Spectroscopy of mesons with bottom quarks

an exploratory study

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OUTLINE

- Motivation and lattice details.
- Tests of the heavy quark discretisation in quarkonium and heavy-light mesons.
- Results
  - The spectrum of excited and exotic states up to J=4.
  - Focus on hybrids - a lightest supermultiplet identified.
- Conclusions & ongoing work.

MOTIVATION

- Expect the renaissance in spectroscopy to continue and flourish with anticipated results from Belle II to come.
- New and unexpected results in charmonium have prompted lattice studies.
- Open question whether a similarly rich spectrum is predicted/discovered in bottomonium.
- Motivates this exploratory study of bottomonium.

Heavy quarks & anisotropic lattices:

- The b quark presents a well-known problem: potentially large discretisation errors which must be removed or controlled.
- Solutions include an EFT approach or extremely fine lattices such that $am_Q < 1$.
- Anisotropic lattices have proven very useful for charm - explore their effectiveness at bottom quark masses.

HEAVY QUARKS AND ANISOTROPIC LATTICES

- Not a new story. Hashimoto, Onogi, Kronfeld and collaborators investigated the practicalities in early 2000s. [PRD64 114503(2001); PRD64 074501 (2001); PRD66 014509 (2002)].
- Investigate discretisation effects via dispersion relations and

$$I = \frac{2\delta M_{hl} - (\delta M_{hh} + \delta M_{ll})}{2M_{2hl}}, \quad \delta M = M_2 - M_1$$

  $I \neq 0 \Rightarrow$ inconsistent binding energies.
- Isolates $\mathcal{O}((a_s p)^2)$ discretisation effects in quarkonium. [NPB (proc suppl) 47 (1996); NPB (proc suppl) 53 (1997)]
- Strategy: monitor dispersion relations and I (via differences in M_2, M_1.
- Noting improvements on earlier studies include stout smearing of spatial links and mass-dependent anisotropy tuning.

LATTICE DETAILS

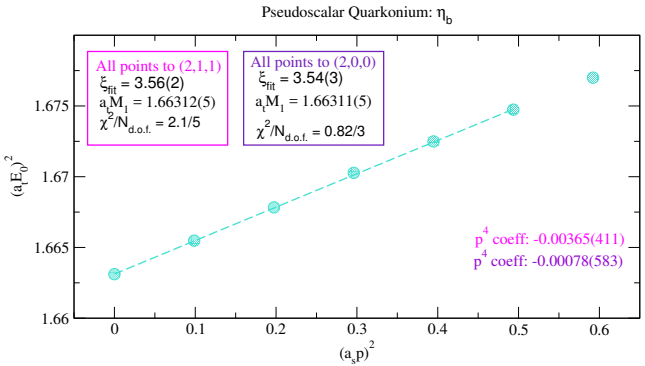
- Symanzik-improved anisotropic gauge action with tree-level tadpole-improved coefficients and $N_f = 2 + 1$.
- Anisotropic clover action with stout-smeared spatial links.
- $\xi = a_s/a_t = 3.5$.
- $a_s \approx 0.12\,\mathrm{fm}$, $a_t^{-1}(m_\Omega) = 5.67(4)\,\mathrm{GeV}$.
- $20^3 \times 128$ volume; also available: $24^3 \times 128$.
- $m_l \sim 400\,\mathrm{MeV}$
- Distillation for quark propagation.
- Operators of definite momenta constructed from up to 3 derivatives in each channel. In general forming a large bases for GEVP (up to 26 operators in e.g. T_1). Heavy-light operators with up to 2 derivates.

Related work:

- Charmonium: JHEP1207 (2012) 126, JHEP 1612 (2016) 089.
- Open charm: JHEP 05 (2013) 021.

THE LATTICE DISPERSION RELATION

- Fermion action has 2 parameters to tune: m_q and ξ.
- Tuned such that $M_{\eta_b}^{latt} = M_{\eta_b}^{expt}$ *and* a relativistic dispersion relation recovered.



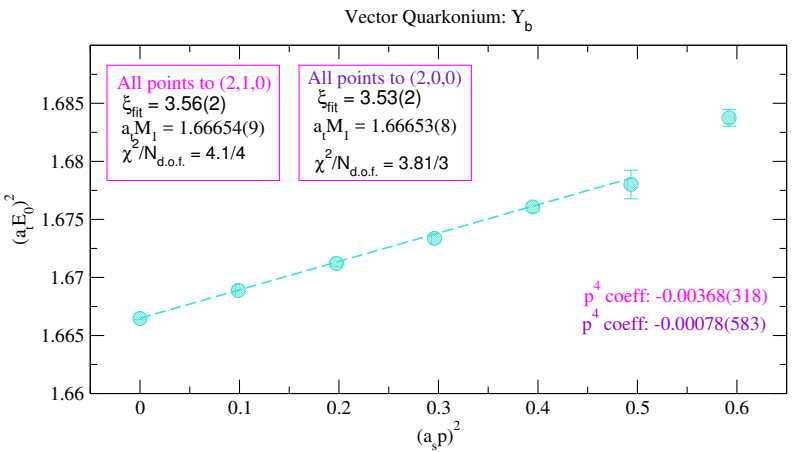
$$(a_tE)^2 = (a_tM)^2 + \left(\frac{2\pi}{\xi L/a_s}\right)^2 n^2.$$

- Momenta to $(2,0,0)$ used in tuning.
- $M_2^{\text{kinetic}} = M_1^{\text{rest}}$ (consistent with Fermilab approach).

- All other masses and dispersion relations are predictions.
- Consider Υ and heavy-light mesons at these tuned parameter values.

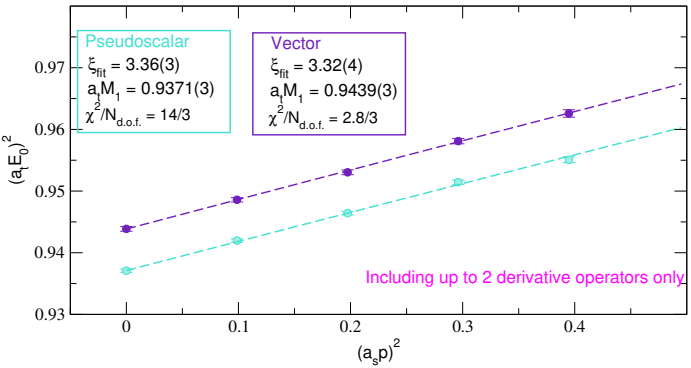
UPSILON

- Using m_b, ξ tuned for (pseudoscalar) η_b.



HEAVY-LIGHT (VECTOR AND PSEUDOSCALAR) MESONS

- Using m_b, ξ tuned for pseudoscalar η_b.



	D	Charmonium	B	Bottomonium
ξ	3.38(3)	3.50(2)	3.36(3)	3.56(2)
M (MeV)	1893(1) [1864.8]		5313(2) [5279.6]	

THE "USUAL" RECIPE FOR (MESON) SPECTROSCOPY

- Construct a basis of local and non-local operators $\bar{\Psi}(x)\Gamma D_i D_j \ldots \Psi(x)$ from *distilled* fields [PRD80 (2009) 054506].
- Build a correlation matrix of two-point functions

$$C_{ij} = \langle 0|\mathcal{O}_i \mathcal{O}_j^\dagger|0\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E_n} e^{-E_n t}$$

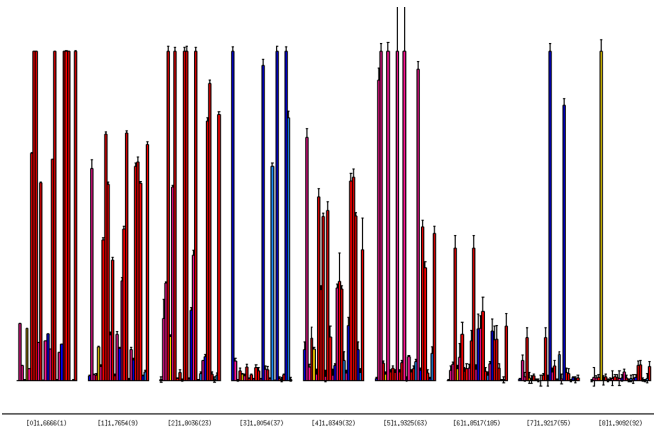
- Ground state mass from fits to $e^{-E_n t}$
- Beyond ground state: Solve generalised eigenvalue problem $C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$

- eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t}\left[1 + O(e^{-\Delta E t})\right]$ - principal correlator
- eigenvectors: related to overlaps $Z_i^{(n)} = \sqrt{2E_n} e^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$

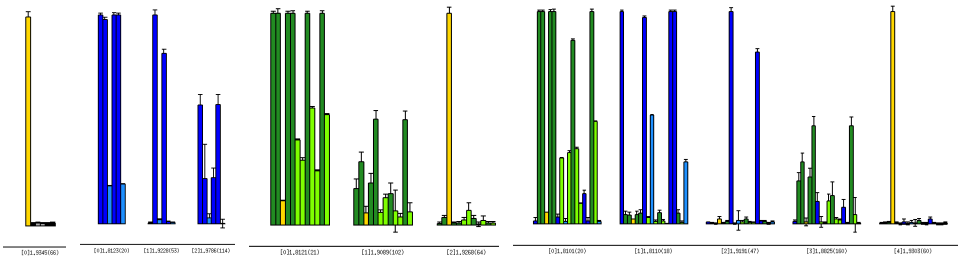
- operators of definite J^{PC} constructed in step 1 are subduced into the relevant irrep
- a subduced irrep carries a "memory" of continuum spin J from which it was subdduced - it **overlaps** predominantly with states of this J.

J	0	1	2	3	4
A_1	1	0	0	0	1
A_2	0	0	0	1	0
E	0	0	1	0	1
T_1	0	1	0	1	1
T_2	0	0	1	1	1

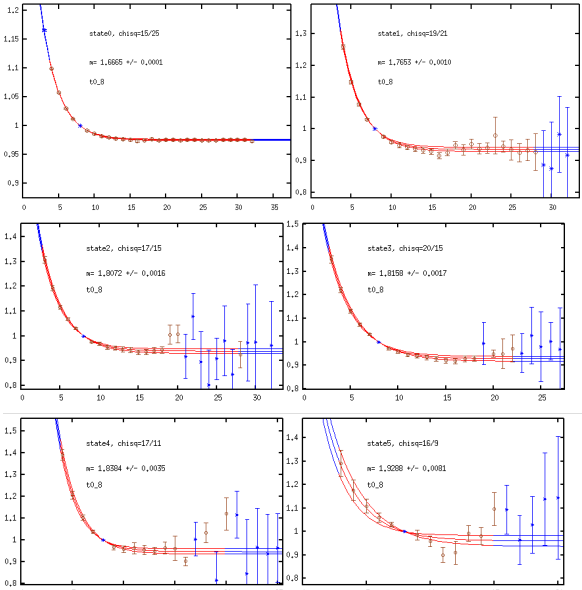
- Using $Z = \langle 0|\Phi|k\rangle$, helps to identify continuum spins
- For high spins, can look for agreement between irreps
- Example: T_1^{---} irrep, with **Spin 1**, **Spin 3** and **Spin 4**.

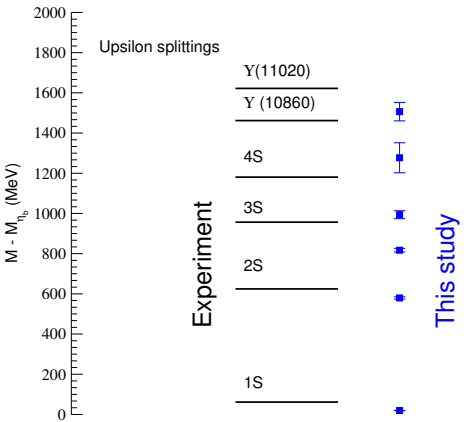


- All polarisations of the spin-4 state are seen
- Spin labelling: **Spin 2**, **Spin 3** and **Spin 4**.

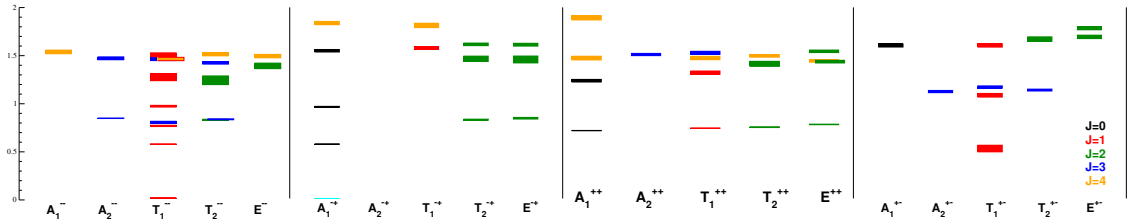


CORRELATORS (UPSILON)

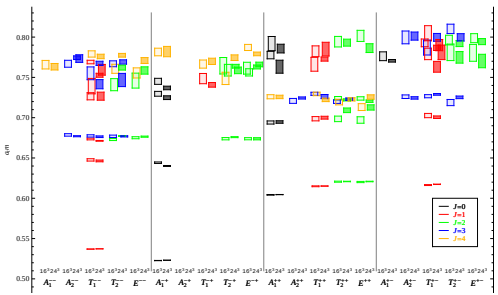




LATTICE SPECTRUM OF BOTTOMIUM ($M - M_{\eta_b}$): PRELIMINARY



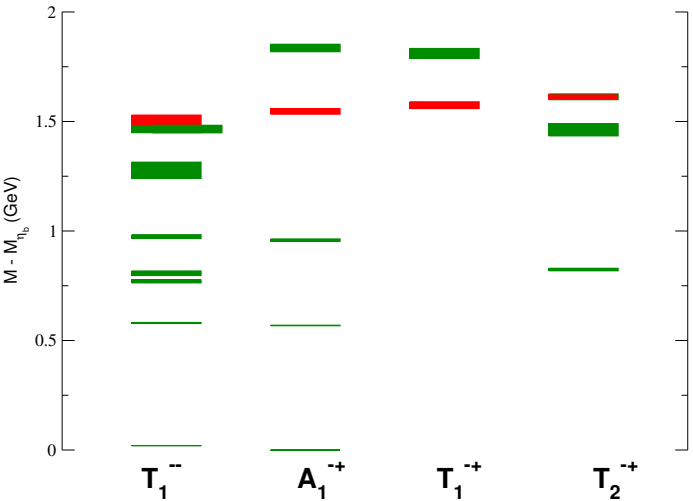
- Broadly similar to pattern of states in charmonium
- Also calculated $B_c = 6276(1)$ MeV [$B_c^{\text{expt}} = 6274.9 \pm 0.8$ MeV].
- Includes exotic channels: $0^{+-}, 1^{-+}, 2^{+-}$.



Charmonium JHEP1207 (2012)

HYBRIDS: PRELIMINARY

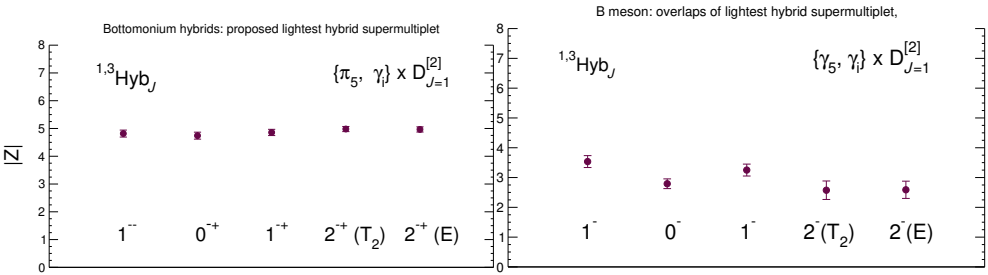
- Evidence of a lightest hybrid supermultiplet in bottomonium



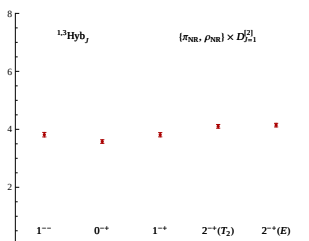
- A similar result found in B meson system.

HYBRIDS

Expect a large overlap with operators $\mathcal{O} \sim F_{\mu\nu}$.



Charmonium

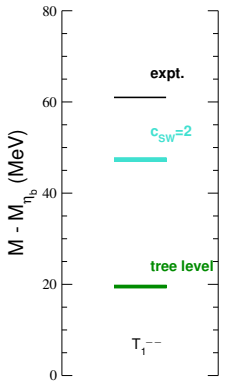


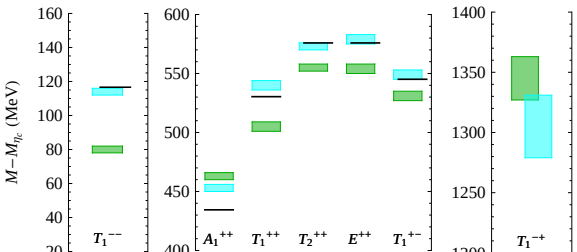
- Similar pattern and structure of hybrid states as seen in light, open-charm and charm (and baryons).
- Splitting to lightest state $\sim$ 1.5GeV - as previously.

SUMMARY & OUTLOOK

- An exploratory study of the spectra of mesons with heavy quarks following a mass-dependent anisotropy tuning is encouraging.
- Following the by-now well-established HadSpec Collaboration recipe a large basis of operators can be constructed, enabled by distillation.
- Preliminary results for bottomonium and B mesons presented:
  - Dispersion relations are consistent between heavy and heavy-light sectors, to large(ish) lattice momenta, for parameters tuned once at η_b.
  - Evidence of a hybrid supermultiplet is found in bottomonium and B mesons, with similar characteristics to those in charmonium, open-charm and light mesons as well as baryons.
  - Rotation breaking effects (at least for lower-lying states) are small.
  - A analogous heavy-light spectrum including hybrids is also determined.
- The study can be extended to larger volumes and lighter pion masses (all available).
- Paving the way for spectroscopy and decays of b-quark mesons.

BACKUP: (BRIEF) INVESTIGATION OF THE EFFECT OF c_{SW}





Note 1: different statistics for the tree-level and $c_{SW} = 2$ results in bottomonium.

Note 2: Same “improved” value of $c_{SW} = 2$ used for charm and bottom.