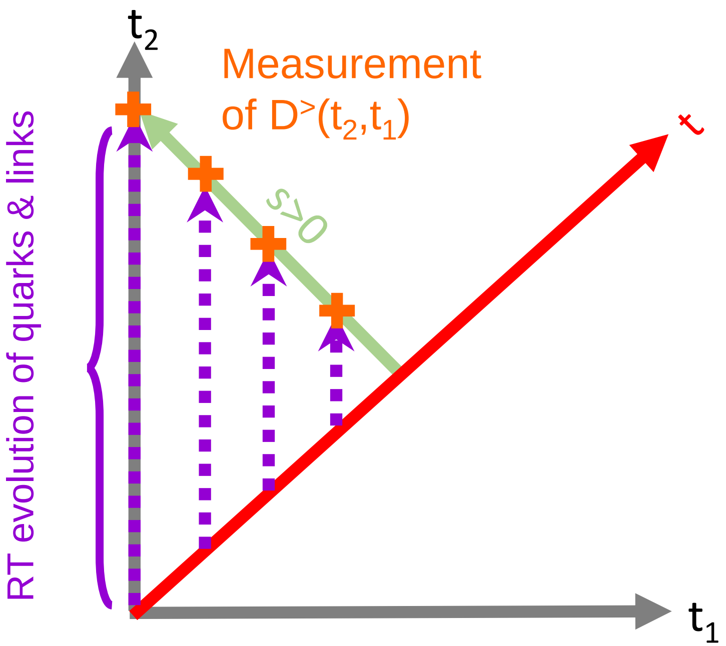

University of Stavanger

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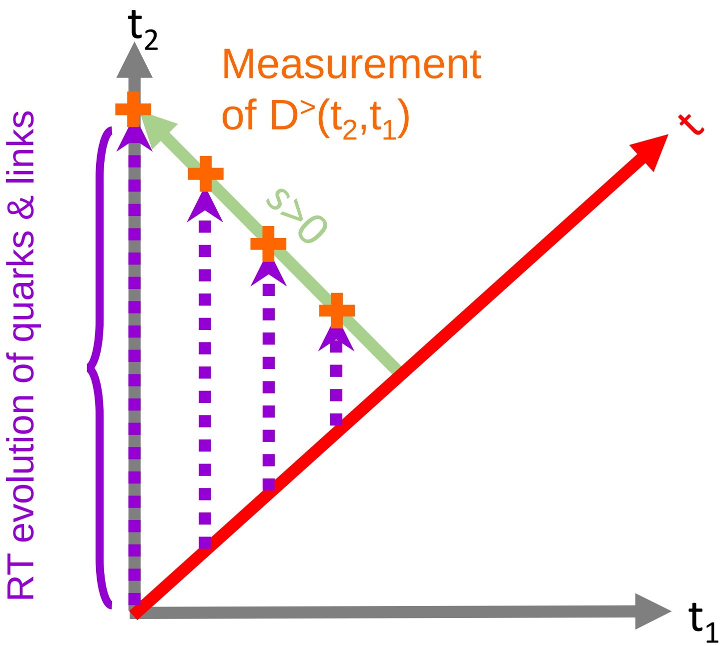
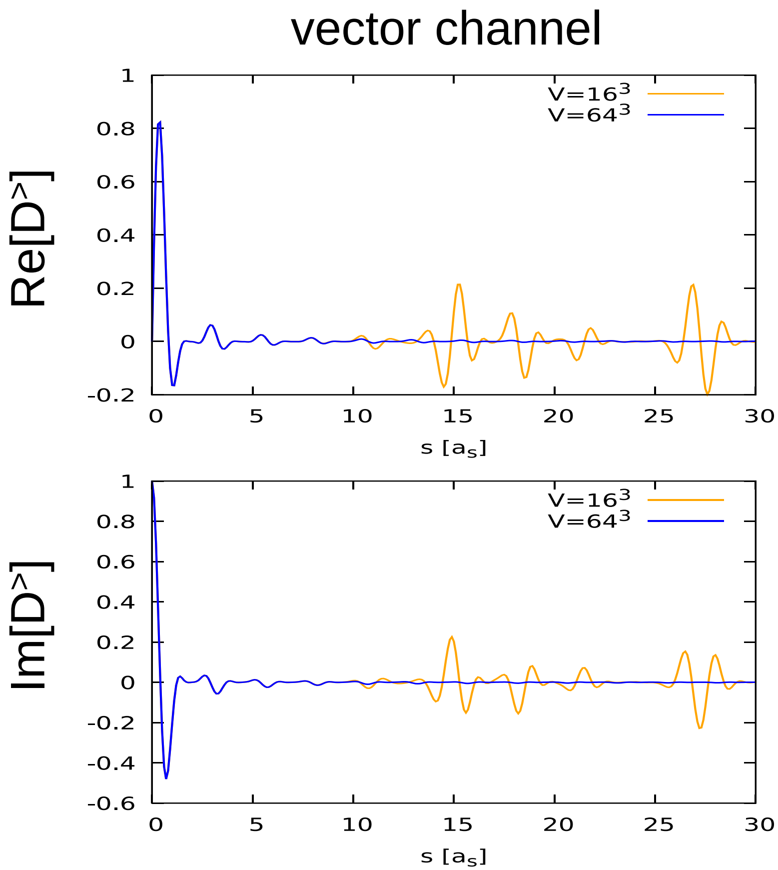


- Spectral function has **explicit t dependence**, signaling real-time evolution of gauge fields

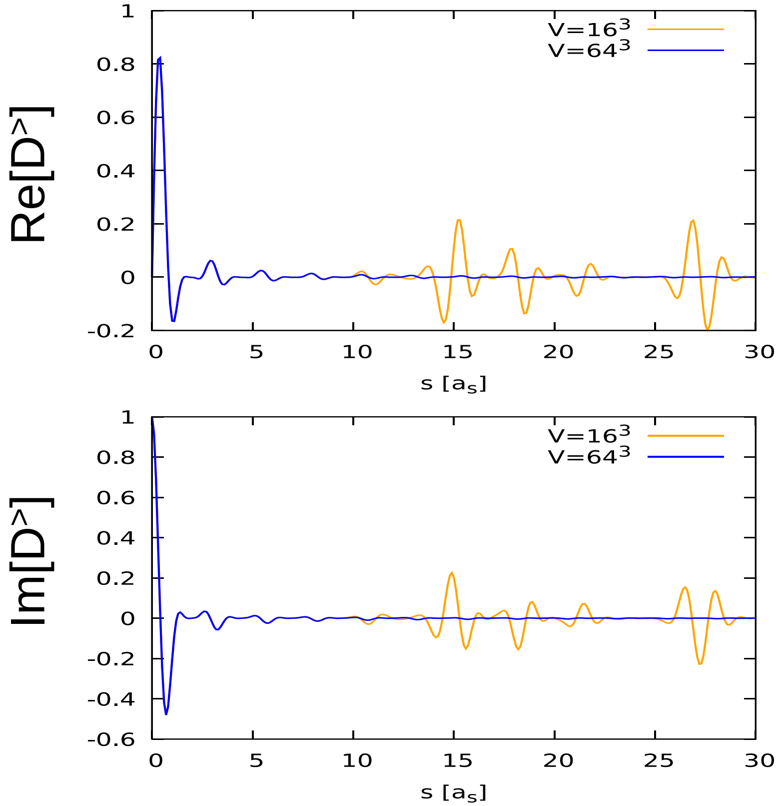
Free theory sanity check

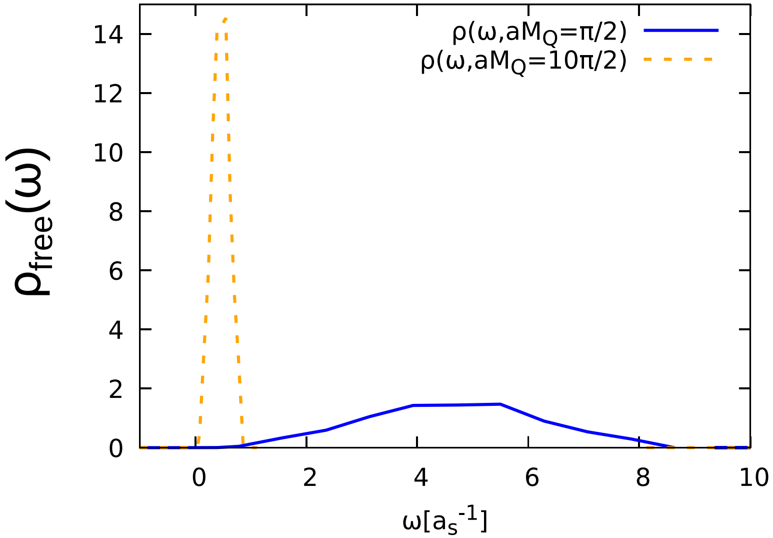


- **Real-time** correlation function is **complex** – finite volume effects as recurrence

Free theory sanity check

vector channel





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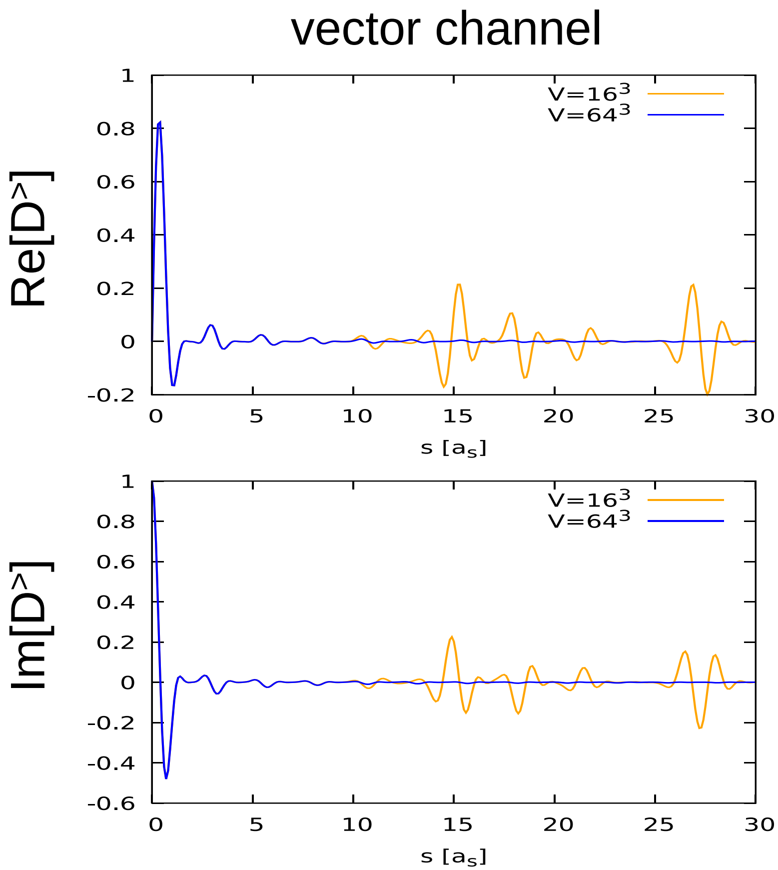

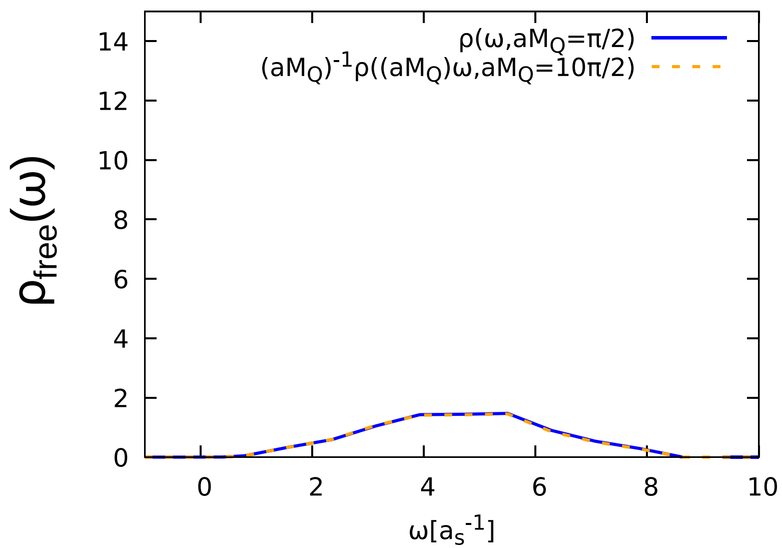

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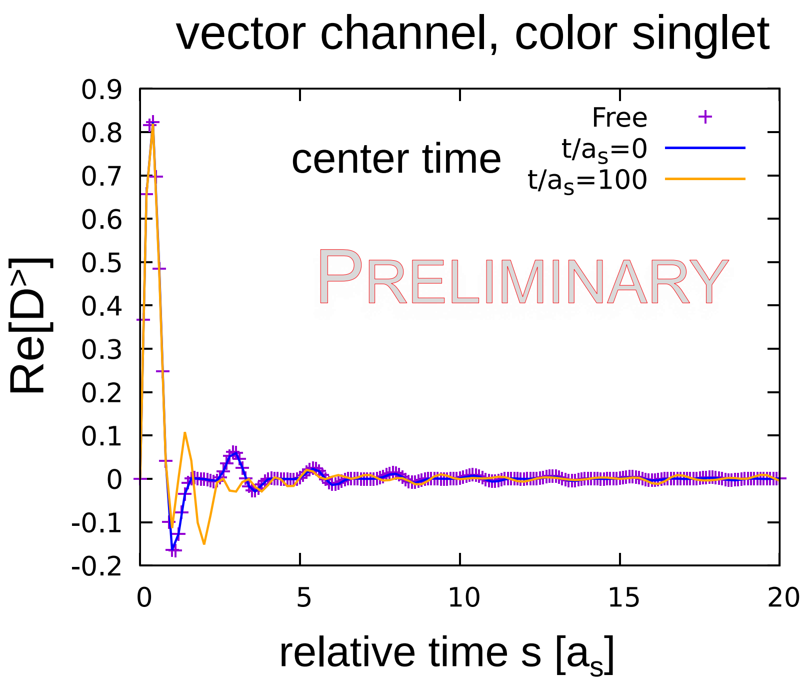
Quarkonium in the Glasma (I)



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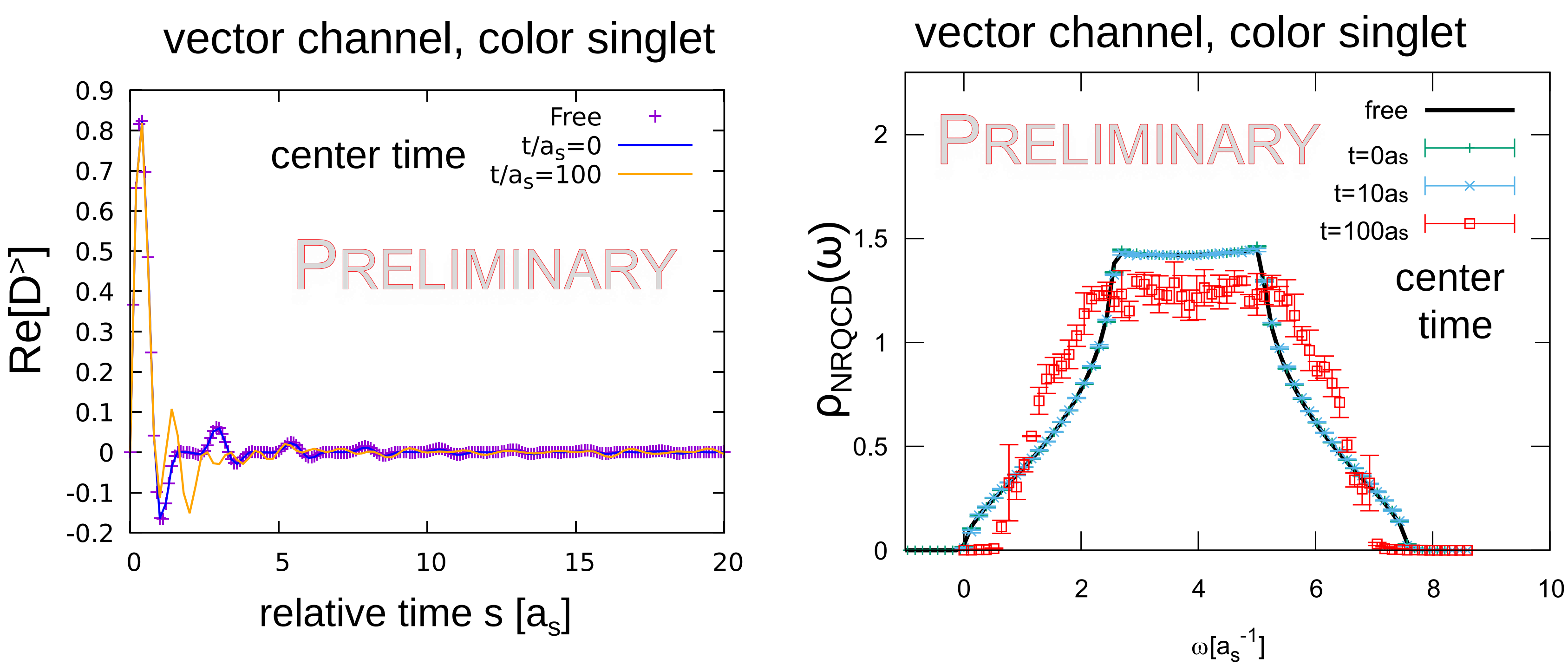

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Q=1
n=1
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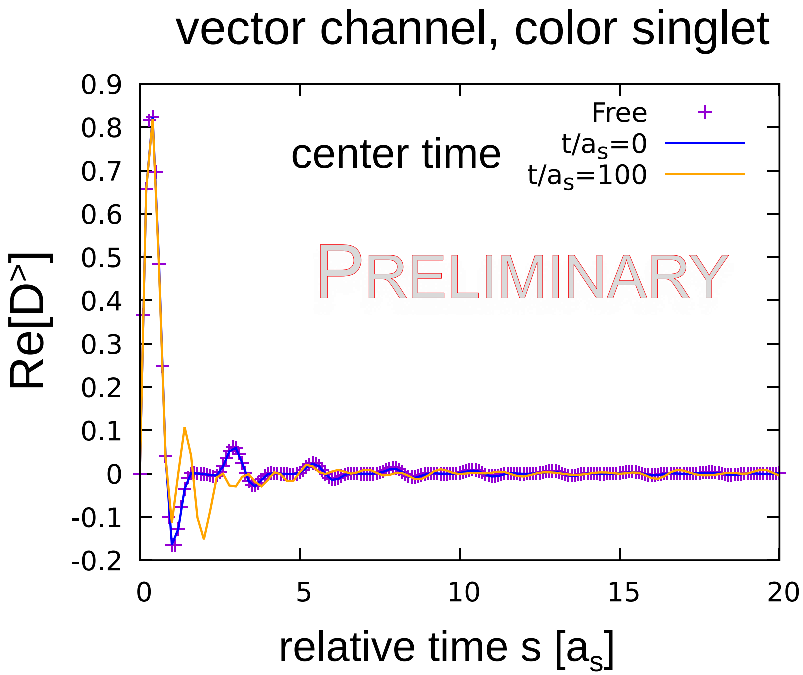

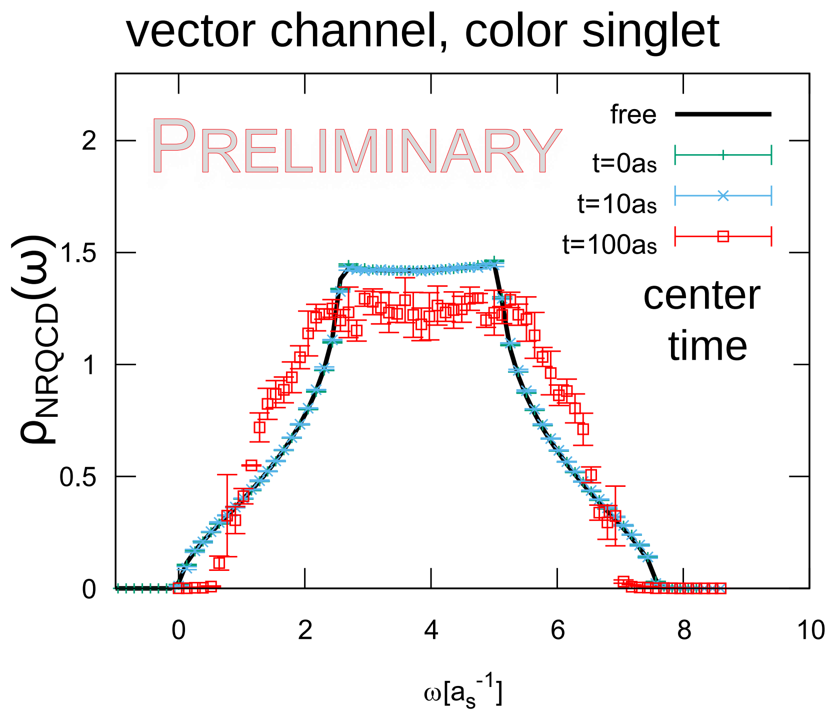

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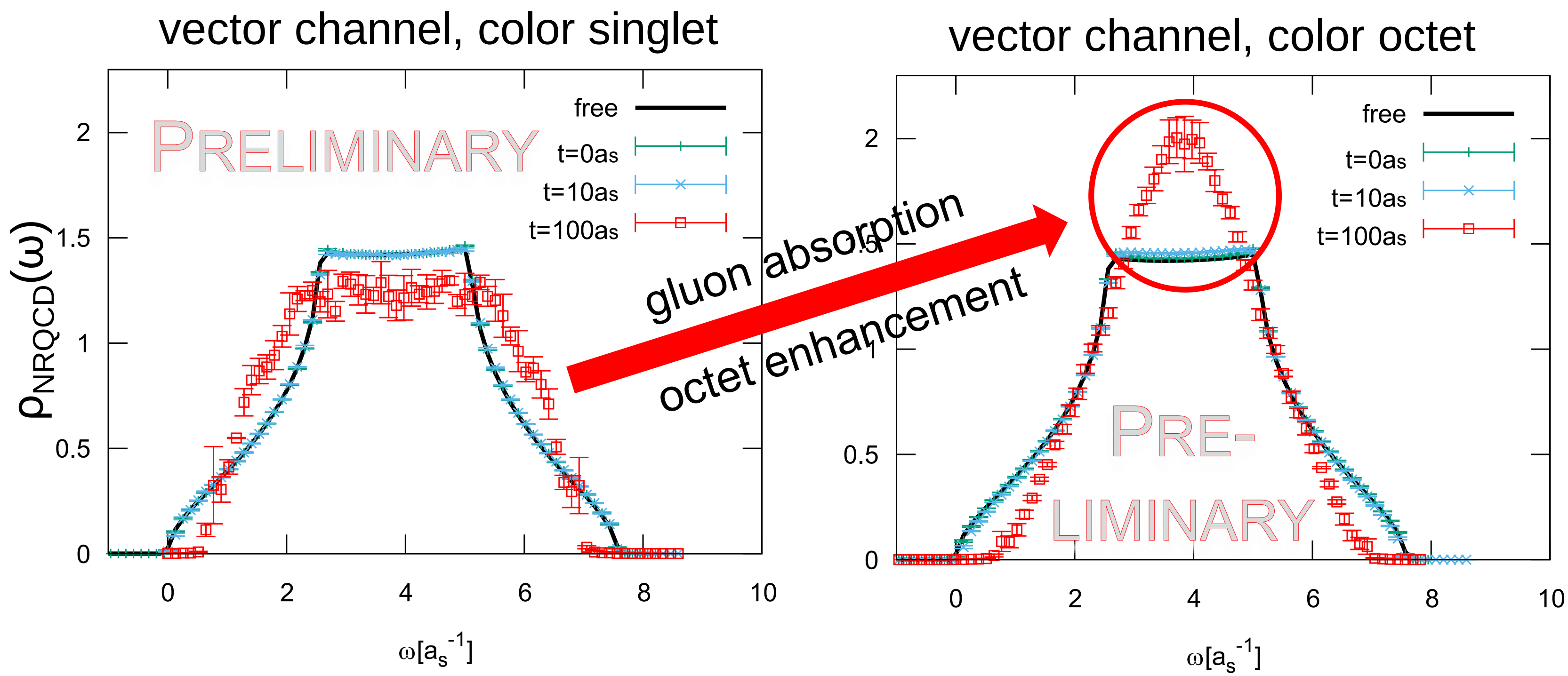
- **Low enegy gluons** do not significantly impact quarks at early times
- Bulk glue effects manifest in the **intermediate (s,t) time physics** of **heavy quarks**
- At the parameters used here, **no signs for binding** into clear resonances

Quarkonium in the Glasma (II)



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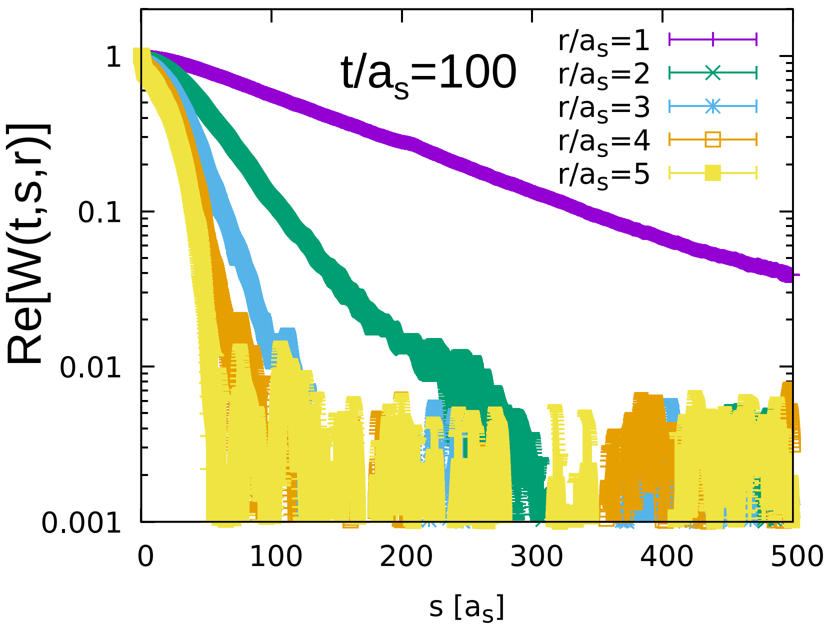
- Reduction of singlet amplitude and broadening understood from **gluon absoprtion**
- **Octet enhancement** from interaction with low enegy gluonic bulk

Understanding the absence of binding

- Consider static quarks via the **non-equilibrium real-time Wilson loop** W(t,s,x)
- Attempt to extract effective **real-time potential** via Wilson loop spectral function
  Re[V] from position of lowest lying peak, **Im[V] from width**, **see Y.Burnier, A. Rothkopf PRD86 (2012) 051503**



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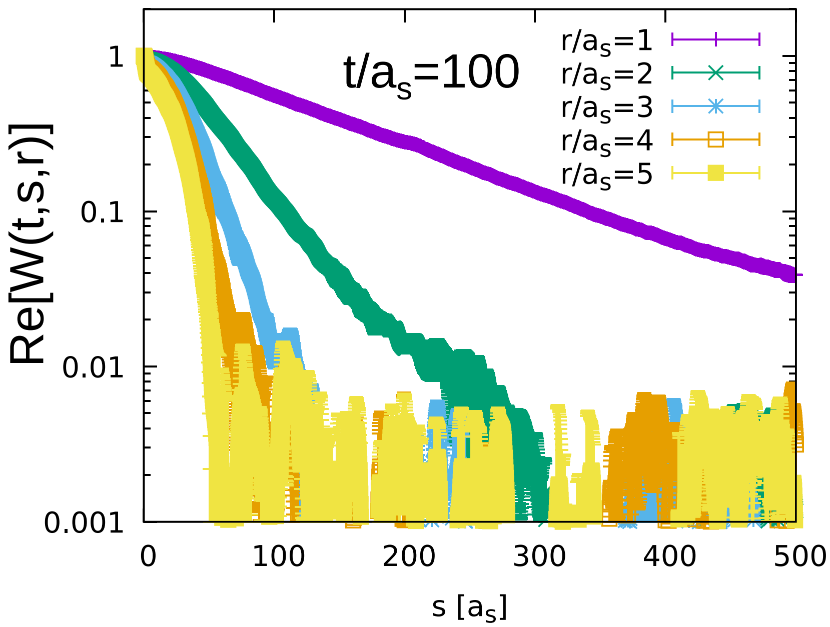

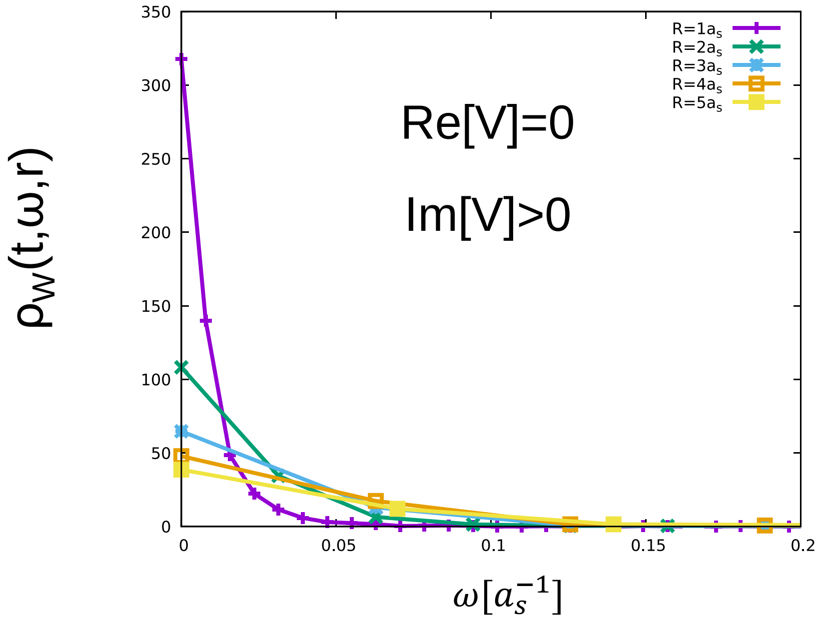

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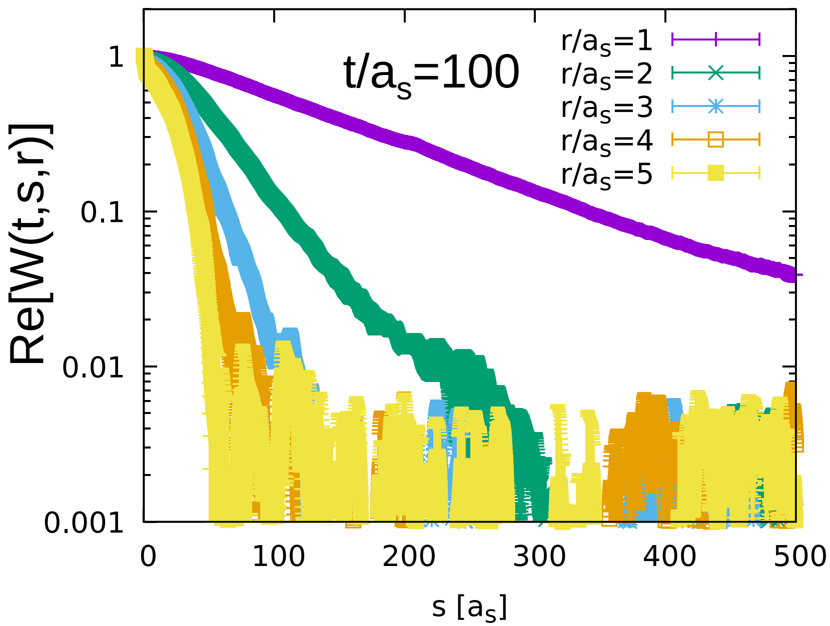

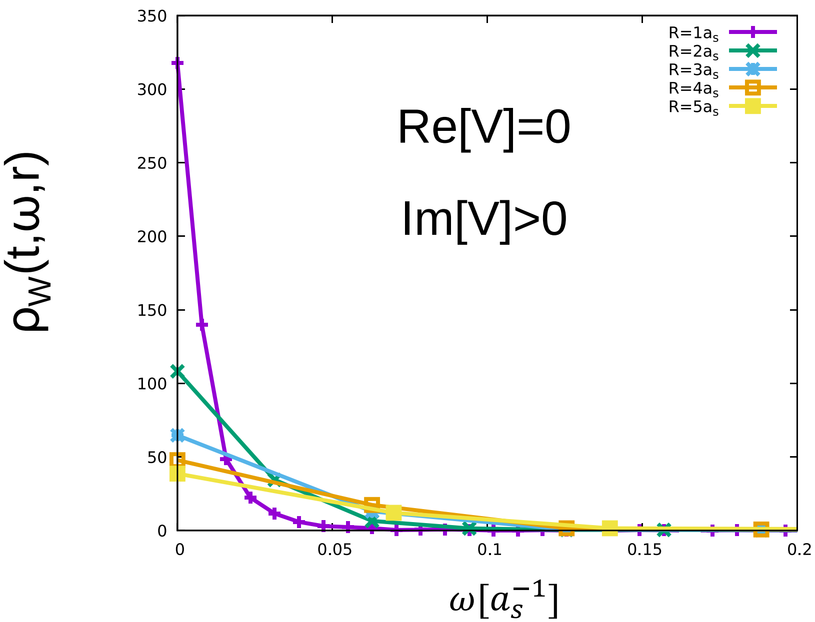

- Similar to results in thermal equilibrium: **no real-part** of the potential emerges
  see M. Laine et.al. JHEP 0709 (2007) 066

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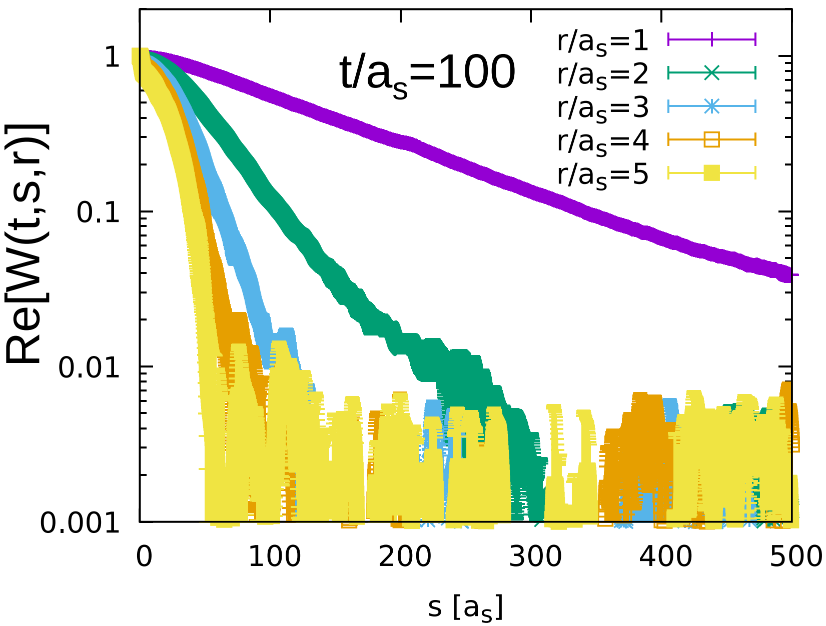

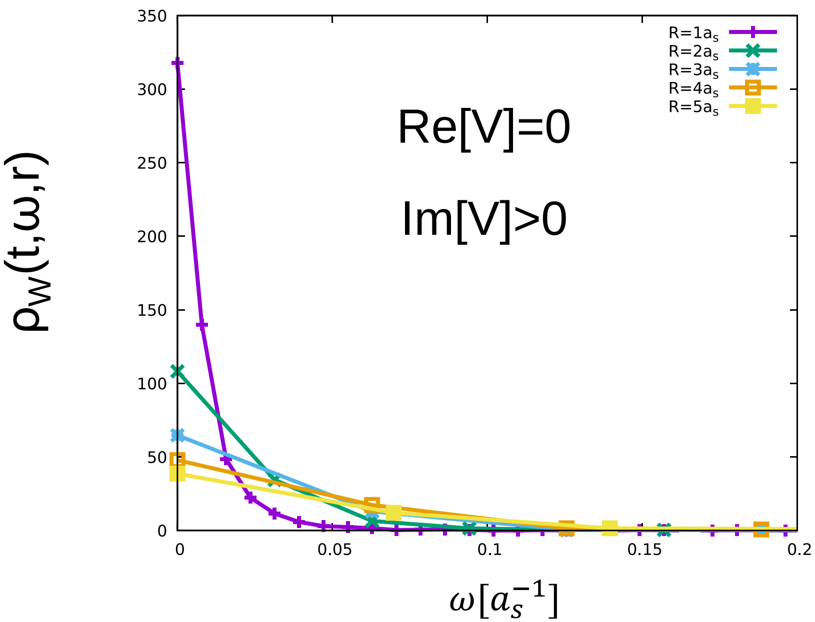

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- **No indications of binding**, not even Coulombic, found out of equilibrium so far

Summary

- Combination of real-time classical statistical simulations for gauge fields with novel stable lattice NRQCD solver
- Direct computation of non-equilibrium real-time quarkonium correlators and spectral functions in Wigner coordinates
- Enhancement in quarkonium colour octet channel and no signs of binding in the singlet channel
- Consistent with absence of a real-part in effective potential
- Need further study at stronger couplings to confirm absence or presence of binding

Thank you for your attention - 謝謝