Pion-Pion Scattering with Elongated Boxes

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Outline

1. Methodology
2. Pion-Pion Scattering
3. Conclusion and Outlook
Methodology

- LQCD gives finite volume energies
- Quantization Conditions (Lüscher’s Formula) give phase shifts
- IAM extrapolates results to physical pion mass
LQCD Details

- $N_f=2$ NHYP Clover Fermions, Luscher Weisz Action
- $\bar{q}q$ and meson-meson interpolators
- Variational Method
- LapH Smearing

[Lüscher and Wolff 1990]
[Blossier et al. 2009]
[Peardon et al. 2009]
Why Spatially Elongated Boxes

- Scan kinematic region of resonances
- $\vec{p}$ quantized in units of $2\pi/L$
- Reduce numerical cost by elongating in a single direction
- Elongation selected specifically for the $\rho$
Symmetry Groups: $O_h, D_{4h}$

<table>
<thead>
<tr>
<th>ensemble</th>
<th>$N_t \times N_x^2 \times N_z$</th>
<th>$\eta$</th>
<th>$a[\text{fm}]$</th>
<th>$N_{\text{cfg}}$</th>
<th>$aM_\pi$</th>
<th>$am_{u/d}^{\text{pcac}}$</th>
<th>$af_\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{E}_1$</td>
<td>$48 \times 24^2 \times 24$</td>
<td>1.00</td>
<td>0.1210(2)(24)</td>
<td>300</td>
<td>0.1931(4)</td>
<td>0.01226(5)</td>
<td>0.0648(8)</td>
</tr>
<tr>
<td>$\mathcal{E}_2$</td>
<td>$48 \times 24^2 \times 30$</td>
<td>1.25</td>
<td>--</td>
<td>--</td>
<td>0.1944(3)</td>
<td>0.01239(4)</td>
<td>0.0651(6)</td>
</tr>
<tr>
<td>$\mathcal{E}_3$</td>
<td>$48 \times 24^2 \times 48$</td>
<td>2.00</td>
<td>--</td>
<td>--</td>
<td>0.1932(3)</td>
<td>0.01227(5)</td>
<td>0.0663(6)</td>
</tr>
<tr>
<td>$\mathcal{E}_4$</td>
<td>$64 \times 24^2 \times 24$</td>
<td>1.00</td>
<td>0.1215(3)(24)</td>
<td>400</td>
<td>0.1378(6)</td>
<td>0.00612(5)</td>
<td>0.0600(10)</td>
</tr>
<tr>
<td>$\mathcal{E}_5$</td>
<td>$64 \times 24^2 \times 28$</td>
<td>1.17</td>
<td>--</td>
<td>378</td>
<td>0.1374(5)</td>
<td>0.00620(4)</td>
<td>0.0600(8)</td>
</tr>
<tr>
<td>$\mathcal{E}_6$</td>
<td>$64 \times 24^2 \times 32$</td>
<td>1.33</td>
<td>--</td>
<td>400</td>
<td>0.1380(5)</td>
<td>0.00619(4)</td>
<td>0.0599(10)</td>
</tr>
</tbody>
</table>
IAM Details

- Exact two body unitarity
- Matches $\chi$PT to NLO
- Correct resonance behavior with $m_\pi$

- We fit all lattice energies, $m_\pi$, $f_\pi$ with cross correlations

References:
- [Truong 1988]
- [Pelaez and Rios 2006]
- [Gómez Nicola, Palaez, Rios 2008]
- [Pelaez and Rios 2010]
I=2 Energy Spectrum

![Graph showing the energy spectrum for various pion-pion scattering channels. The graph includes lines labeled with different combinations of indices like [000], [002], [001], [000], [102], [101], [100], and [000]. The horizontal axis represents $\eta$, and the vertical axis represents $aE$. The graph highlights the energy levels for different channels and their relationships.](image-url)
Finite Volume Spectrum

$I = 0, \ A_1^+$

$I = 1, \ A_2^-$

$I = 2, \ A_1^+$

1803.02897

1605.03993

1905.10202
\[ \cot \delta(E) \approx \mathcal{Z}(E) \quad \chi^2_{dof} \approx 2.7 \]
Experimental data from [Protopopescu 1973]
Physical Quantities

Picture adapted from [Helmes et al. 2019]

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<th></th>
<th>Single Channel</th>
<th>IAM Global</th>
<th>Exp.</th>
</tr>
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<tbody>
<tr>
<td>(m_\rho) [MeV]</td>
<td>720(1) (- i120.8(8))</td>
<td>740(^+3_{-4}) (- i69(^+2_{-1}))</td>
<td>775.26(.25) (- i149.1(8))</td>
</tr>
<tr>
<td>(m_\sigma) [MeV]</td>
<td>440(^+10_{-15}) (- i240(^+20_{-20}))</td>
<td>450(^+3_{-3}) (- i230(^+7_{-5}))</td>
<td>449(^+22_{-16}) (- i275(^+12_{-12}))</td>
</tr>
</tbody>
</table>
Previous Rho Discrepancy

\[ m_\rho [\text{MeV}] \]

\[ m_{\pi\pi} [\text{MeV}] \]

- \( \triangle \) Lang \textit{et al.}
- \( \square \) JLAB
- \( \diamond \) Bali \textit{et al.}
- \( \circ \) this work
Conclusions

- Pion-pion scattering complete up to inelastic threshold
- Elongated boxes give cheap access to different energies
- $\sigma$ resonance determined more precisely with global fit
- Rho resonance still below expected value

Outlook

- $\pi^+\pi^+\pi^+$ - Predictions Done - [Mai 2019]
- $a_1(1260)$ with 3 pion operators
Effective Mass

![Graph showing effective mass as a function of time normalized by lattice spacing (t/a). The graph features data points connected by lines, with a zoomed-in inset showing additional detail at specific time points.](image)

- The x-axis represents time normalized by lattice spacing (t/a).
- The y-axis represents the effective mass (am_{eff}).
- The graph includes data points at different values of t/a, showing a trend of decreasing effective mass with increasing time normalized by lattice spacing.
Non-Interacting Poles
Distribution of LEC's