Pion-Pion Scattering with Elongated Boxes

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Outline

Methodology

- LQCD gives finite volume energies
- Quantization Conditions(Lüscher's Formula) give phase shifts
- IAM extrapolates results to physical pion mass

LQCD Details

- N_f=2 NHYP Clover Fermions, Luscher Weisz Action
- $\bar{q}q$ and meson-meson interpolators
- Variational Method
- LapH Smearing

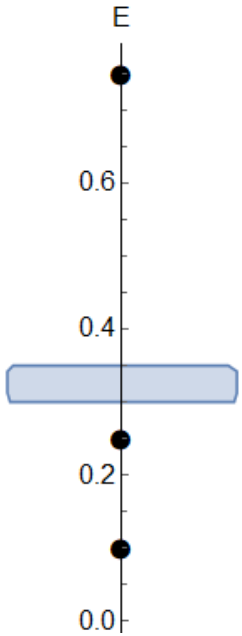
[Lüscher and Wolff 1990]

[Blossier et al. 2009]

[Peardon et al. 2009]

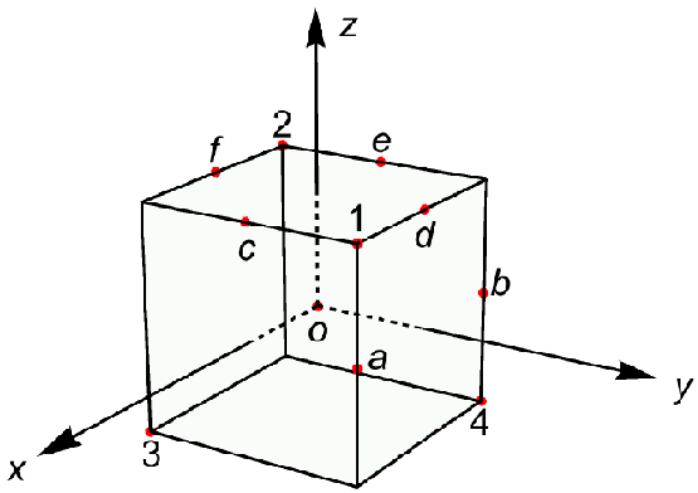
Why Spatially Elongated Boxes

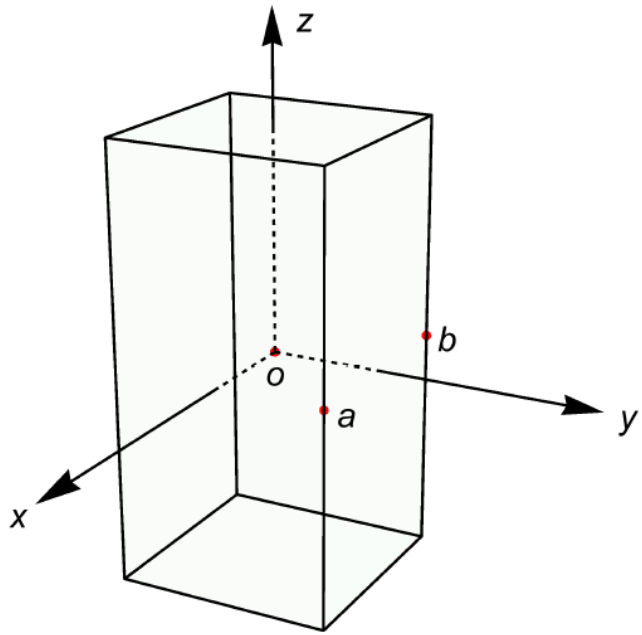
- Scan kinematic region of resonances
- $\vec{p}$ quantized in units of $2\pi/L$
- Reduce numerical cost by elongating in a single direction
- Elongation selected specifically for the ρ



Ensembles

Symmetry Groups: $\mathcal{O}_h$, $\mathcal{D}_{4h}$





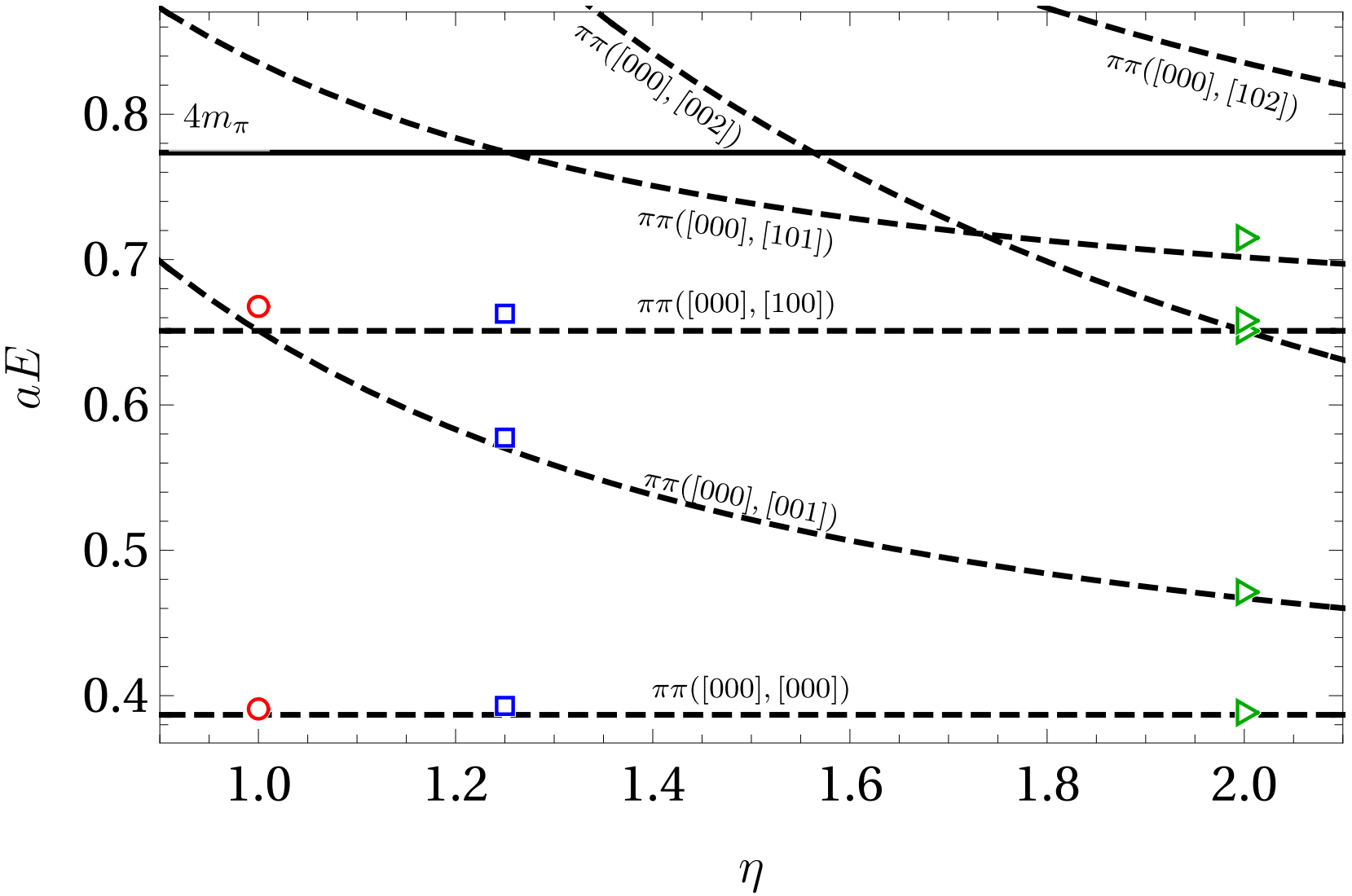
ensemble	$N_t \times N_{x,y}^2 \times N_z$	η	a[fm]	N_{cfg}	aM_π	$am_{u/d}^{pcac}$	af_π
$\mathcal{E}_1$	$48 \times 24^2 \times 24$	1.00	0.1210(2)(24)	300	0.1931(4)	0.01226(5)	0.0648(8)
$\mathcal{E}_2$	$48 \times 24^2 \times 30$	1.25	–	–	0.1944(3)	0.01239(4)	0.0651(6)
$\mathcal{E}_3$	$48 \times 24^2 \times 48$	2.00	–	–	0.1932(3)	0.01227(5)	0.0663(6)
$\mathcal{E}_4$	$64 \times 24^2 \times 24$	1.00	0.1215(3)(24)	400	0.1378(6)	0.00612(5)	0.0600(10)
$\mathcal{E}_5$	$64 \times 24^2 \times 28$	1.17	–	378	0.1374(5)	0.00620(4)	0.0600(8)
$\mathcal{E}_6$	$64 \times 24^2 \times 32$	1.33	–	400	0.1380(5)	0.00619(4)	0.0599(10)

IAM Details

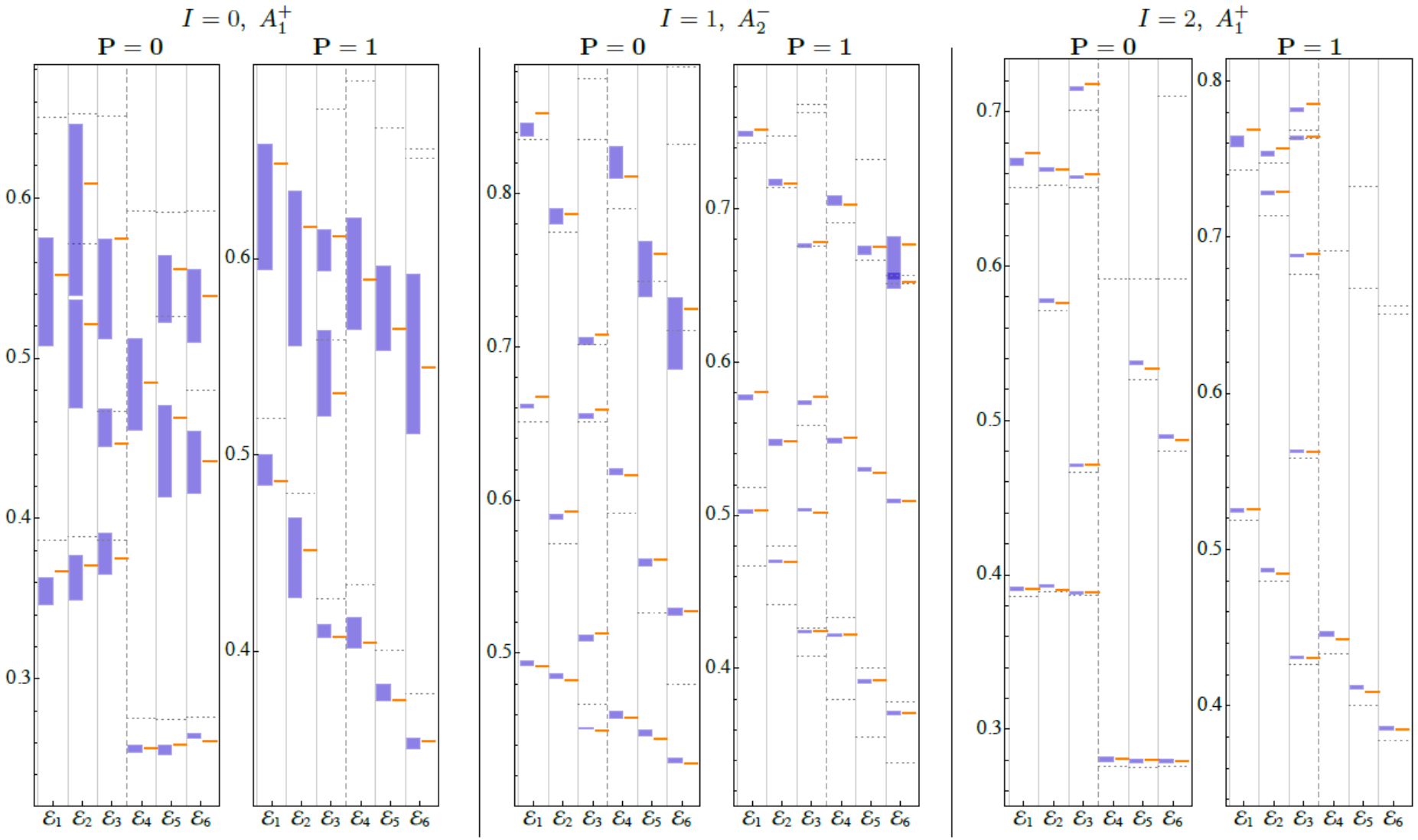
[Truong 1988]
[Pelaez and Rios 2006]
[Gómez Nicola, Palaez, Rios 2008]
[Pelaez and Rios 2010]

- Exact two body unitarity
- Matches χPT to NLO
- Correct resonance behavior with m_π
- We fit all lattice energies, m_π, f_π with cross correlations

I=2 Energy Spectrum

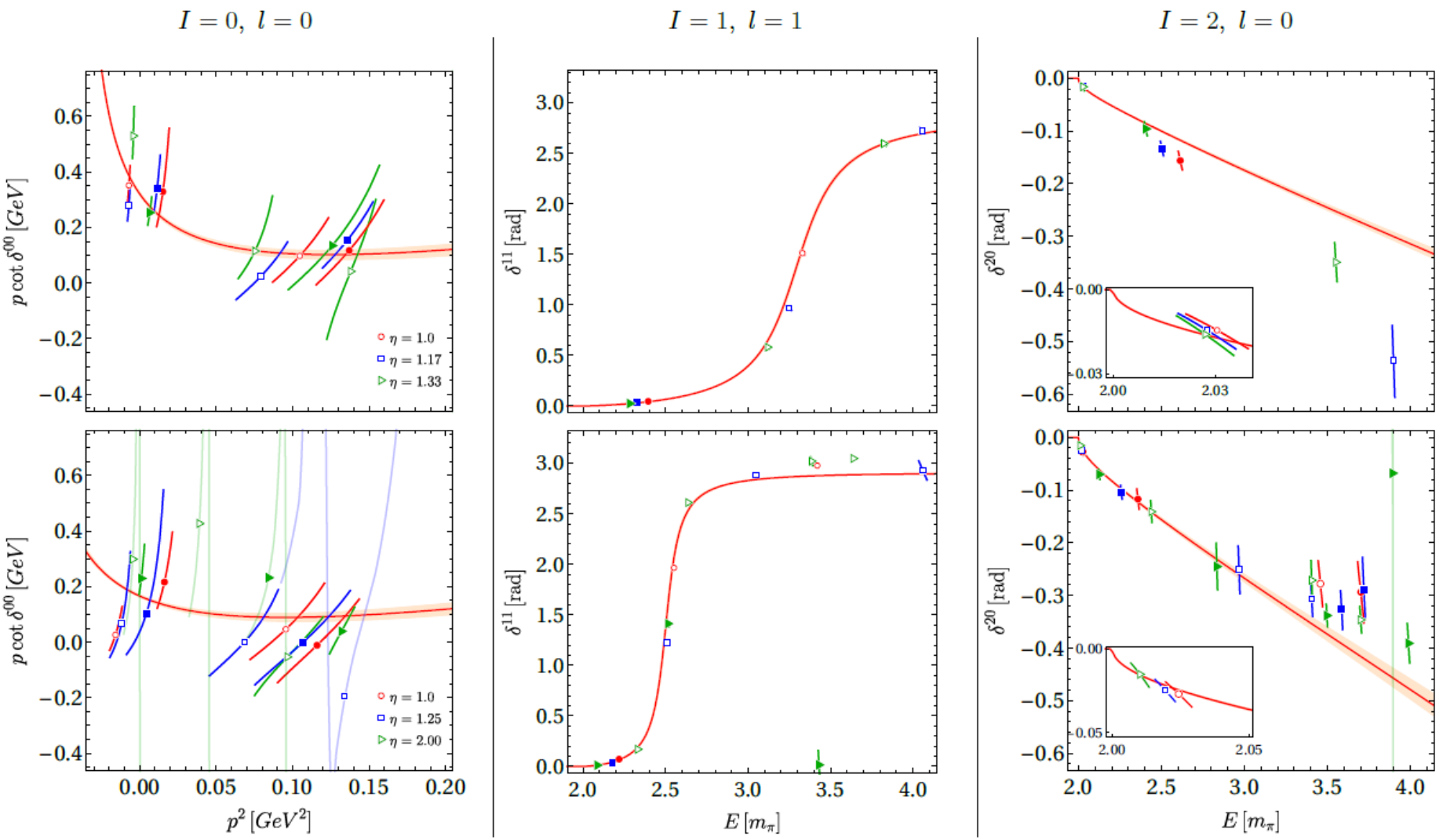


Finite Volume Spectrum



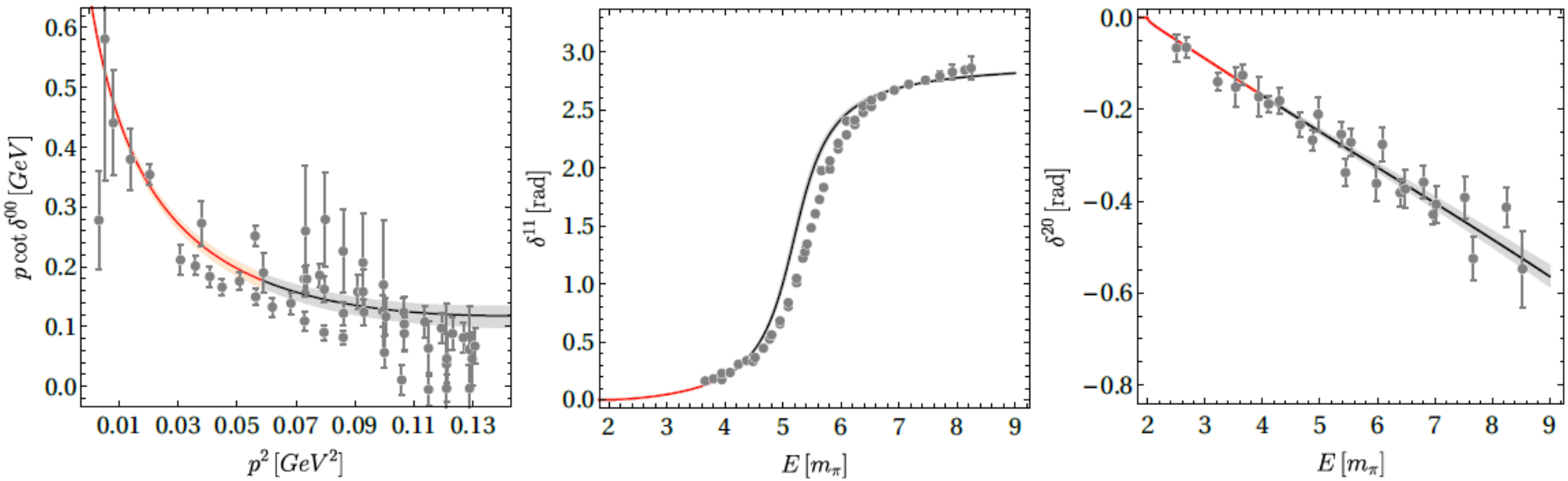
1803.02897 1605.03993 1905.10202

Phase Shifts & IAM Fit Result



$$\cot \delta(E) \approx \mathcal{Z}(E) \qquad \chi^2_{\mathrm{dof}} \approx 2.7$$

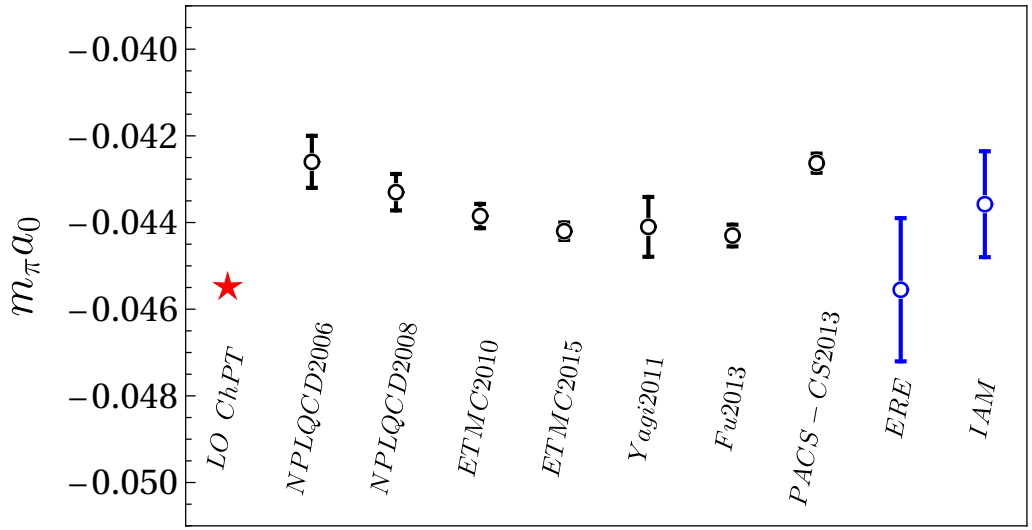
Physical Pion Mass Extrapolation



Experimental data from [Protopopescu 1973]

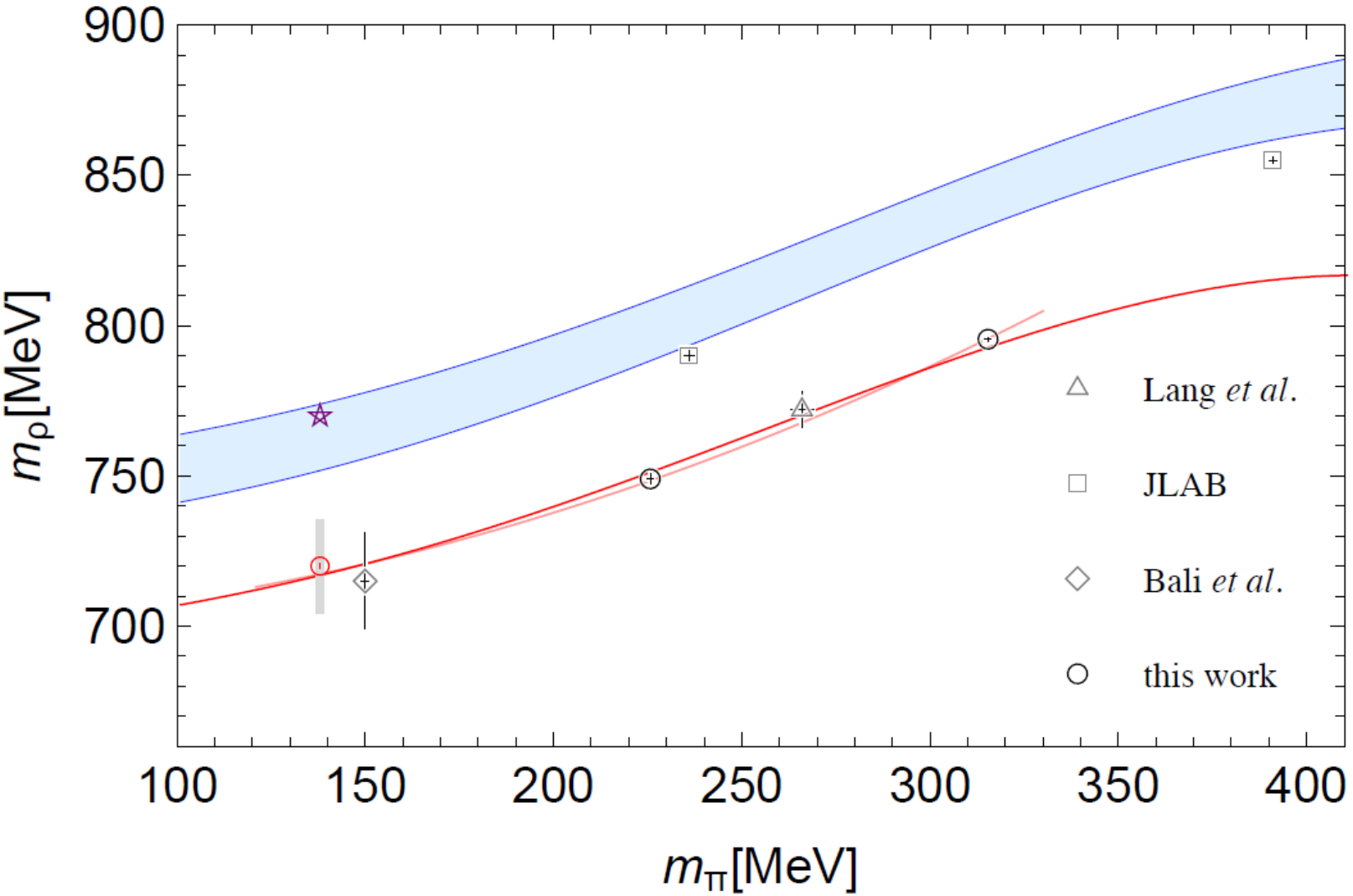
Physical Quantities

Picture adapted from [Helmes et al. 2019]



	Single Channel	IAM Global	Exp.
m_ρ [MeV]	$720(1) - i120.8(8)$	$740^{+3}_{-4} - i69^{+2}_{-1}$	$775.26(.25) - i149.1(8)$
m_σ [MeV]	$440^{+10}_{-15} - i240^{+20}_{-20}$	$450^{+3}_{-3} - i230^{+7}_{-5}$	$449^{+22}_{-16} - i275^{+12}_{-12}$

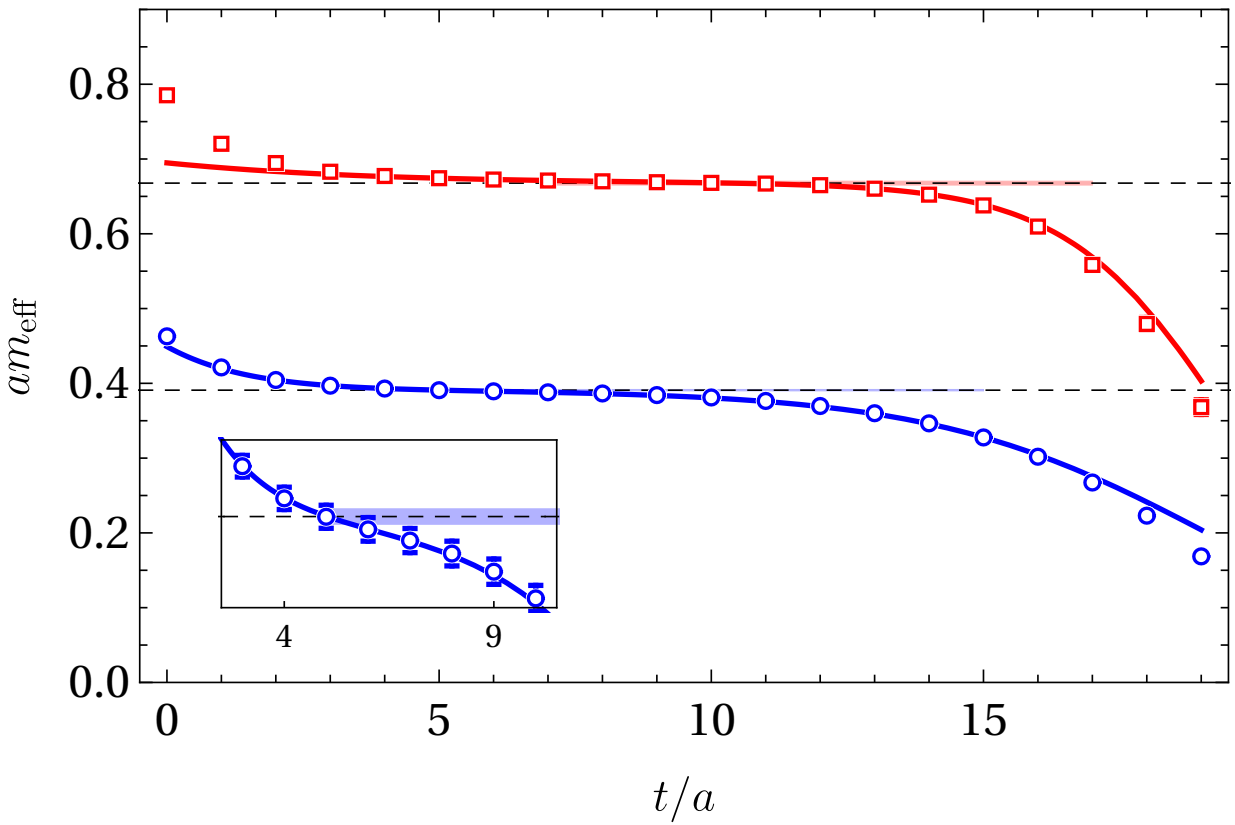
Previous Rho Discrepancy



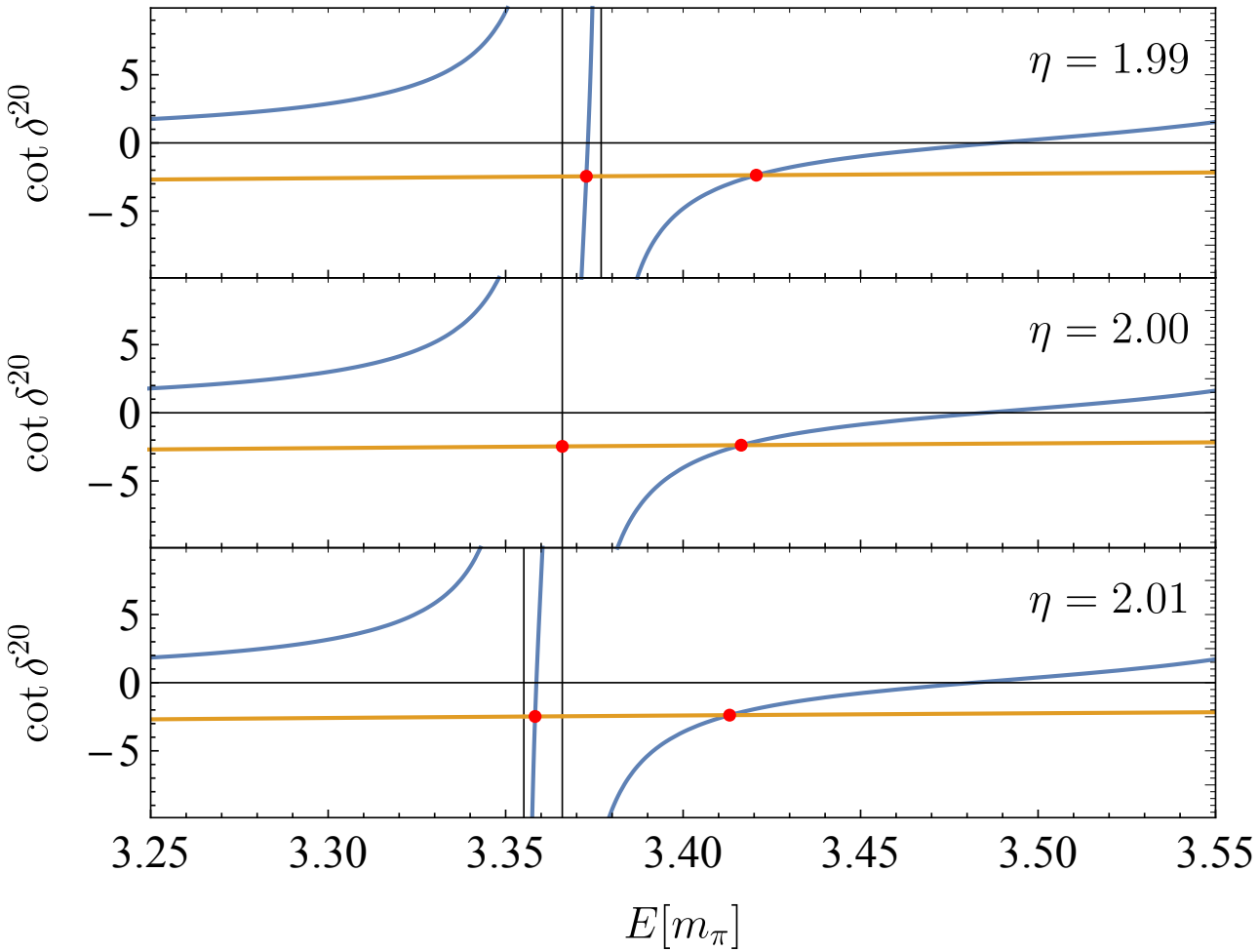
Conclusion and Outlook

- Conclusions
  - Pion-pion scattering complete up to inelastic threshold
  - Elongated boxes give cheap access to different energies
  - σ resonance determined more precisely with global fit
  - Rho resonance still below expected value
- Outlook
  - $\pi^+\pi^+\pi^+$ - Predictions Done - [Mai 2019]
  - $a_1(1260)$ with 3 pion operators

Effective Mass



Non-Interacting Poles



Distribution of LEC's