Study of Finite Size Effect on Hadron Masses and Decay Constants with \((5.4\text{fm})^4\) and \((10.8\text{fm})^4\) Lattices at the Physical Point in 2+1 Flavor QCD

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Plan of talk

• PACS Collaboration Members
• Simulation Parameters for 2+1 Flavor QCD
• Quick Review of Results for Pseudoscalar Sector
• Results for Vector Meson and Baryon Sectors
  – Comparison btw 128\(^4\) and 64\(^4\)
  – Ω baryon
  – Determination of Physical Point
• Summary
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Simulation Parameters for 2+1 Flavor QCD (1)

- Wilson-clover quark action + Iwasaki gauge action
- Stout smearing with $\alpha=0.1$ and $N_{\text{smear}}=6$
- NP $C_{SW}=1.11$ determined by SF
- $\beta=1.82 \Rightarrow a^{-1} \approx 2.33$ GeV
- Lattice size=$128^4 \Rightarrow \sim (11 \text{ fm})^4$
- Hopping parameters: $(\kappa_{ud}, \kappa_{s})=(0.126117, 0.1247902)$
  $\Rightarrow m_\pi \approx 135$ MeV, $m_\pi L \approx 7.5$
- Simulation algorithm
  - $(\text{HB})^2$DDHMC w/ active link for ud quarks,
  - RHMC w/ $N_{\text{RHMC}}=8; [0:00025; 1:85]$ for s quark
  - Block size=16x16x8x64
  - HB parameters: $(\rho_1, \rho_2)=(0.9997, 0.9940)$
  - Multi-time scale integrator: $(N_0, N_1, N_2, N_3, N_4)=(8, 2, 2, 2, 22)$
  - trajectory length: $\tau=1.0$
  - Chronological inverter guess
  - Solver: mixed precision nested BiCGStab
Simulation Parameters for 2+1 Flavor QCD (2)

- Wilson-clover quark action + Iwasaki gauge action
- Stout smearing with $\alpha=0.1$ and $N_{\text{smear}}=6$
- NP $C_{SW}=1.11$ determined by SF
- $\beta=1.82 \Rightarrow a^{-1} \approx 2.33 \text{ GeV}$
- Lattice size $= 64^4 \Rightarrow \sim (5.5 \text{ fm})^3$ spatial volume
- Hopping parameters: $(\kappa_{ud}, \kappa_{s})=(0.126117, 0.1247902)$
  $\Rightarrow m_\pi \approx 138 \text{ MeV}, m_\pi L \approx 7.6$

- Simulation algorithm
  - $(HB)^2$DDHMC w/ active link for ud quarks,
  - UVPHMC w/ $N_{\text{poly}}=350$ for s quark
  - Block size $= 8 \times 8 \times 16 \times 32$
  - HB parameters: $(\rho_1, \rho_2)=(0.9997, 0.9940)$
  - Multi-time scale integrator: $(N_0, N_1, N_2, N_3, N_4)=(8, 2, 2, 2, 12)$
  - Trajectory length: $\tau=1.0$
  - Chronological inverter guess
  - Solver: mixed precision nested BiCGStab
Clear finite size effect on $\pi$ meson mass

$m_{\pi}(L=64)$ is heavier than $m_{\pi}(L=128)$ by 2.1(8)\%

$m_{ud}(L=64)$ is heavier than $m_{ud}(L=128)$ by 4.8(1.6)\%

It is hard to detect the finite size effect on $K$ meson mass
Finite Size Effect on PS Sector (2)

PACS, PRD 99(2019)014504

\[ \pi \] meson decay const \hspace{1cm} \[ K \] meson decay const \hspace{1cm} ratio

2σ difference after tuning \( m_{\pi}(L=64)=m_{\pi}(L=128) \)

\( f_{\pi}(L=64) \) is smaller than \( f_{\pi}(L=128) \) by 0.66(33)%

\( f_{K}(L=64) \) is smaller than \( f_{K}(L=128) \) by 0.26(13)%

The deviation becomes smaller for \( f_{K}/f_{\pi} \)
Results for Vector Meson Sector

Measurements (128\(^4\): 7679, 64\(^4\): 25573) were carried out by AMA method

Finite size effects are negligible within error bars
No plateau is observed
Effective masses go below the experimental resonance value in the large time region

⇒ Need appropriate treatment to extract the energy levels
Results for Octet Baryon Sector

Finite size effects are negligible within error bars
Reasonable plateau regions are observed
Finite size effects seem negligible
No plateau similar to vector mesons
Similar signal with other decuplet baryons
Plateau in the large time region \( t \gtrsim 16 \)?
Finite size effect in \( t \gtrsim 16 \)?

\[ \Rightarrow \text{Need to check with high statistics employing a different source and a different solver algorithm} \]
Precision measurement with BCC Ball Source

• Ball sources are filled in 3D spatial lattice with the body-centered-crystal (BCC) structure

• BCC Ball source is a variant of grid source
  
  Li et al., PRD82(2010)114501; PRD88(2013)014503

• \((n_{\text{ball}},r_{\text{ball}}) = (128,13.9)\) for \(128^4\) lattice
• \((n_{\text{ball}},r_{\text{ball}}) = (16,13.9)\) for \(64^4\) lattice

• Exponential smearing: \(A\exp(-Br)\) w/ \((A=1.0,B=0.06)\)

• Solver: mixed precision nested BiCGStab
• Strict tolerance \(|Dx-b|/|b|<10^{-15}\)
• No. measurements: 2560 for \(128^4\) and 25600 for \(64^4\)
**Ω Baryon with BCC Ball Source**

Error bar is significantly reduced with bcc ball source method.

No plateau before signal is lost beyond \( t \approx 20 \)

⇒ impossible to extract \( \Omega \) baryon mass with single exponential fit
Possible Mixing States on the Lattice

<table>
<thead>
<tr>
<th>Channel $^2O$ rep.</th>
<th>$j$</th>
<th>$j'$</th>
<th>nearby two-body state</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$ A$_1$</td>
<td>0</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>$K$ A$_1$</td>
<td>0</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>$\rho$ G$_1$</td>
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<td>3</td>
<td>$\pi\pi$</td>
</tr>
<tr>
<td>$K^*$ G$_1$</td>
<td>1</td>
<td>3</td>
<td>$K\pi$</td>
</tr>
<tr>
<td>$\phi$ G$_1$</td>
<td>1</td>
<td>3</td>
<td>$KK$</td>
</tr>
<tr>
<td>$N$ T$_1$</td>
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<td>7/2</td>
<td></td>
</tr>
<tr>
<td>$\Lambda$ T$_1$</td>
<td>1/2</td>
<td>7/2</td>
<td></td>
</tr>
<tr>
<td>$\Sigma$ T$_1$</td>
<td>1/2</td>
<td>7/2</td>
<td></td>
</tr>
<tr>
<td>$\Xi$ T$_1$</td>
<td>1/2</td>
<td>7/2</td>
<td></td>
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<tr>
<td>$\Delta$ H</td>
<td>3/2</td>
<td>5/2</td>
<td>$N\pi$</td>
</tr>
<tr>
<td>$\Sigma^*$ H</td>
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<td>$\Lambda\pi$</td>
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<td>$\Xi^*$ H</td>
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<td>5/2</td>
<td>$\Xi\pi$</td>
</tr>
<tr>
<td>$\Omega$ H</td>
<td>3/2</td>
<td>5/2</td>
<td>$\Xi K$</td>
</tr>
</tbody>
</table>

Possible contaminations to decuplet baryon ($j=3/2$):
- higher spin states of $j' = 5/2$
- $\Xi K$ state ($m_\Xi + m_K - m_\Omega \approx 140$ MeV)
Determination of Physical Point on $128^4$ lattice (1)

Physical inputs: $m_\pi$, $m_K$, $m_\Xi$

Taylor expansion around the simulation point in terms of $m_{ud}$ and $m_s$

\begin{align*}
    m_{\pi}^2 &= m_{\pi}^2 \big|_{\text{org}} + \frac{\partial m_{\pi}^2}{\partial m_{ud}} \big|_{\text{org}} (m_{ud} - m_{ud} \big|_{\text{org}}) \\
    &\quad + \frac{\partial m_{\pi}^2}{\partial m_s} \big|_{\text{org}} (m_s - m_s \big|_{\text{org}}), \\
    m_{K}^2 &= m_{K}^2 \big|_{\text{org}} + \frac{\partial m_{K}^2}{\partial m_{ud}} \big|_{\text{org}} (m_{ud} - m_{ud} \big|_{\text{org}}) \\
    &\quad + \frac{\partial m_{K}^2}{\partial m_s} \big|_{\text{org}} (m_s - m_s \big|_{\text{org}}), \\
    m_{\Xi} &= m_{\Xi} \big|_{\text{org}} + \frac{\partial m_{\Xi}}{\partial m_{ud}} \big|_{\text{org}} (m_{ud} - m_{ud} \big|_{\text{org}}) \\
    &\quad + \frac{\partial m_{\Xi}}{\partial m_s} \big|_{\text{org}} (m_s - m_s \big|_{\text{org}}). 
\end{align*}

Coefficients are determined from the reweighted results on $64^4$ lattice
Determination of Physical Point on $128^4$ lattice (2)

Reweighting point in $m_{ud}$ – $m_s$ plane

$\Delta m_{ud}: 4.4\%$, $\Delta m_s: \pm 1.6\%$
Reweighted data are well described by the linear expansion
Determination of Physical Point on $128^4$ lattice (4)

Physical point is consistent with simulation point on $128^4$ lattice

The scale is determined to be $1/a=2.3162(44)$ GeV
Summary

2+1 flavor QCD simulation at the physical point on $128^4$ and $64^4$ lattices

• Clear finite size effect for PS meson sector

• Hard to detect finite size effects for vector meson and baryon sectors

• $\Omega$ baryon seems “unstable” on the lattice

• Simulation point was successfully tuned to the physical point

Future plan

• Investigation of cut-off effects with finer lattice

• Calculation of various physical quantities