$I = 3/2$ $N\pi$ scattering and the $\Delta(1232)$ resonance on $N_f = 2 + 1$ CLS ensembles using the stochastic LapH method

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17 June 2019

SDU

CP3

"We"

- John Bulava, University of Southern Denmark
- Colin Morningstar, Carnegie Mellon Univeristy
- Ben Hörz, Berkeley Lab

. . . and me

The $\Delta(1232)$ resonance

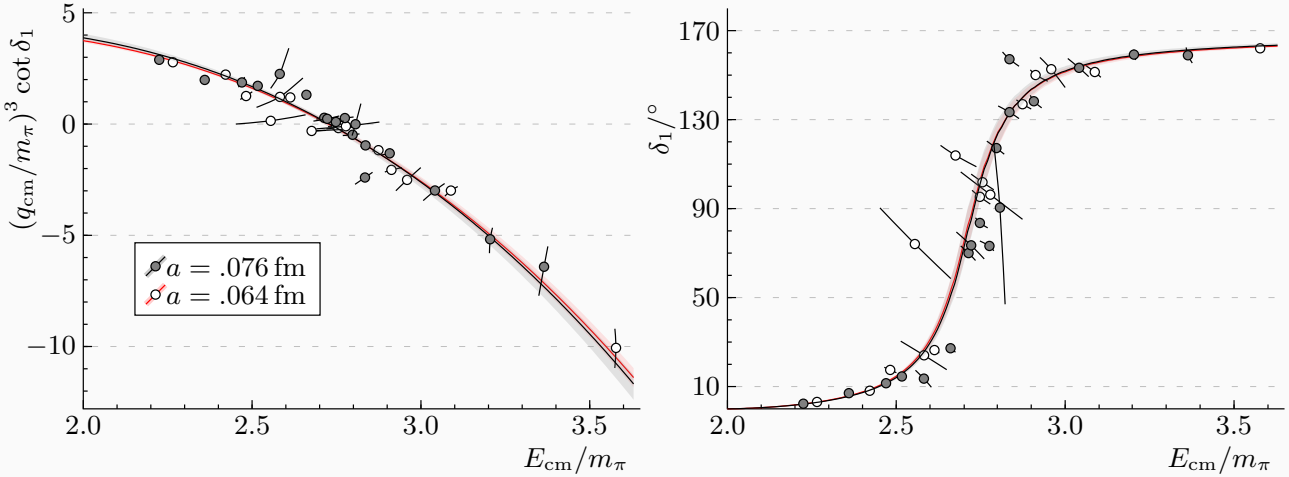
p-wave resonance in $N\pi$ scattering

$I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$

$m_\Delta \approx 1232\text{MeV}$, $\Gamma \approx 117\text{MeV}$

Motivation

- ν-nucleus scattering experiments: Relies on knowledge of $N\Delta(1232)$ transition form factors. $N\pi$ scattering amplitudes are needed for this.
- Success in the meson-meson sector: $\pi\pi$ scattering and many others



Alvarez-Ruso et al. (2018)

CWA, John Bulava, Ben Hörz, Colin Morningstar, Nucl Phys B939 (2019)

Scattering on a Euclidean lattice

Quantization condition: "Solve" the equation

$$\det\left(\tilde{K}^{-1} - B^{\mathbf{P},\Lambda}\right) = 0$$

- $\tilde{K}^{-1} = p^{L'+\frac{1}{2}} K^{-1} p^{L+\frac{1}{2}}$ and $S = (1+iK)(1-iK)^{-1}$
- $\tilde{K}^{-1}$ and B are block diagonal in the $|\Lambda\lambda nJLSa\rangle$ basis, with blocks labelled by $\mathbf{P}$, Λ.
- In $N\pi$ scattering, $\tilde{K}^{-1}$ is diagonal (J and parity conservation)

$$\begin{pmatrix} (\frac{1}{2},0) & 0 & 0 & 0 & 0 \\ 0 & (\frac{1}{2},1) & 0 & 0 & 0 \\ 0 & 0 & (\frac{3}{2},1) & 0 & 0 \\ 0 & 0 & 0 & (\frac{3}{2},2) & 0 \\ 0 & 0 & 0 & 0 & (\frac{5}{2},2) \end{pmatrix}$$

Strategy: Solve the quantisation condition for each block separately, truncating at some $L_{\max}$.

$$(J,L) = p^{2L+1}\cot(\delta_{J,L})$$

Lüscher (1986,1991),..., Morningstar et al. (2017)

Scattering on a Euclidean lattice

Setting $L_{\max} = 1$:

$\mathbf{P} = \mathbf{0}$, $\Lambda = H_g$:

$$p^3 \cot \delta_{\frac{3}{2},1} = R_{00}$$

$\mathbf{P} = (0, 0, 1)$, $\Lambda = G_2$:

$$p^3 \cot \delta_{\frac{3}{2},1} = R_{00} - \frac{\sqrt{5}}{5} R_{20}$$

- Similar expressions for $(\Lambda, \mathbf{P}) = (F_1, (1,1,1)), (F_2, (1,1,1)), (G_2, (0,0,2))$
- d-wave contributions need to be assessed

Göckeler et al. (2012)

Scattering on a Euclidean lattice

$\mathbf{P} = (0, 0, 1)$, $\Lambda = G_1$:

J'	L'	J	L	B
$\frac{1}{2}$	0	$\frac{1}{2}$	0	R_{00}
$\frac{1}{2}$	0	$\frac{1}{2}$	1	$-\frac{\sqrt{3}}{3} R_{10}$
$\frac{1}{2}$	0	$\frac{3}{2}$	1	$\frac{\sqrt{6}}{3} R_{10}$
$\frac{1}{2}$	1	$\frac{1}{2}$	1	R_{00}
$\frac{1}{2}$	1	$\frac{3}{2}$	1	$-\frac{\sqrt{10}}{3} R_{20}$
$\frac{3}{2}$	1	$\frac{3}{2}$	1	$R_{00} + \frac{\sqrt{5}}{5} R_{20}$

Morningstar et al. (2017)

Stochastic LapH method

Stochastic LapH allows flexible and economic correlator construction:

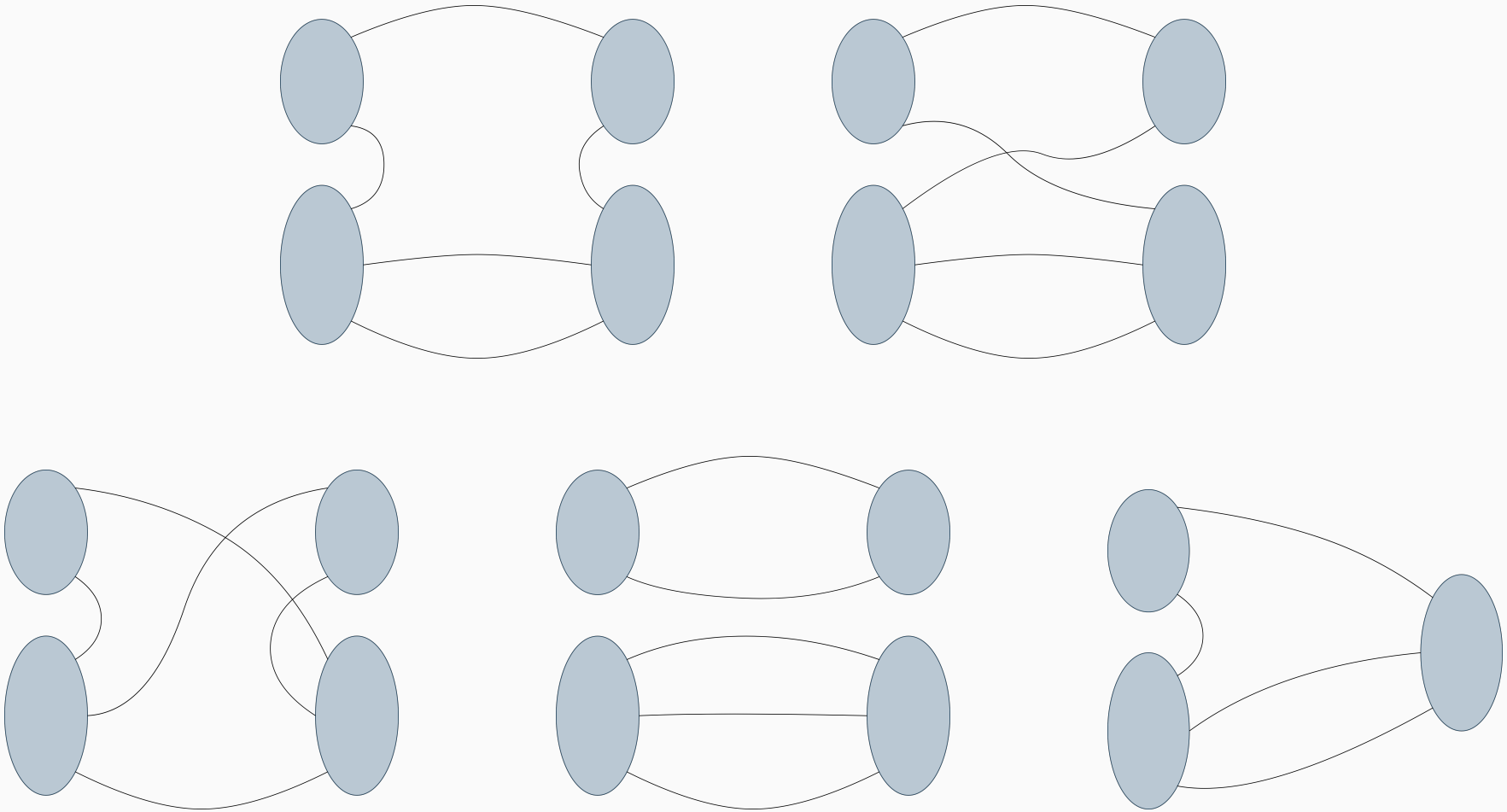
$$C(t_F - t_0) = \Big\langle \mathcal{B}^{[abc]}(\varphi_1, \varphi_2, \varphi_3; t_F) \times \Big(\mathcal{B}^{[abc]}(\rho_1, \rho_2, \rho_3; t_0) - \mathcal{B}^{[acb]}(\rho_1, \rho_3, \rho_2; t_0) + \dots \Big) \Big\rangle_{U,\rho}$$

with

$$\mathcal{B}^{[d_1 d_2 d_3]}(\varphi_1, \varphi_2, \varphi_3; t) = c_{\alpha\beta\gamma} \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}} \epsilon_{abc} \varphi_{a\alpha}^{r_1[d_1]}(\mathbf{x}, t) \varphi_{b\beta}^{r_2[d_2]}(\mathbf{x}, t) \varphi_{c\gamma}^{r_3[d_3]}(\mathbf{x}, t)$$

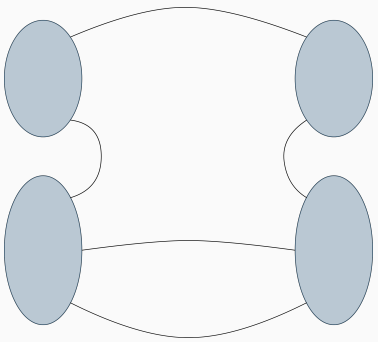
Morningstar et al. (2011)

Wick contractions



Wick contractions

$$\tilde{C}(t, t_0) = \mathcal{M}^{ab}(t_0)\mathcal{B}^{acd}(t_0)\mathcal{M}^{be}(t)\mathcal{B}^{cde}(t)$$



~ 40 diagrams per correlator

$\sim 20 \times 2$ time separations per diagram

$O(10)$ terms in each $N\pi$ operator

$\sim 100 - 1000$ correlators

- Custom built tensor contraction routines with highly optimised BLAS libraries
- Order is important: the difference between the slowest and fastest way of contracting this diagram is a factor 10

Operator basis and fitting

For each $(\Lambda, \mathbf{P})$ we have $N_{\text{op}} \approx 5$

Operators look like $N(\mathbf{p}_N^2)\pi(\mathbf{p}_\pi^2)$ or $\Delta(\mathbf{P})$

Fitting ansatz:

$$R_n(t) = \frac{\hat{C}_n(t)}{C_\pi(\mathbf{p}_\pi^2, t) C_N(\mathbf{p}_N^2, t)} = Ae^{-\Delta E_n t}$$

where

$$\hat{C}_n(t) = (v_n(t_0, t_d), C(t) v_n(t_0, t_d))$$

$$C(t_d) v_n = \lambda_n C(t_0) v_n$$

Michael (1985), Lüscher and Wolff (1990)

Lattices

- CLS $N_\mathrm{f} = 2 + 1$ ensembles
- Wilson clover fermions

ID	β	a(fm)	$L^3 \times T$	m_π, m_K(MeV)	N_conf	N_{t_0}
N401	3.46	0.075	$48^3 \times 128$	$280, 460$	275	2
D200	3.55	0.065	$64^3 \times 128$	$200, 480$	559	2

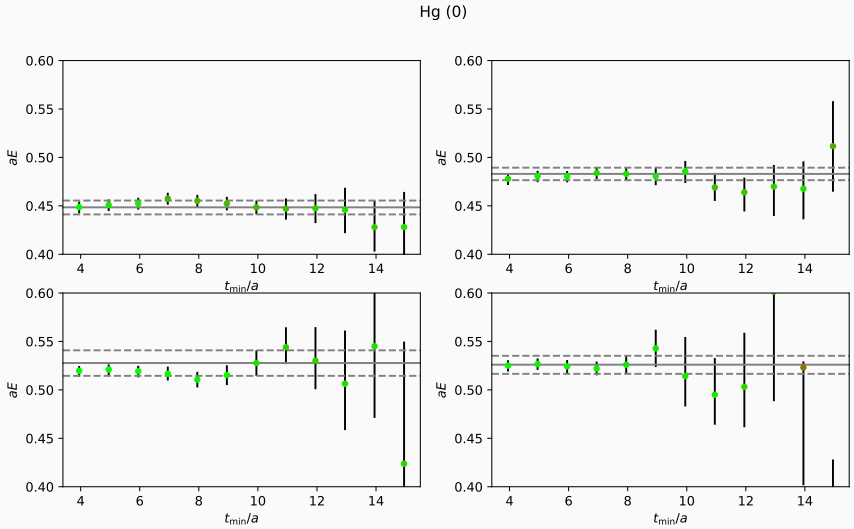
Bruno et al. (2015), RQCD Collaboration (2016)

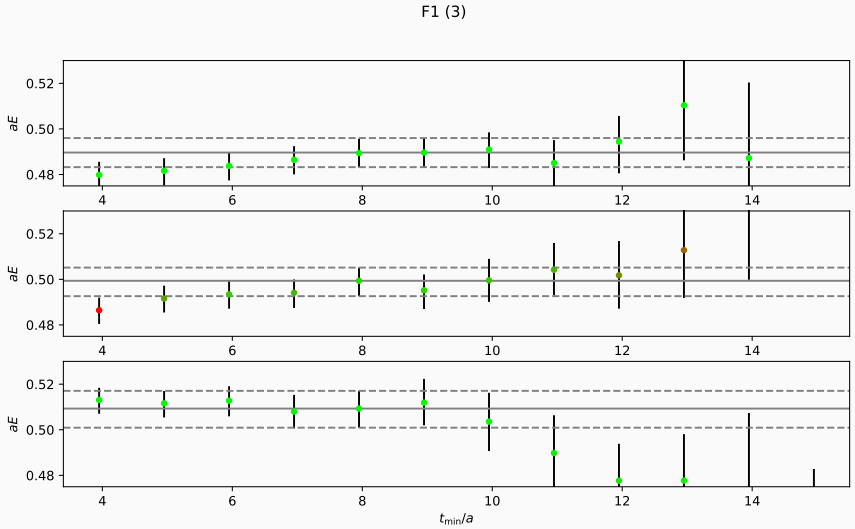
Stochastic LapH method: Parameters

ID	(ρ, n_ρ)	N_{ev}	dilution scheme	N_R^{fix}	N_R^{rel}
N401	$(0.1, 25)$	320	$(\mathrm{TF}, \mathrm{SF}, \mathrm{LI16})_F(\mathrm{TI8}, \mathrm{SF}, \mathrm{LI16})_R$	5	1
D200	$(0.1, 36)$	448	$(\mathrm{TF}, \mathrm{SF}, \mathrm{LI8})_F(\mathrm{TI8}, \mathrm{SF}, \mathrm{LI8})_R$	7	3

Results: t_{min} plots, D200

$$(L = 4.2\text{fm}, a = 0.065\text{fm}, m_\pi = 200\text{MeV})$$





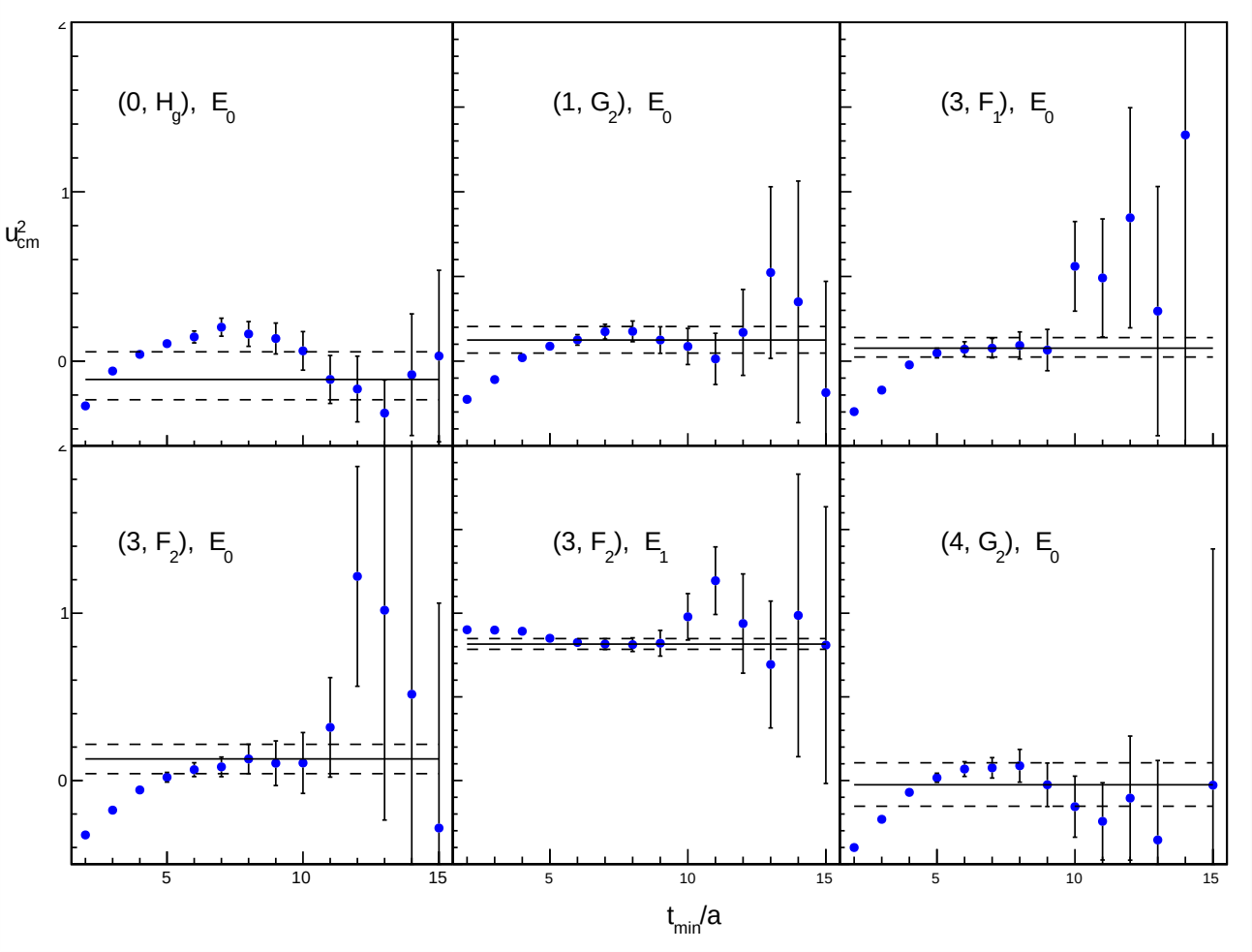
Fit $R_n(t) = Ae^{-\Delta E_n t}$ from t_{min}/a to t_{max}/a.

Plots show the fitted value of aE for varying t_{min}/a with t_{max}/a kept fixed (at $t_{\text{max}}/a = 22$ here)

Results: $t_{\min}$ plots, N401

$$(L = 3.6\text{fm}, a = 0.075\text{fm}, m_\pi = 280\text{MeV})$$

$$u_{cm}^2 = \frac{L^2 q_{cm}^2}{(2\pi)^2}$$



CWA, John Bulava, Ben Hörz, Colin Morningstar, Phys. Rev. D97 (2018), 014506

First results: N401

$$(L = 3.6\text{fm}, a = 0.075\text{fm}, m_\pi = 280\text{MeV})$$

Fit to

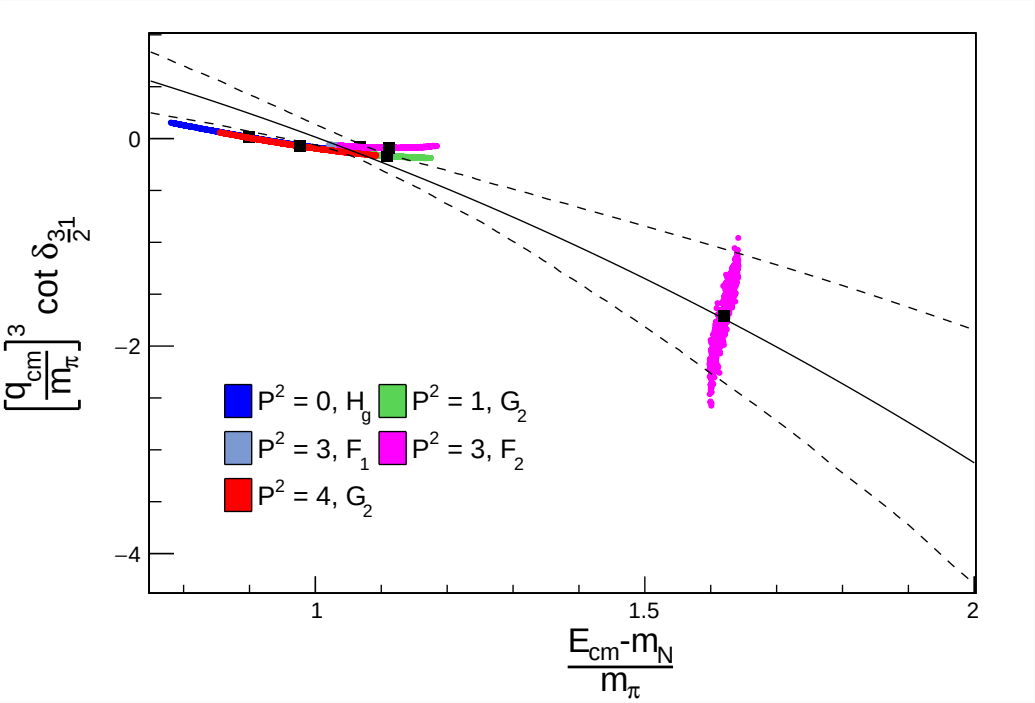
$$\left(\frac{q_{cm}}{m_\pi}\right)^3 \cot\delta_{\frac{3}{2},1} = \left(\frac{m_\Delta^2}{m_\pi^2} - \frac{E_{cm}^2}{m_\pi^2}\right)\frac{6\pi E_{cm}}{g^2 m_\pi}$$

$$\frac{m_\Delta}{m_\pi} = 4.74(5)$$

$$g = 19(5)$$

$$\chi^2/\text{d.o.f} = 1.11$$

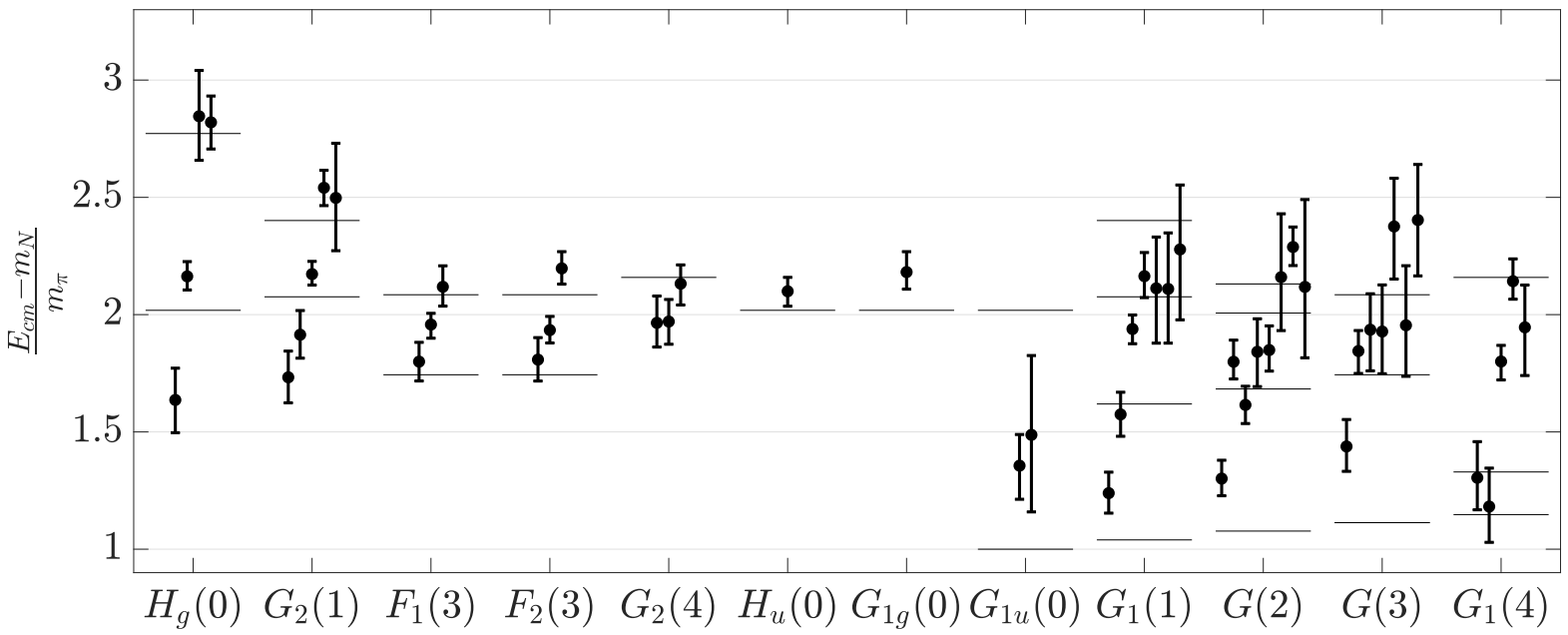
Insensitive to d-wave contributions



CWA, John Bulava, Ben Hörz, Colin Morningstar, Phys. Rev. D97 (2018), 014506

New results: D200

$(L = 4.2\text{fm}, a = 0.065\text{fm}, m_\pi = 200\text{MeV})$



Non-interacting energies from arXiv:1905.09291 Harris, von Hippel, Junnarkar, Meyer, Ottnad, Wilhelm, Wittig, Wrang

New results: D200

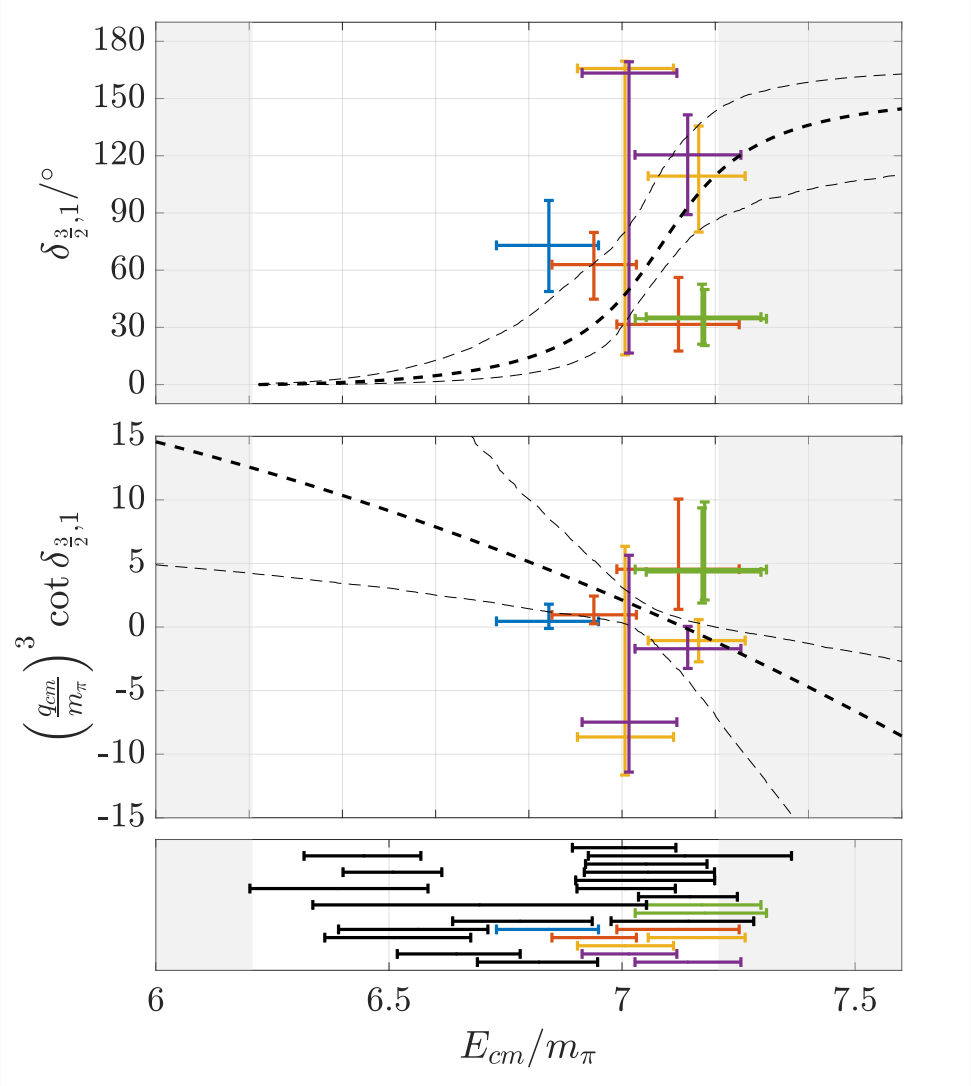
$(L = 4.2\text{fm}, a = 0.065\text{fm}, m_\pi = 200\text{MeV})$

- Simultaneous s- and p-wave fit
- Including s-wave dominated irreps helps!

$$\frac{m_\Delta}{m_\pi} = 7.13(9)$$

$$g = 11(6)$$

$$\chi^2/\text{d.o.f} = 0.8$$



Outlook

- N401: Ramping up the statistics
- D200: Better dilution and more statistics: Expected reduction in correlator errors by a factor 5(!) (Also: stay tuned for NN scattering)
- D101 ($L \approx 5.5$fm): Hopefully more levels in the entire elastic range.

Thank you!