Recent developments in LQCD studies on tetraquarks

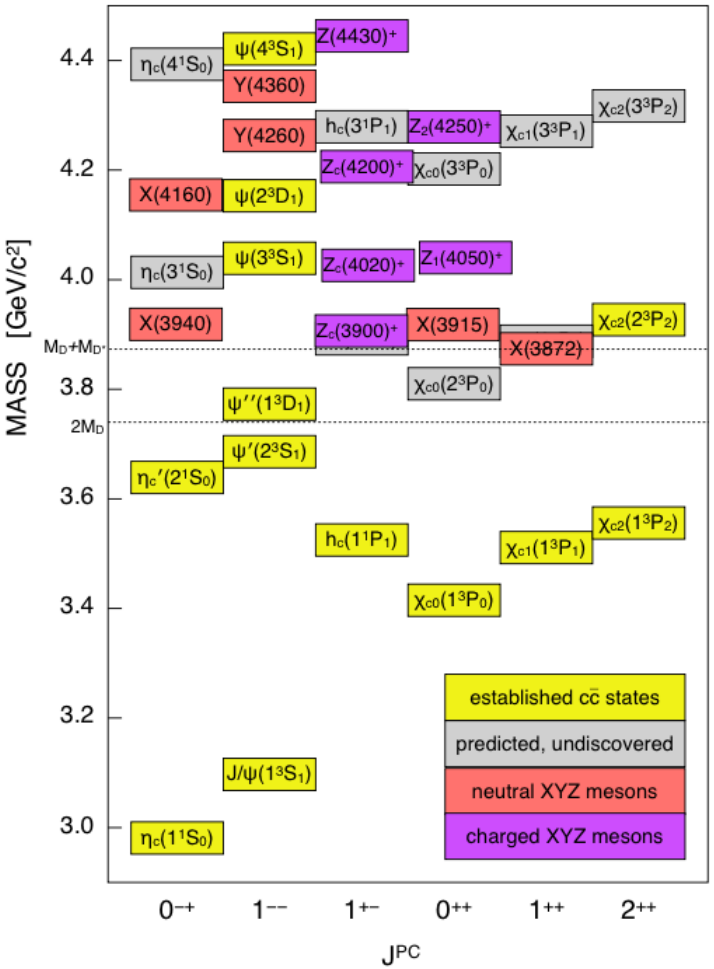
Anthony Francis

Special thanks to
R. J. Hudspith, R. Lewis, K. Maltman

Lattice 2019
The 37th International Symposium on Lattice Field Theory

Wuhan, 17.06.2019

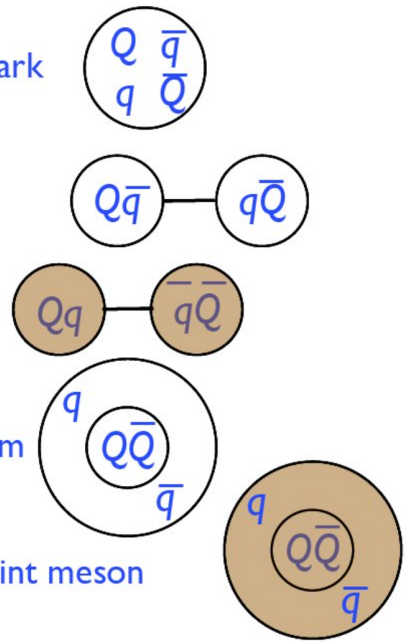
*Mitchell, Olsen *Ali



Models for XYZ Mesons

Quarkonium Tetraquarks

- compact tetraquark
- meson molecule
- diquark-onium
- hadro-quarkonium
- quarkonium adjoint meson



- $\mathcal{O}(15)$ new heavy flavor states discovered.
  - → Some expected: Charmonia, bound and resonant
  - → Some unexpected: Exotica. Tetraquarks?
- Many models and interpretations exist. Need lattice insight!

Approaches on the lattice

On the lattice there are four methods followed:

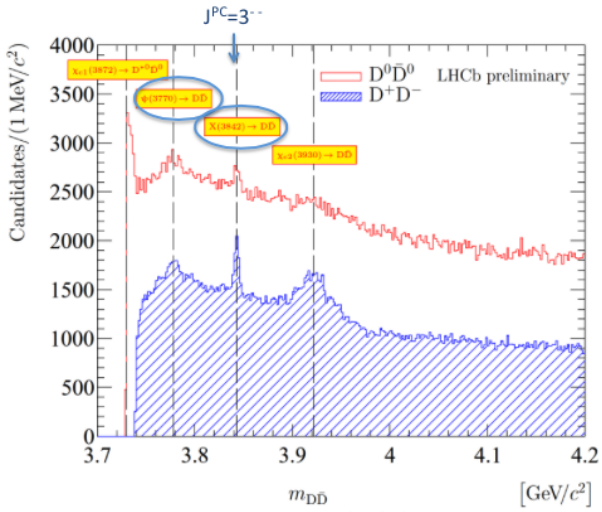
- **Studies using static quarks** *(not covered here)*
  Fitted potentials used to predict bound states and resonances.
  *Bicudo et al. ('17,'17) in the $ud\bar{b}\bar{b}$ system
- **HAL QCD method**
  Lattice potentials studied for scattering properties.
- **Finite volume energy levels**
  Lattice energies equated to (un)observed states.
- **Scattering analysis**
  Lattice energies studied in terms of scattering phase shifts.

Visit session "Hadron Spectroscopy and Interactions" today 14:20-16:00 (Mon, 17.06.) for more!

- "Z_b tetraquark channel and $B\bar{B}^*$ interaction" *Sasa Prelovsek
- "Heavy four-quark and six-quark states from lattice QCD" *Nilmani Mathur
- "Exploration of a singly-bottom tetraquark on 2+1 flavour lattices" *Brian Colquhoun

Charmonia, e.g. $\Psi(3770)$ and $X(3842)$

*LHCb ('19)



*Piemonte et al. ('19)

Charmonia with $J^{PC} = 1^{--}$ and 3^{--} $\rightarrow D\bar{D}$ scattering in partial waves $l = 1, 3$

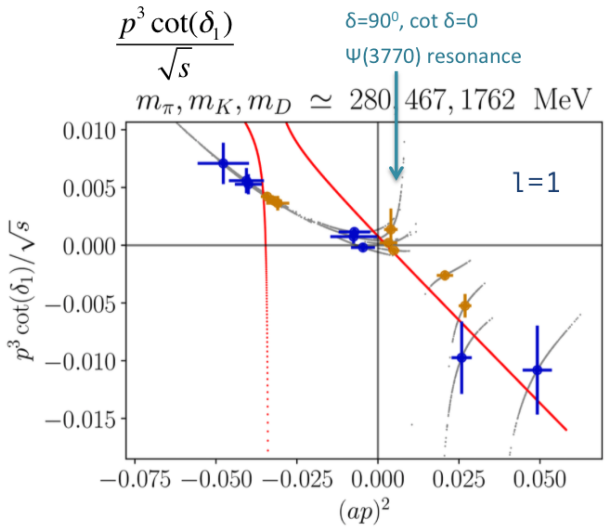
- CLS $n_f = 2+1$, $m_\pi \approx 280 MeV$, $m_K \approx 467 MeV$
- $m_D = 1762, 1927 MeV$

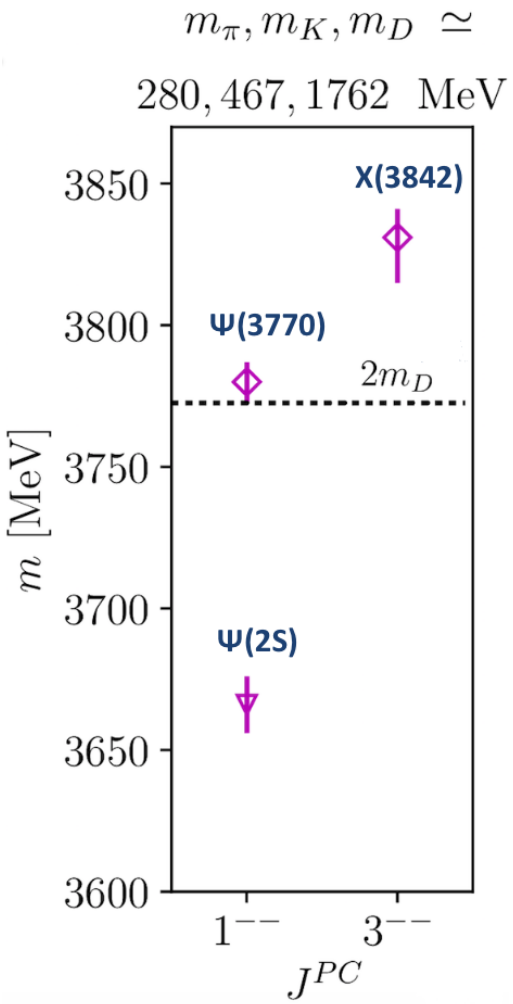
Fits to phase shifts using Breit-Wigner forms.

$l = 1$ (double pole preferred):

$$\frac{p^3 \cot(\delta_1)}{\sqrt{s}} = \Psi(2S) + \Psi(3770)$$

$l = 3$: $\frac{p^7 \cot(\delta_3)}{\sqrt{s}} = X(3842)$





Lattice $c\bar{c}$ spectrum including strong transitions to $D\bar{D}$: $J^{PC} = 1^{--}$ and 3^{--} states identified (1 bound, 2 resonant).

Exotica, like $Z_c(3900)$

As example: The $Z_c(3900)$ could be a charged $c\bar{c}u\bar{d}$ tetraquark ($J^{PC} = 1^{+-}$).

Goal: Same kind of clarity as for the conventional charmonia.

Lattice status:

*HadronSpectrum Coll. ('17)

Most recent calculation with large basis of meson-meson and tetraquark operators.

$\Rightarrow$ Currently no significant deviations from a spectrum with only weak interactions and no resonance present.

*HAL QCD Coll. ('18)

Most recent calculation using coupled channel HAL QCD method.

$\Rightarrow$ Strong transition potential between $\pi J/\Psi$ and $D\bar{D}^*$ indicates Z_c is possibly a threshold cusp.

Pending studies eagerly awaited.

Phenomenologically interesting: $\bar{b}\bar{b}bb$ tetraquark

Multiple pheno. models predict fully heavy (bottom) tetraquark states below the corresponding $2\,\bar{b}b$ thresholds.

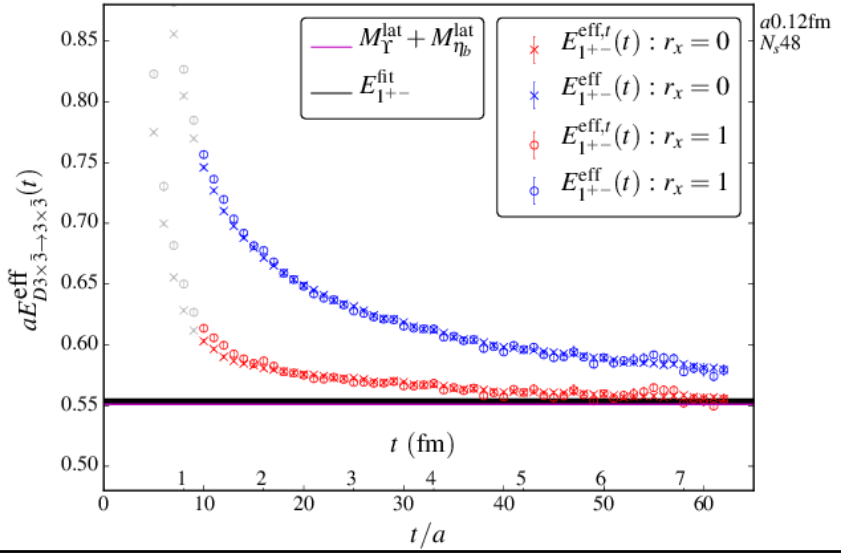
*Hughes et al. ('18)

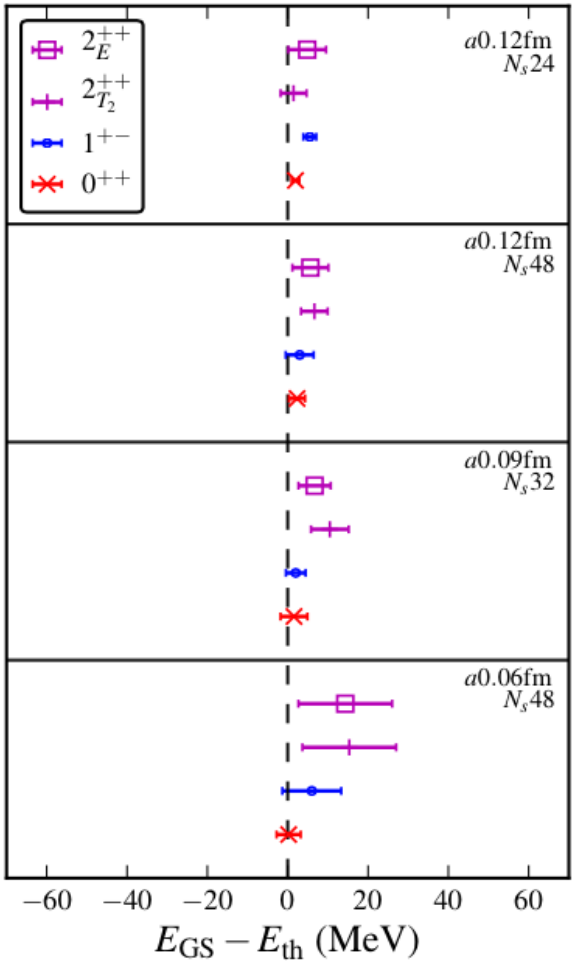
Calculation using NRQCD in 0^{++}, 1^{+-} and 2^{++} channels.

- MILC $n_f = 2+1+1$, coarse, fine, superfine
- 4 ensembles, 1 with m_π =phys

$\Rightarrow$ No binding found.

Diquark-Antidiquark





Wrap up of situation:

- Many heavy states in experiment lacking theoretical understanding
- **Conventional charmonia**: lattice work identified *resonances*, $\Psi(3770)$, $X(3842)$, and bound state $\Psi(2S)$. *Piemonte et al. ('19)
- Some exotica could be **tetraquarks**: lattice work has not been able to clearly identify *hidden heavy tetraquarks* yet.
  $\rightarrow$ some indication that $Z_c(3900)$ might not be a resonance at all.
  *HadronSpectrum Coll ('17), *HAL QCD Coll. ('18)
- Lattice indicates $\bar{b}\bar{b}bb$ tetraquarks are not bound. *Hughes et al. ('18)
  $\rightarrow$ Also not detected in experiment (searches at LHCb, CMS).

In the following:

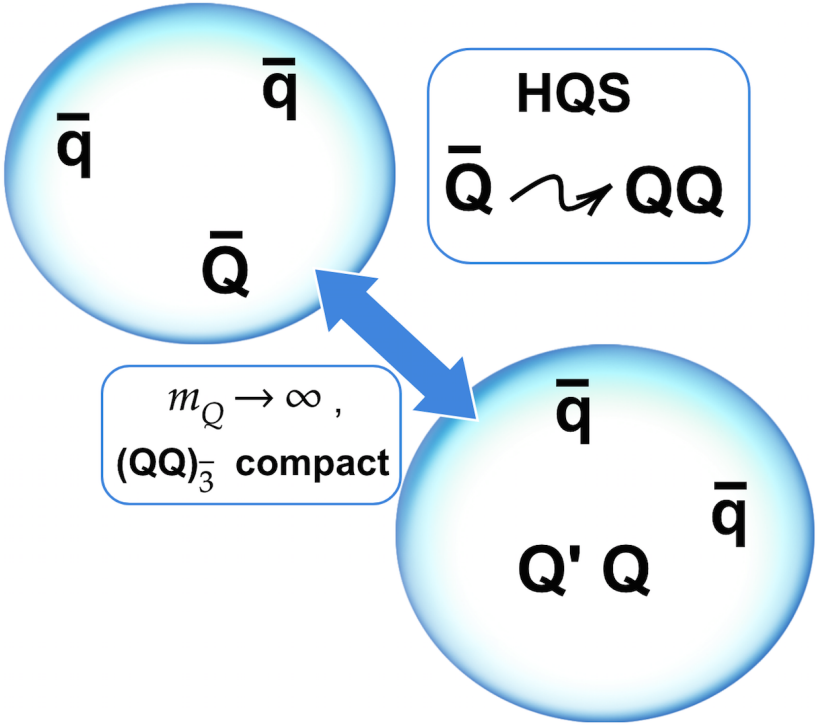
- A simple(r) tetraquark with two heavy (c, b) and two light (ℓ, s) quarks
- Lattice evidence for $ud\bar{b}\bar{b}$, $\ell s\bar{b}\bar{b}$ and more.
- Qualitative description of the lattice data by a simple model.
- Not observed experimentally (yet). $\rightarrow$ Difficulty: two b's.

A case for doubly heavy tetraquarks:

The heavy hadron spectrum suggests a binding mechanism for doubly heavy ground state tetraquarks, $qq'\bar{Q}\bar{Q}'$ ($J^P = 1^+$).

Observations in Q and q:

- HQS: Q-spin decouples (good approx. for $Q = b$)
- $[\bar{Q}\bar{Q}]_3^{m_Q \to \infty}$ becomes compact
- $[\bar{Q}\bar{Q}]_3 \leftrightarrow Q$ relates $qq'Q$ & $qq'\bar{Q}\bar{Q}$

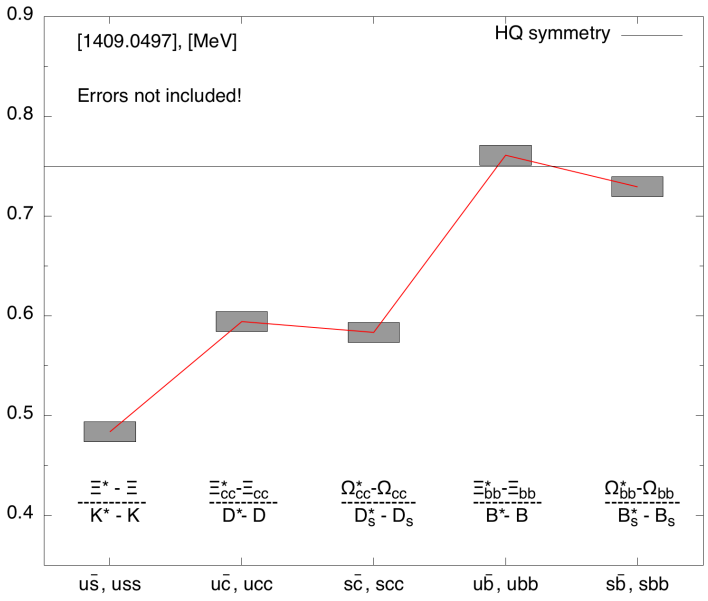


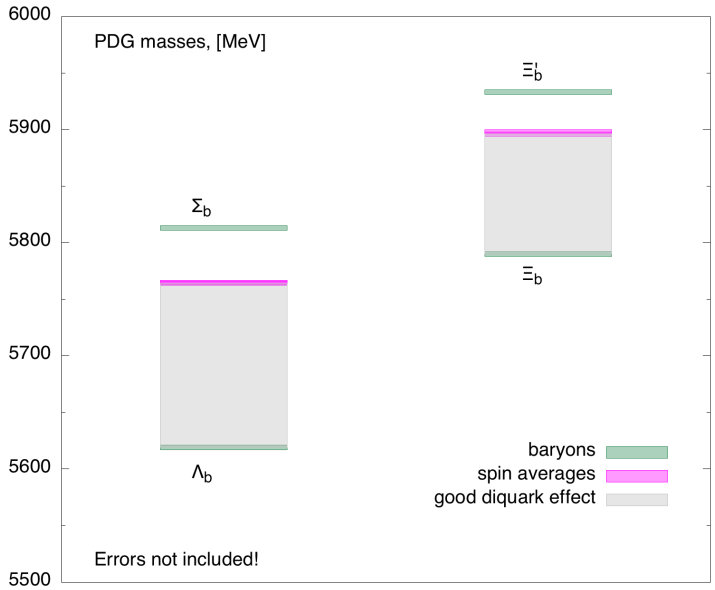
- Diquarks: q's prefer to be in $\{qq\}_{\bar{3}}$
  *Jaffe ('05)
- $\{qq\} = (qC\gamma_5 q)$ lightest
  *Alexandrou et al. ('06)
- $m(\{ud\}) < m(\{us\})$

Question: Combining $\overbrace{(qC\gamma_5 q')}^{\{qq\}}\overbrace{(\bar{Q}C\gamma_i\bar{Q}')}^{[\bar{Q}\bar{Q}]}$ diquarks, do they form stable tetraquarks, e.g. $ud\bar{b}\bar{b}$, $\ell s\bar{b}\bar{b}$, $ud\bar{c}\bar{b}$?

Answer in the simple HQS-GDQ picture: yes

$\rightarrow$ Single-b baryon as analogous system to tetraquark.





HQS: $[\bar{Q}\bar{Q}]$ behaves like single Q:

- Good approx. in $(\Xi^*_{bb} - \Xi_{bb})/(B^* - B)$ and $(\Omega^*_{bb} - \Omega_{bb})/(B^*_s - B_s)$

"Good" diquark effect, use $qq'b$ spectrum as guide:

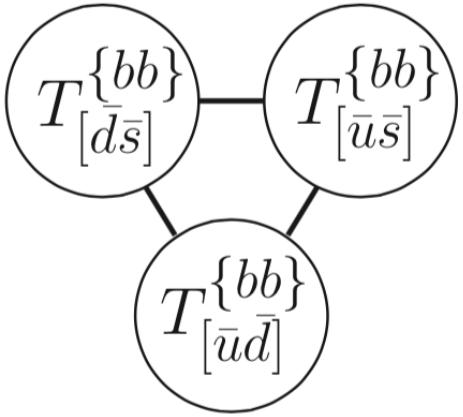
- $\{ud\}$: $\Lambda_b - B_{sp} \sim -145\text{MeV} \leftrightarrow [ud]$: $\Sigma_b - B_{sp} \sim 49\text{MeV}$
- $\{\ell s\}$: $\Xi_b - B_{sp} - \sim -106\text{MeV} \leftrightarrow [\ell s]$: $\Xi'_b - B_{sp} \sim 36\text{MeV}$

$B_{sp} = \frac{1}{4}[\,3B_{s=0} + B_{s=1}\,] \sim$ spin averaged "threshold"

Old idea: Stable multiquarks pointed out previously *Ader et al. ('82); *Manohar, Wise ('93); ...

Renewed interest from phenomenology

*Karliner, Rosner ('17); *Eichten, Quigg ('17); *Czarnecki, Leng, Voloshin ('18); *Mehen ('17); *Maiani ('19); ...



Past lattice *Guerrieri et al. ('15); *Bicudo, Wagner et al ('11-'19); Bali, Herzegger ('11); ...

$\Rightarrow$ These studies typically identify $ud\bar{b}\bar{b}$ $J^P = 1^+$ as favorable channel.

HQS-GDQ picture, consequences for $qq'\bar{Q}'\bar{Q}$ tetraquarks:

- $J^P = 1^+$ *ground state* tetraquark below meson-meson threshold
- Deeper binding with heavier quarks in the $\bar{Q}'\bar{Q}$ diquark
- Deeper binding for lighter quarks in the qq' diquark

Goal: $\Delta E = E_{\text{tetra}} - E_{\text{meson-meson}}$, e.g. in $ud\bar{b}\bar{b}$, $\ell s\bar{b}\bar{b}$ and others
$\Rightarrow$ Verify, quantify predictions of binding mechanism in mind

Direct lattice calculation of doubly heavy tetraquarks

Step I: Set up a basis of operators, here $J^P = 1^+$

Diquark-Antidiquark:

$$D = \left((q_a)^T (C\gamma_5) q'_b \right) \times \left[\bar{Q}_a (C\gamma_i) (\bar{Q}'_b)^T - a \leftrightarrow b \right]$$

Dimeson: $$M = (\bar{b}_a \gamma_5 u_a)\,(\bar{b}_b \gamma_i d_b) - (\bar{b}_a \gamma_5 d_a)\,(\bar{b}_b \gamma_i u_b)$$

Step II: Solve the GEVP and fit the energies

$$F(t) = \begin{pmatrix} G_{DD}(t) & G_{DM}(t) \\ G_{MD}(t) & G_{MM}(t) \end{pmatrix}, \quad F(t)\nu = \lambda(t) F(t_0)\nu\,, \quad \lambda(t) = A e^{-\Delta E (t - t_0)}$$

*$\Delta E = E_{\text{tetra}} - E_{\text{thresh}}$ in case of binding correlator $(C_{\mathcal{O}_1 \mathcal{O}_2}(t))/(C_{PP}(t) C_{VV}(t))$.

Most use these operators, but a larger basis has been worked out:

$I = 0$				$I = \frac{1}{2}$	
A_1^+	T_1^+	E^+	T_2^+	A_1^+	T_1^+
$\delta^{a_0}_{6,1} \bar{\delta}^{b_0}_{\bar{6},3}$	$\delta^{b_1}_{\bar{3},1} \bar{\delta}^{a_0}_{3,3}$	$\delta^{a_1}_{6,1} \bar{\delta}^{b_1}_{\bar{6},3}$	$\delta^{a_1}_{6,1} \bar{\delta}^{b_1}_{\bar{6},3}$	$\delta^{b_0}_{\bar{3},1} \bar{\delta}^{b_0}_{3,\bar{6}}$	$\delta^{a_1}_{6,1} \bar{\delta}^{b_0}_{\bar{6},3}$
$\delta^{a_1}_{6,1} \bar{\delta}^{b_1}_{\bar{6},3}$	$\delta^{\rho_2}_{\bar{3},1} \bar{\delta}^{\pi_2}_{3,3}$	$\delta^{b_1}_{\bar{3},1} \bar{\delta}^{a_1}_{3,3}$	$\delta^{b_1}_{\bar{3},1} \bar{\delta}^{a_1}_{3,3}$	$\delta^{b_1}_{\bar{3},1} \bar{\delta}^{a_1}_{3,3}$	$\delta^{b_1}_{\bar{3},1} \bar{\delta}^{a_0}_{3,3}$
$\delta^{b_0}_{\bar{3},1} \bar{\delta}^{a_0}_{3,3}$	$\delta^{\rho}_{\bar{3},1} \bar{\delta}^{\pi_2}_{3,3}$	$D^{[110]}_{A_2} D^{*[110]}_{B_2}$	$D^{[100]}_{A_2} D^{*[100]}_{E_2}$	$\delta^{\pi}_{6,1} \bar{\delta}^{\pi}_{\bar{6},\bar{6}}$	$\delta^{\rho}_{\bar{3},1} \bar{\delta}^{\pi}_{3,3}$
$\delta^{b_1}_{\bar{3},1} \bar{\delta}^{a_1}_{3,3}$	$\delta^{\rho}_{\bar{3},1} \bar{\delta}^{\pi}_{3,3}$		$D^{[110]}_{A_2} D^{*[110]}_{A_1}$	$\delta^{\rho}_{\bar{3},1} \bar{\delta}^{\rho}_{3,\bar{6}}$	$\delta^{\rho}_{\bar{3},1} \bar{\delta}^{\rho}_{3,\bar{6}}$
$D^{[000]}_{A_1} D(2S)^{[000]}_{A_1}$	$D^{[000]}_{A_1} D^{*[000]}_{T_1}$		$D^{[110]}_{A_2} D^{*[110]}_{B_1}$	$D^{[000]}_{A_1} D_{s\,A_1}^{[000]}$	$D^{[000]}_{A_1} D^{*[000]}_{s\,T_1}$

*HadronSpectrum Coll. ('17)

Roadmap:

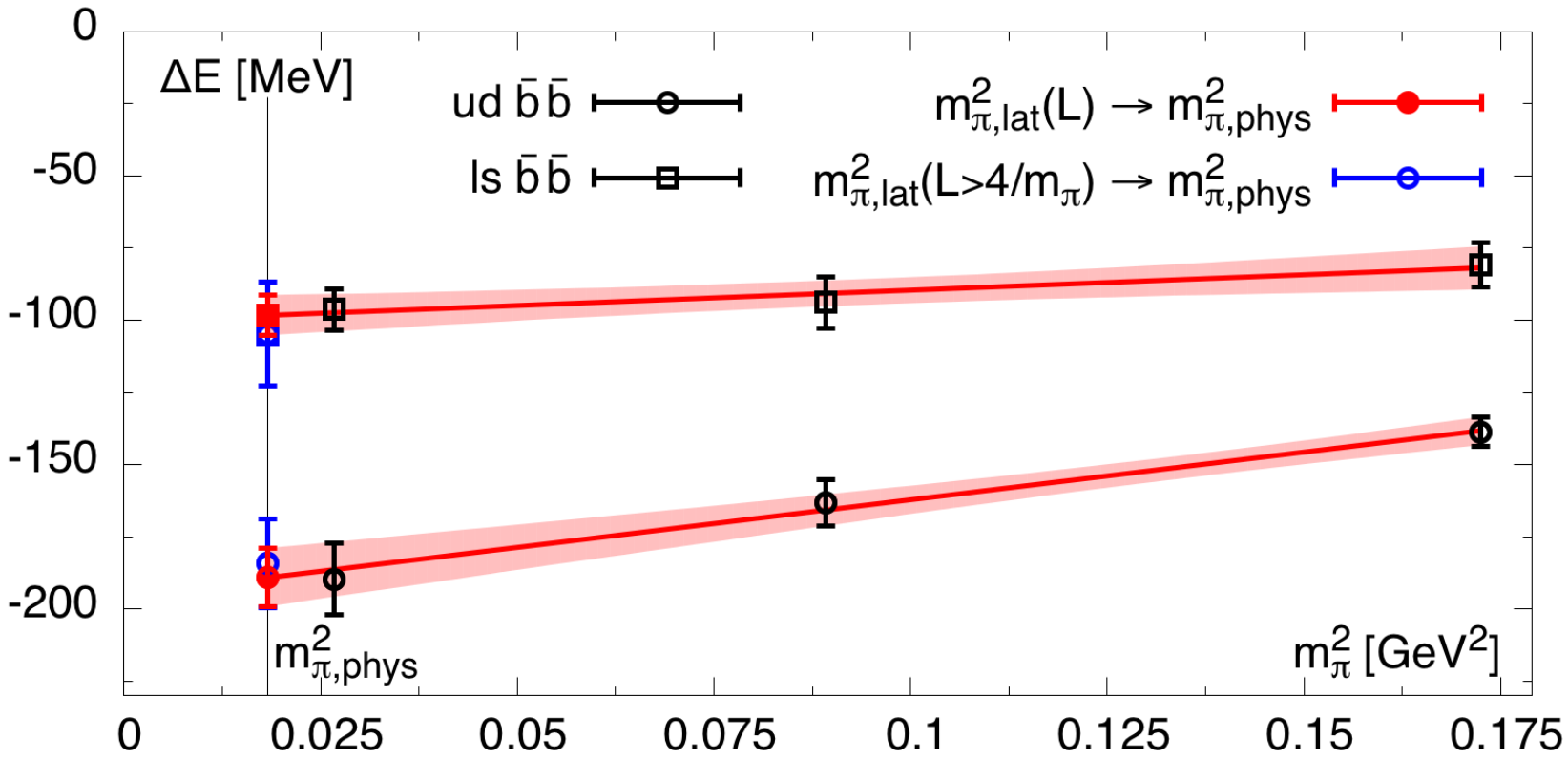
- Determine $\Delta E_{tetra} \Rightarrow$ Establish ground state
- Quark mass dependence $qq', \bar{Q}\bar{Q}' \Rightarrow$ Verify, quantify predictions
- Finite volume effects $\Rightarrow$ Scattering or stable state
- Energy level systematics $\Rightarrow$ Precision studies

Currently four lattice studies focused on energy levels:

1. Junnarkar, Mathur, Padmanath ('18)
2. Leskovec, Meinel, Plaumer, Wagner ('19)
3. HadronSpectrum Coll. ('17)
4. AF, Hudspith, Lewis, Maltman ('17), ('18)

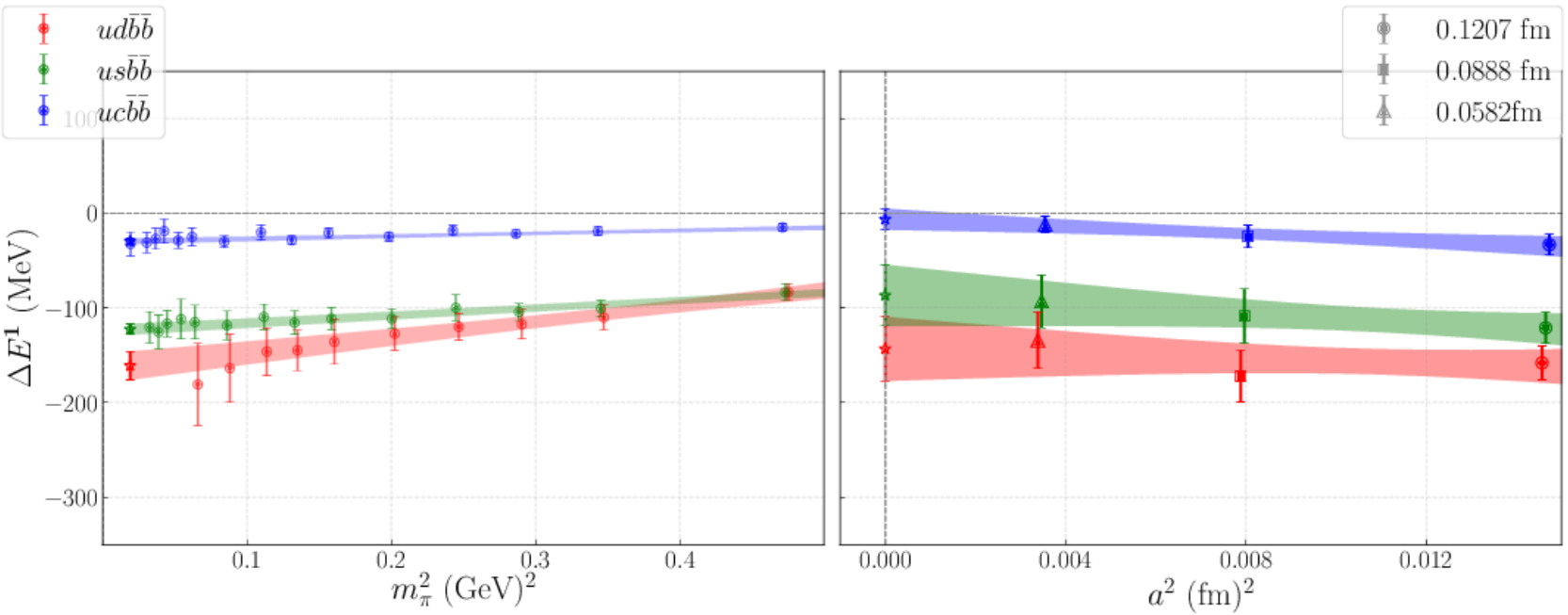
	1.	2.	3.	4.
Configs.	MILC	RBC/UKQCD	HadSpec	PACS-CS
N_{ens}, N_{alat}	25,3	5,3	1,1	3,1
m_π[MeV]	153-689†	139-431	391	164-415
L[fm]	~ 2.80	2.65-5.48	1.92	2.88
ℓ, s-quarks	Overlap	DMW	ani.-Clover	Clover
c-quark	Fermilab		ani.-Clover	Tsukuba
b-quark	NRQCD	NRQCD		NRQCD
Props	gf-wall	smeared	distilled	gf-wall
Ops (N_{ops})	local (2,3)	non-local sink (5)	non-local (full)	local (2,3)

Francis et al. ('17)



- *Bound ground state* tetraquark below meson-meson threshold ✓
- Deeper binding with heavier $\bar{Q}'\bar{Q}$ diquarks
- Deeper binding for lighter quarks in the qq' diquark ✓

Junnarkar et al. ('18)



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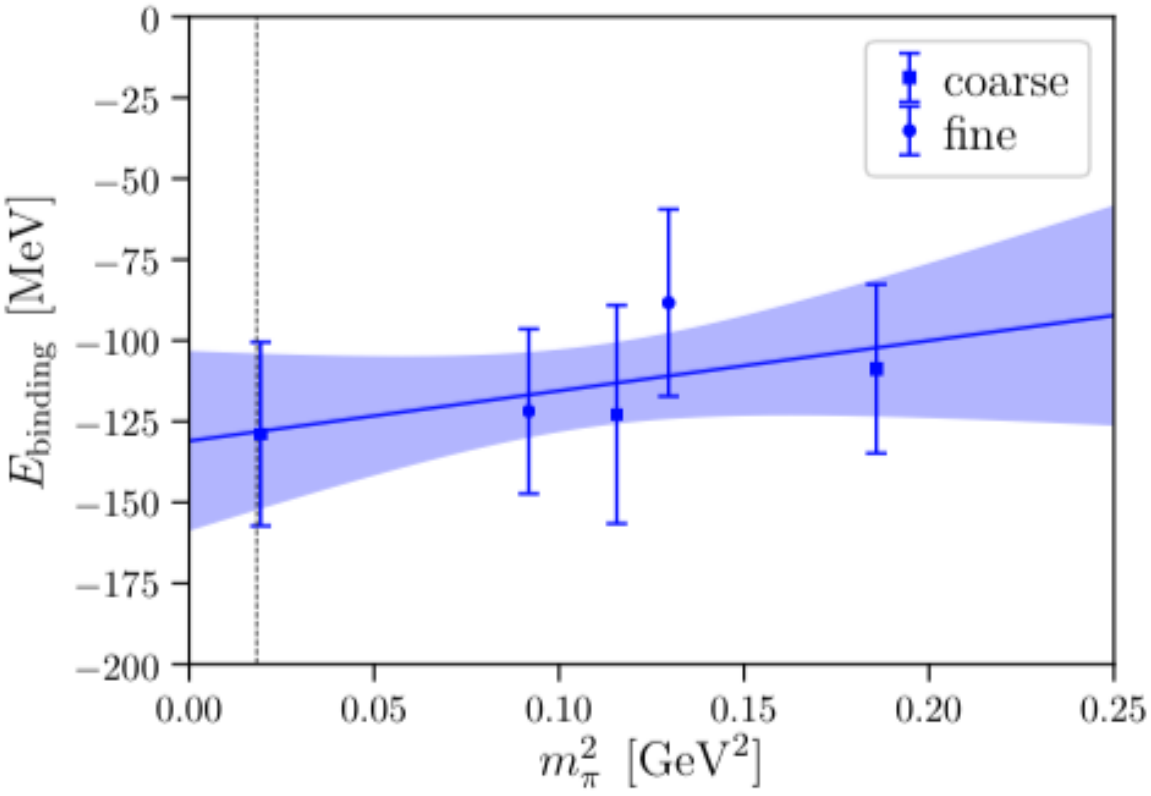
Leskovec et al. ('19)



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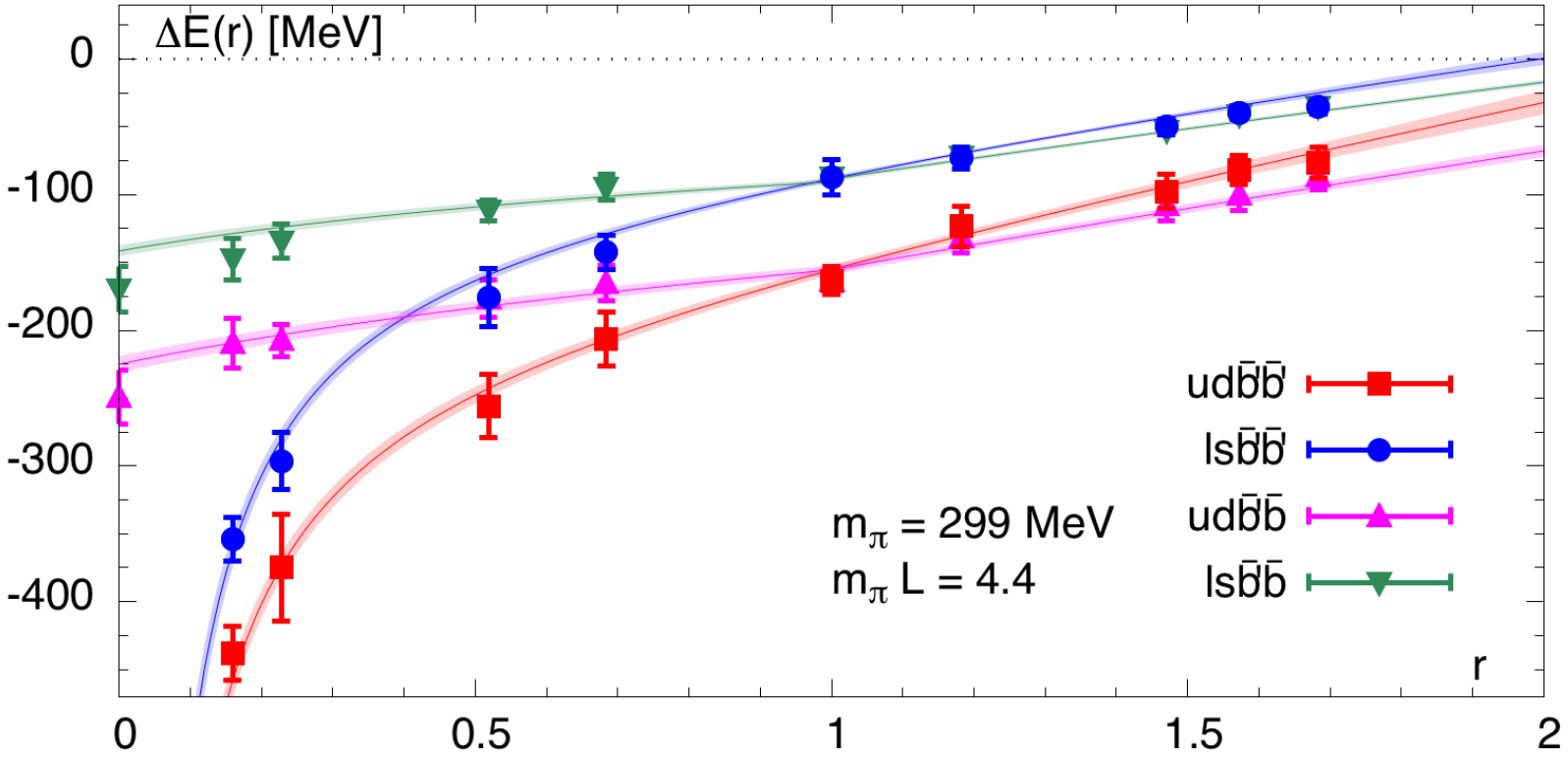
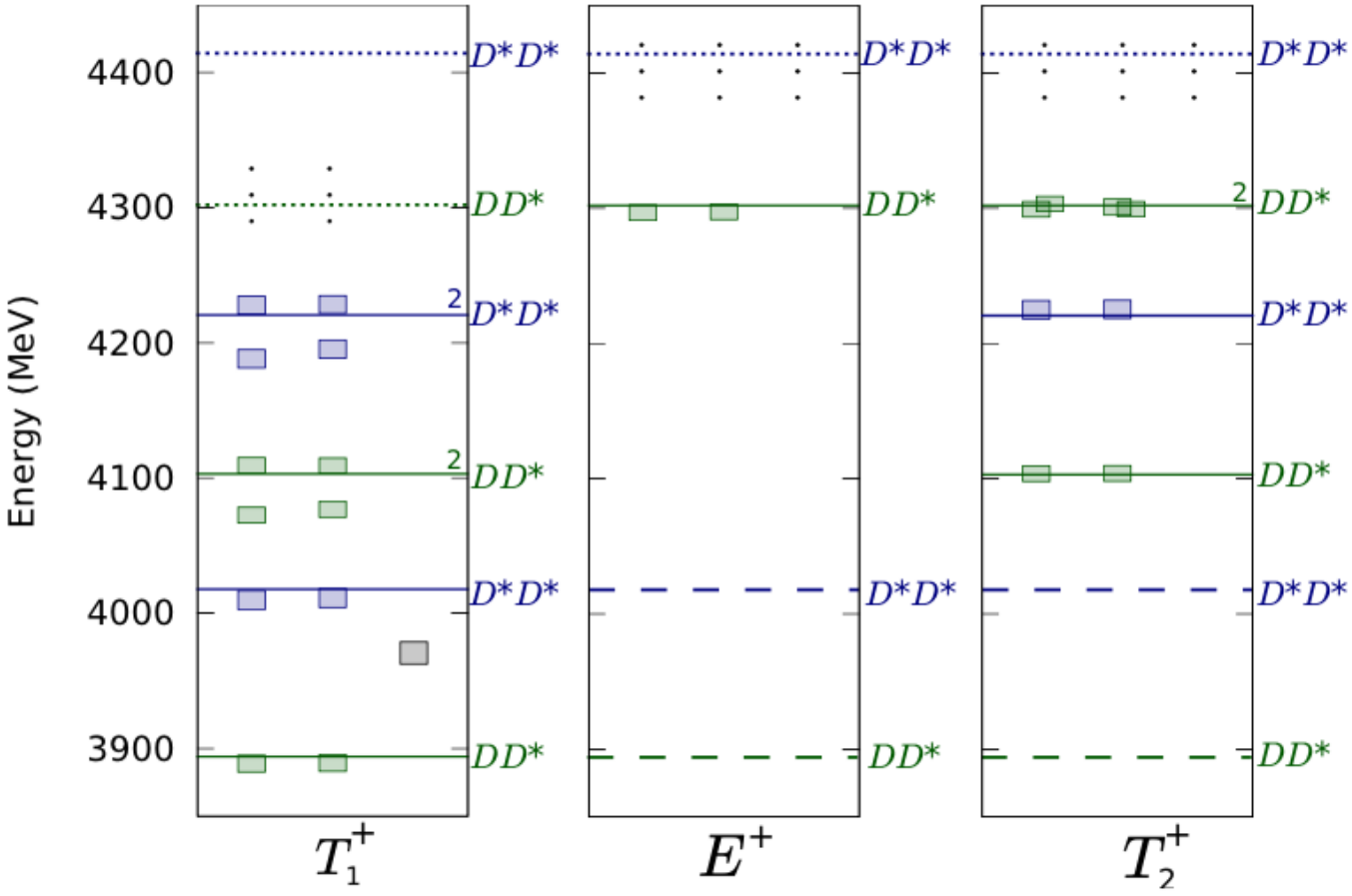


Scan in $m_{b'}$ maps out the heavy quark mass dependence.
$\Rightarrow$ Most likely bound at m_c: $ud\bar{c}\bar{b}$, only just (un)bound: $ud\bar{c}\bar{c}$

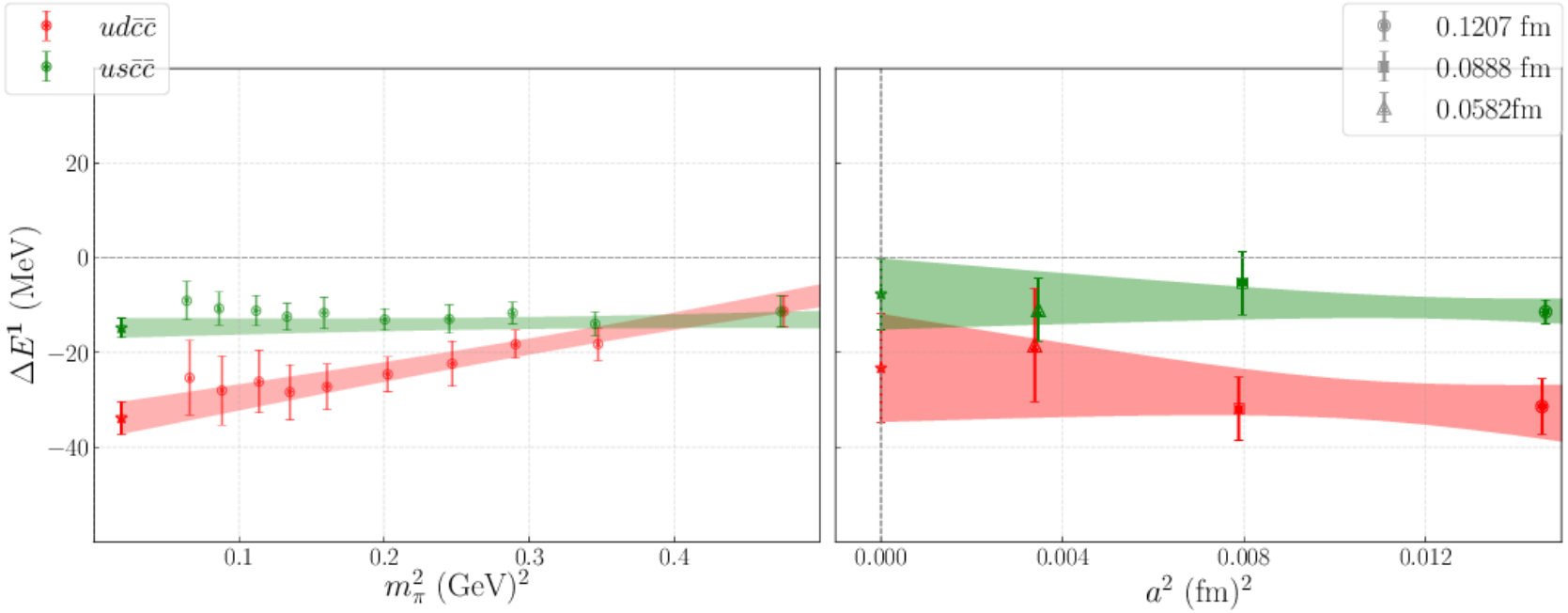
- *Bound ground state* tetraquark below meson-meson threshold ✓
- Deeper binding with heavier $\bar{Q}'\bar{Q}$ diquarks ✓
- Deeper binding for lighter quarks in the qq' diquark ✓

HadronSpectrum ('17)



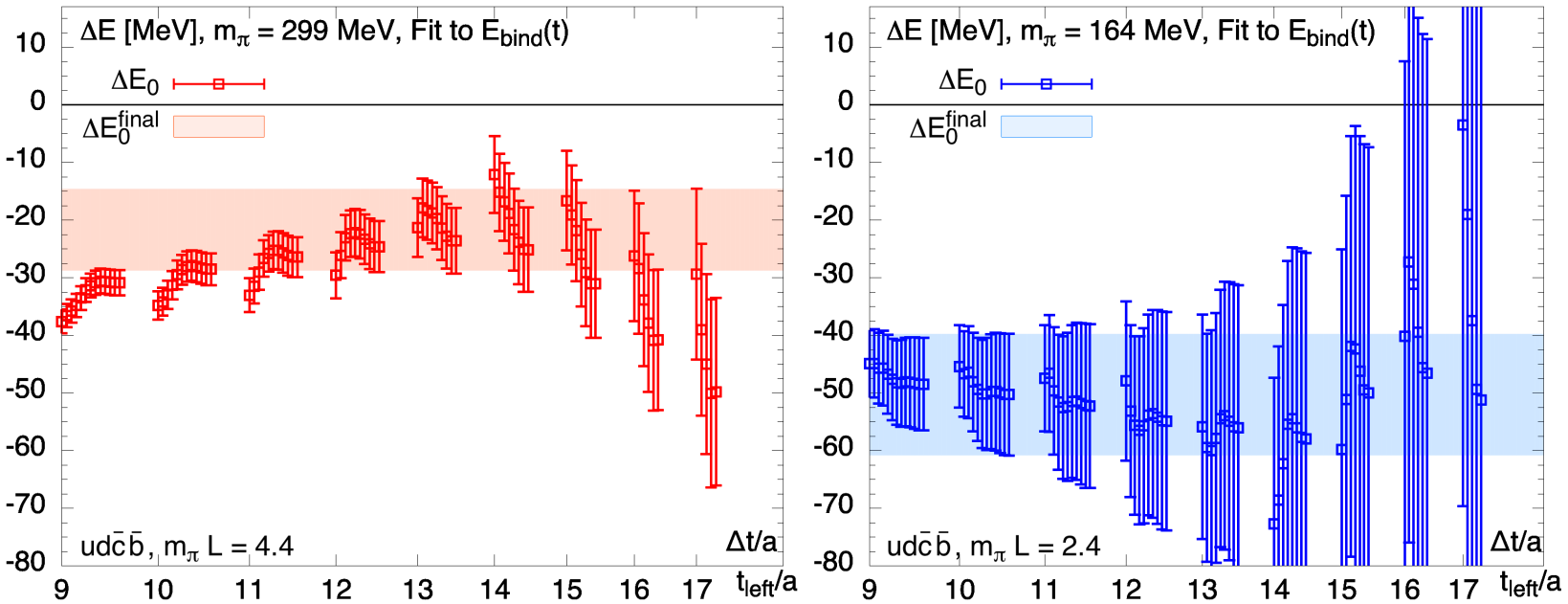
$\Rightarrow$ No clear signs of bound $ud\bar{c}\bar{c}$ at $m_\pi = 391$MeV

Junnarkar et al. ('18)



*Recall: gf-wall correlators approach from below and have no positive definite spectral-decomp.

$\Rightarrow$ Binding in $ud\bar{c}\bar{c}$ at the physical point $\Delta E_{ud\bar{c}\bar{c}} = 23(11)$



$\Rightarrow$ Binding in $ud\bar{c}\bar{b}$ at $m_\pi = 299$ and 164MeV. (Increasing with decreasing m_π)

Calculation indeed reveals evidence for doubly heavy tetraquarks:

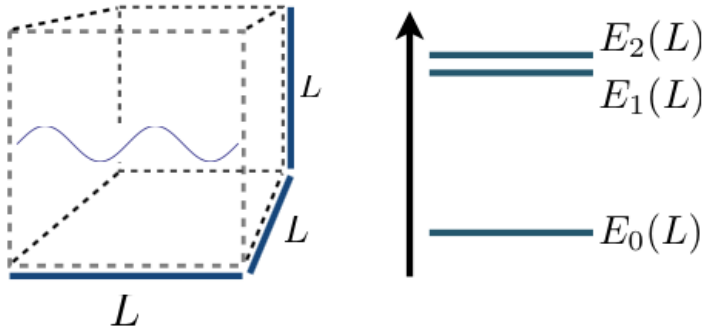
- $\Delta E_{ud\bar{b}\bar{b}} \simeq 189(13)$ MeV and $\Delta E_{ls\bar{b}\bar{b}} \simeq 98(10)$ MeV (our work)
- $\Delta E_{ud\bar{c}\bar{b}} \simeq 15 - 61$ MeV (above)
- $\Delta E_{ud\bar{c}\bar{c}} \simeq 23(11)$ MeV or unbound (two groups)

Finite volume corrections

Large energy shifts are possible due to the finite lattice volume.

Scenario I: Scattering state
The finite volume energy belongs to a scattering state, the corrections go as

$$E_{b,L} \sim E_{b,\infty} \cdot \left[1 + \frac{a}{L^3} + \mathcal{O}(\frac{1}{L^4})\right]$$



*M. Hansen

Scenario II: Stable state
The corrections are exponentially suppressed with $\kappa = \sqrt{E_{b,\infty}^2 + p^2}$

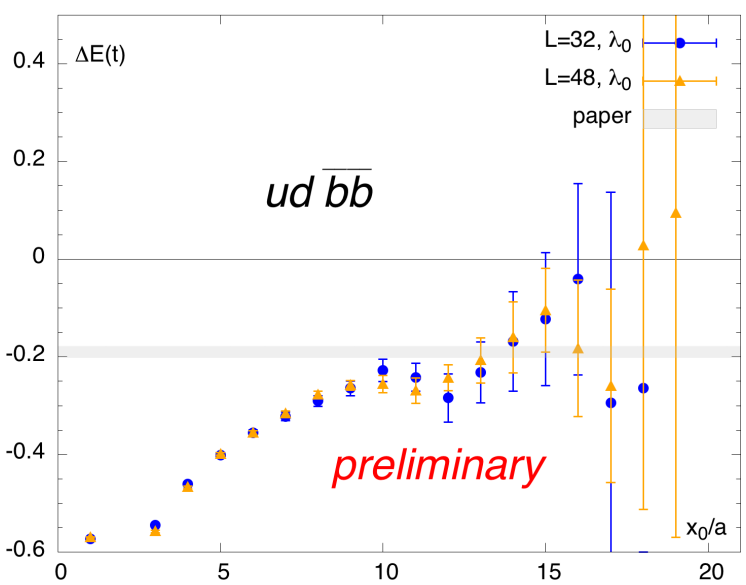
$$E_{b,L} \sim E_{b,\infty} \cdot \left[1 + Ae^{-\kappa L}\right]$$

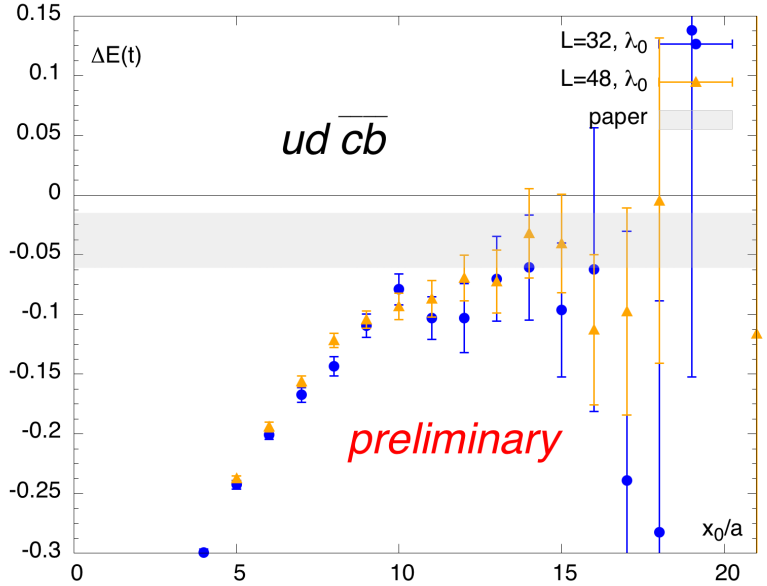
An in-depth study of volume effects is absolutely important and gives insight into the nature of the states observed.

*New work by Colquhoun, AF, Hudspith, Lewis, Maltman.

κ_l	L	T	m_π[MeV]	$m_\pi L$	L[fm]	n_{conf}	status
0.13781	32	64	164	2.4	2.88	80	preliminary
	48	64		3.6	4.32	130	preliminary
	64	64		4.8	5.76	32	pending

⇒ New volumes for a well understood/tuned setup. (add. $m_\pi \simeq 180, 200$MeV)





- Good agreement is a sign of stable scenario†. †See e.g. Beane et al. ('17) [1705.09239].
- Similar signs in first scattering analysis* (2 point ERE). *Leskovec et al. ('19)

Further work needed!

Experimental detection possibilities

$J^P = 1^+$ doubly heavy tetraquarks are a new type of exotic predicted in QCD. Many possible decay channels exist, examples:

$ud\bar{b}\bar{b} \longrightarrow B^+D^0$	$us\bar{b}\bar{b} \longrightarrow B^+D_s^0$	$ud\bar{c}\bar{b} \longrightarrow \bar{D}^0\bar{D}^0$
$\longrightarrow J/\psi B^+K^0$	$\longrightarrow B_s\bar{D}^+$	$us\bar{c}\bar{b} \longrightarrow \pi^-K^+B^0$
	$\longrightarrow J/\psi B_sK^+$	$ds\bar{c}\bar{b} \longrightarrow D^-B^+\gamma$

Highest experimental detection probability at LHCb. *Gershon, Poluetkov

- Have presented an inclusive strategy to search for double beauty hadron production at the LHC through the signature of displaced B_c^- mesons
  - Well suited for search at LHCb
  - Still highly challenging, but seems to have better potential than exclusive searches
- More detailed study needed to understand LHCb sensitivity
  - Depends on bbx production cross-sections, inclusive branching fractions to final states containing B_c^- mesons, masses and lifetimes
  - More theory work on these would be welcome

arXiv:1810.06657

Reviewed lattice studies on doubly heavy tetraquarks:

1. Junnarkar, Mathur, Padmanath ('18)
2. Leskovec, Meinel, Plaumer, Wagner ('19)
3. HadronSpectrum Coll. ('17)
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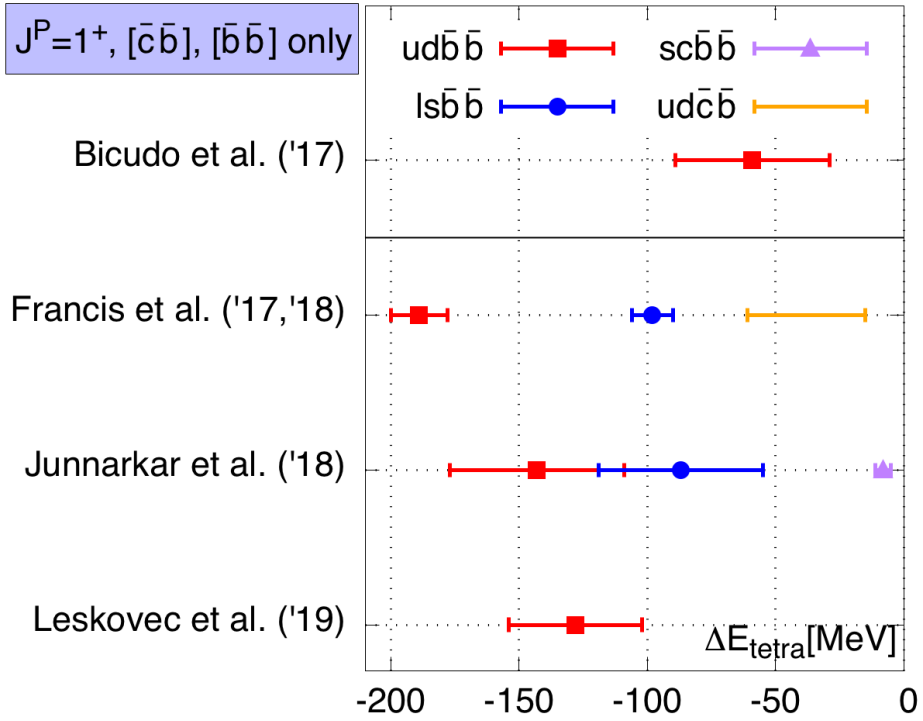
Ops (N_{ops})	local (2,3)	non-local sink (5)	non-local (full)	local (2,3)
$J^P = 1^+$	$ud\bar{b}\bar{b}$, $ud\bar{c}\bar{c}$ $\ell s\bar{b}\bar{b}$, $\ell s\bar{c}\bar{c}$ $\ell c\bar{b}\bar{b}$, $sc\bar{b}\bar{b}$	$ud\bar{b}\bar{b}$	$ud\bar{c}\bar{c}$	$ud\bar{b}\bar{b}$ $\ell s\bar{b}\bar{b}$ $ud\bar{c}\bar{b}$
$J^P = 0^+$	$uu\bar{b}\bar{b}$, $uu\bar{c}\bar{c}$ $ss\bar{b}\bar{b}$, $ss\bar{c}\bar{c}$ $cc\bar{b}\bar{b}$		$ud\bar{c}\bar{c}$	

† Not all masses used for every channel.

(candidate): observed by more than 1 group (candidate): unbound result
(candidate): observed by 1 group (candidate): not confirmed by more than 1 group

Prospects and summary

- Direct calculations reveal evidence of $ud\bar{b}\bar{b}$, $\ell s\bar{b}\bar{b}$ $J^P = 1^+$ tetraquarks.
- Broad agreement with the intuitive binding mechanism.
- Binding in $ud\bar{c}\bar{b}$, $sc\bar{b}\bar{b}$, $ud\bar{c}\bar{c}$, $\ell s\bar{c}\bar{c}$ requires further study.



- First scattering and volume scaling analyses show signs that $ud\bar{b}\bar{b}$ but also $\ell s\bar{b}\bar{b}$ and $ud\bar{c}\bar{b}$ are stable states. A clear statement is premature.
- Systematics need to be better controlled:
  - excited state contamination, operator bases
  - chiral limit (especially as deeper binding for lighter π's)
  - discretisation effects, continuum limit
- Outlook for experimental detection (1806.09288, 1810.06657)

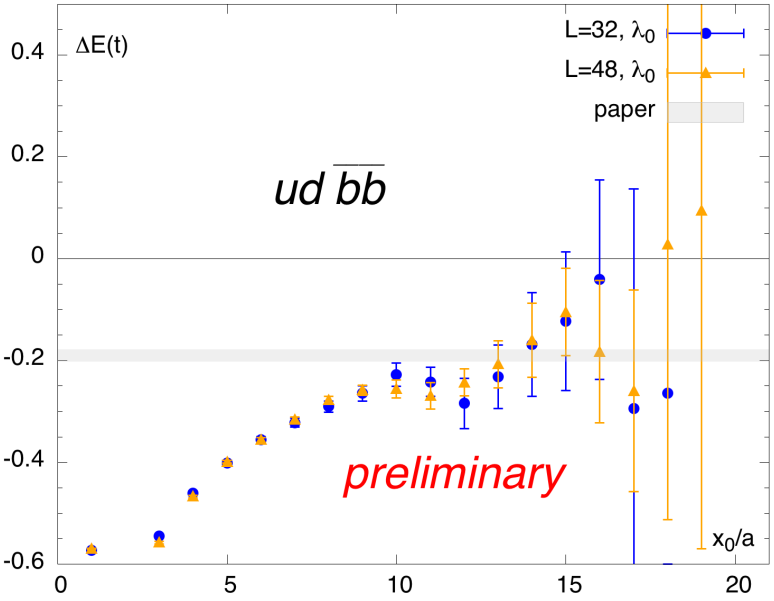
Exciting prospects and an interesting challenge!

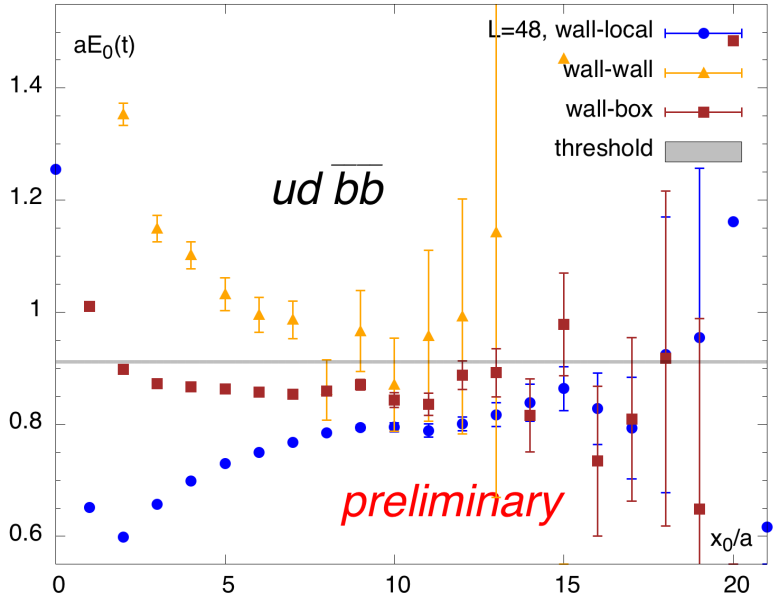
Thank you for your attention.

Appendix

Solidifying conclusions

*New work by Colquhoun, AF, Hudspith, Lewis, Maltman





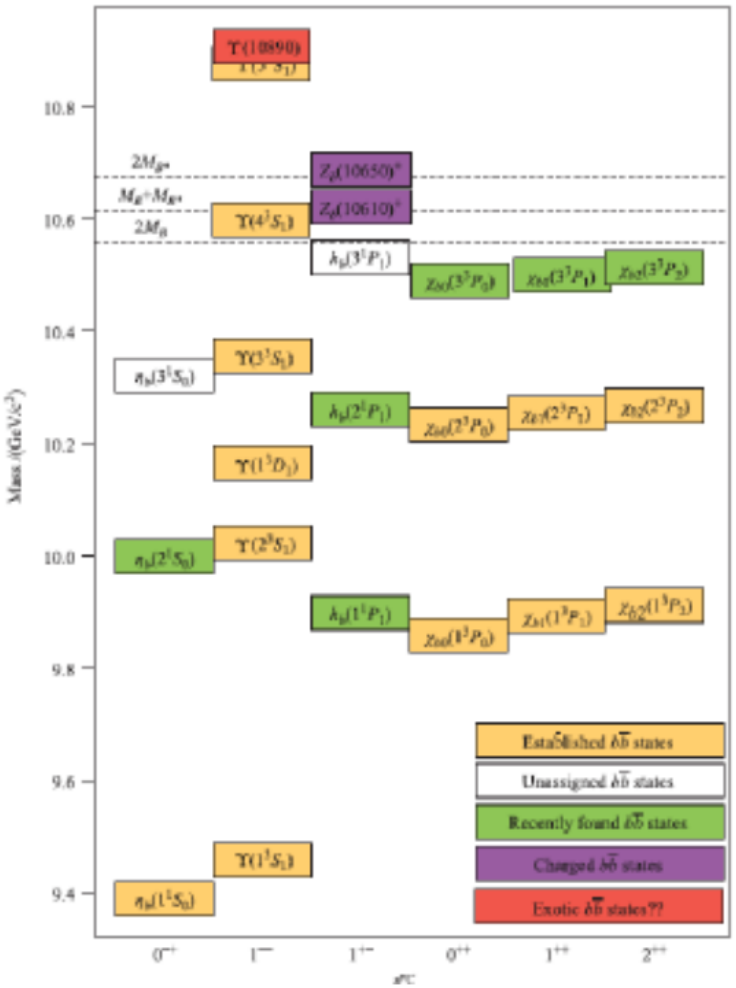
Finite volume scaling
$\rightarrow$ stable states in QCD?

To Do: Further statistics and study is needed to firmly establish this conclusion.

Wall-local correlators
$\rightarrow$ approach to ground state from below. Systematic?

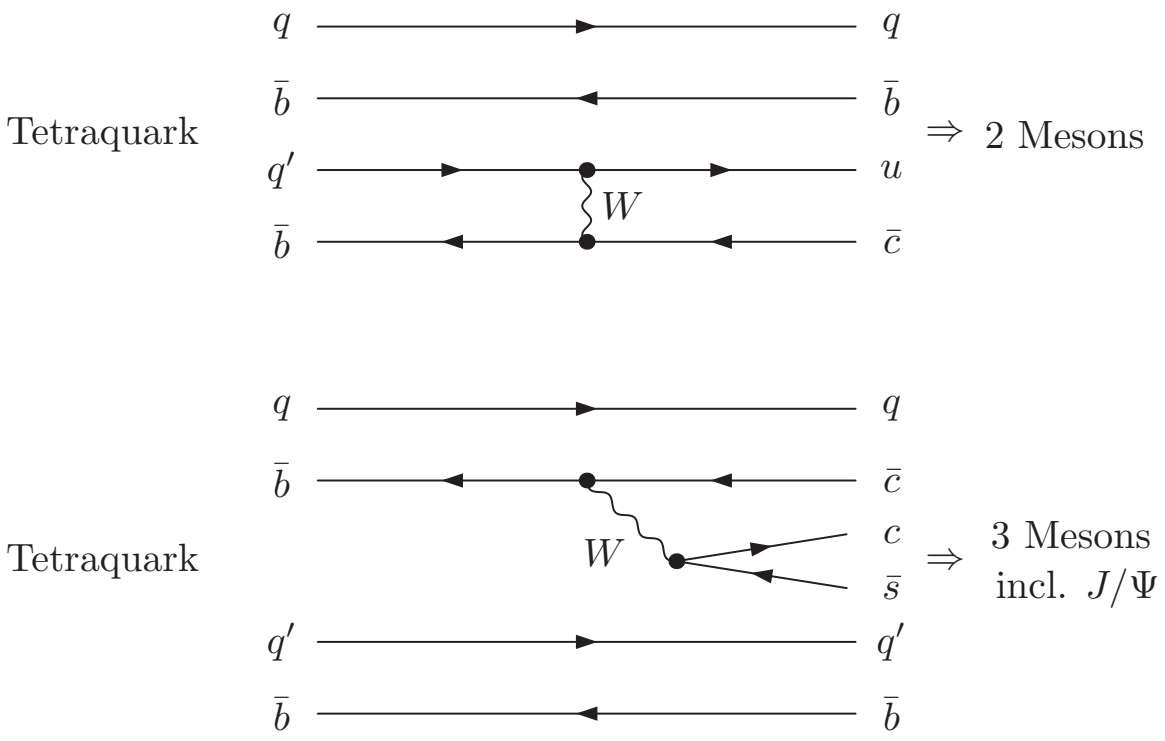
To Do: Extend and include correlators that approach from above, e.g. wall-box.

New states and exotica in b-sector



Detection possibilities in experiment: $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$

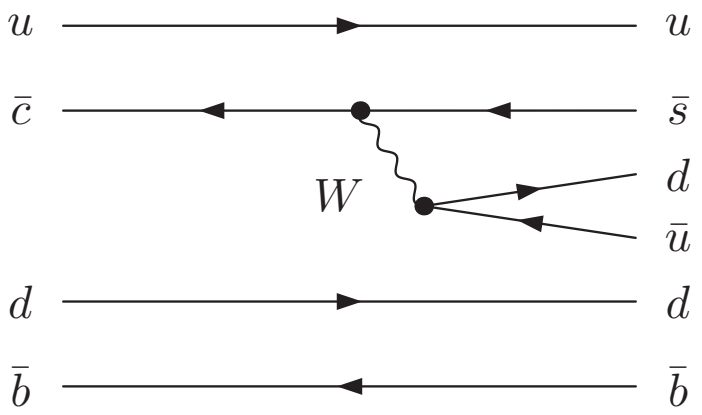
With such deep ΔE, both $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ tetraquarks decay only weakly



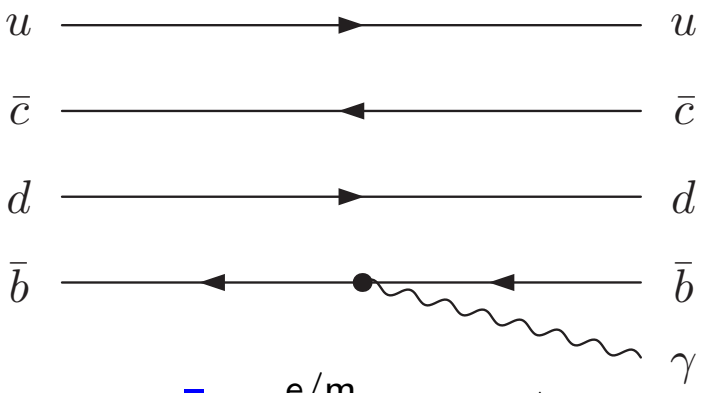
$ud\bar{b}\bar{b}$	$\to$	B^+D^0
	$\to$	$J/\psi B^+K^0$
$us\bar{b}\bar{b}$	$\to$	$B^+D_s^0$
	$\to$	$B_s\bar{D}^+$
	$\to$	$J/\psi B^+\phi$
	$\to$	$J/\psi B_sK^+$
$ds\bar{b}\bar{b}$	$\to$	$B^+D_s^-$
	$\to$	$B_s\bar{D}^0$
	$\to$	$J/\psi B^0\phi$
	$\to$	$J/\psi B_sK^0$

Detection possibilities in experiment: $ud\bar{c}\bar{b}$

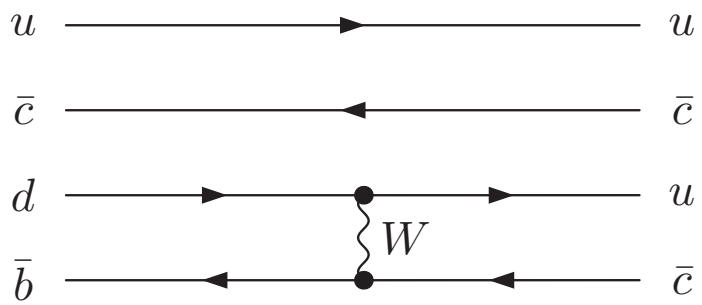
At this point $ud\bar{c}\bar{b}$ could decay only weakly or also electromagnetically



$us\bar{c}\bar{b} \xRightarrow{\text{weak}} (\pi^- K^+ B^0)$



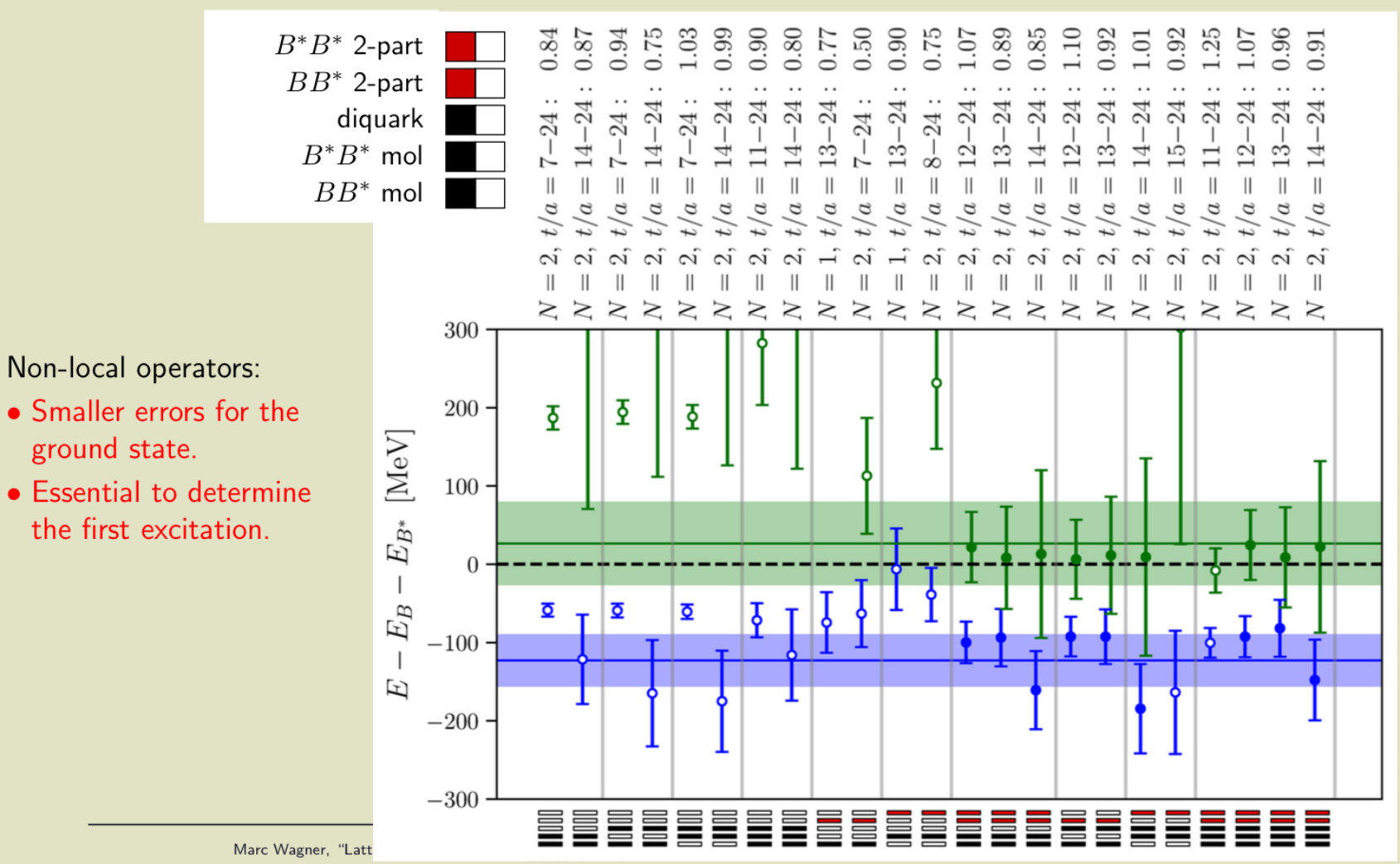
$ds\bar{c}\bar{b} \xRightarrow{\text{e/m}} (D^- B^+ \gamma)$



$ud\bar{c}\bar{b} \xRightarrow{\text{weak}} (\bar{D}^0 \bar{D}^0)$

Non-local operators

Marc Wagner at QWG '19, results from Leskovec et al. ('19)



Phenomenological model

$\bar{b}'\bar{b}'$:

$$\Delta E_{ud\bar{b}'\bar{b}'} = \frac{C_0}{2r} + C_1^{ud} + C_2^{ud}(2r) + (23\ \text{MeV})\, r,$$
$$\Delta E_{\ell s\bar{b}'\bar{b}'} = \frac{C_0}{2r} + C_1^{\ell s} + C_2^{\ell s}(2r) + (24\ \text{MeV})\, r$$

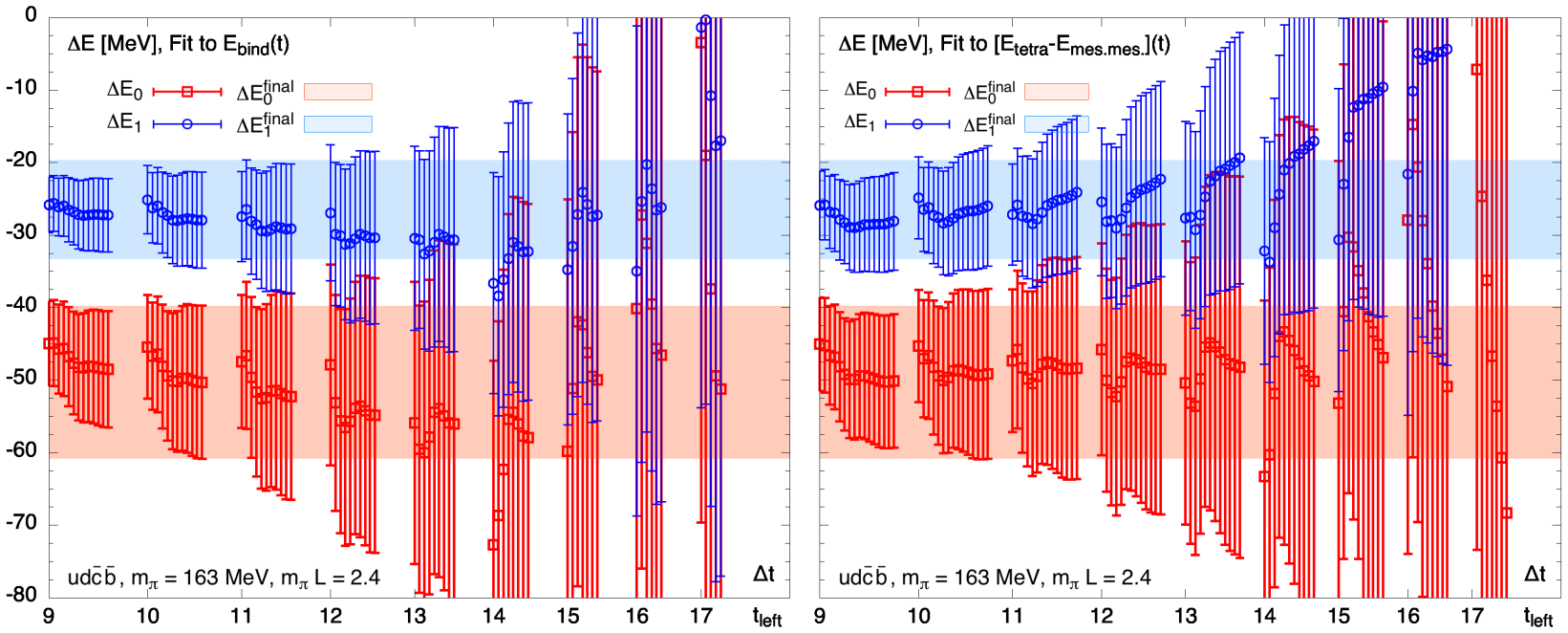
$\bar{b}'\bar{b}$, $r < 1$:

$$\Delta E_{ud\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{ud} + C_2^{ud}(1+r) + (34\ \text{MeV} - 11\ \text{MeV}\, r)\,,$$
$$\Delta E_{\ell s\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{\ell s} + C_2^{\ell s}(1+r) + (34\ \text{MeV} - 12\ \text{MeV} r)$$

$\bar{b}'\bar{b}$, $r > 1$:

$$\Delta E_{ud\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{ud} + C_2^{ud}(1+r) + (34\ \text{MeV}\, r - 11\ \text{MeV})\,,$$
$$\Delta E_{\ell s\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{\ell s} + C_2^{\ell s}(1+r) + (36\ \text{MeV}\, r - 11\ \text{MeV})$$

Energy of $ud\bar{c}\bar{b}$ at m_π[MeV] = 164



$ud\bar{c}\bar{b}$ comparison with effective masses