The light baryon spectrum in the continuum limit

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with

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Outline

Motivation

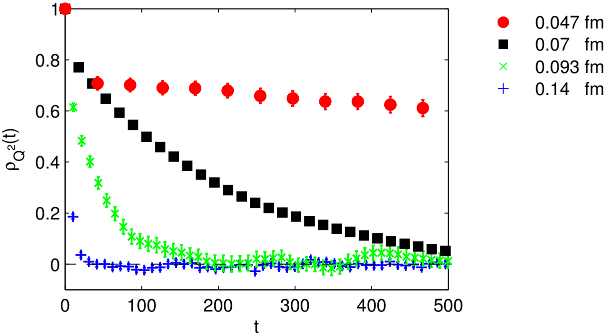
- Scale determination
- Check the validity range of SU(3) BChPT
- Determine LECs: F, D, σ terms etc.
- Preparatory step for the determination of SM parameters

Outline

- CLS simulations
- Chiral and continuum limit extrapolations
- Results
- Summary

The continuum limit

Generic problem: critical slowing down of local updating algorithms:

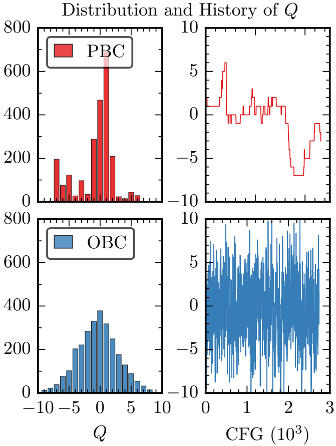


Autocorrelation function for the squared topological charge $Q^2 \propto \left[\int \mathrm{d}^4x\, F\widetilde{F}\right]^2$ versus Monte Carlo time.

S Schaefer, R Sommer, F Virotta, 1009.5228

Open boundary conditions in time

OBC in time [S Schaefer, M Lüscher, 1105.4749] allow the flow of topological objects (instantons) into and out of the lattice.



SU(7) gauge theory.

$a \approx 0.094$ fm.

Problem becomes worse at large N_c:

Instanton action: $8\pi^2 N_c/\lambda$.

Higher cost to create an (anti)instanton!

A Amato, G Bali, B Lucini, 1512.00806

Disadvantage: Breaking of translational invariance in time near the boundaries. $\Rightarrow$ Discard part of the simulated volume.

Coordinated Lattice Simulations (CLS)

CLS members/groups at

- HU Berlin
- CERN
- TC Dublin
- Mainz
- UA Madrid
- Milano Bicocca
- Münster
- Odense/CP3 Origins
- Regensburg
- Roma I + II
- Wuppertal
- DESY/Zeuthen

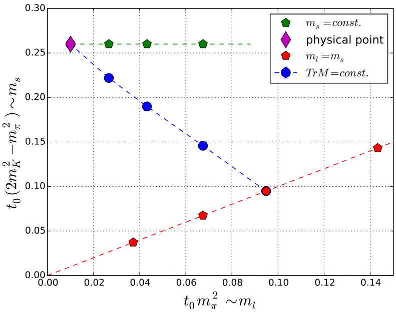
Coordinated generation of gauge ensembles using openQCD
`https://luscher.web.cern.ch/luscher/openQCD/`
[M Lüscher, S Schaefer, 1206.2809].

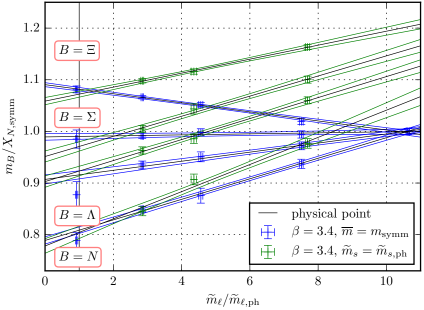
$N_f = 2 + 1$ flavours of non-perturbatively order-a improved Wilson fermions on tree level Symanzik improved glue.

Keep it simple and local: no smeared action etc.

Simulation strategy

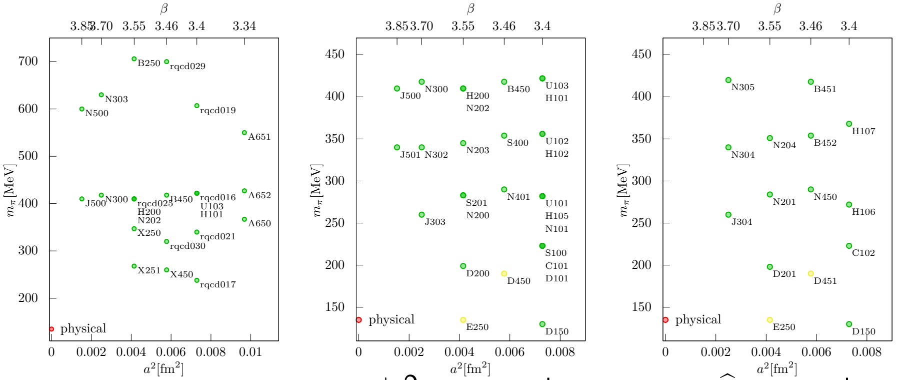
Simulate along $m_s + 2m_\ell = \text{const}$ [QCDSF+UKQCD: W Bietenholz et al, 1003.1114], **and** $\widehat{m}_s \approx \text{const}$ [G Bali et al, 1606.09039; 1702.01035], enabling Gell-Mann–Okubo/SU(3) and SU(2) ChPT extrapolations.





(Right: old, linear unconstrained baryon mass fits at fixed $a \approx 0.086\,\text{fm}$.)

Ensemble overview

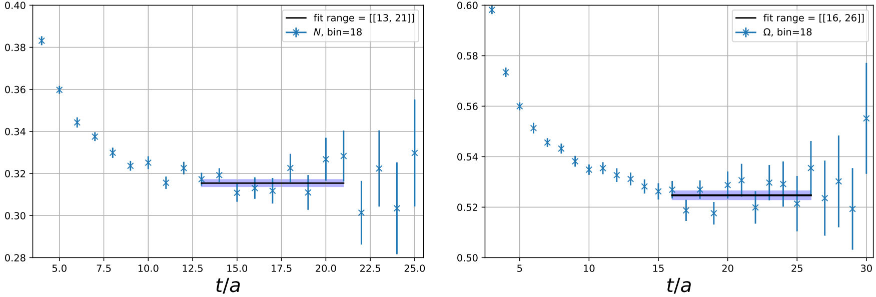


E: $192 \cdot 96^3$, J: $192 \cdot 64^3$, D: $128 \cdot 64^3$, N: $128 \cdot 48^3$, C: $96 \cdot 48^3$,
S: $128 \cdot 32^3$, H: $96 \cdot 32^3$, B: $64 \cdot 32^3$, U: $128 \cdot 24^3$.

??5? and rqcd0?? have PBC.

Typically 6000 – 10000 MDUs.

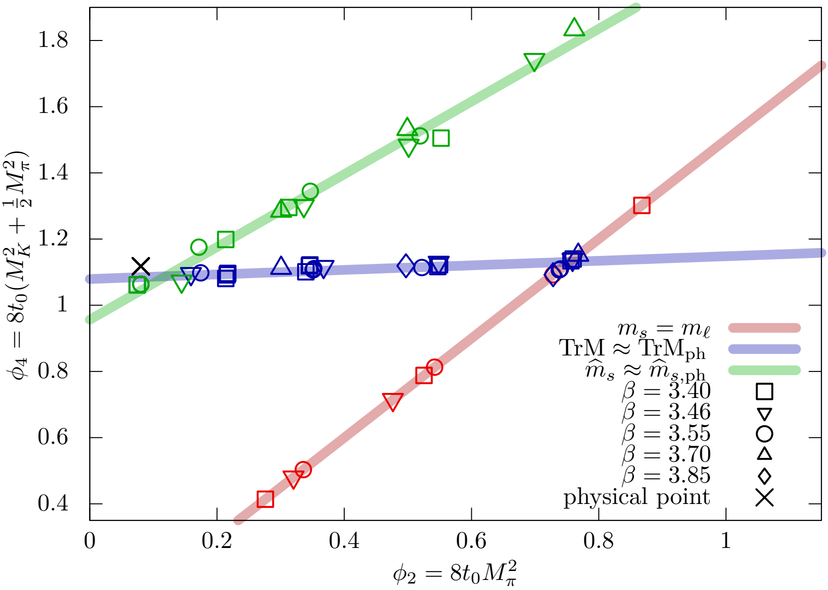
Effective masses for m_N and m_Ω on D200



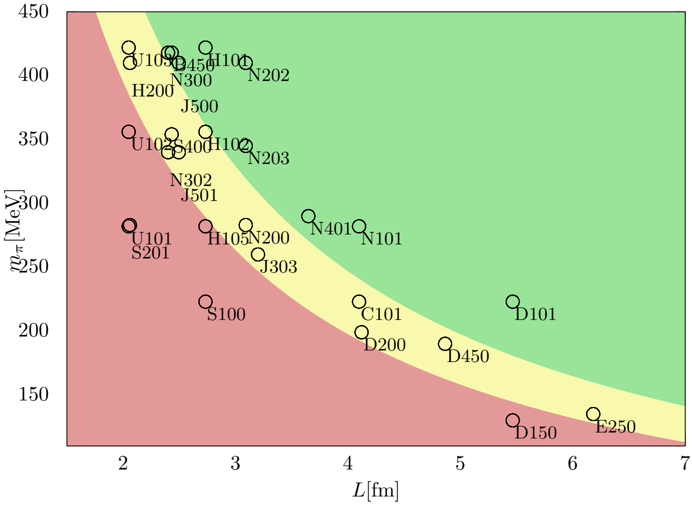
$a \approx 0.064\,\text{fm},\ M_\pi \approx 190\,\text{MeV},\ M_K \approx 480\,\text{MeV}$

We realized different bin sizes and extrapolated the integrated autocorrelation time in 1/binsize to arrive at the final results.

Tr M vs. $m_u = m_d = m_\ell$



Volumes



$M_\pi L < 4$, $4 \leq M_\pi L < 5$, $M_\pi L \geq 5$.

Finite size effects

Pseudoscalar meson masses in SU(3) ChPT:

$$M_\pi^2(L) = M_\pi^2\left[1 + \frac{1}{2}h(\lambda_\pi, M_\pi^2) - \frac{1}{6}h(\lambda_{\eta_8}, M_{\eta_8}^2)\right],$$
$$M_K^2(L) = M_K^2\left[1 + \frac{1}{3}h(\lambda_{\eta_8}, M_{\eta_8}^2)\right],$$

$$h(\lambda_M, M_M^2) = \frac{4M_M^2}{(4\pi F_0)^2}\sum_{\mathbf{n}\neq\mathbf{0}}\frac{K_1(\lambda_M|\mathbf{n}|)}{\lambda_M|\mathbf{n}|}, \quad \lambda_M = LM_M.$$

Octet baryon masses in covariant SU(3) BChPT:

$$m_B(L) = m_B + \frac{2m_0}{(4\pi F_0)^2}\sum_{M\in\{\pi,K,\eta_8\}} g_{O,M}M_M^2\int_0^\infty \mathrm{d}x\sum_{\mathbf{n}\neq\mathbf{0}}K_0\left(\lambda_M|\mathbf{n}|\sqrt{1-x+\frac{m_0^2}{M_M^2}x^2}\right).$$

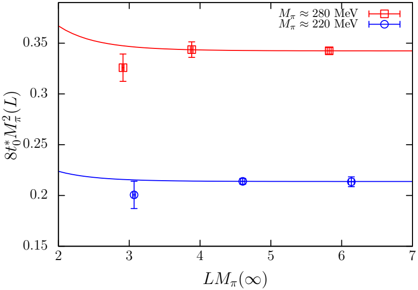
m_0 and F_0: octet baryon mass and pion decay constant in the chiral limit. $g_{B,M}$ are known, quadratic functions of F and D, e.g., $g_{N,\pi} = (3/2)(D+F)^2 = (3/2)\mathring{g}_A^2$.

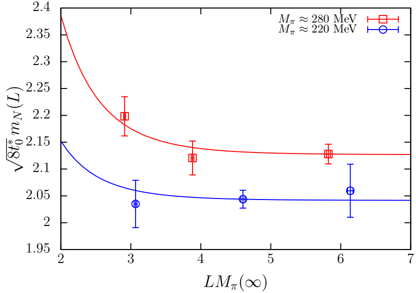
FSE: expectation vs. data

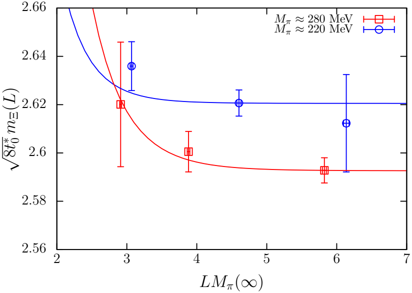
Literature values for F_0, F, D, m_0.

In some cases pion FSE can be as big as half the statistical error.

On our volumes baryon mass FSE are negligible.







Order-a improvement

For an order-a improved spectrum, it is sufficient to improve the action:

$$S^{(0)}_{\text{lattice}} = S_{\text{continuum}} + \underbrace{aS^{(1)} + a^2 S^{(2)} + \ldots}_{\text{unwanted "physics" at scales } \sim 1/a} \quad .$$

We subtract $aS^{(1)}$ from both sides. Three types of improvement terms:

1. Lagrangian counter term $\propto i c_{SW} a \bar{\psi}_f \sigma_{\mu\nu} F_{\mu\nu} \psi_f$
   Known non-perturbatively [J Bulava, S Schaefer, 1304.7093].
2. Quark masses: [T Bhattacharya et al, hep-lat/0511014]

$$\mathrm{Tr}\,\widehat{M} = Z_m r_m [\mathrm{Tr}\,M + a d_m \mathrm{Tr}\,(M^2) + a\bar{d}_m (\mathrm{Tr}\,M)^2]\,,$$

$$\widehat{m}_s - \widehat{m}_\ell = Z_m[(m_s - m_\ell) + a b_m (m_s^2 - m_\ell^2) + a\bar{b}_m \mathrm{Tr}\,M(m_s - m_\ell)]\,.$$

   Relevant for determinations of renormalized quark masses.
   b_m known non-perturbatively [P Korcyl, G Bali, 1607.07090].
3. Improvement of the coupling $g^2 \mapsto g^2(1 + a b_g \mathrm{Tr}\,M/N_f)$.
   Problem if $\mathrm{Tr}\,M = m_s + 2m_\ell \neq \text{const}$.

The effect of b_g cancels from ratios of hadron masses at each fixed $(\beta,\ \kappa_\ell,\ \kappa_s)$. Therefore, we extrapolate the combination $\sqrt{8t_0}m_B$.

Continuum limit extrapolation

Define

$$\overline{m} = \frac{1}{3}\mathrm{Tr}\, M = \frac{1}{3}(2m_\ell + m_s)\,, \quad \delta m = m_s - m_\ell\,.$$

Then

$$\overline{M}^2 := \frac{1}{3}(2M_K^2 + M_\pi^2) \approx 2B_0\overline{m}\,, \quad \delta M^2 := 2(M_K^2 - M_\pi^2) \approx 2B_0\delta m\,.$$

We rescale into the dimensionless quantities

$$\overline{\mathbb{M}} = \sqrt{8t_0}\,\overline{M}\,, \quad \delta\mathbb{M} = \sqrt{8t_0}\delta M\,, \quad \mathrm{m}_B = \sqrt{8t_0}m_B\,, \quad \mathbb{a} = \frac{a}{\sqrt{8t_0^*}}\,,$$

where $B \in \{N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega\}$.
t_0^* is t_0 at the point where $\phi_4 = 1.11 = (3/2)\phi_2$
[M Bruno, T Korzec, S Schaefer, 1608.08900].
$t_0^* = t_{0,\mathrm{ph}}$, up to $\mathcal{O}(a)$ effects. Continuum limit extrapolation:
(term independent of mass, term $\propto \overline{M}^2 \sim \overline{m}$ and terms $\propto \delta M^2 \sim \delta m$)

$$\mathrm{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, \mathbb{a}) = \mathrm{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, 0)\left[1 + c\,\mathbb{a}^2 + \bar{c}\,\mathbb{a}^2\overline{\mathbb{M}}^2 + \delta c_B\,\mathbb{a}^2\delta\mathbb{M}^2\right]$$

Chiral extrapolation

Our parametrizations include:

Linear = NLO SU(3) BChPT:

$$\mathbb{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, 0) = \mathbb{m}_0 + \overline{\mathbb{b}}\, \overline{\mathbb{M}}^2 + \delta\mathbb{b}_B\, \delta\mathbb{M}^2\,,$$

where $\mathbb{m}_0 = m_0\sqrt{8t_{0,\mathrm{ch}}}$ and the $\mathbb{b}$ differ from the standard $b/\sqrt{8t_{0,\mathrm{ch}}}$ parameters by $\mathcal{O}(a)$ effects and $\overline{\mathbb{b}}$ also by the quark mass dependence of t_0 [O Bär, M Golterman, 1312.4999] (through $t_{0,\mathrm{ph}}/t_{0,\mathrm{ch}}$).

SU(3) constraints:

$$\delta b_N = \frac{2}{3}(3b_F - b_D), \quad \delta b_\Sigma = \frac{4}{3}b_D, \quad \delta b_\Xi = -\frac{2}{3}(3b_F + b_D), \quad \delta b_\Lambda = -\frac{4}{3}b_D.$$

$\Rightarrow$ 10 parameters: $\mathbb{m}_0$, $\overline{\mathbb{b}}$, $\mathbb{b}_D$, $\mathbb{b}_F$, c, $\bar{c}$, δc^B to fit 4 baryon masses on a large set of ensembles (at present over 100 data points).

SU(3) HBChPT and BChPT in EOMS at NNLO:

only 2 additional parameters: $\mathbb{F}$, $\mathbb{D}$ (total of $6+4+2=12$).

Decuplet. NLO: 9 parameters, NNLO: 10 parameters.

Chiral extrapolation 2

Order p^3 (NNLO), octet baryons

$$\mathbb{m}_B(\mathbb{M}_\pi, \mathbb{M}_\mathbb{K}, 0) = \mathbb{m}_0 + \overline{\mathbb{b}}\ \overline{\mathbb{M}}^2 + \delta\mathbb{b}_B\ \delta\mathbb{M}^2 + \mathbb{g}_{B,\pi} f_O\left(\frac{\mathbb{M}_\pi}{\mathbb{m}_0}\right) + \mathbb{g}_{B,K} f_O\left(\frac{\mathbb{M}_K}{\mathbb{m}_0}\right) + \mathbb{g}_{B,\eta_8} f_O\left(\frac{\mathbb{M}_{\eta_8}}{\mathbb{m}_0}\right).$$

BChPT in EOMS regularization:

$$f_O(x) = -2x^3\left[\sqrt{1-\frac{x^2}{4}}\arccos\left(\frac{x}{2}\right) + \frac{x}{2}\ln(x)\right].$$

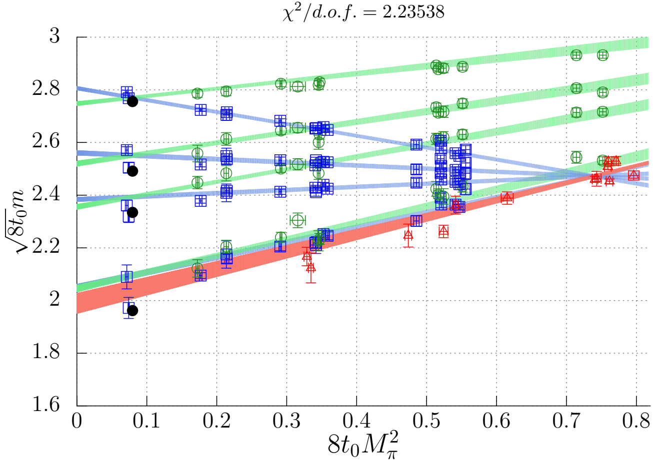
HBChPT:

$$f_O(x) = -\pi x^3.$$

In preparation: effect of decuplet baryon loops within small scale expansion.

Order p^2 BChPT (preliminary)

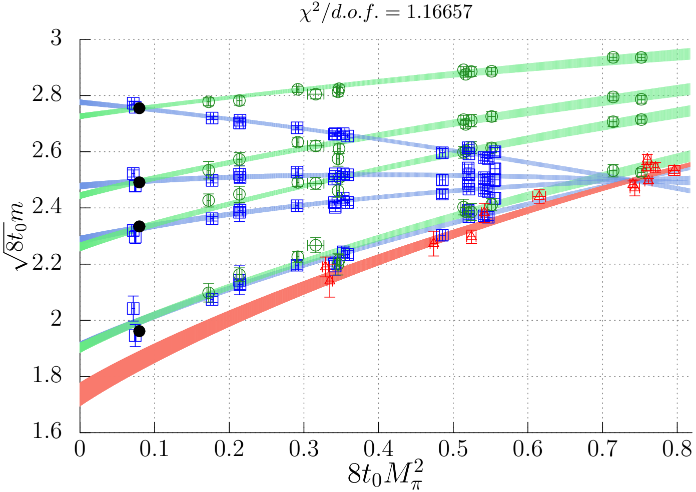
N, Λ, Σ, Ξ, $\hat{m}_s \approx \hat{m}_{s,\text{ph}}$, $m_s + 2m_\ell \approx$ phys., $m_s = m_\ell$



Data projected to $a = 0$ and along quark mass trajectories according to the fit.
Scale set using $\sqrt{8t_0^*} = 0.413\,\text{fm}$. Black circles: experiment.

Order p^3 covariant BChPT (preliminary)

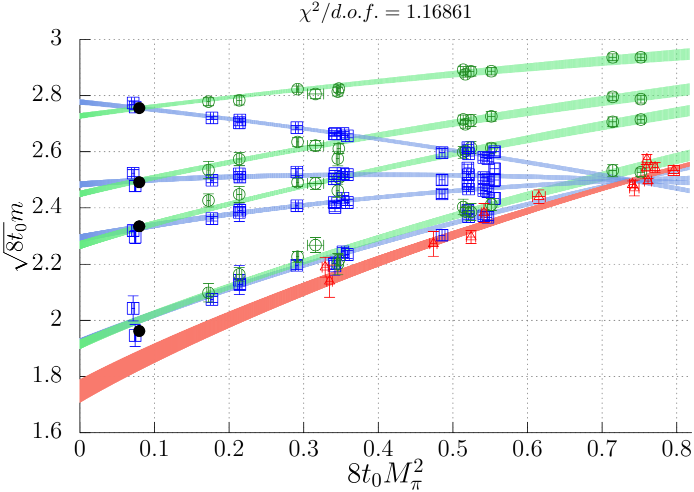
N, Λ, Σ, Ξ, $\hat{m}_s \approx \hat{m}_{s,\text{ph}}$, $m_s + 2m_\ell \approx$ phys., $m_s = m_\ell$



Scale determined from $m_\Xi = 1316.9(3)$ MeV: $\sqrt{8t_{0,\text{ph}}} = 0.4128(22)$ fm.

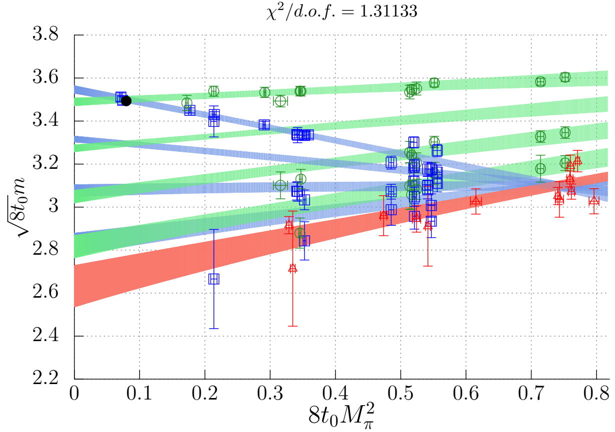
Order p^3 HBChPT (preliminary)

N, Λ, Σ, Ξ, $\hat{m}_s \approx \hat{m}_{s,\text{ph}}$, $m_s + 2m_\ell \approx$ phys., $m_s = m_\ell$



Scale determined from $m_\Xi = 1316.9(3)$ MeV: $\sqrt{8t_{0,\text{ph}}} = 0.4129(22)$ fm.

Δ, Σ^*, Ξ^*, Ω, $\hat{m}_s \approx \hat{m}_{s,\text{ph}}$, $m_s + 2m_\ell \approx$ phys., $m_s = m_\ell$

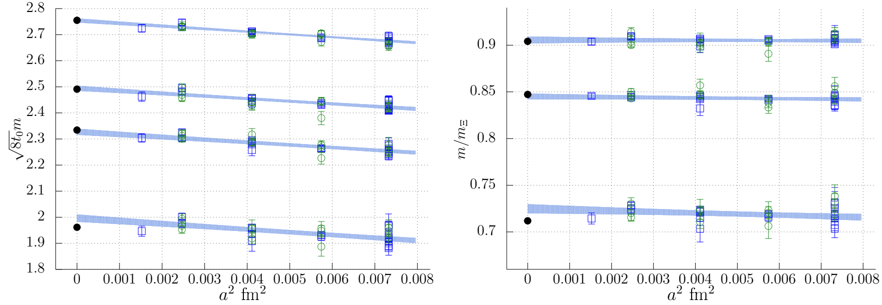


Scale set to $\sqrt{8t_{0,\text{ph}}} = 0.413\,\text{fm}$. Ω is spot on.

Problem: strong decays (unstable baryons not shown).

Plan: covariant BChPT including octet baryon loops.

The continuum limit (order p^3 BChPT, preliminary)



$\sqrt{8t_{0,\text{ph}}} = 0.413(6)\,\text{fm}$ from $F_\pi + 2F_K$ on a subset of CLS ensembles.
[ALPHA: M Bruno et al, 1608.08900; 1706.03821]

$\sqrt{8t_{0,\text{ph}}} = 0.414(7)\,\text{fm}$ from m_Ω.
[BMWc: S Borsanyi et al, 1203.4469]

Our result: $\sqrt{8t_{0,\text{ph}}} = 0.4128(22)(??)\,\text{fm}$.
(systematics not yet fully determined)

σ terms (preliminary)

Results on σ_s are parametrization-, not data-driven since we do not vary m_s near the physical m_ℓ.

Pion σ terms can be determined with more confidence:

$$\sigma_{\pi N} = 41(2)(2)(?)\,\text{MeV}, \qquad \sigma_{\pi\Lambda} = 29(2)(1)(?)\,\text{MeV},$$
$$\sigma_{\pi\Sigma} = 23(1)(1)(?)\,\text{MeV}, \qquad \sigma_{\pi\Xi} = 13(1)(0)(?)\,\text{MeV}.$$

Errors are statistical and difference between BChPT and HBChPT.
The fit range dependence and impact of other parametrizations are yet to be investigated.

LECs seem reasonably stable but $D/F > 2$:
BChPT parametrization describes the data vs.
BChPT describes the data?

This question can be addressed, varying fit ranges and combining with results on baryon structure.

Summary

- Several limits need to be taken:
  $t \to \infty$, $m_q \to m_q^{\text{phys}}$, $V = a^4 N_t N_s^3 \to \infty$, $a \to 0$.
- Wilson fermions are theoretically clean.
- Chiral symmetry will be restored in the continuum limit.
  Drawback: unlike for overlap fermions that have a chiral symmetry at $a > 0$, order a improvement is needed and operator mixing is more involved.
- Within CLS we implement full order a improvement and vary a^2 by a factor ≈ 5. This is possible using open boundary conditions in time.
- At $a^{-1} \gtrsim 4\,\text{GeV}$ the physical point will require $N_s = 128$. This is expensive. Instead, we carry out joint extrapolations along two quark mass trajectories, only realizing the physical point for $a^{-1} \lesssim 3\,\text{GeV}$.
- Here I showed some results on the baryon spectrum.
- Soon: SU(3) and SU(2) LECs, light and charmed hadron spectroscopy etc.