The b_1 resonance from lattice QCD

David Wilson

based on work with the **Hadron Spectrum Collaboration**:
Antoni Woss, Christopher Thomas, Jo Dudek,
Robert Edwards, David Wilson - **arXiv:1904.04136**

hadspec
hadspec.org

Lattice 2019
Wuhan

Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

Science Foundation Ireland sfi For what's next

THE ROYAL SOCIETY

‘natural’ spin-parities: $J^P = 0^+, 1^-, 2^+, \ldots$

e.g.: $\sigma(600)$, $\rho(770)$, $f_2(1270)$

seen in pseudoscalar-pseudoscalar scattering

‘unnatural’ spin-parities: $J^P = 0^-, 1^+, 2^-, \ldots$

e.g.: $\pi, a_1(1260), a_2(1320)$
$b_1(1235)$,

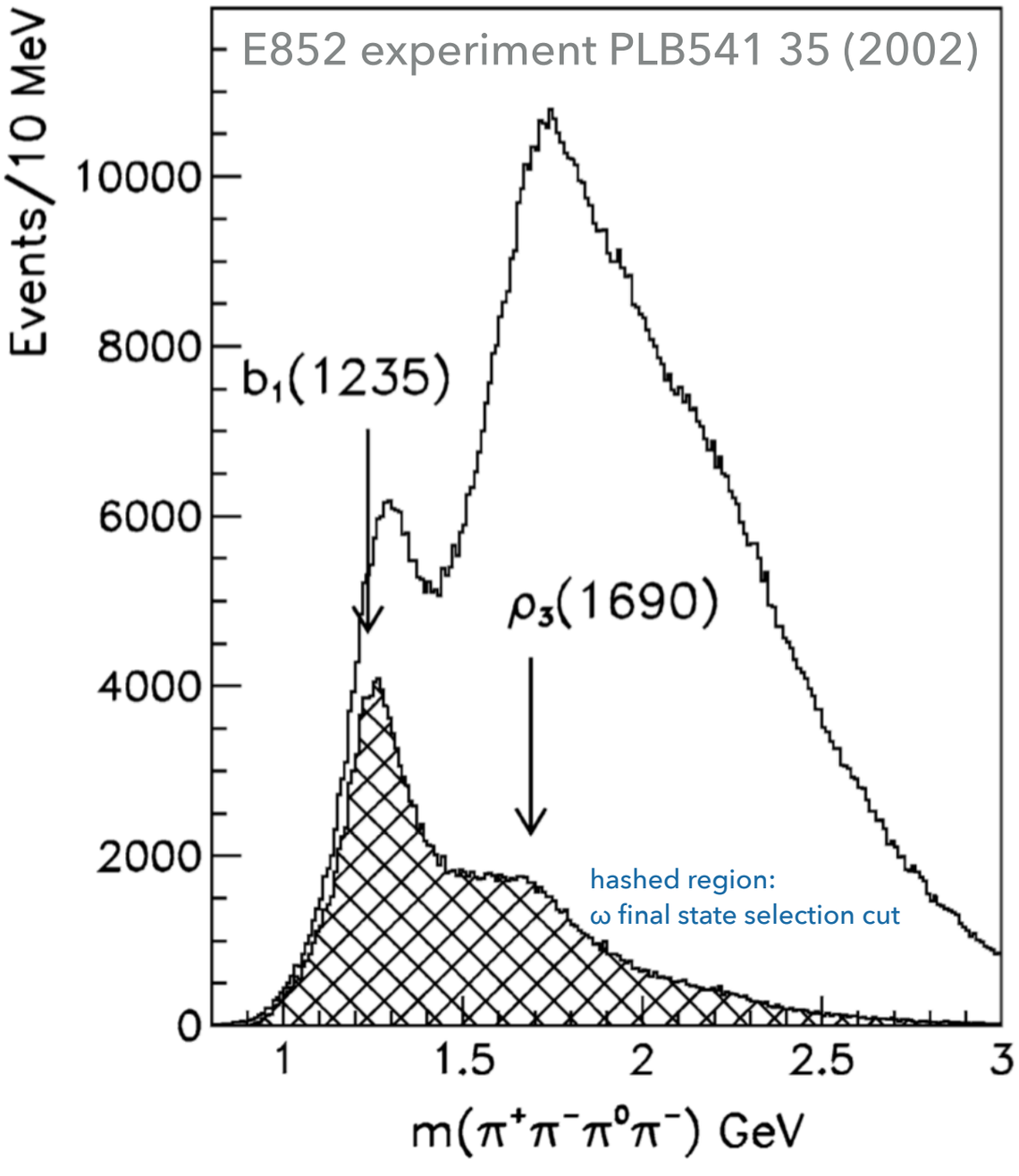
needs something more than pseudoscalar-pseudoscalar
e.g.: pseudoscalar-vector scattering or three-body scattering

experimentally the $b_1(1235)$ is the lightest axial vector resonance

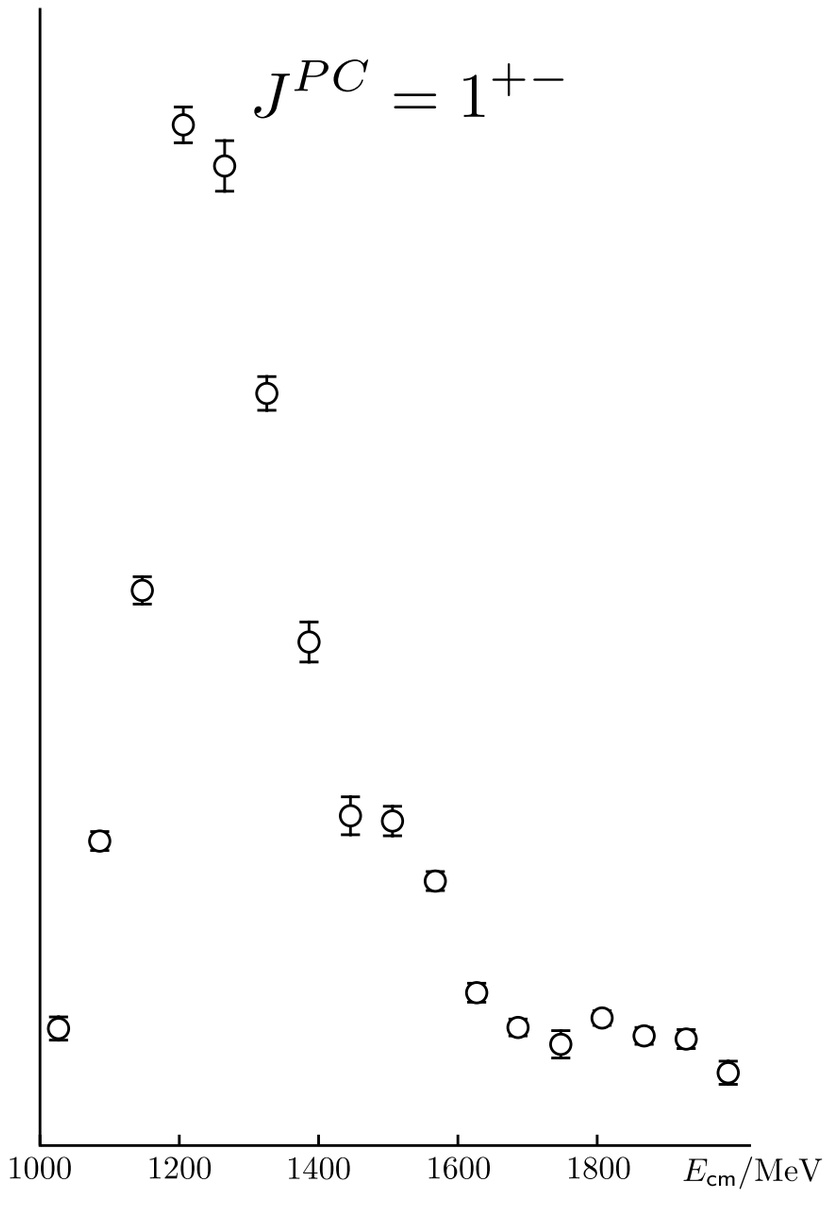
important step on the road to understanding highly excited states
- e.g.: hybrids

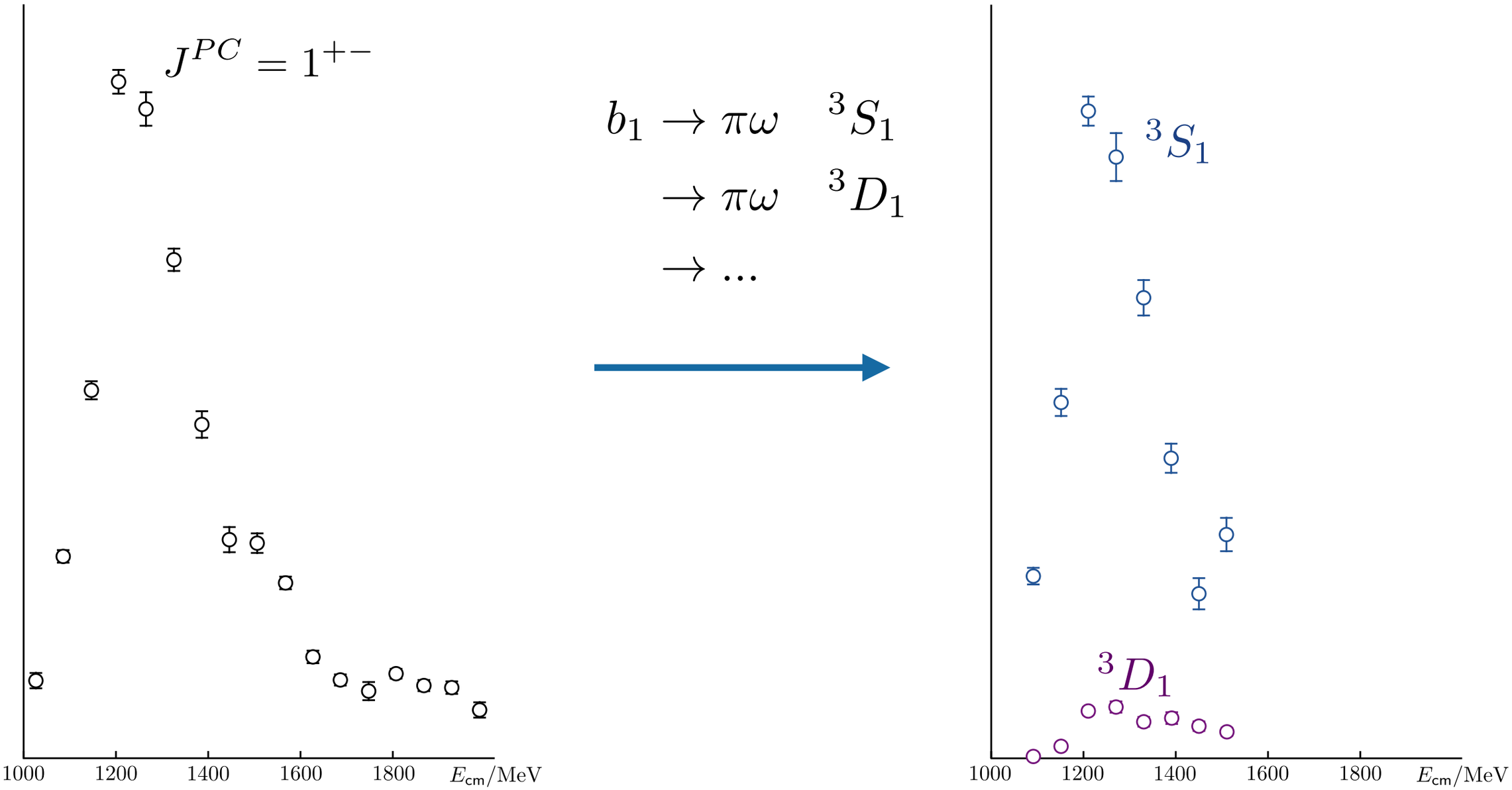
In the quark model:

b_1 $\quad q\bar{q}\,(^1P_1)$ $\quad J^{PC} = 1^{+-}$ $\quad \to \pi\omega \to \pi(\pi\pi\pi)$ $\quad \to \ldots$



partial wave analysis





E852 experiment PLB541 35 (2002)

In the quark model:

b_1 $q\bar{q}\left({}^1P_1\right)$ $J^{PC} = 1^{+-}$ $\rightarrow \pi\omega \rightarrow \pi(\pi\pi\pi)$ $\rightarrow \ldots$

cf: a_1 $q\bar{q}\left({}^3P_1\right)$ $J^{PC} = 1^{++}$ $\rightarrow \pi\pi\pi$ $\rightarrow \ldots$

working at a heavier than physical pion mass:

$$m_\pi = 391 \text{ MeV}$$
$$m_\omega = 881 \text{ MeV}$$

$m_\omega < 3m_\pi \quad \rightarrow$ ω is stable

earlier studies:
Lang et al JHEP 04 162 (2014)
Michael & McNeile PRD 73 074506

optimised operator formed from the eigenvector from the variational method, e.g.:

$$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$$

local qq-like constructions

$$\bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D}\ \psi$$

two-hadron constructions

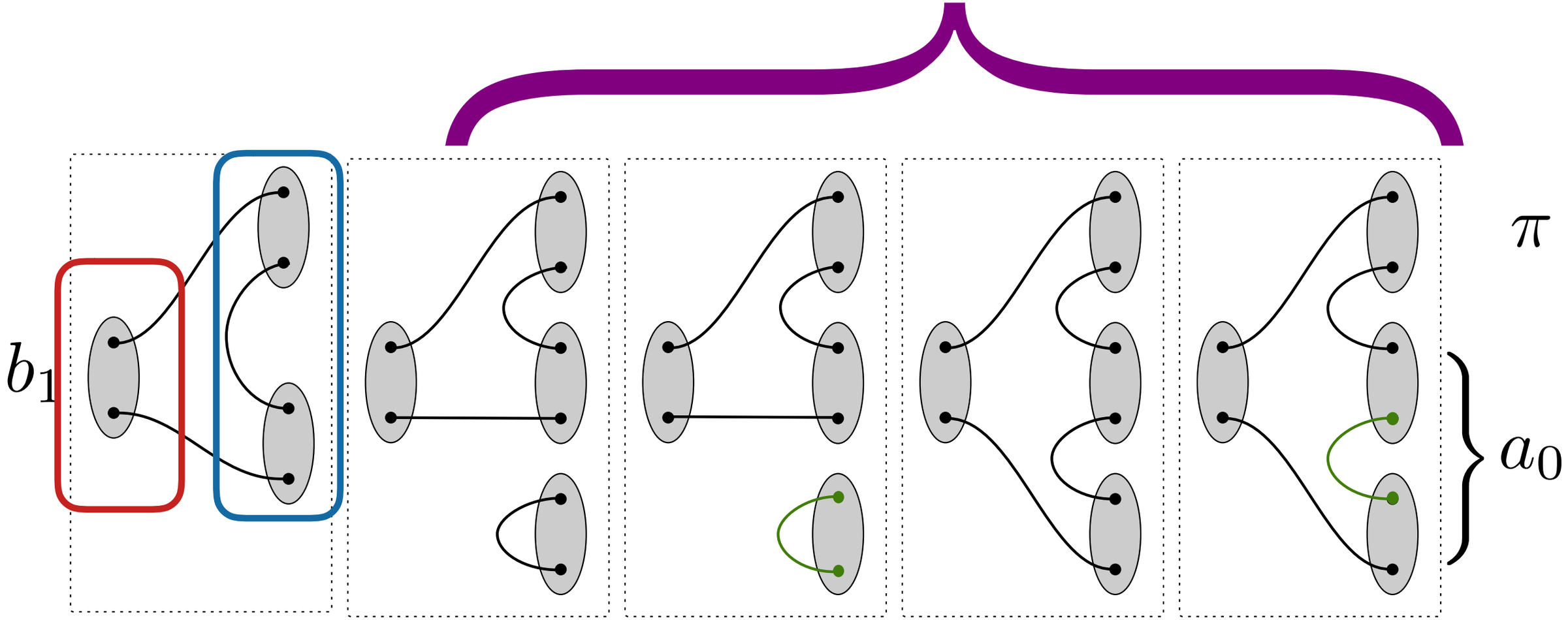
$$\sum_{\vec{p}_1+\vec{p}_2\in\vec{p}} C(\vec{p}_1,\vec{p}_2;\vec{p})\Omega_\pi(\vec{p}_1)\ \Omega_\omega(\vec{p}_2)$$

three-hadron constructions

$$\sum_{\vec{p}_1+\vec{p}_2\in\vec{p}} C(\vec{p}_1,\vec{p}_2\ \vec{p})\Omega_\pi(\vec{p}_1)\ \Omega_{a_0}(\vec{p}_2)$$

these “two-hadron” parts constructed from a_0, ρ, K* variational analysis

contains both meson-meson and local qq-like constructions



3 volumes: 2 - 3 fm
$L/a_s = 16, 20, 24$ $T/a_t = 128$ $m_\pi L \sim 4 - 6$
anisotropic action: 3.5 finer spacing in time

Symanzik-improved Wilson-Clover fermions

Distillation (Peardon *et al* 2009) to efficiently handle the many wick contractions

heavier-than-physical light quark masses
- suppresses multi-body channels

$m_\pi = 391$ MeV

$m_K = 549$ MeV

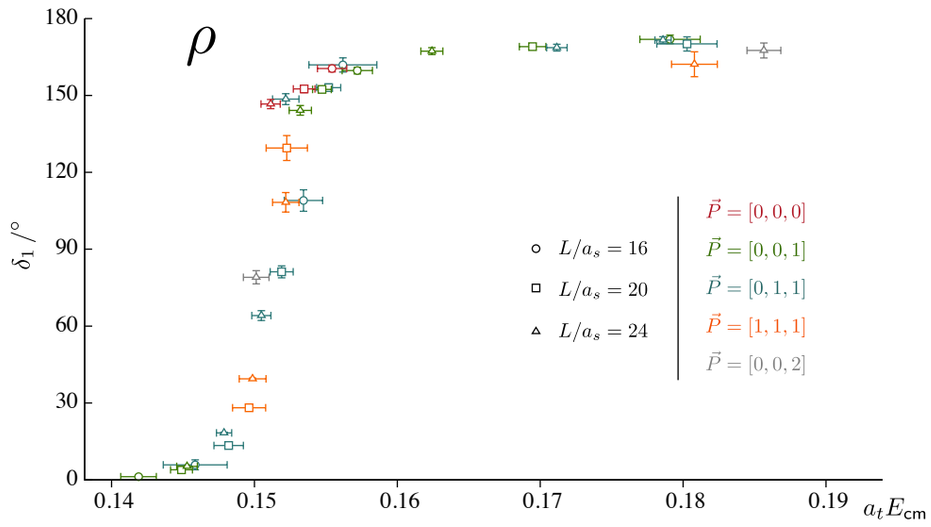
$m_\eta = 587$ MeV

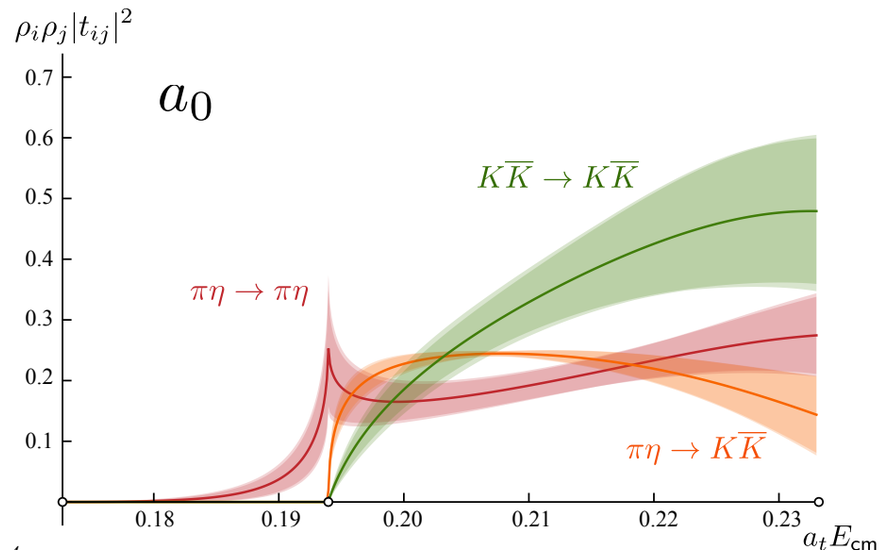
$m_\omega = 881$ MeV

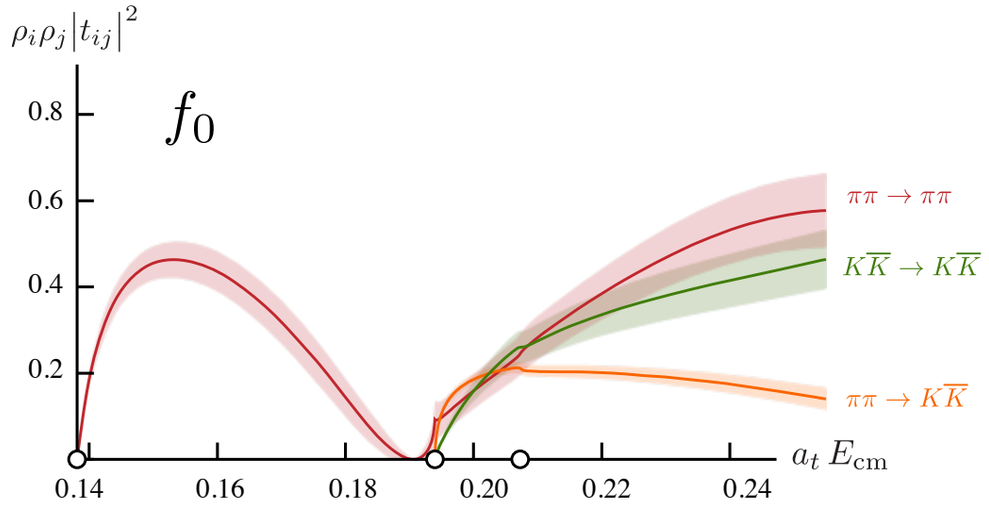
$m_\phi = 1017$ MeV

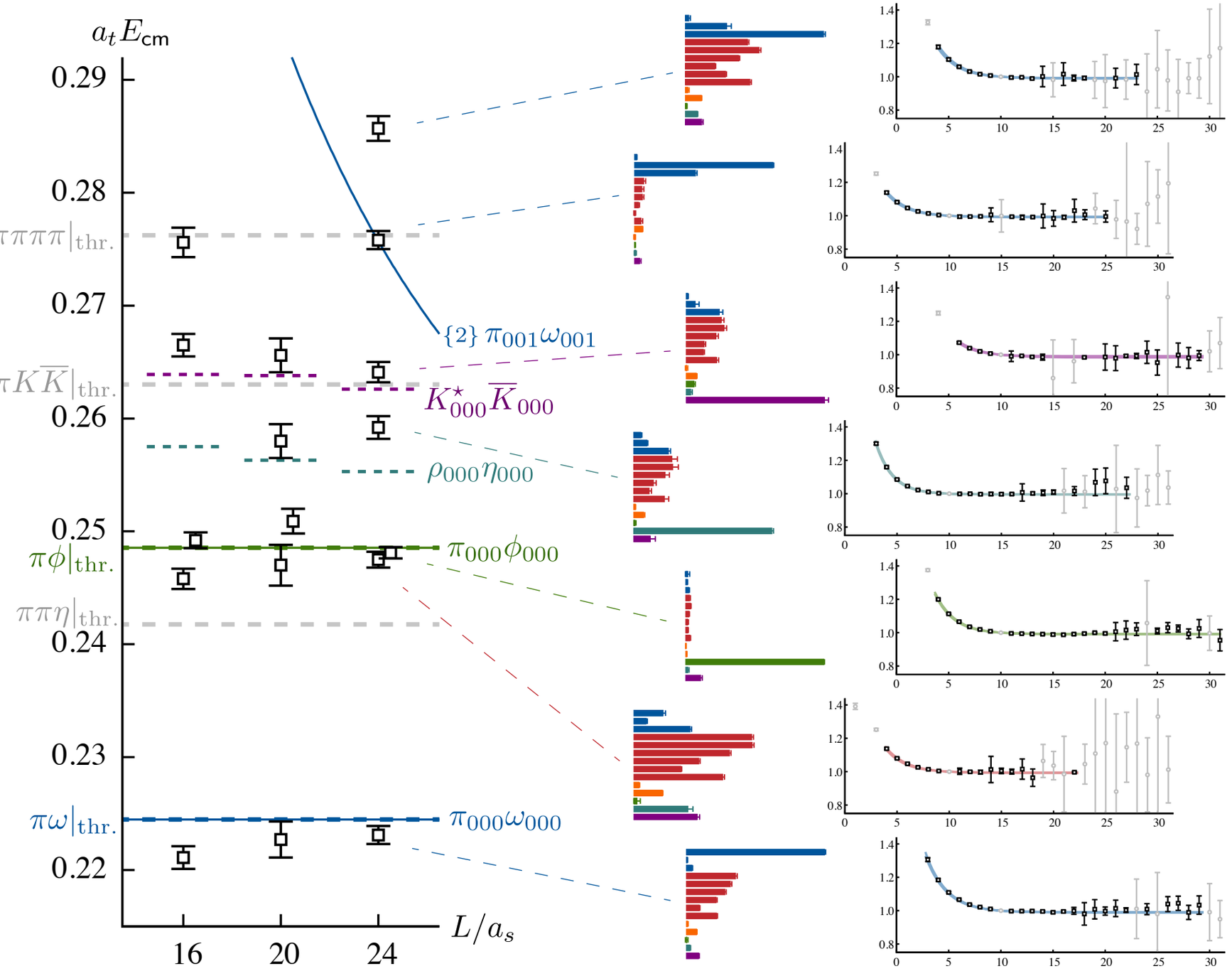
use moving frames
- different QC, more energies obtained

used in many calculations to-date

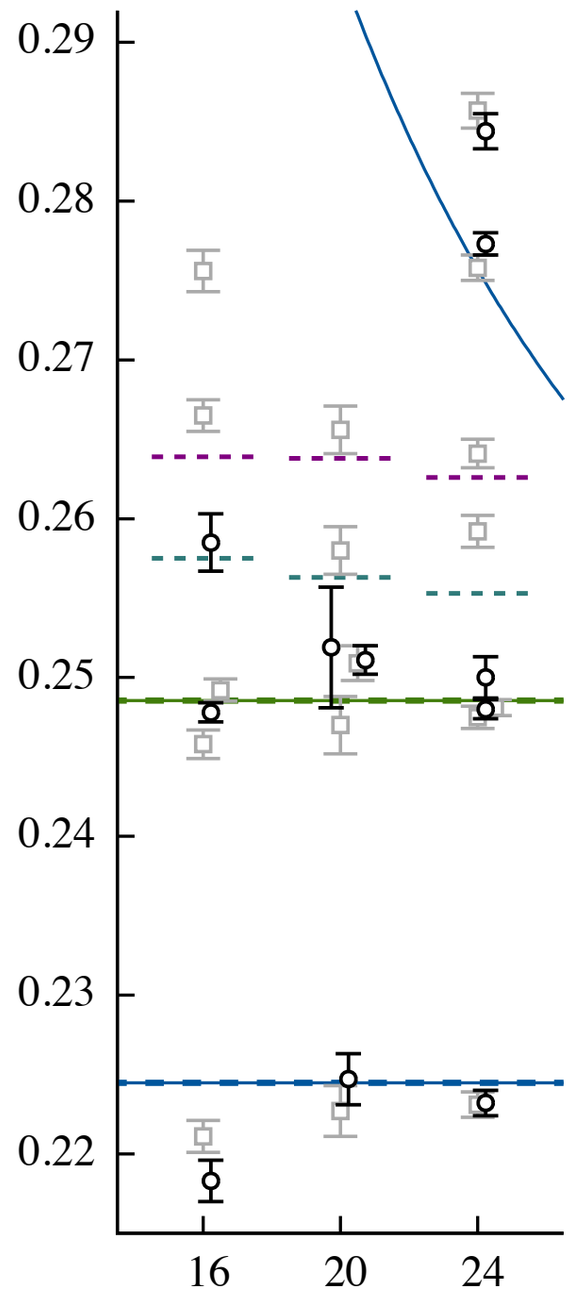


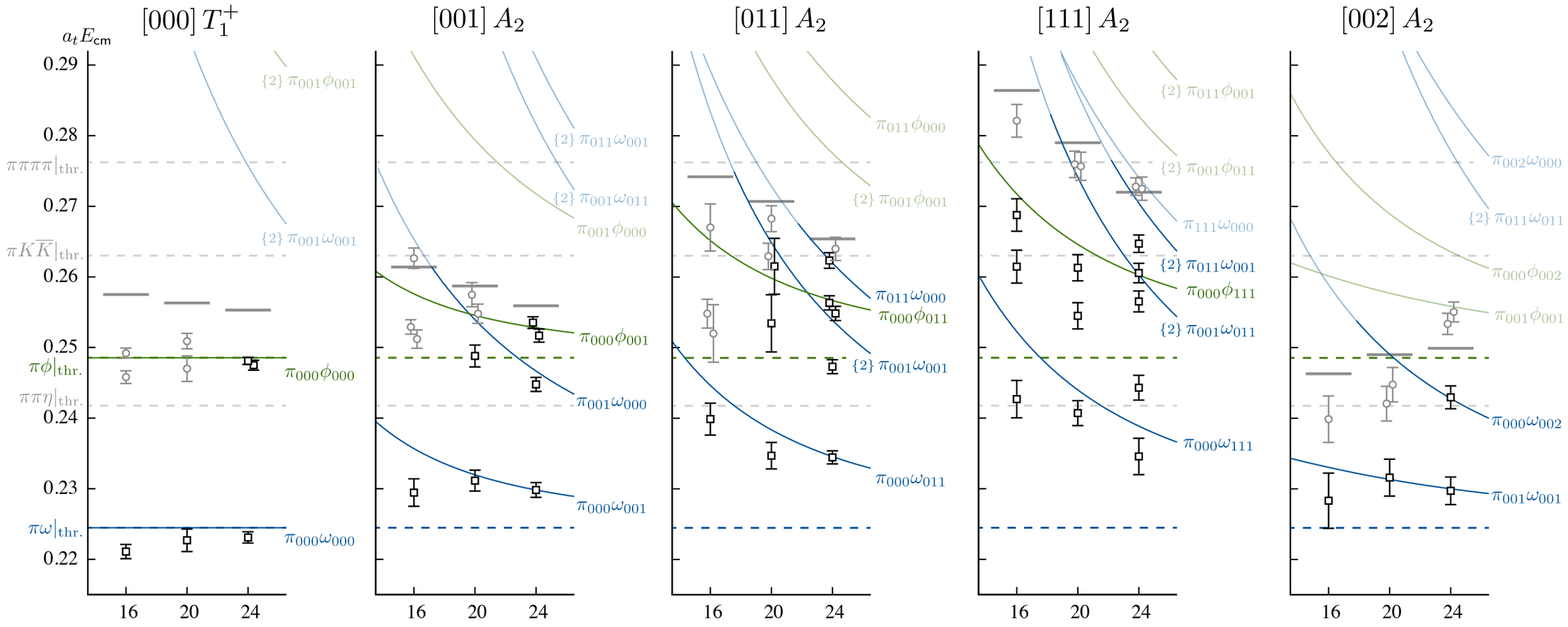


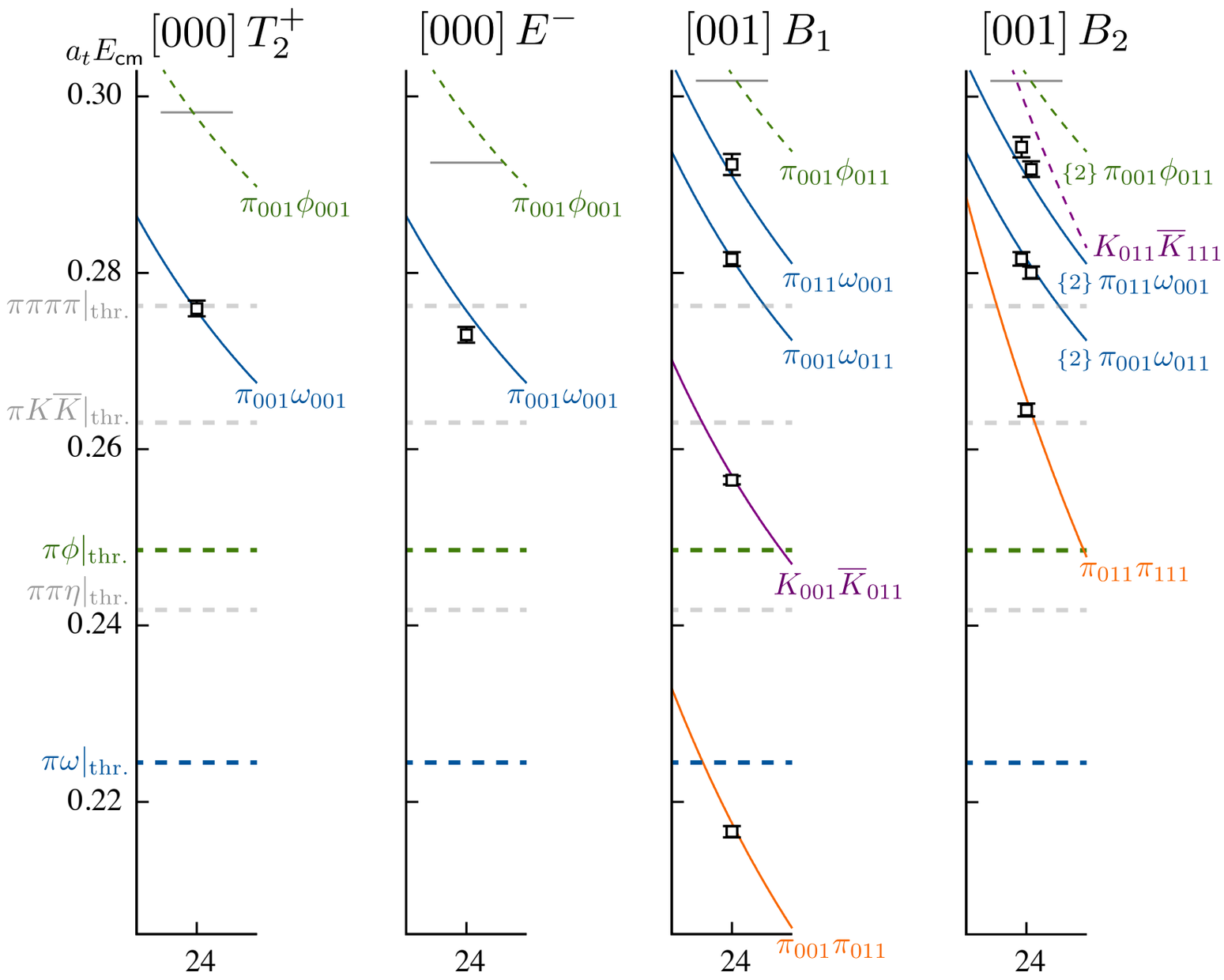




excluding $\rho\eta$, $K^\star\overline{K}$ ops.




$[000]\ T_1^+$
$[001]\ A_2$
$[011]\ A_2$
$[111]\ A_2$
$[002]\ A_2$
$a_t E_{\rm cm}$
0.29
0.28
0.27
0.26
0.25
0.24
0.23
0.22
$\pi\pi\pi\pi|_{\rm thr.}$
$\pi K\overline{K}|_{\rm thr.}$
$\pi\phi|_{\rm thr.}$
$\pi\pi\eta|_{\rm thr.}$
$\pi\omega|_{\rm thr.}$
16
20
24
$\{2\}\,\pi_{001}\phi_{001}$
$\{2\}\,\pi_{001}\omega_{001}$
$\pi_{000}\phi_{000}$
$\pi_{000}\omega_{000}$
$\{2\}\,\pi_{011}\omega_{001}$
$\{2\}\,\pi_{001}\omega_{011}$
$\pi_{001}\phi_{000}$
$\pi_{000}\phi_{001}$
$\pi_{001}\omega_{000}$
$\pi_{000}\omega_{001}$
$\pi_{011}\phi_{000}$
$\{2\}\,\pi_{001}\phi_{001}$
$\pi_{011}\omega_{000}$
$\pi_{000}\phi_{011}$
$\{2\}\,\pi_{001}\omega_{001}$
$\pi_{000}\omega_{011}$
$\{2\}\,\pi_{011}\phi_{001}$
$\{2\}\,\pi_{001}\phi_{011}$
$\pi_{111}\omega_{000}$
$\{2\}\,\pi_{011}\omega_{001}$
$\pi_{000}\phi_{111}$
$\{2\}\,\pi_{001}\omega_{011}$
$\pi_{000}\omega_{111}$
$\pi_{002}\omega_{000}$
$\{2\}\,\pi_{011}\omega_{011}$
$\pi_{000}\phi_{002}$
$\pi_{001}\phi_{001}$
$\pi_{000}\omega_{002}$
$\pi_{001}\omega_{001}$


$a_t E_{\text{cm}}$
$[000]\, T_2^+$
$[000]\, E^-$
$[001]\, B_1$
$[001]\, B_2$
0.30
0.28
0.26
0.24
0.22
$\pi\pi\pi\pi|_{\text{thr.}}$
$\pi K\overline{K}|_{\text{thr.}}$
$\pi\phi|_{\text{thr.}}$
$\pi\pi\eta|_{\text{thr.}}$
$\pi\omega|_{\text{thr.}}$
$\pi_{001}\phi_{001}$
$\pi_{001}\omega_{001}$
$\pi_{001}\phi_{011}$
$\pi_{011}\omega_{001}$
$\pi_{001}\omega_{011}$
$K_{001}\overline{K}_{011}$
$\pi_{001}\pi_{011}$
$\{2\}\,\pi_{001}\phi_{011}$
$K_{011}\overline{K}_{111}$
$\{2\}\,\pi_{011}\omega_{001}$
$\{2\}\,\pi_{001}\omega_{011}$
$\pi_{011}\pi_{111}$
24
24
24
24

Direct extension of the elastic quantization condition

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E) \cdot \boldsymbol{t}(E) \cdot (\mathbf{1} + i\boldsymbol{\mathcal{M}}(E, L))\right] = 0$$

phase space

infinite volume scattering *t*-matrix

known finite-volume functions

Elastic scattering: Lüscher 1986,1991
Generalised to moving frames: Gottlieb, Rummukainen 1995
Unequal masses: Prelovsek, Leskovec 2012

Many derivations of the coupled-channel extension, **all in agreement**:
He, Feng, Liu 2005 - two channel QM, strong coupling
Hansen & Sharpe 2012 - field theory, multiple two-body channels
Briceño & Davoudi 2012 - strongly-coupled Bethe-Salpeter amplitudes
Guo et al 2012 - Hamiltonian & Lippmann-Schwinger

Also derivations in specific channels, or for a specific parameterization of the interactions like NREFT, chiral PT, Finite Volume Hamiltonian, etc.

Briceño 2014 - Generalised to scattering of particles with non-zero spin, and spin-½.

Significant steps towards a general 3-body quantization condition are being made

Extracting the t-matrix

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L))\right] = 0 \qquad \mathbf{t} = \begin{pmatrix} \pi\pi\to\pi\pi & \pi\pi\to K\bar{K} \\ K\bar{K}\to\pi\pi & K\bar{K}\to K\bar{K} \end{pmatrix}$$

determinant condition:
- several unknowns at each value of energy
- energy levels typically do not coincide
- underconstrained problem for a single energy

one solution: use energy dependent parameterizations
- Constrained problem when #(energy levels) > #(parameters)
- Essential amplitudes respect unitarity of the S-matrix

$$\mathbf{S}^\dagger\mathbf{S} = \mathbf{1} \quad \rightarrow \quad \mathrm{Im}\,\mathbf{t}^{-1} = -\boldsymbol{\rho} \qquad \rho_{ij} = \delta_{ij}\frac{2k_i}{E_{\mathrm{cm}}}$$

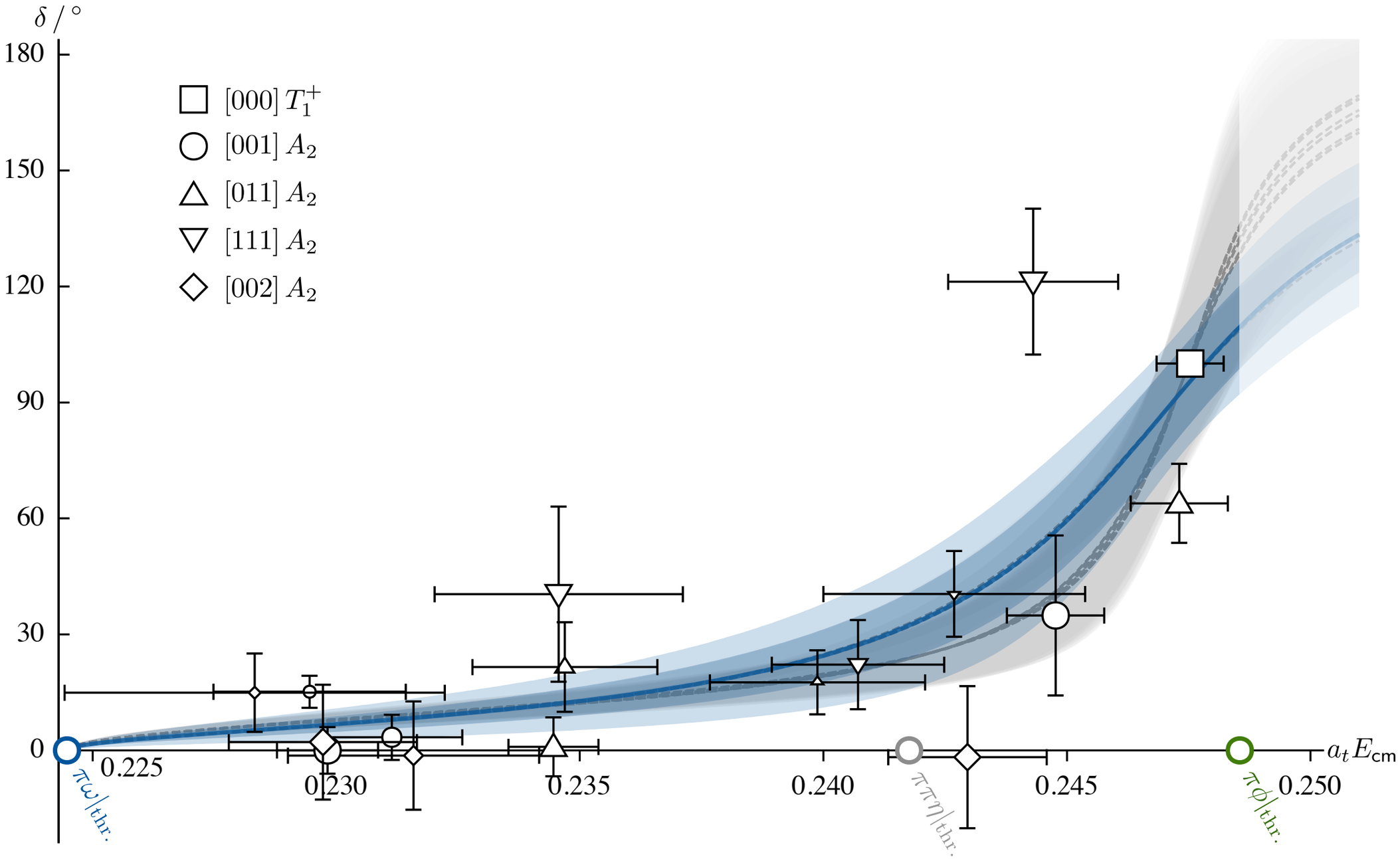
K-matrix approach: $\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho}$

Chew-Mandelstam phase space: $\mathbf{t}^{-1} = \mathbf{K}^{-1} + \boldsymbol{I}$

use a dispersion relation to generate a real part from iρ

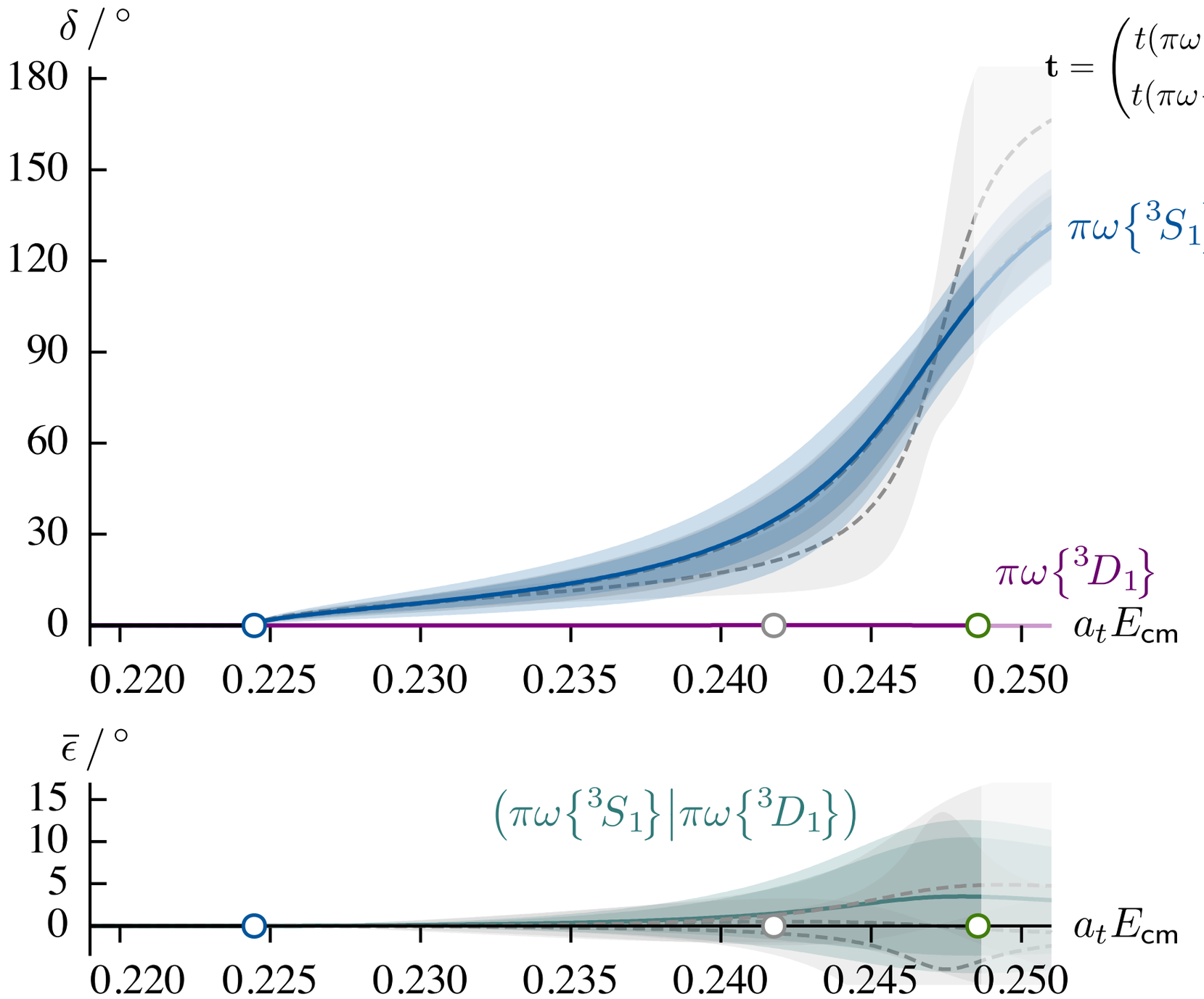
- any form real for real energies is valid
- we use a broad selection of K-matrices
- neglects left-hand cut

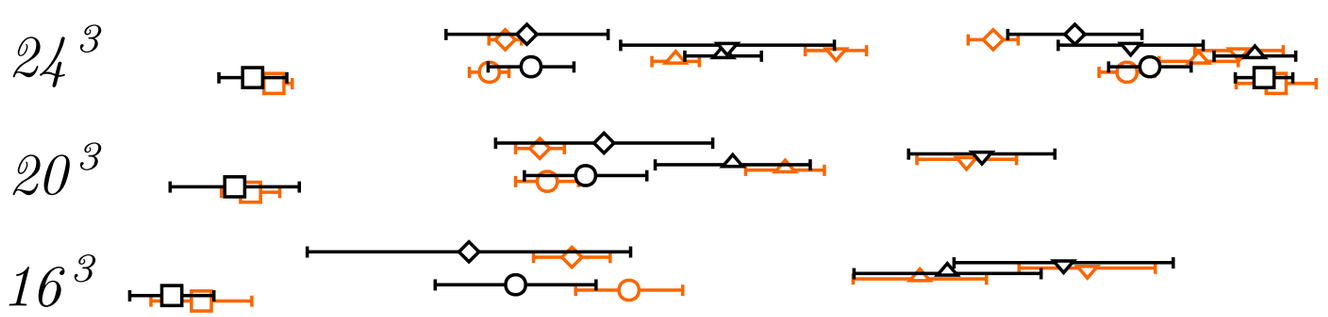
$$t(\pi\omega\{^3S_1\}|\pi\omega\{^3S_1\})$$



e.g.: single-channel K-matrix with one pole: $\chi^2/N_{\text{dof}} = \dfrac{15.1}{20-2} = 0.84$

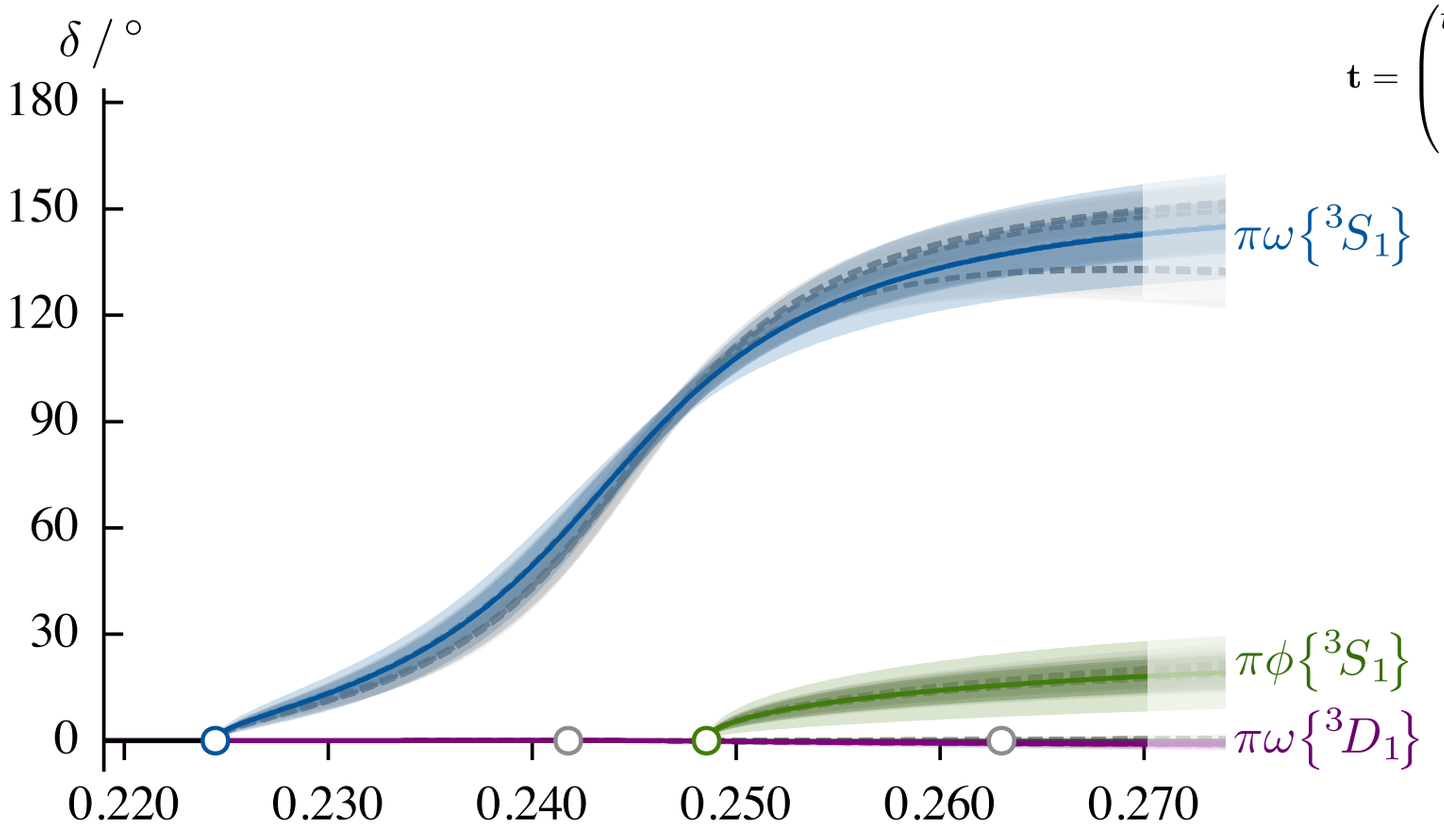
$$\mathbf{t} = \begin{pmatrix} t(\pi\omega\{^3S_1\}|\pi\omega\{^3S_1\}) & t(\pi\omega\{^3S_1\}|\pi\omega\{^3D_1\}) \\ t(\pi\omega\{^3S_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3D_1\}|\pi\omega\{^3D_1\}), \end{pmatrix}$$



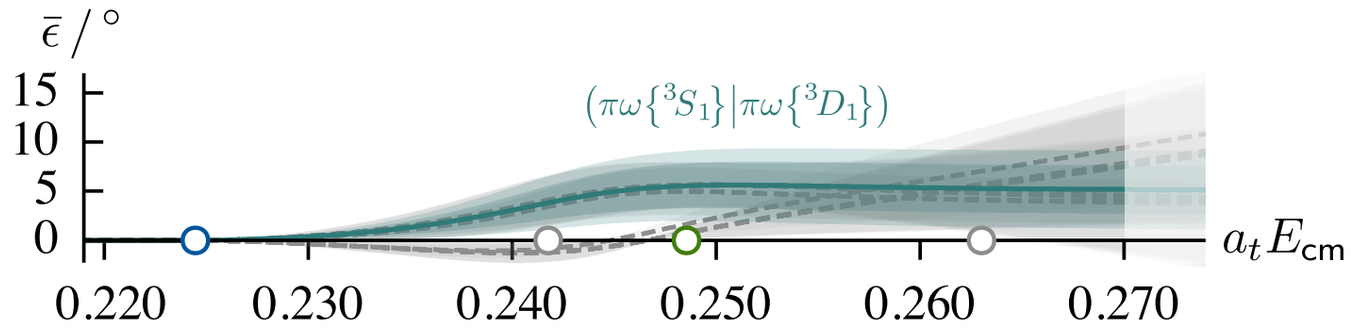


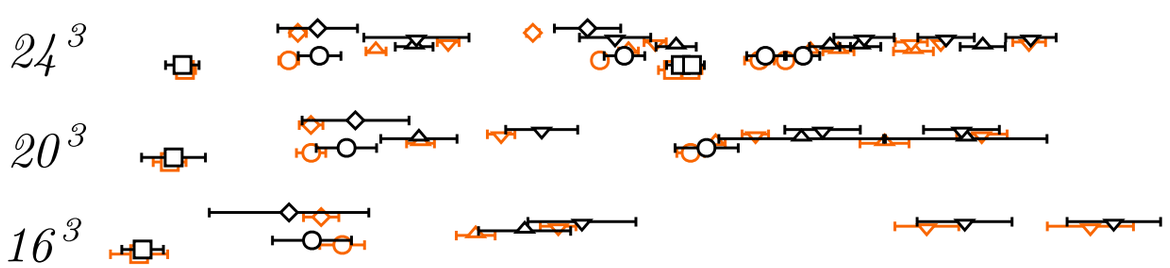
e.g.: two-channel K-matrix with one pole:

$$\chi^2/N_{\mathrm{dof}} = \frac{14.9}{20-3} = 0.87$$



$$\mathbf{t} = \begin{pmatrix} t(\pi\omega\{^3S_1\}|\pi\omega\{^3S_1\}) & t(\pi\omega\{^3S_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3S_1\}|\pi\phi\{^3S_1\}) \\ & t(\pi\omega\{^3D_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3D_1\}|\pi\phi\{^3S_1\}) \\ & & t(\pi\phi\{^3S_1\}|\pi\phi\{^3S_1\}) \end{pmatrix}$$

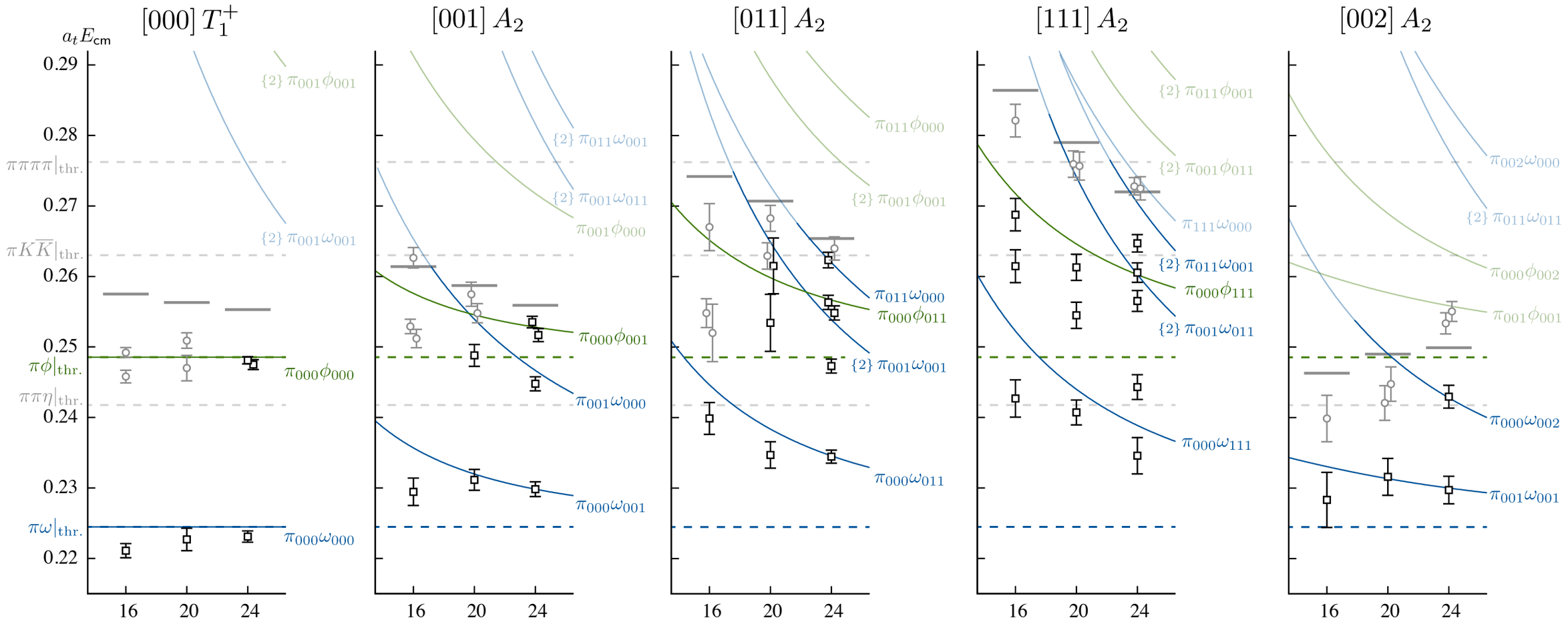




e.g.: three-channel K-matrix with a pole and constants:

$$\begin{aligned} m &= (0.2465 \pm 0.0007 \pm 0.0001) \cdot a_t^{-1} \\ g_{\pi\omega\{^3S_1\}} &= (0.106 \pm 0.007 \pm 0.007) \cdot a_t^{-1} \\ g_{\pi\omega\{^3D_1\}} &= (1.08 \pm 0.47 \pm 0.28) \cdot a_t \\ \gamma^{(0)}_{\pi\omega\{^3S_1\},\pi\omega\{^3S_1\}} &= -0.35 \pm 0.19 \pm 0.18 \\ \gamma^{(0)}_{\pi\phi\{^3S_1\},\pi\phi\{^3S_1\}} &= 0.90 \pm 0.24 \pm 0.27 \end{aligned} \qquad \begin{bmatrix} 1 & -0.05 & 0.05 & -0.01 & -0.23 \\ & 1 & 0.70 & -0.54 & -0.06 \\ & & 1 & -0.39 & -0.06 \\ & & & 1 & 0.22 \\ & & & & 1 \end{bmatrix}$$

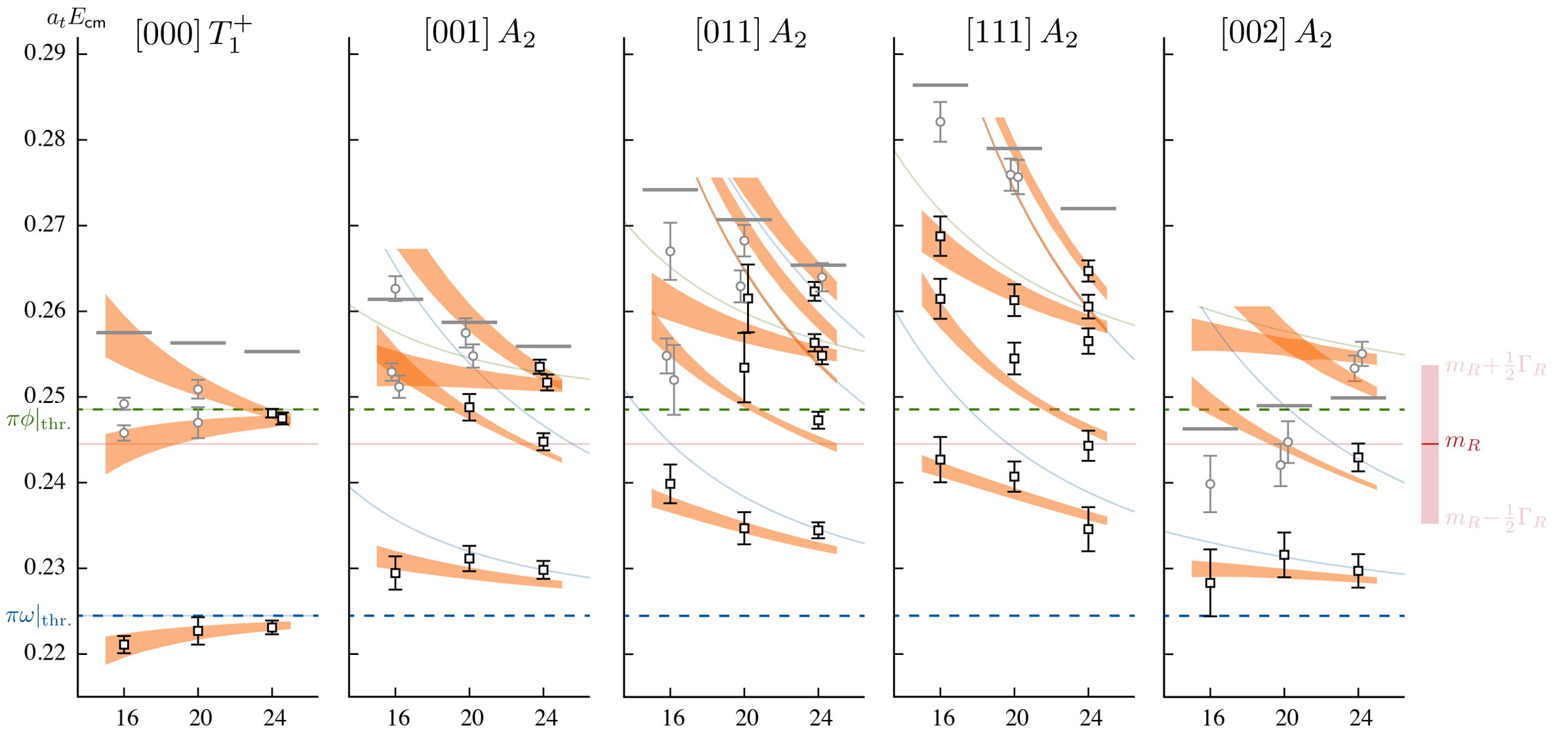
$$\chi^2/N_{\text{dof}} = \tfrac{36.8}{36-5} = 1.19.$$


$[000]\ T_1^+$
$[001]\ A_2$
$[011]\ A_2$
$[111]\ A_2$
$[002]\ A_2$
$a_t E_{\rm cm}$
0.29
0.28
0.27
0.26
0.25
0.24
0.23
0.22
$\pi\pi\pi\pi|_{\rm thr.}$
$\pi K\overline{K}|_{\rm thr.}$
$\pi\phi|_{\rm thr.}$
$\pi\pi\eta|_{\rm thr.}$
$\pi\omega|_{\rm thr.}$
16
20
24
$\{2\}\,\pi_{001}\phi_{001}$
$\{2\}\,\pi_{001}\omega_{001}$
$\pi_{000}\phi_{000}$
$\pi_{000}\omega_{000}$
$\{2\}\,\pi_{011}\omega_{001}$
$\{2\}\,\pi_{001}\omega_{011}$
$\pi_{001}\phi_{000}$
$\pi_{000}\phi_{001}$
$\pi_{001}\omega_{000}$
$\pi_{000}\omega_{001}$
$\pi_{011}\phi_{000}$
$\{2\}\,\pi_{001}\phi_{001}$
$\pi_{011}\omega_{000}$
$\pi_{000}\phi_{011}$
$\{2\}\,\pi_{001}\omega_{001}$
$\pi_{000}\omega_{011}$
$\{2\}\,\pi_{011}\phi_{001}$
$\{2\}\,\pi_{001}\phi_{011}$
$\pi_{111}\omega_{000}$
$\{2\}\,\pi_{011}\omega_{001}$
$\pi_{000}\phi_{111}$
$\{2\}\,\pi_{001}\omega_{011}$
$\pi_{000}\omega_{111}$
$\pi_{002}\omega_{000}$
$\{2\}\,\pi_{011}\omega_{011}$
$\pi_{000}\phi_{002}$
$\pi_{001}\phi_{001}$
$\pi_{000}\omega_{002}$
$\pi_{001}\omega_{001}$

levels from the determinant

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot(\mathbf{1}+i\boldsymbol{\mathcal{M}}(E,L))\right] = 0$$

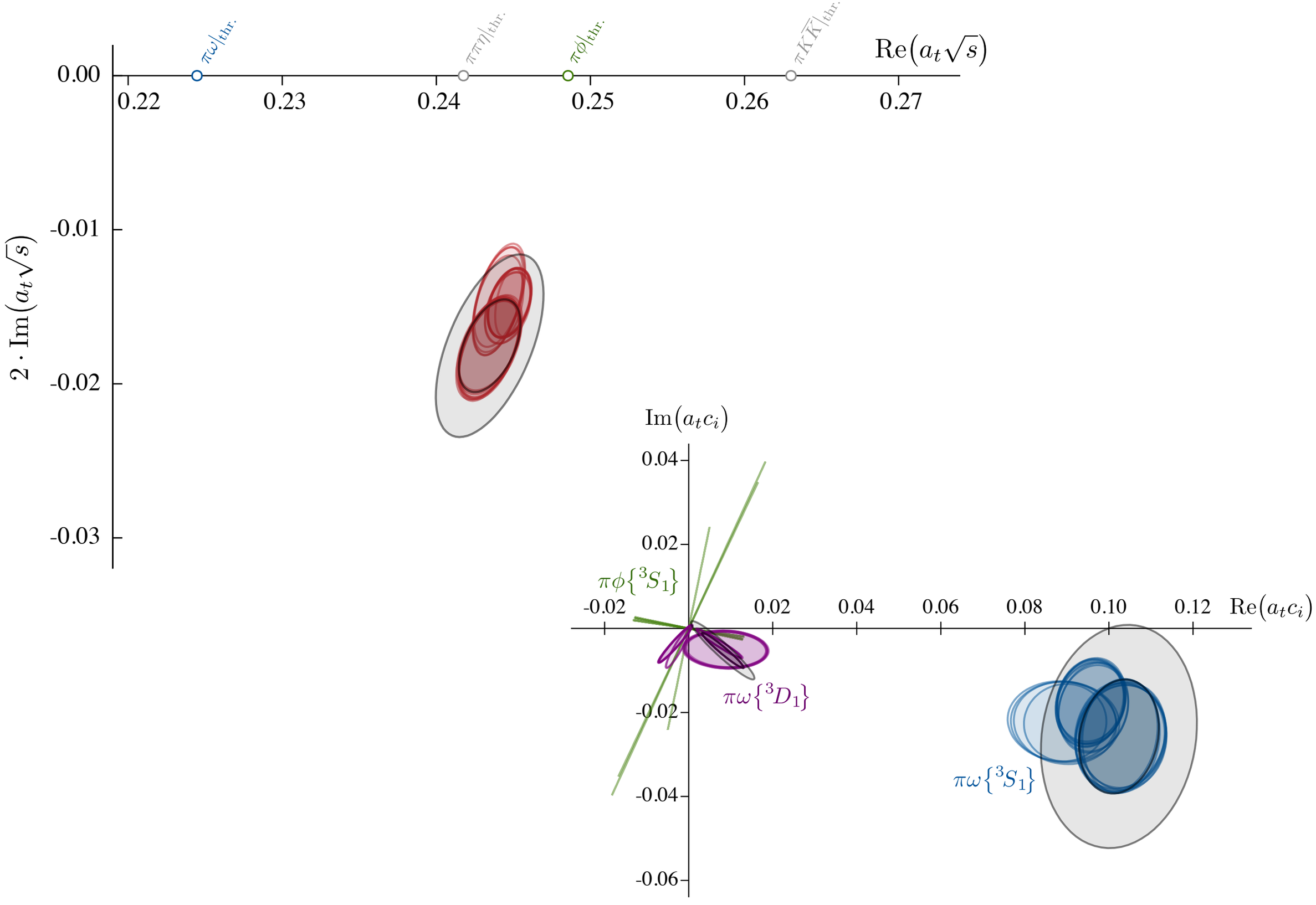
$$\mathbf{t} = \begin{pmatrix} t(\pi\omega\{^3S_1\}|\pi\omega\{^3S_1\}) & t(\pi\omega\{^3S_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3S_1\}|\pi\phi\{^3S_1\}) \\ & t(\pi\omega\{^3D_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3D_1\}|\pi\phi\{^3S_1\}) \\ & & t(\pi\phi\{^3S_1\}|\pi\phi\{^3S_1\}) \end{pmatrix}$$

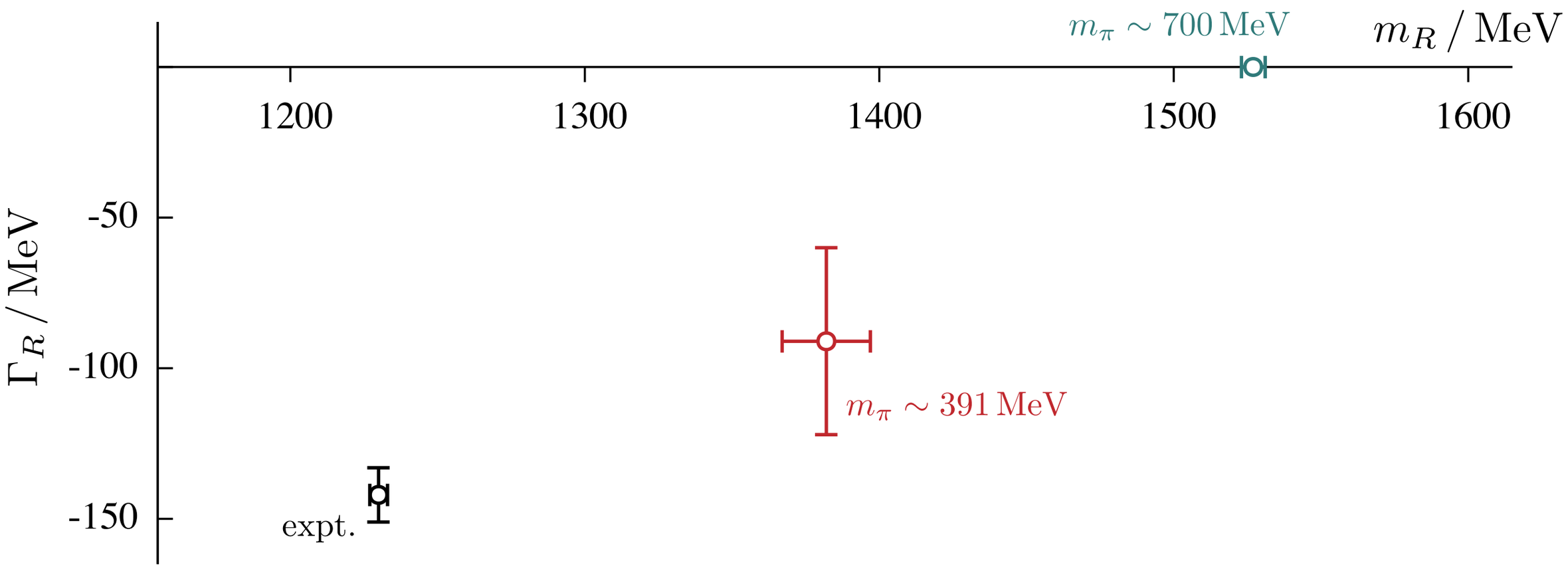


$$\begin{aligned}
m &= (0.2465 \pm 0.0007 \pm 0.0001)\cdot a_t^{-1} \\
g_{\pi\omega\{^3S_1\}} &= (0.106 \pm 0.007 \pm 0.007)\cdot a_t^{-1} \\
g_{\pi\omega\{^3D_1\}} &= (1.08 \pm 0.47 \pm 0.28)\cdot a_t \\
\gamma^{(0)}_{\pi\omega\{^3S_1\},\pi\omega\{^3S_1\}} &= -0.35 \pm 0.19 \pm 0.18 \\
\gamma^{(0)}_{\pi\phi\{^3S_1\},\pi\phi\{^3S_1\}} &= 0.90 \pm 0.24 \pm 0.27
\end{aligned}$$

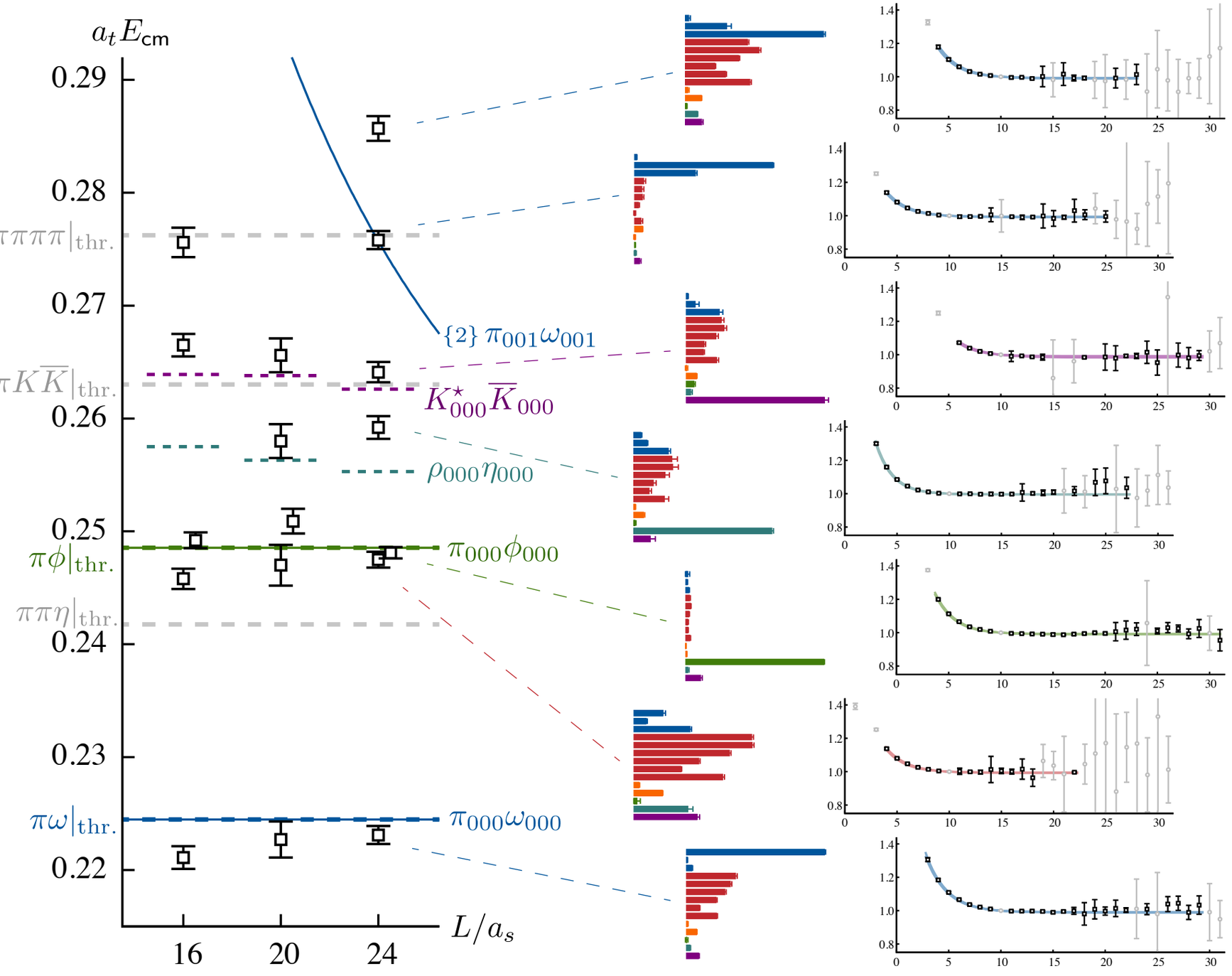
$$\begin{bmatrix} 1 & -0.05 & 0.05 & -0.01 & -0.23 \\ & 1 & 0.70 & -0.54 & -0.06 \\ & & 1 & -0.39 & -0.06 \\ & & & 1 & 0.22 \\ & & & & 1 \end{bmatrix}$$

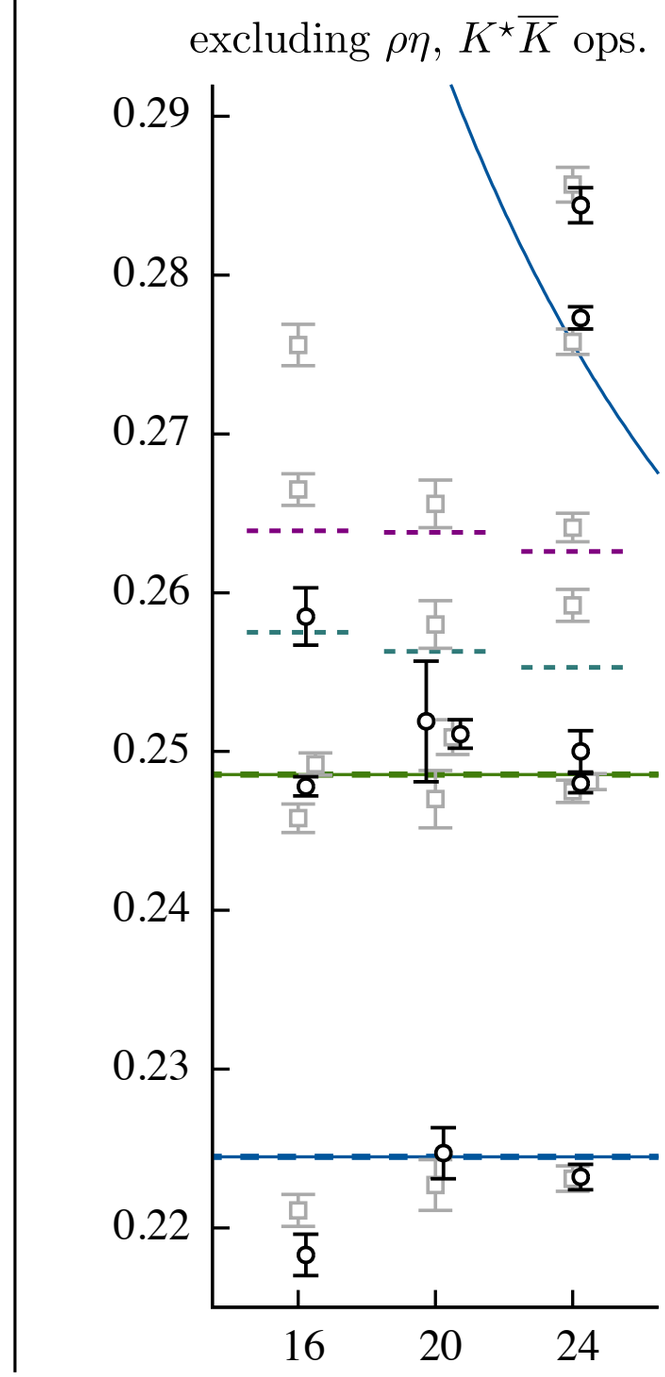
$$\chi^2/N_{\text{dof}} = \tfrac{36.8}{36-5} = 1.19.$$


$\pi\omega|_{\text{thr.}}$
$\pi\pi\eta|_{\text{thr.}}$
$\pi\phi|_{\text{thr.}}$
$\pi K\bar{K}|_{\text{thr.}}$
$\text{Re}(a_t\sqrt{s})$
0.00
0.22
0.23
0.24
0.25
0.26
0.27
$2\cdot\text{Im}(a_t\sqrt{s})$
-0.01
-0.02
-0.03
$\text{Im}(a_t c_i)$
0.04
0.02
-0.02
-0.04
-0.06
$\text{Re}(a_t c_i)$
-0.02
0.02
0.04
0.06
0.08
0.10
0.12
$\pi\phi\{^3S_1\}$
$\pi\omega\{^3D_1\}$
$\pi\omega\{^3S_1\}$


$m_\pi \sim 700\,\mathrm{MeV}$
m_R / MeV
1200
1300
1400
1500
1600
-50
-100
-150
Γ_R / MeV
$m_\pi \sim 391\,\mathrm{MeV}$
expt.

3-body energies



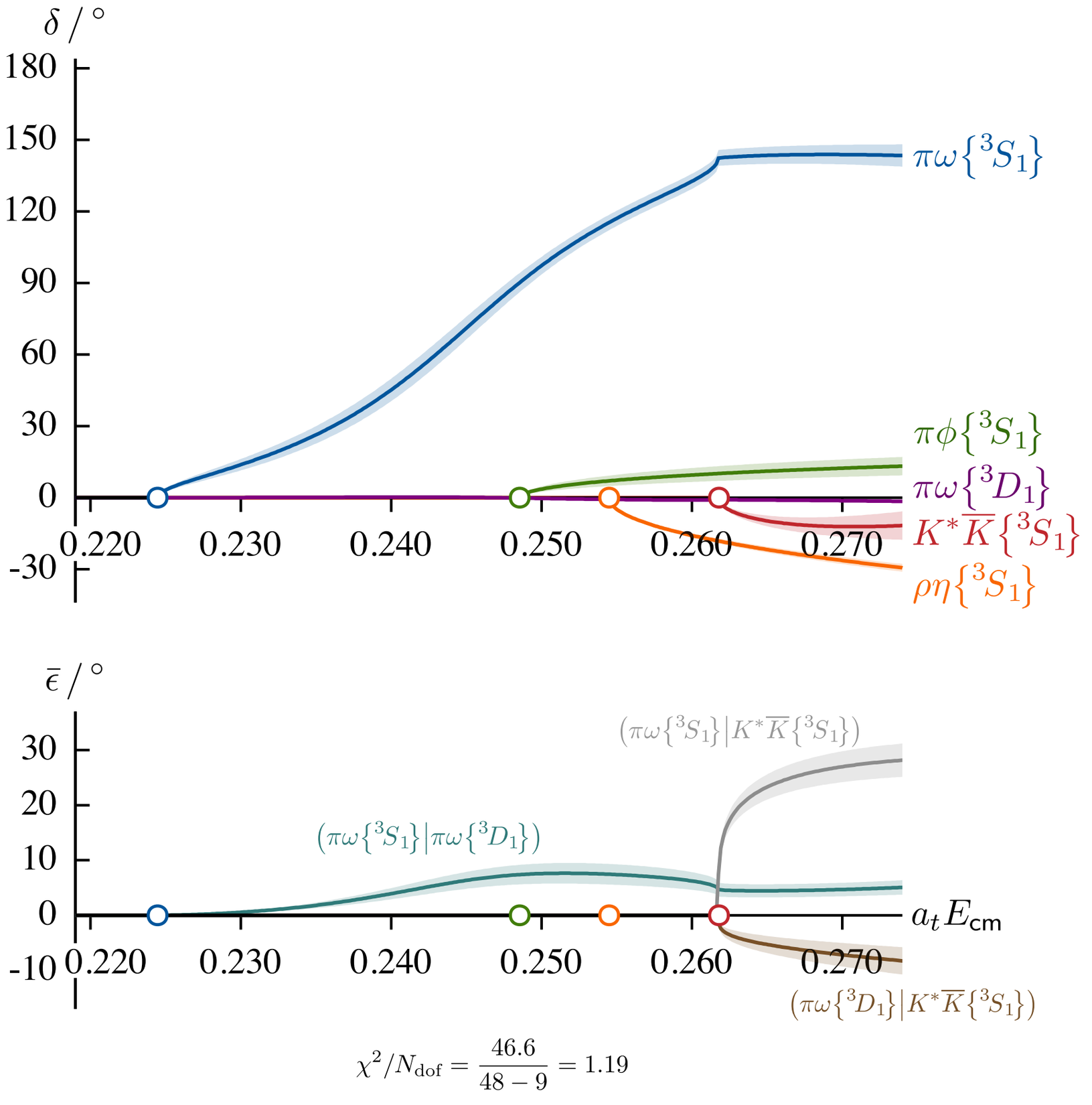


5 coupled amplitudes

treating ρ, K* as stable states

useful to check that the πω, πφ amplitudes are insensitive to the effects of these nearby channels

resonance pole position is robust



$$\chi^2/N_{\mathrm{dof}} = \frac{46.6}{48-9} = 1.19$$

summary

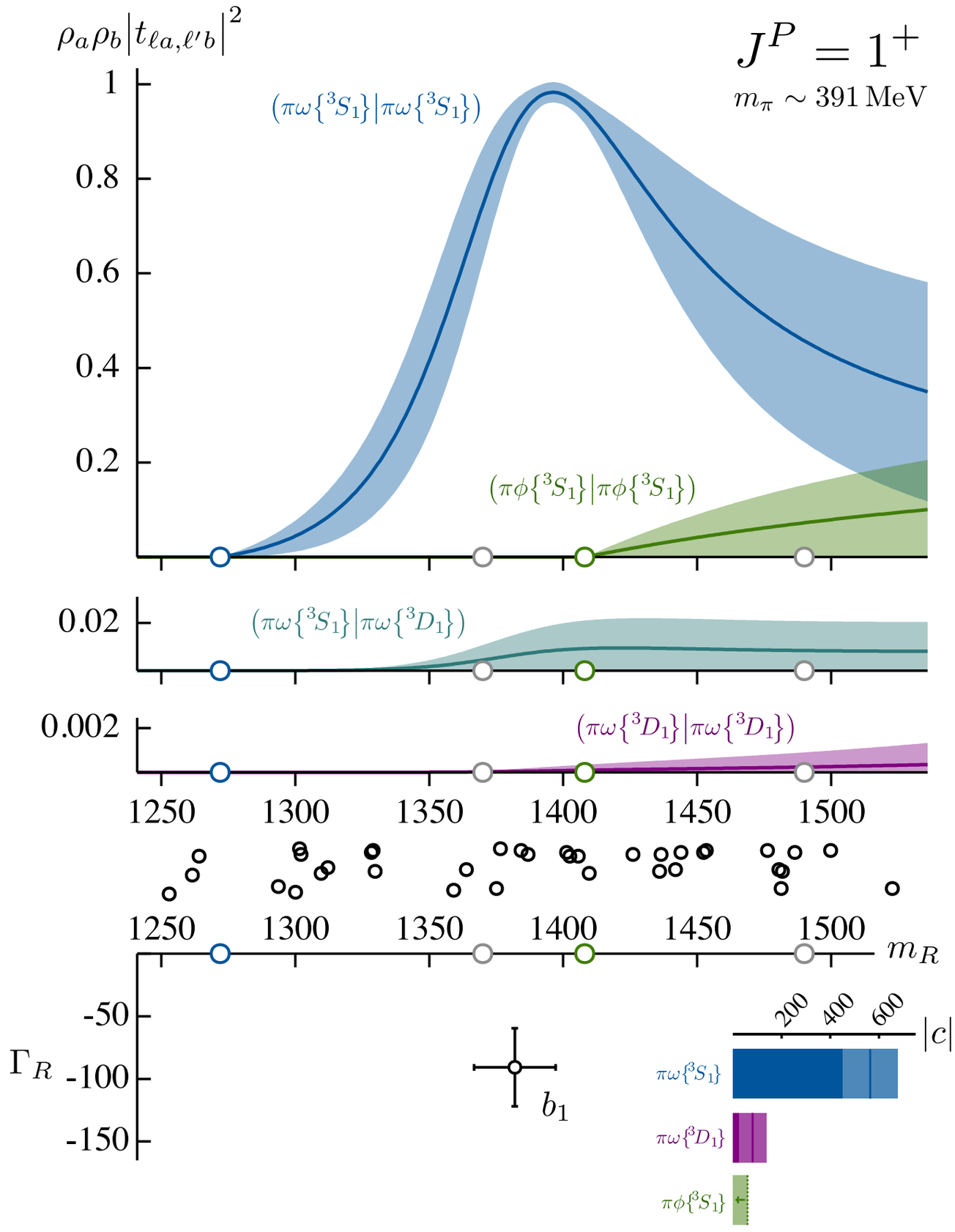
we have computed the b_1 resonance in dynamically coupled partial waves at $m_\pi = 391$ MeV

it has all the hallmarks of the experimental b_1 - a broad resonance pole with a dominant $^3S_1$ and subleading $^3D_1$ components

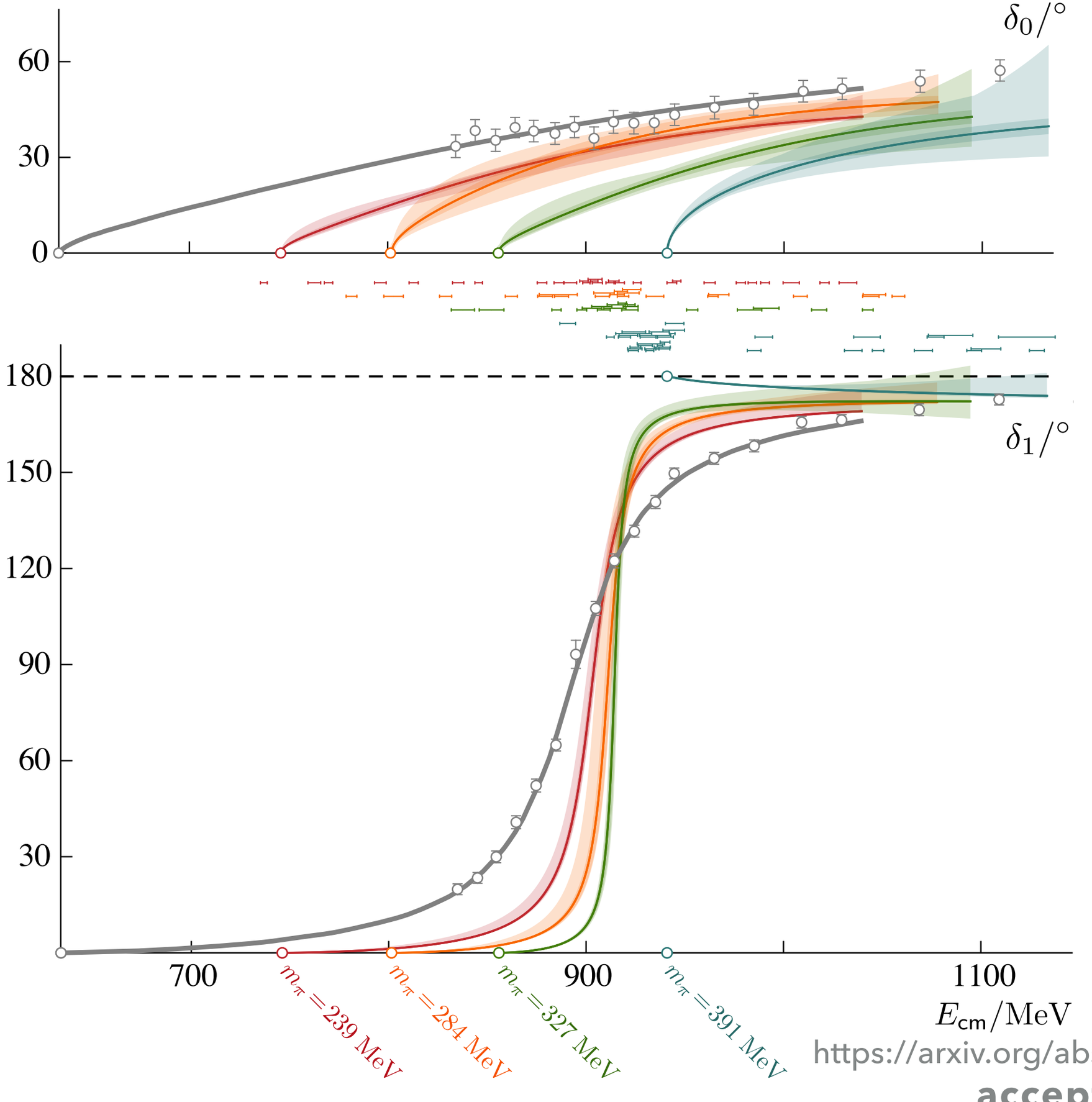
3-body operators were used in extracting the spectrum

3 and 5 coupled-channel systems were obtained

a consistent pole position was found



backup



https://arxiv.org/abs/1904.03188

accepted in PRL