# Towards the spectrum of flavour-diagonal pseudoscalar mesons in QCD+QED

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*QCDSF Collaboration*

*Lattice 2019, June 17 - June 22, 2019,*
*Wuhan, China*

# CSSM/QCDSF/UKQCD Collaborations

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- **Z. Kordov (Adelaide)**
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- R. Horsley (Edinburgh)
- Y. Nakamura (RIKEN, Kobe)
- H. Perlt (Leipzig)
- P. Rakow (Liverpool)
- G. Schierholz (DESY)
- H. Stüben (Hamburg)
- R. Young (Adelaide)

# Motivation

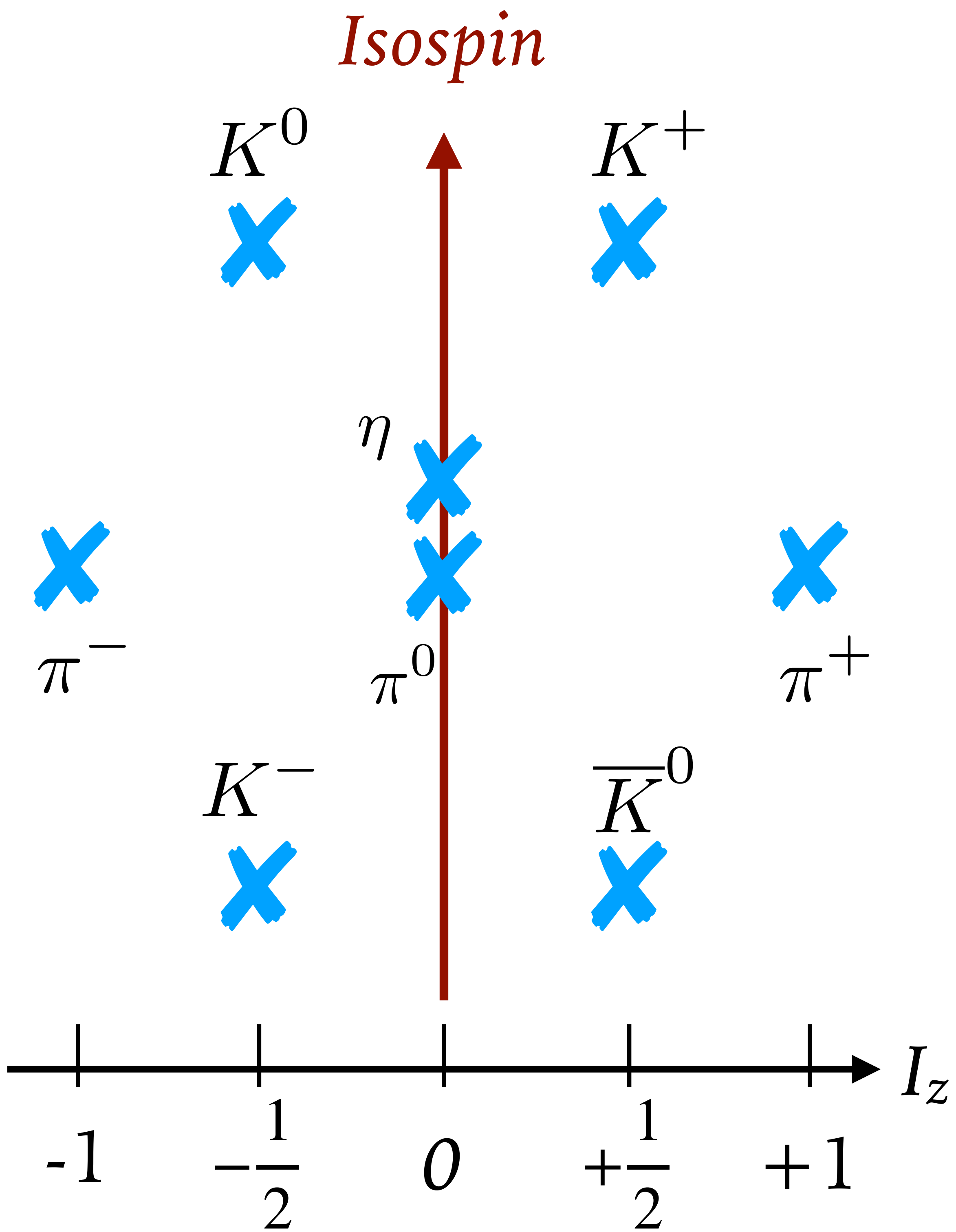
- Mixing of flavour-neutral $\eta$-$\eta'$ due to $SU(3)_f$ breaking
  - how are physically observed mass eigenstates are formed from $SU(3)_f$ octet and singlet components?
  - are there gluonic components?
  - implications for eg. CKM studies using

$$B^0_{d/s} \to J/\psi\eta^{(\prime)}$$

➡ help identify new physics contributions to $B_s^0 - \bar{B}_s^0$ mixing

  - already received much attention in Lattice QCD
- Allowing for isospin-breaking (naturally broken with QED)
  - $\pi^0$ can also mix
  - not yet studied on lattice

# SU(2) subgroups — Isospin ($u \leftrightarrow d$)



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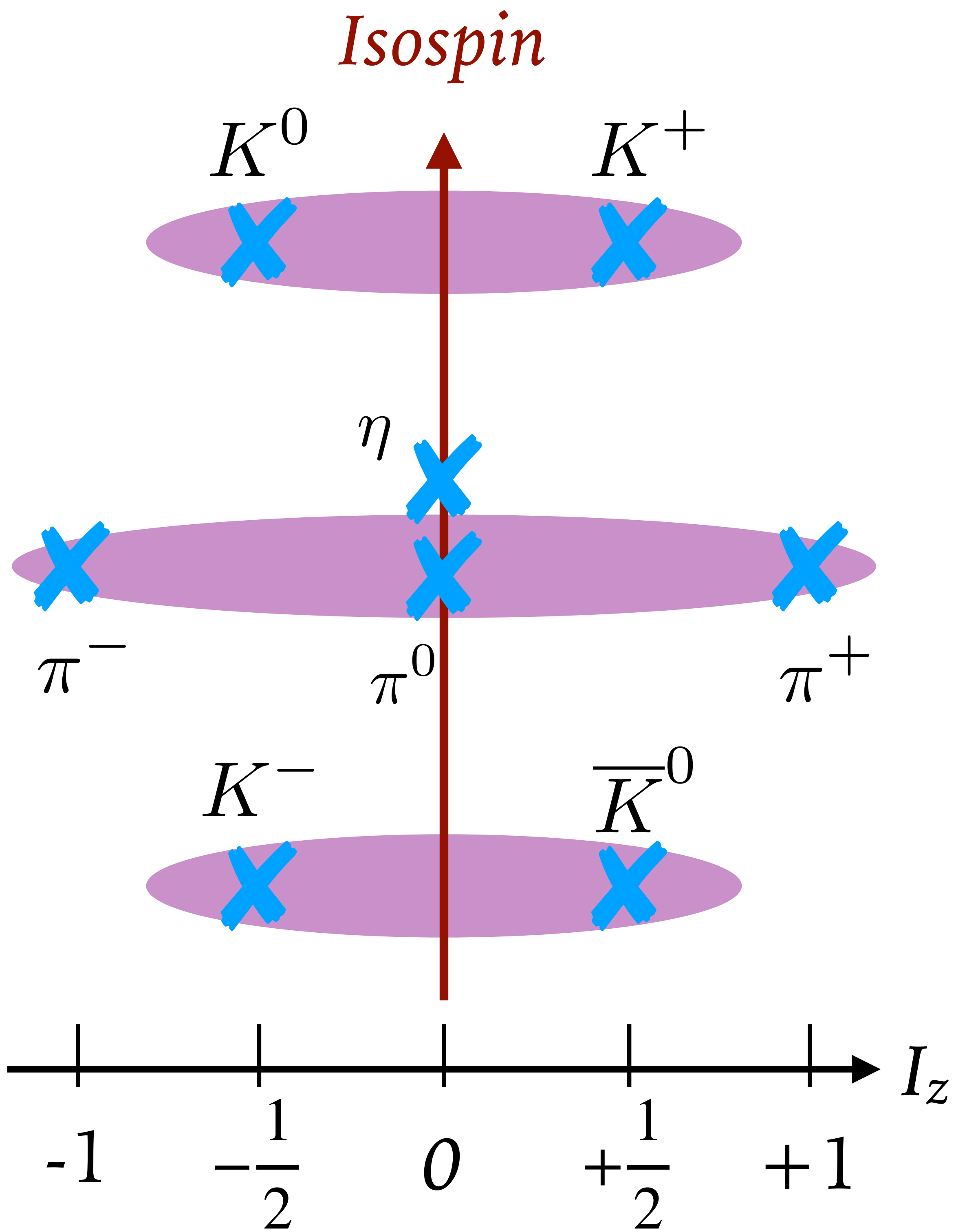

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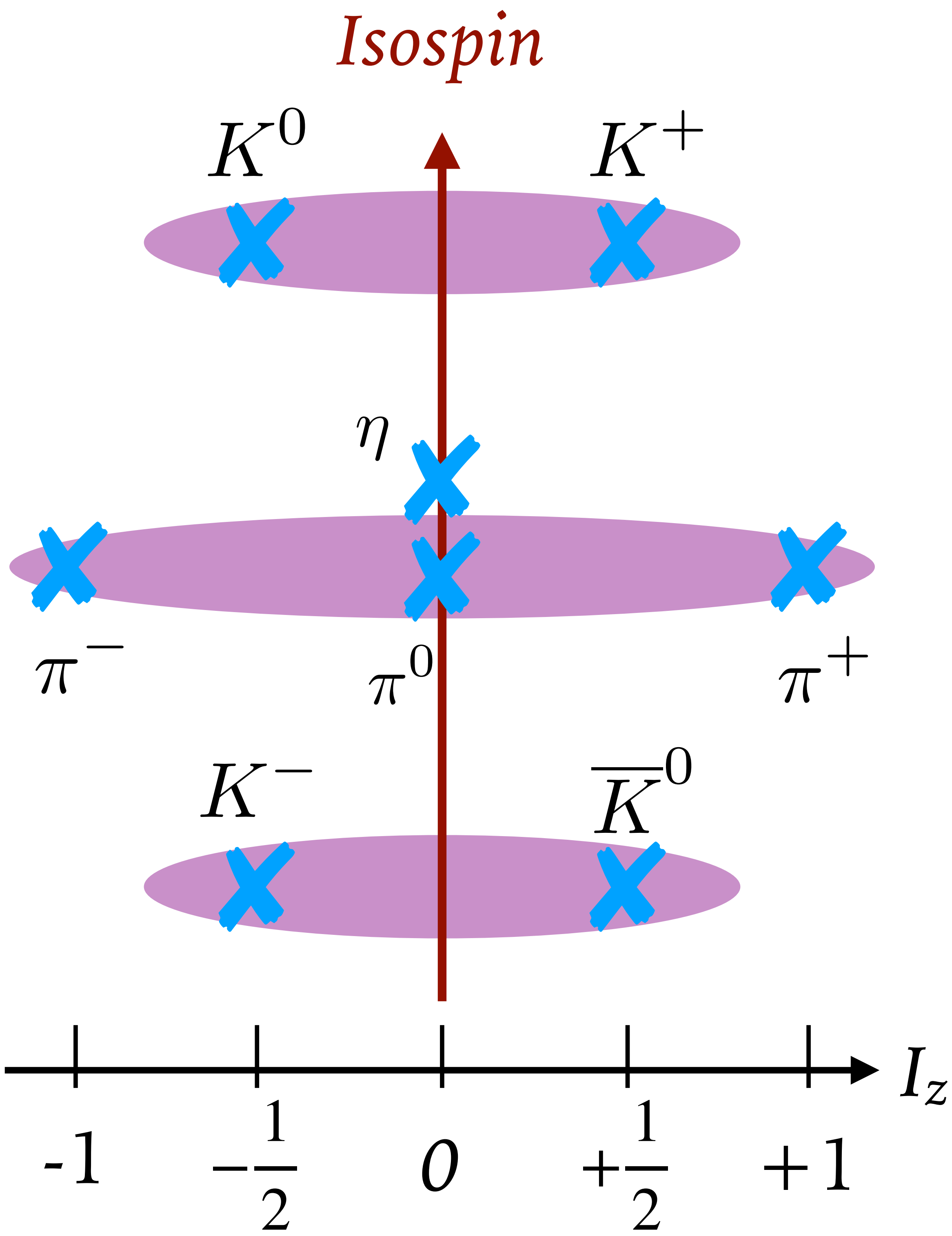
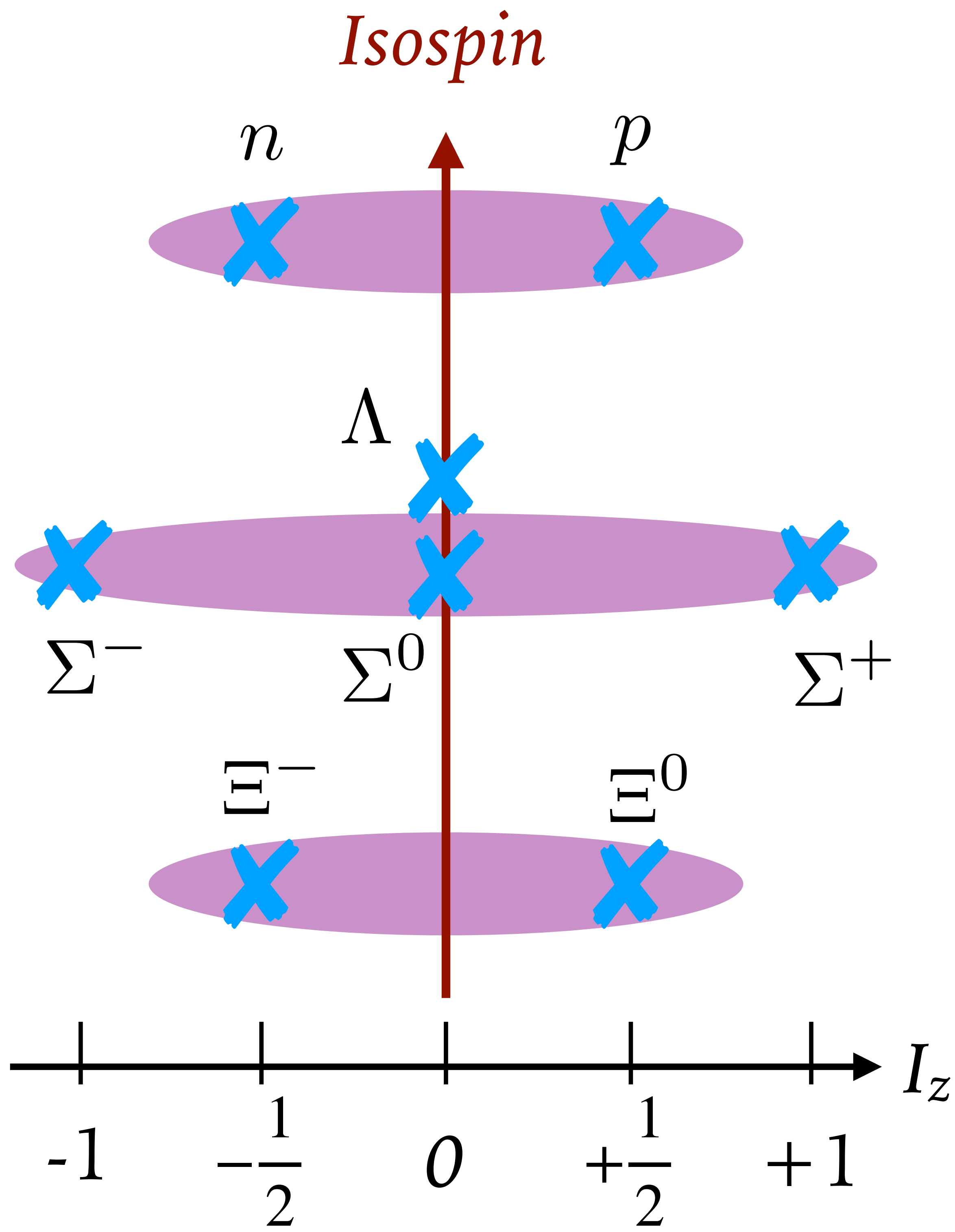
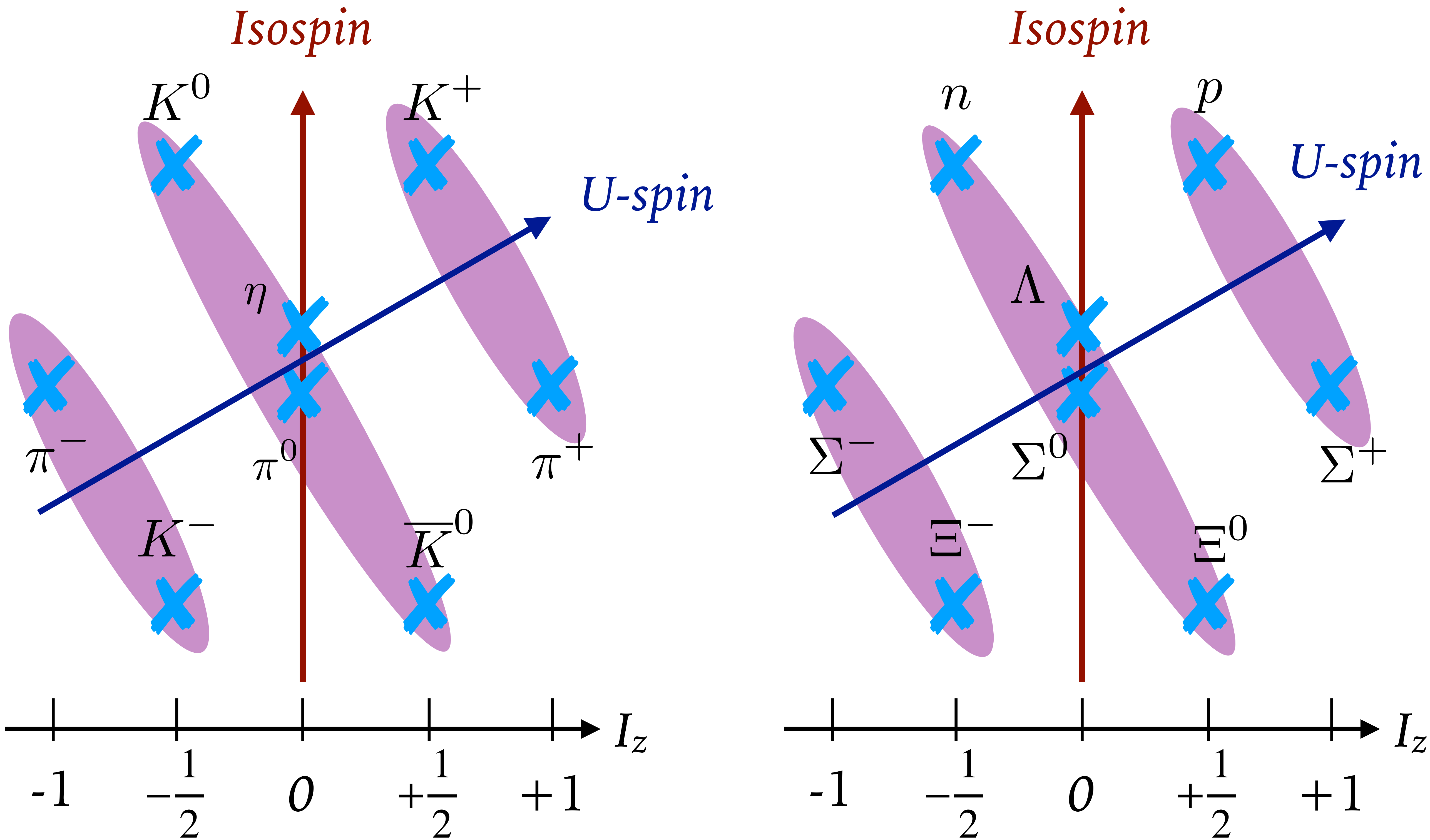
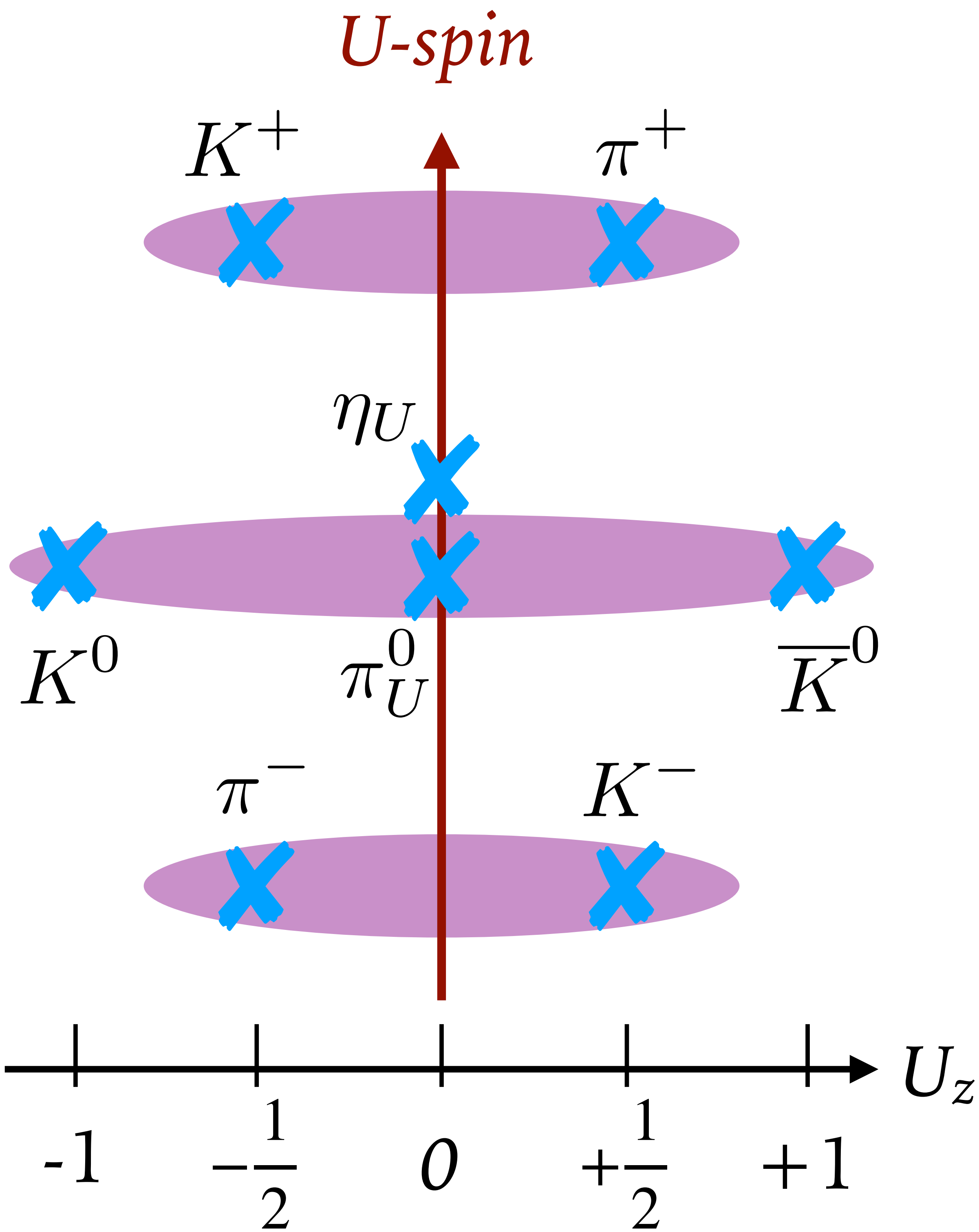


# SU(2) subgroups — U-spin $(d \leftrightarrow s)$



# SU(2) subgroups — U-spin $(d \leftrightarrow s)$




*Isospin basis:*

$$\pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$

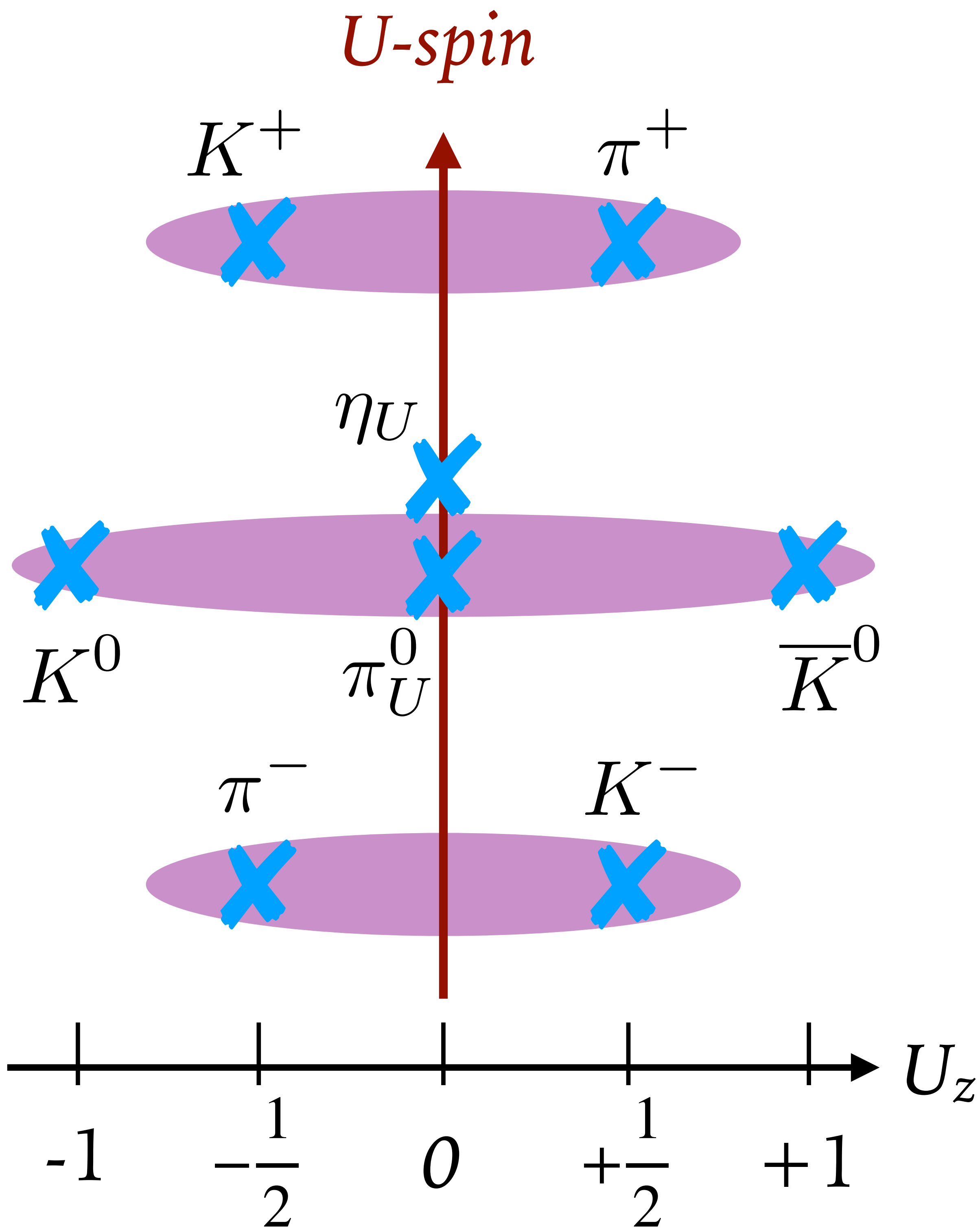
$$\eta = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$$

*U-spin basis:*

$$\pi^0_U = \frac{1}{\sqrt{2}}\left(d\bar{d} - s\bar{s}\right)$$

$$\eta_U = \frac{1}{\sqrt{6}}\left(d\bar{d} + s\bar{s} - 2u\bar{u}\right)$$

# SU(2) subgroups — U-spin $(d \leftrightarrow s)$




- Note:
  - all U-spin multiplets have same electric charge
  - Natural starting point for QCD+QED simulations
  - SU(3) broken naturally by quark charges
    $Q_u = +\frac{2}{3},\ Q_d = Q_s = -\frac{1}{3}$
  - Breaking purely EM if masses tuned to be same
  - How to achieve this?

# Lattice QCD+QED set-up

**QCDSF, JHEP 1604, 093 (2016)**

- $SU(3)_f$ symmetric point?
  - QCD: trivial — input $am_u = am_d = am_s$ ➡ $m_u^R = m_d^R = m_s^R$
  - +QED: with $Q_u = +\frac{2}{3},\ Q_d = Q_s = -\frac{1}{3}$

$$am_u = am_d = am_s \Rightarrow m_u^R \neq m_d^R = m_s^R$$

- Define the *"Dashen Scheme"*
  - Tune quark masses to $SU(3)_{sym}$ point via $m_\pi^{u\bar{u}} = m_\pi^{d\bar{d}} = m_\pi^{s\bar{s}}$
    - *n : 0* $m_\pi^{n\bar{n}} = 408(3)\ \text{MeV}$
    - *d : -1/3* $m_\pi^{d\bar{d}} = 409(1)\ \text{MeV}$
    - *u : +2/3* $m_\pi^{u\bar{u}} = 407(3)\ \text{MeV}$

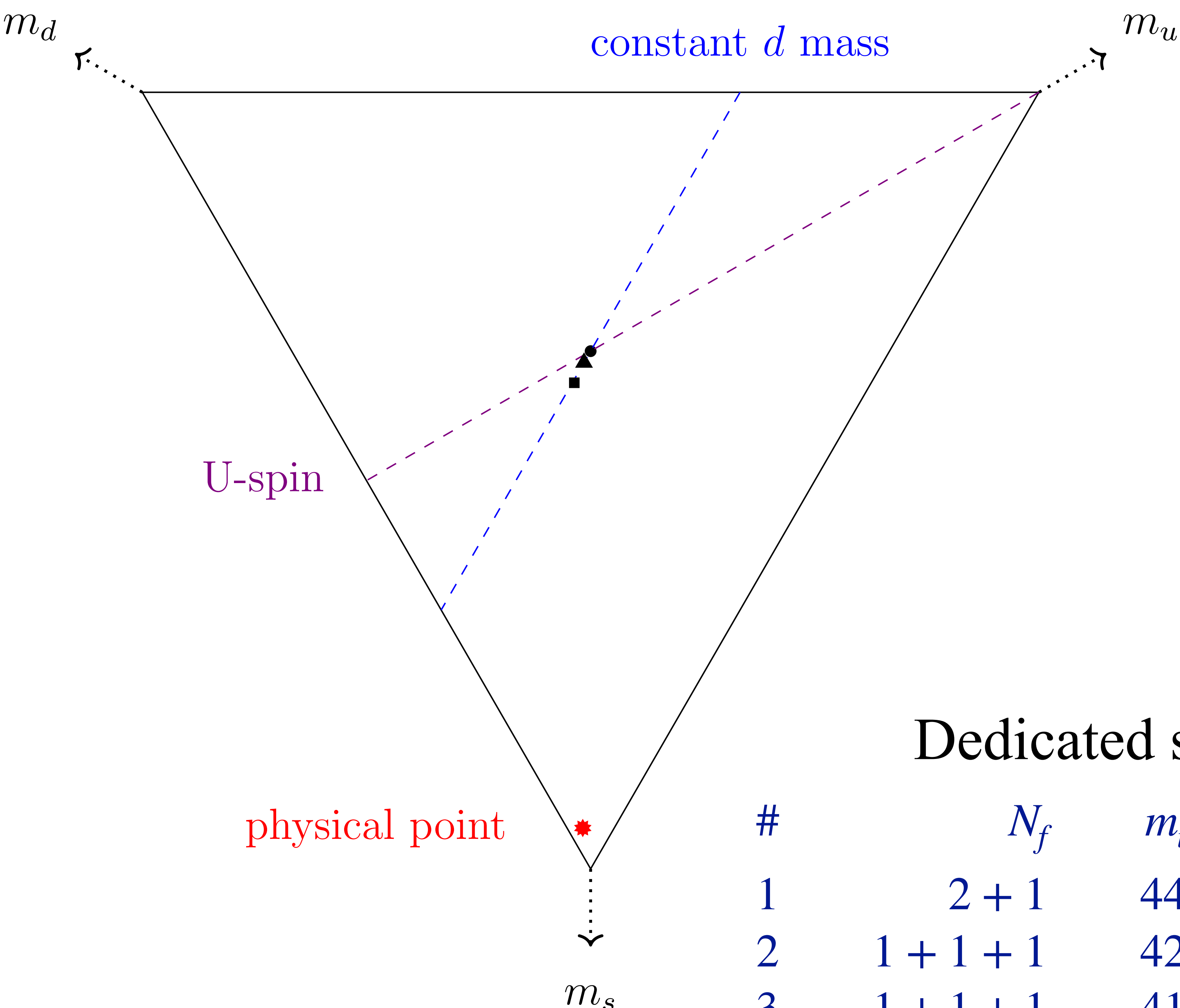
*$V=32^3x64$, $a=0.068fm$*

- $N_f = 2+1$ $O(a)$-improved Clover ("SLiNC")
- Tree-level Symanzik gluon action
- Non-compact QED with $\alpha_{QED} = 0.1$

*gauge-fixing of Uno & Hayakawa (2008) — on valence quarks*

# Lattice QCD+QED set-up

***V=24³x48, a=0.068fm*** $\alpha_{\text{QED}} = \frac{e^2}{4\pi} \simeq 0.1$




Triangle defined for fixed singlet mass

$m_u + m_d + m_s = const.$

Dedicated simulations with $\kappa_d$ fixed

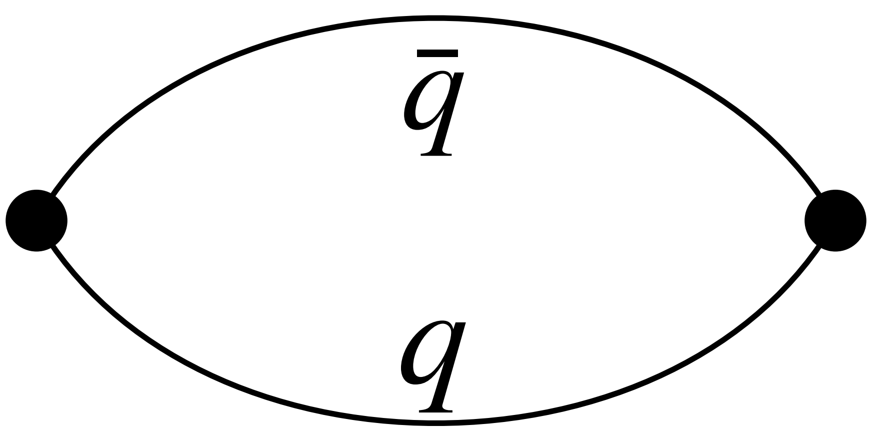
| # | $N_f$ | $m_{u\bar{u}}$ | $m_{d\bar{d}}$ | $m_{s\bar{s}}$ | $m_{\pi^+}$ | $m_{K^+}$ |
|---|---|---|---|---|---|---|
| 1 | 2 + 1 | 440 | 415 | 415 | 440 | 440 |
| 2 | 1 + 1 + 1 | 425 | 410 | 420 | 430 | 435 |
| 3 | 1 + 1 + 1 | 415 | 425 | 450 | 435 | 450 |

*~2000 trajectories each*

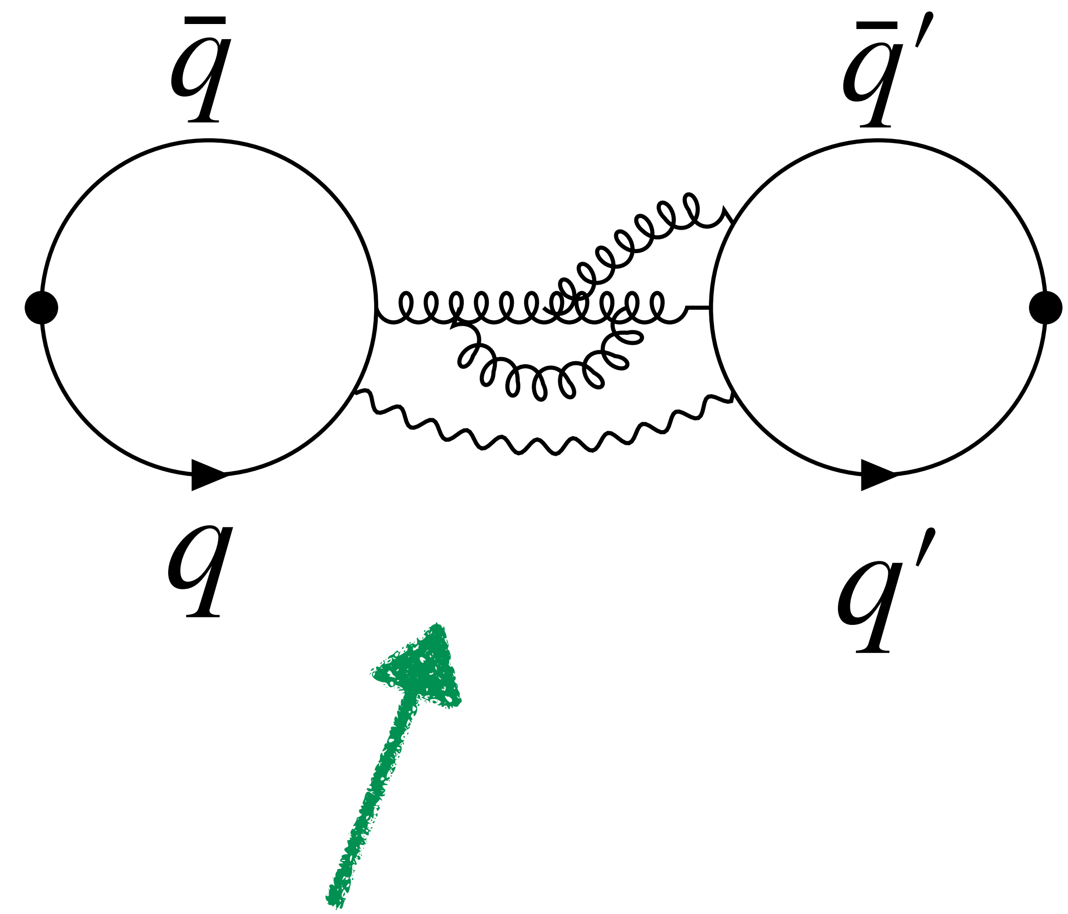
# Flavour-neutral mesons

- Two types of Wick contractions

*Connected*




*Disconnected*



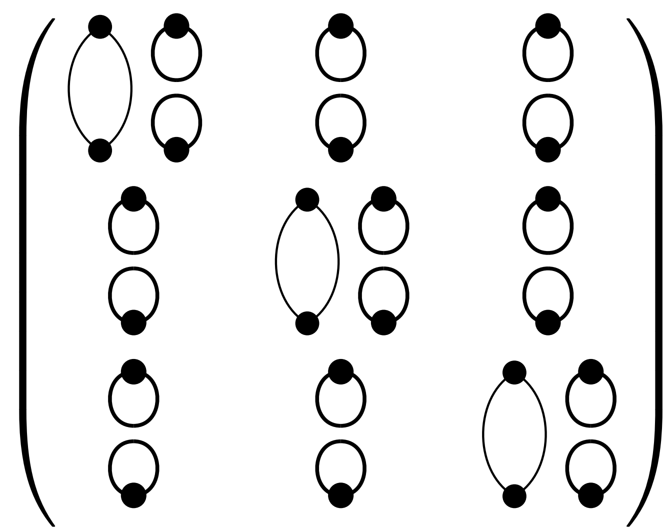

*allows for flavour off-diagonal contributions*

# Flavour-neutral mesons

- Construct a correlation matrix

Sink: $u\bar{u}$ $d\bar{d}$ $s\bar{s}$

$C_{qq'} =$ Source: $u\bar{u}$, $d\bar{d}$, $s\bar{s}$



- Diagonalise ➡ physical flavour-neutral mesons (*state label* $\alpha$)

$$C^{\alpha} = v_q^{\alpha\dagger} C_{qq'} v_{q'}^{\alpha}$$

  - e.g. with SU(3)$_f$ symmetry in isospin basis

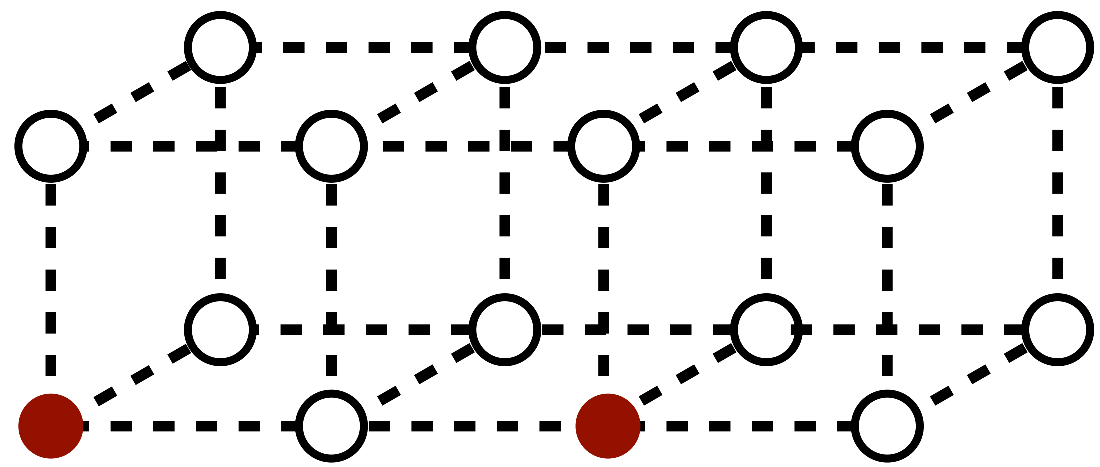
$$v^{\pi^0} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad v^{\eta} = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \quad v^{\eta'} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

# Disconnected diagrams

- Stochastic $Z_2$ noise with colour, spin, time-dilution
- Spatial dilution implemented via cubic interlacing ($2^3$)

*Interlaced source #*

1



# Disconnected diagrams

- Stochastic $Z_2$ noise with colour, spin, time-dilution
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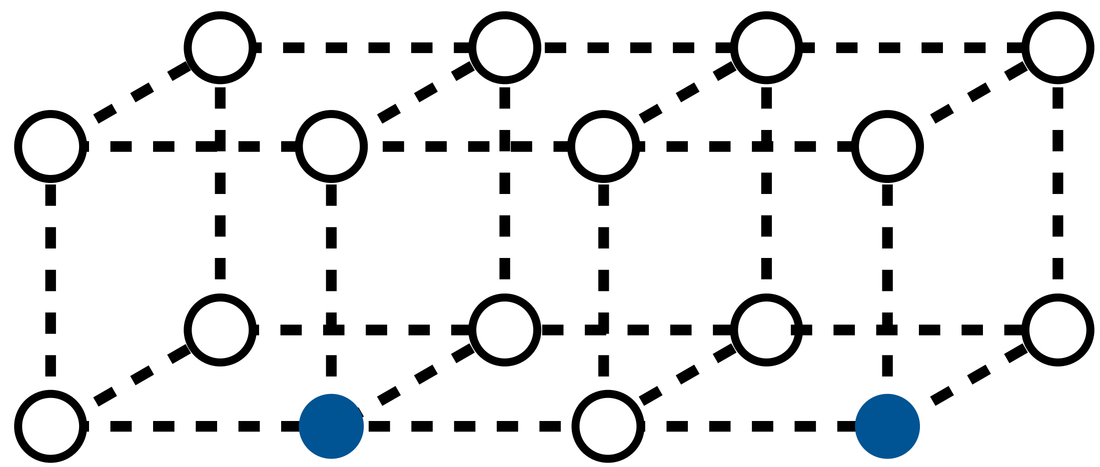
2



# Disconnected diagrams

- Stochastic $Z_2$ noise with colour, spin, time-dilution
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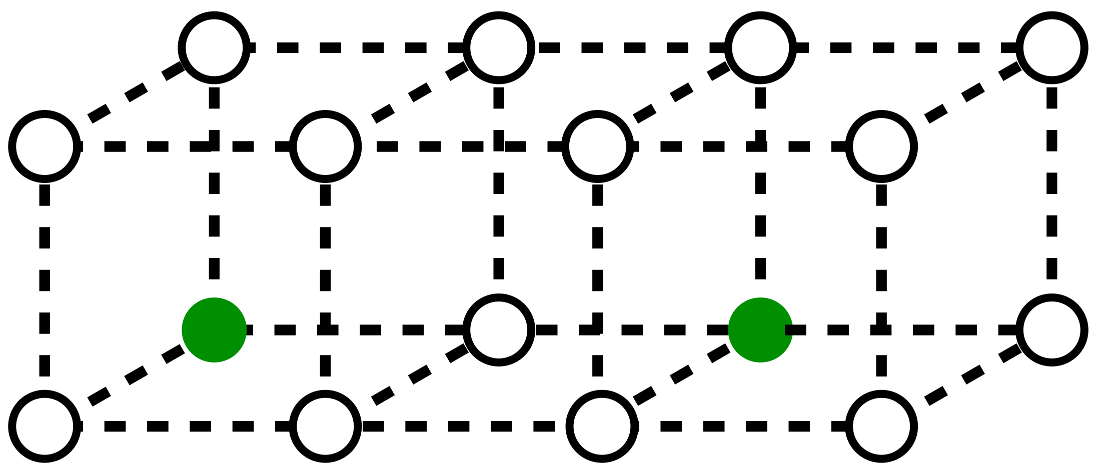
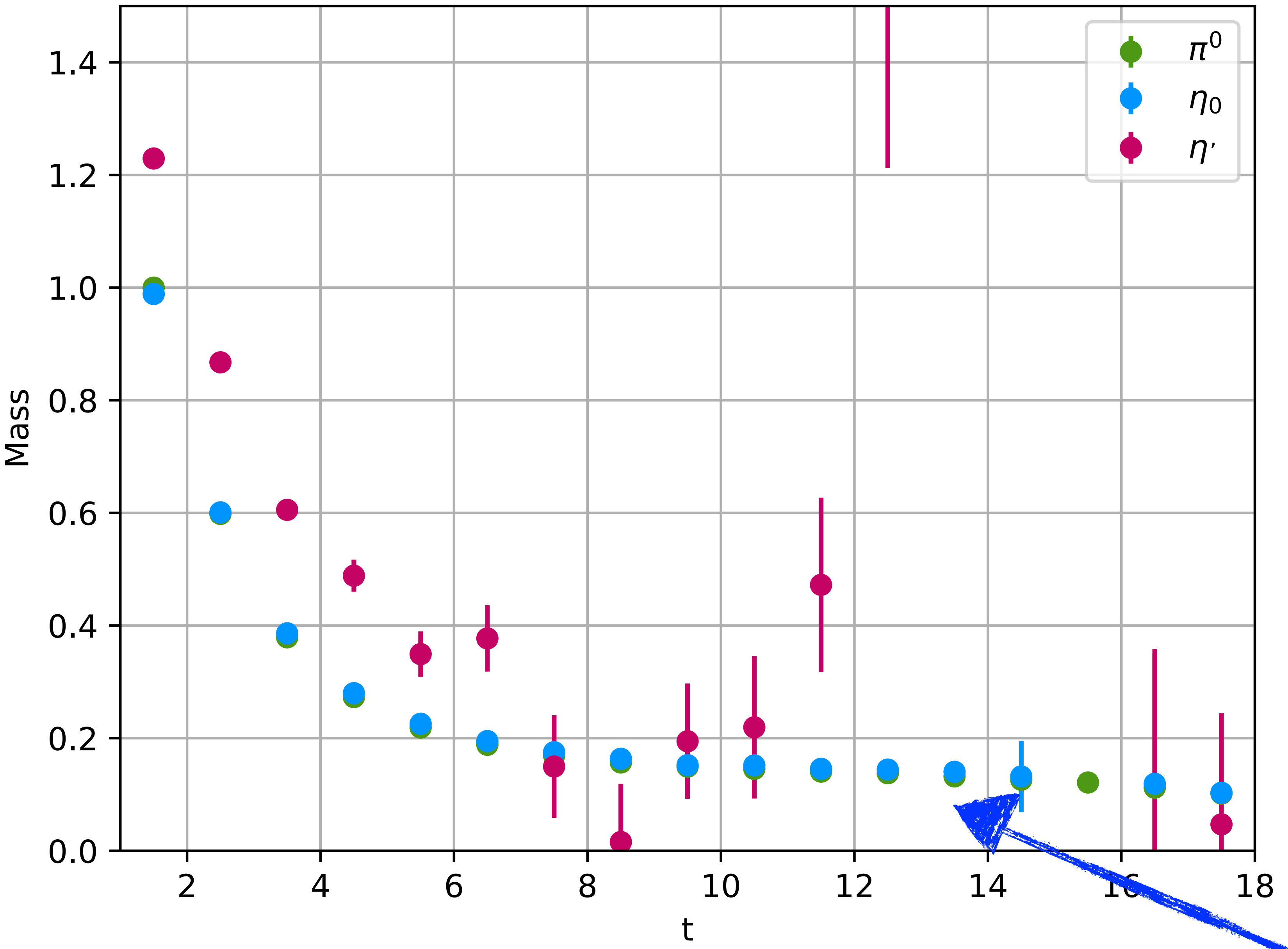
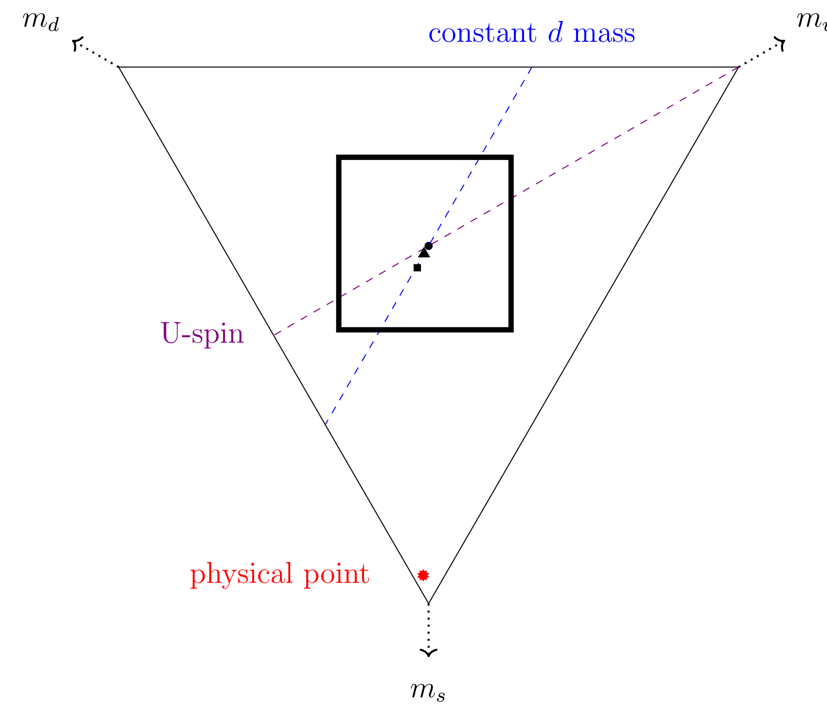
3

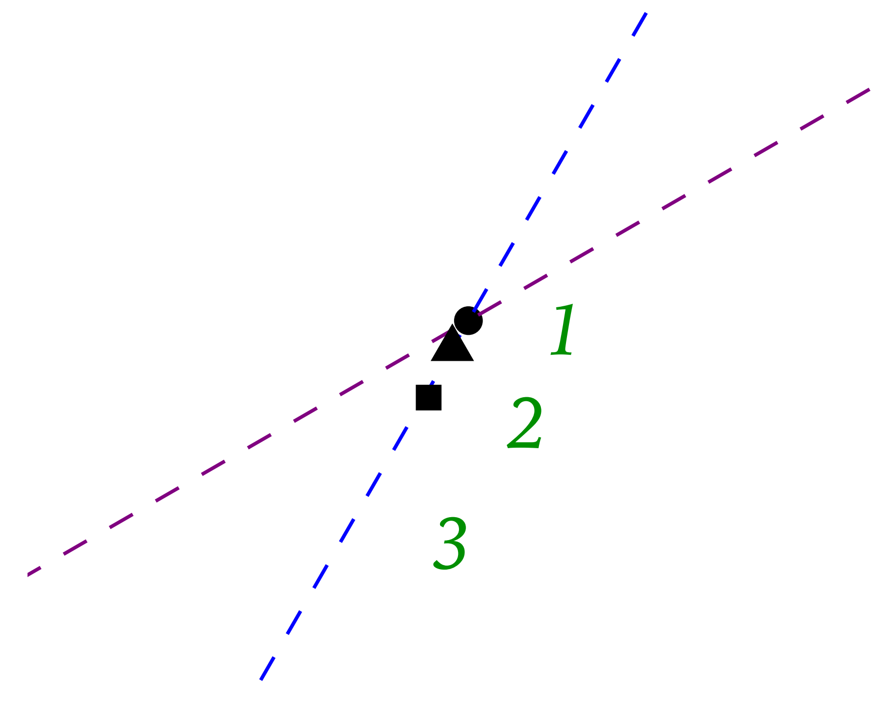


*Diagonalised state (using t=4,5): Effective mass*

| *Ensemble 1* ($m_d = m_s$) | $m_{u\bar{u}}$ | $m_{d\bar{d}}$ | $m_{s\bar{s}}$ |
|---|---|---|---|
| | 440 | 415 | 415 |



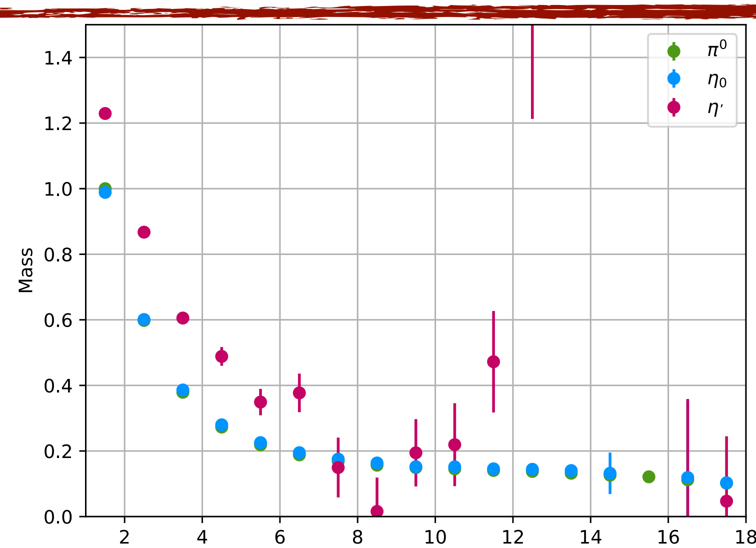



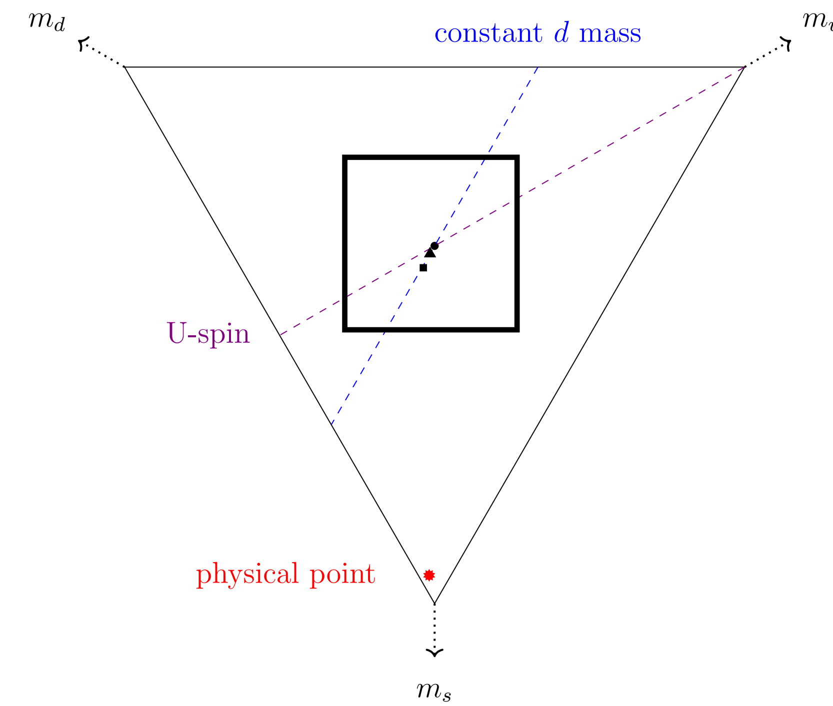

*Clean signal for $\pi^0$, $\eta$*

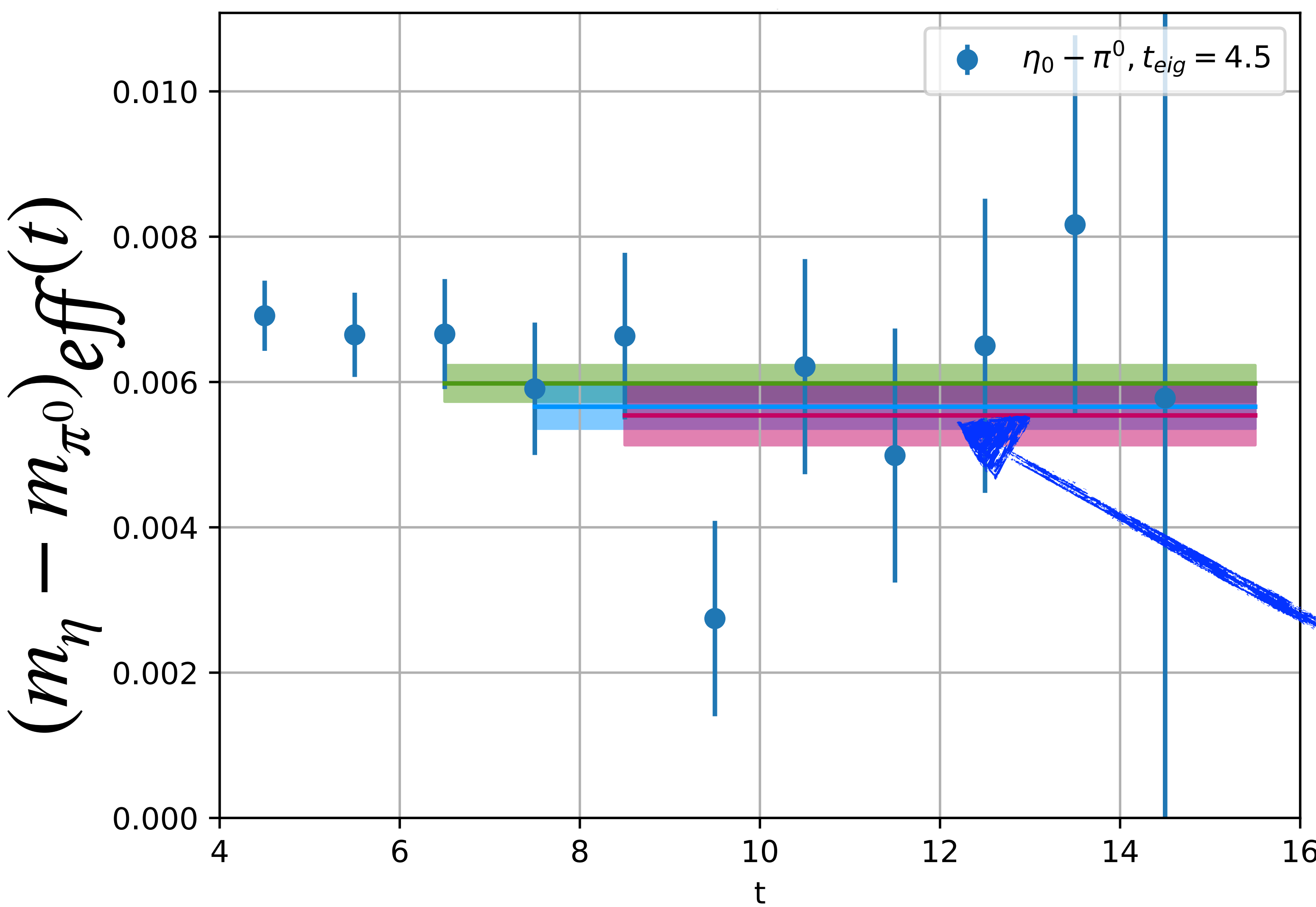
## *Diagonalised state (using t=4,5): Effective mass*

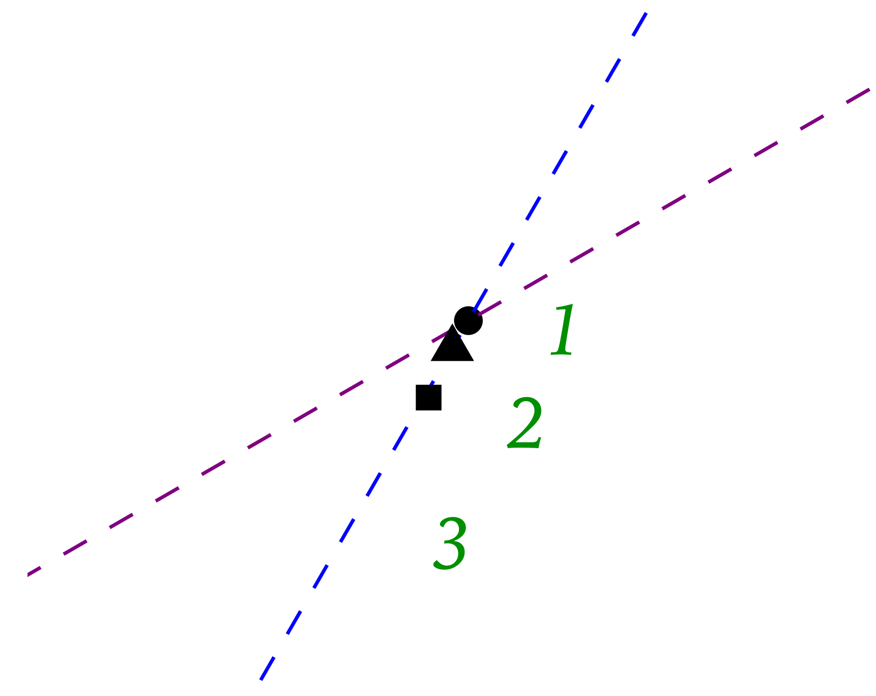
*Ensemble 1 ($m_d=m_s$)*

| $m_{u\bar{u}}$ | $m_{d\bar{d}}$ | $m_{s\bar{s}}$ |
|---|---|---|
| 440 | 415 | 415 |



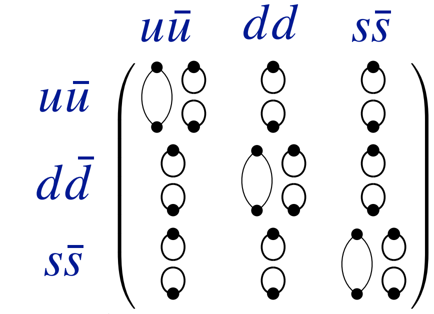

*Clean signal for $\pi^0$-$\eta$ splitting*

*~17(2) MeV*

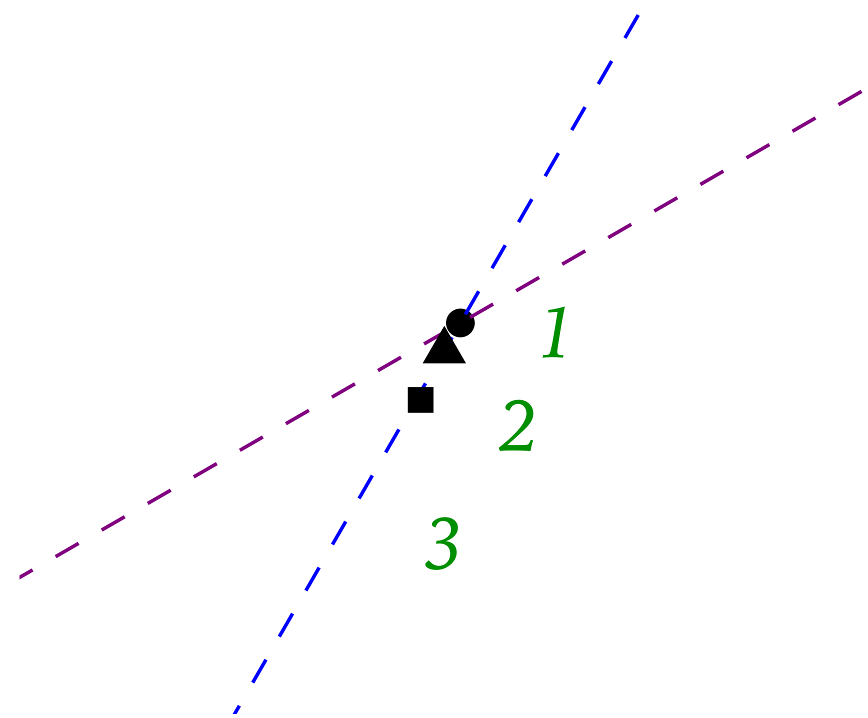
## *Diagonalised state: Ensemble 1 ($m_d = m_s < m_u$)*

| $m_{u\bar{u}}$ | $m_{d\bar{d}}$ | $m_{s\bar{s}}$ |
|---|---|---|
| 440 | 415 | 415 |

$$C^{\alpha} = v_q^{\alpha\dagger} C_{qq'} v_{q'}^{\alpha}$$




$$v^1 = \begin{pmatrix} 0.000(1) \\ 0.707(1) \\ -0.707(1) \end{pmatrix} \quad v^2 = \begin{pmatrix} -0.828(1) \\ 0.397(1) \\ 0.397(1) \end{pmatrix} \quad v^3 = \begin{pmatrix} 0.561(2) \\ 0.585(1) \\ 0.585(1) \end{pmatrix}$$



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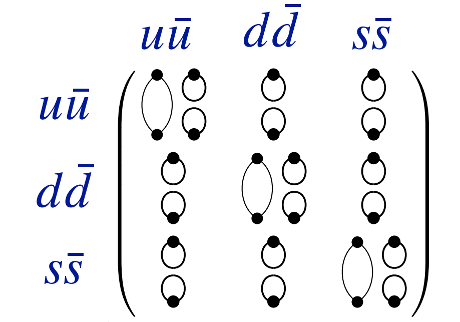
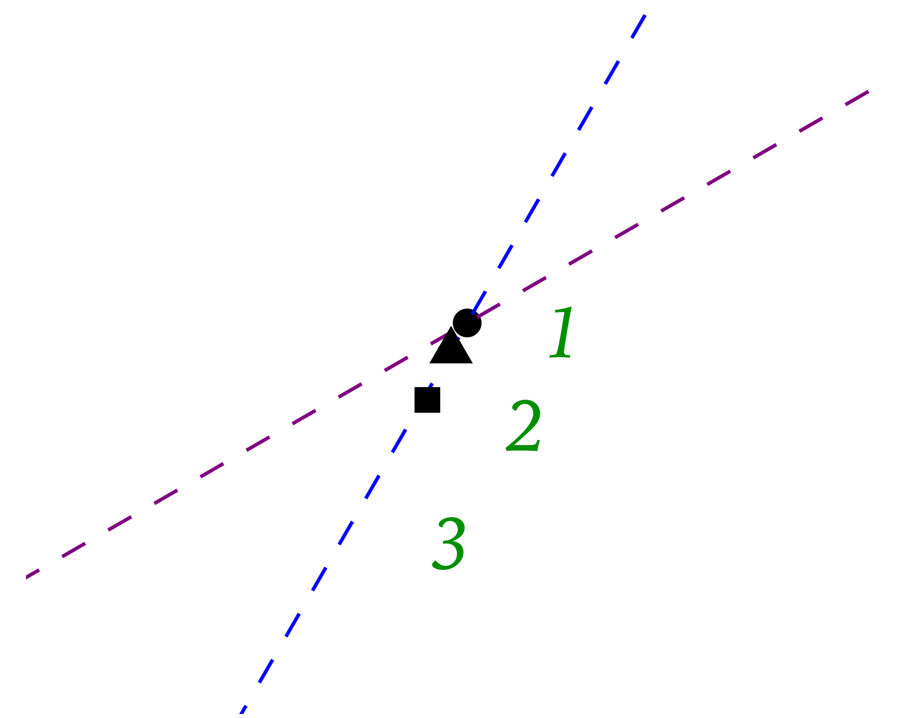



$$v^1 = \begin{pmatrix} 0.000(1) \\ 0.707(1) \\ -0.707(1) \end{pmatrix} \quad v^2 = \begin{pmatrix} -0.828(1) \\ 0.397(1) \\ 0.397(1) \end{pmatrix} \quad v^3 = \begin{pmatrix} 0.561(2) \\ 0.585(1) \\ 0.585(1) \end{pmatrix}$$

$$v^{\pi_U^0} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \quad v^{\eta_U} = \frac{1}{\sqrt{6}} \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \quad v^{\eta'} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$



## Diagonalised state: Ensemble 1 ($m_d = m_s < m_u$)

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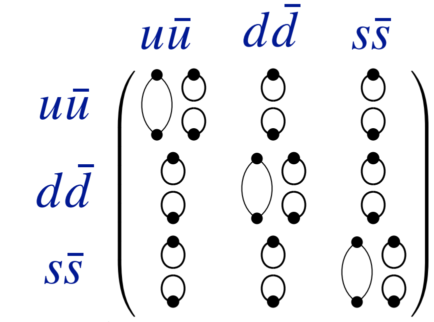
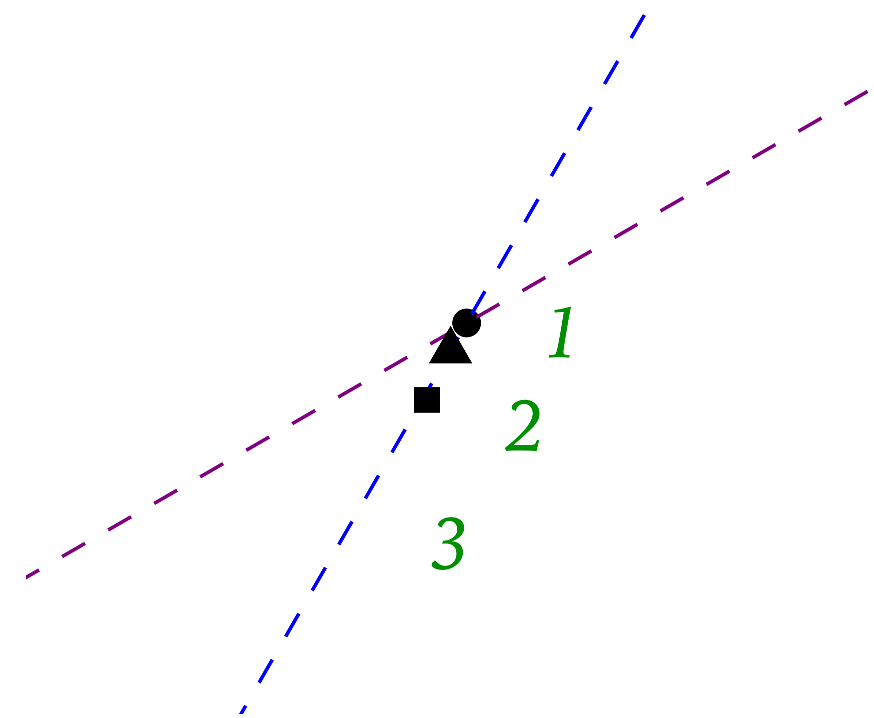



$$v^1 = \begin{pmatrix} 0.000(1) \\ 0.707(1) \\ -0.707(1) \end{pmatrix} \quad v^2 = \begin{pmatrix} -0.828(1) \\ 0.397(1) \\ 0.397(1) \end{pmatrix} \quad v^3 = \begin{pmatrix} 0.561(2) \\ 0.585(1) \\ 0.585(1) \end{pmatrix}$$

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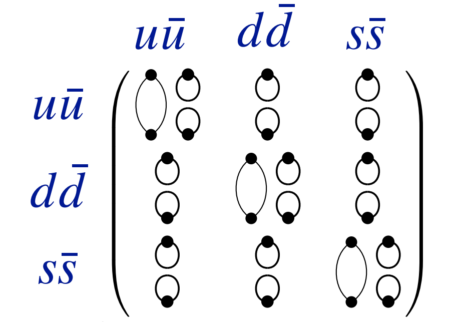
$$v^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0.000(1) \\ 1.000(2) \\ -1.000(2) \end{pmatrix} \quad v^2 = \frac{1}{\sqrt{6}} \begin{pmatrix} -2.028(3) \\ 0.972(2) \\ 0.972(2) \end{pmatrix} \quad v^3 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0.972(3) \\ 1.013(2) \\ 1.013(2) \end{pmatrix}$$

## *Diagonalised state: Ensemble 1* ($m_d = m_s < m_u$)

| $m_{u\bar{u}}$ | $m_{d\bar{d}}$ | $m_{s\bar{s}}$ |
|---|---|---|
| 440 | 415 | 415 |

$$C^{\alpha} = v_q^{\alpha\dagger} C_{qq'} v_{q'}^{\alpha}$$

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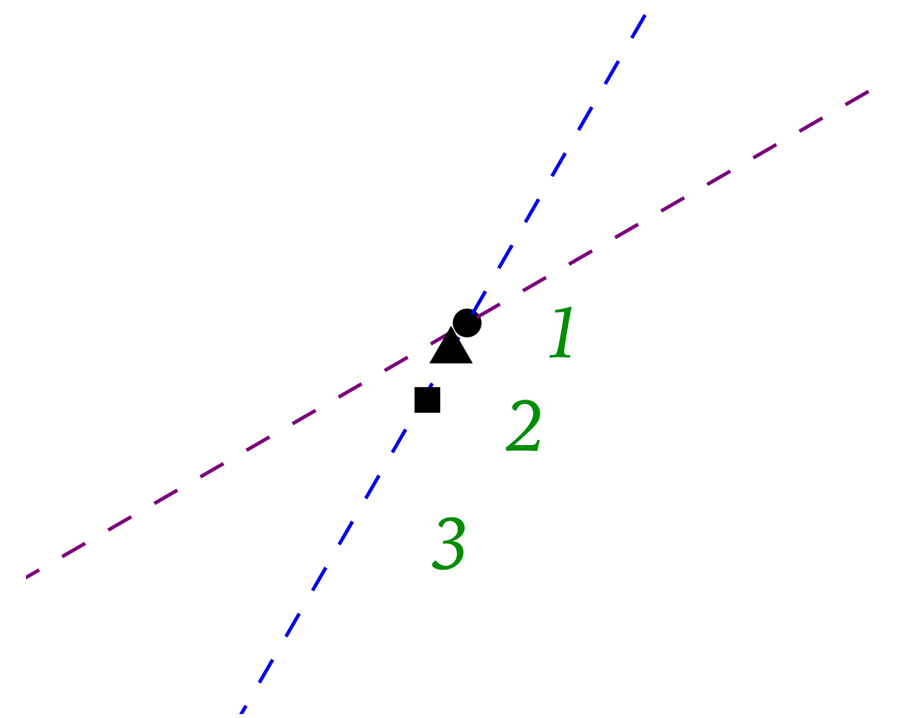


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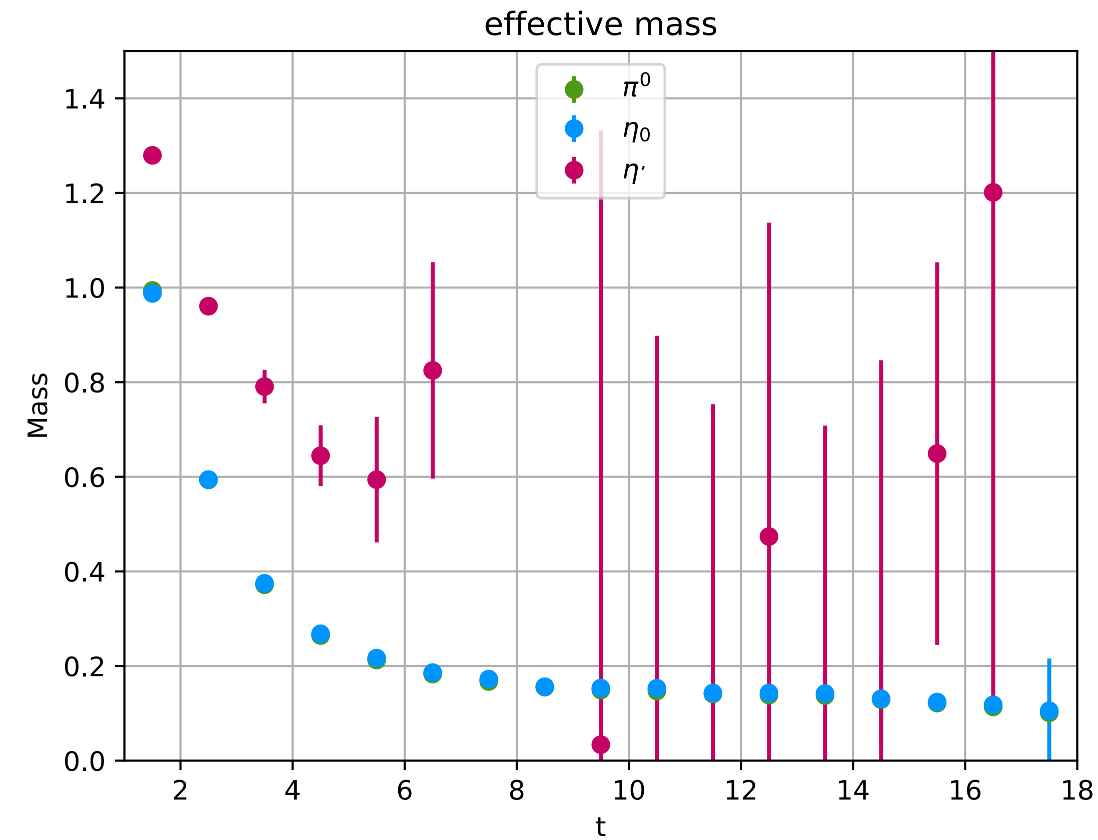
$$v^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0.000(1) \\ 1.000(2) \\ -1.000(2) \end{pmatrix} \quad v^2 = \frac{1}{\sqrt{6}} \begin{pmatrix} -2.028(3) \\ 0.972(2) \\ 0.972(2) \end{pmatrix} \quad v^3 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0.972(3) \\ 1.013(2) \\ 1.013(2) \end{pmatrix}$$

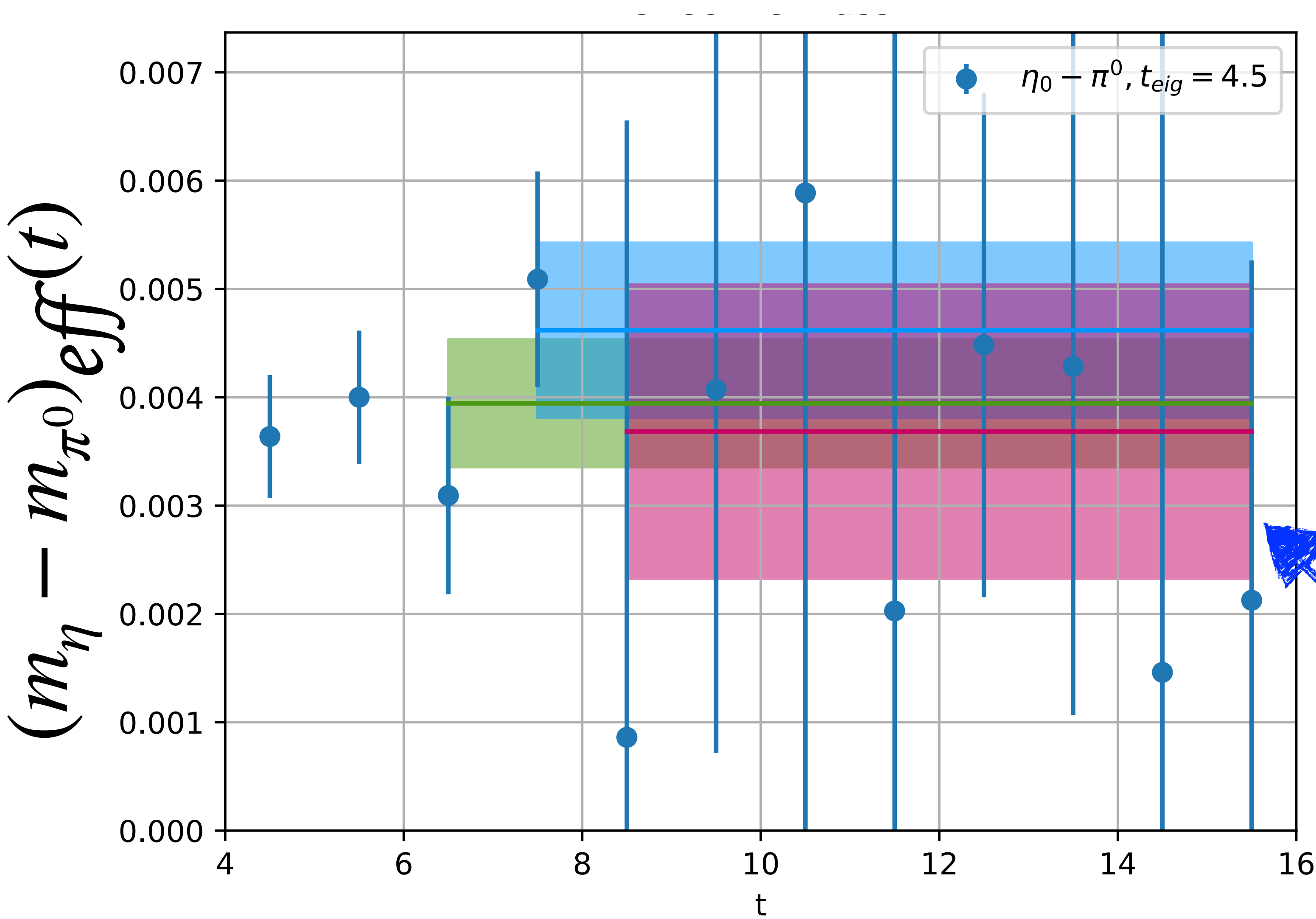
| | | *state 1* | *state 2* | *state 3* |
|---|---|---|---|---|
| | Basis, $\phi$ | $\langle \phi \mid \pi^0 \rangle^2$ | $\langle \phi \mid \eta \rangle^2$ | $\langle \phi \mid \eta' \rangle^2$ |
| Ensemble 1 | $\pi_U^0$ | 1.0000(1) | 0.0000(1) | 0.0000(1) |
| | $\eta_U$ | 0.0000(1) | 0.999623(83) | 0.000377(83) |
| | $\eta'_U$ | 0.0000(1) | 0.000377(83) | 0.999623(83) |

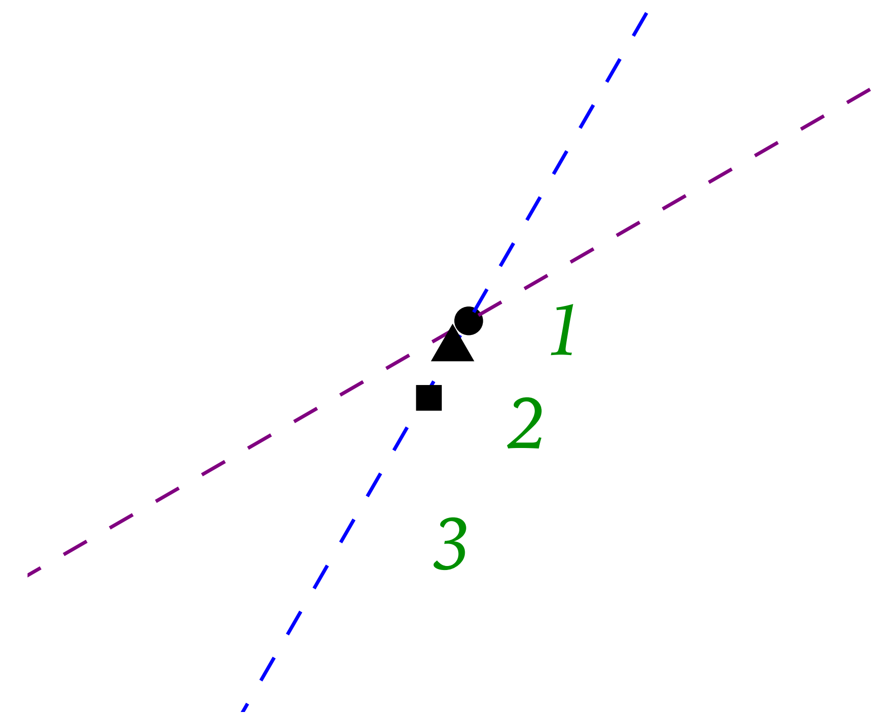
*Diagonalised state (using t=4,5): Effective mass*

*Ensemble 2 ($m_d<m_s$)*

| $m_{u\bar{u}}$ | $m_{d\bar{d}}$ | $m_{s\bar{s}}$ |
|---|---|---|
| 425 | 410 | 420 |



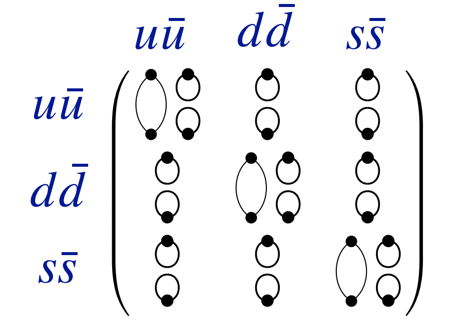

*Reduced signal for $\pi^0$-$\eta$ splitting*

12(4) MeV

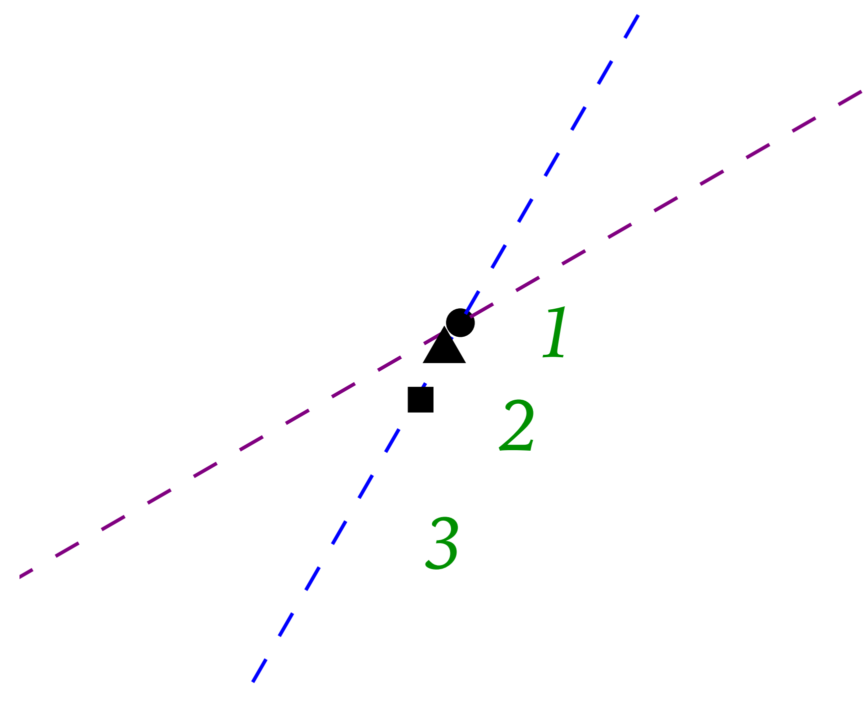
*Diagonalised state: Ensemble 2 ($m_d < m_s$)*

| $m_{u\bar{u}}$ | $m_{d\bar{d}}$ | $m_{s\bar{s}}$ |
|---|---|---|
| 425 | 410 | 420 |

$$C^{\alpha} = v_q^{\alpha\dagger} C_{qq'} v_{q'}^{\alpha}$$




$$v^1 = \begin{pmatrix} 0.41(9) \\ -0.80(1) \\ 0.43(7) \end{pmatrix} \quad v^2 = \begin{pmatrix} 0.73(5) \\ 0.00(2) \\ 0.68(5) \end{pmatrix} \quad v^3 = \begin{pmatrix} 0.544(3) \\ 0.595(1) \\ 0.591(2) \end{pmatrix}$$




$$v^{\eta_V} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \quad v^{\pi_V^0} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad v^{\eta'} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

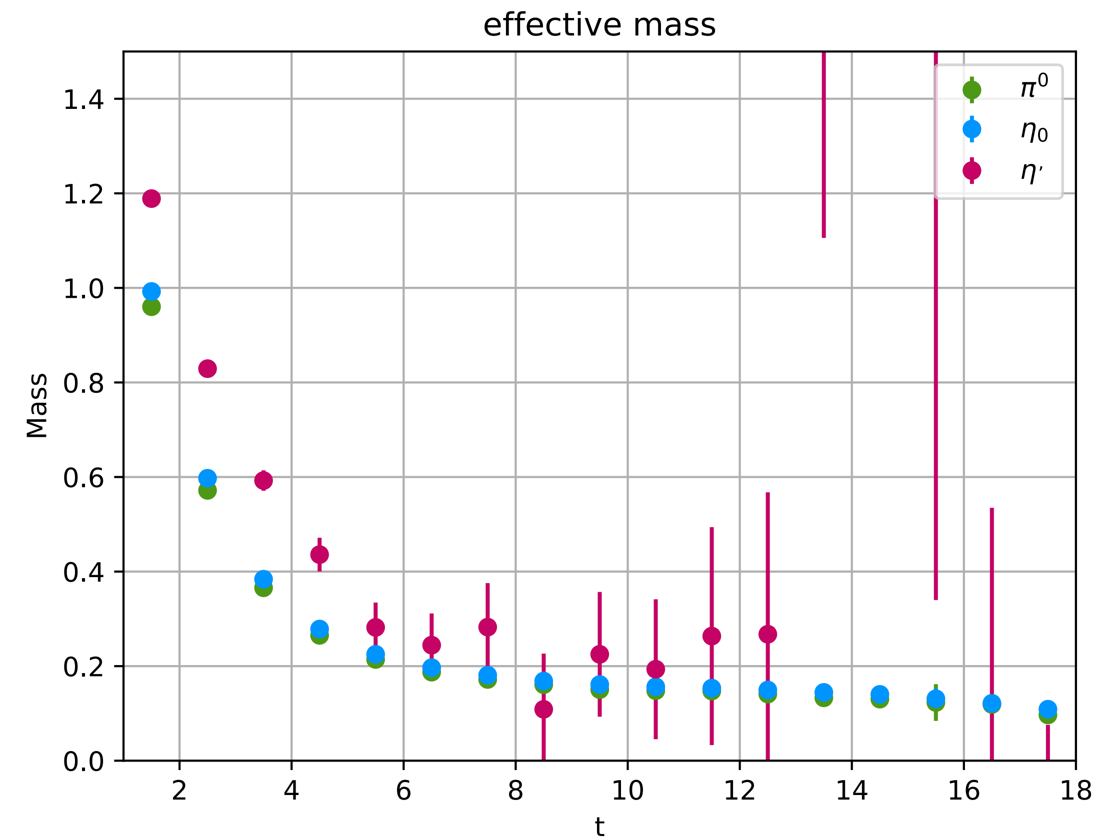
$$v^1 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1.00(23) \\ -1.97(3) \\ 1.06(18) \end{pmatrix} \quad v^2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1.04(7) \\ 0.00(3) \\ -0.96(6) \end{pmatrix} \quad v^3 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0.943(5) \\ 1.031(2) \\ 1.024(3) \end{pmatrix}$$

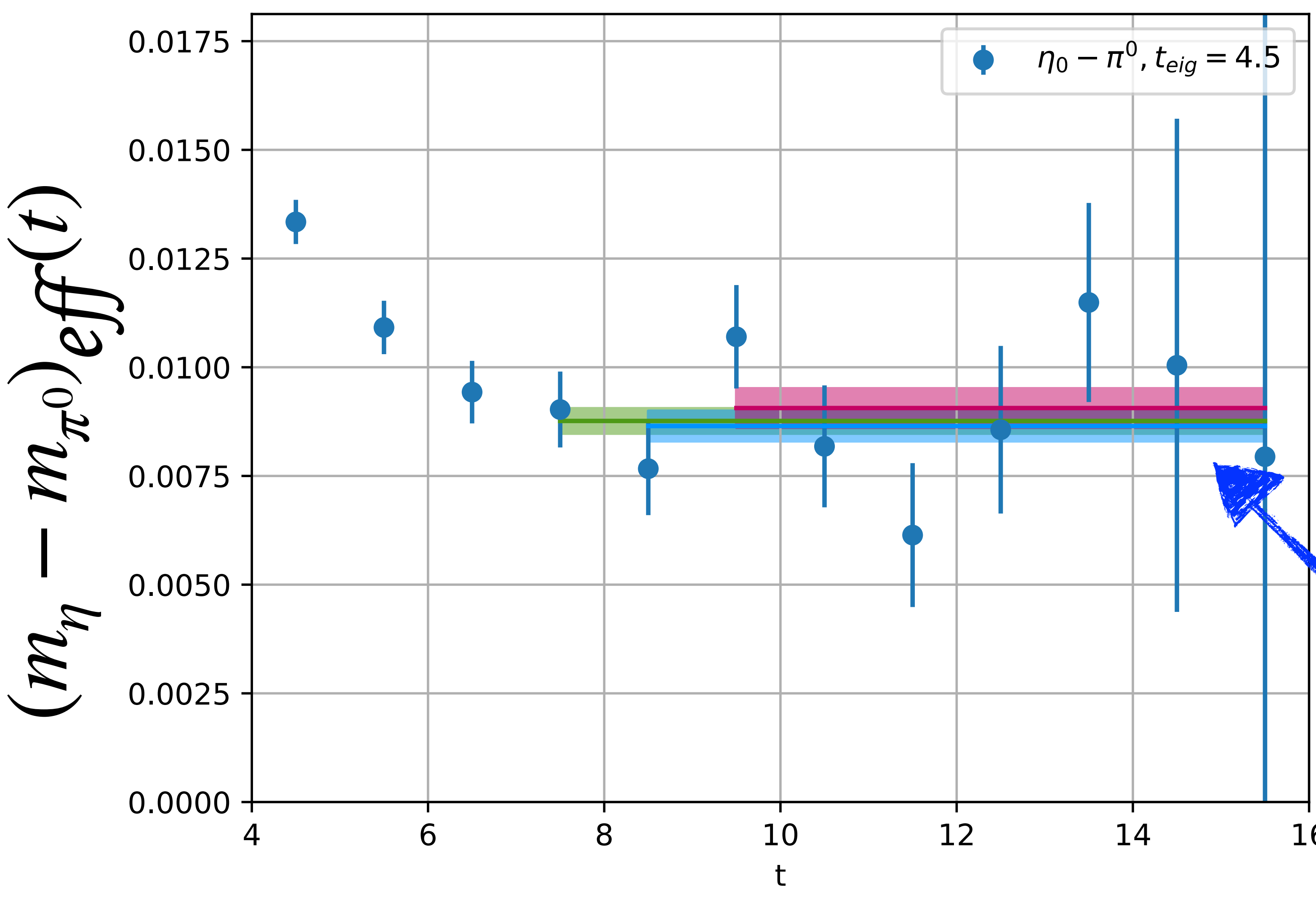
| | | *state 1* | *state 2* | *state 3* |
|---|---|---|---|---|
| | Basis, $\phi$ | $\langle \phi \mid \pi^0 \rangle^2$ | $\langle \phi \mid \eta \rangle^2$ | $\langle \phi \mid \eta' \rangle^2$ |
| Ensemble 2 | $\eta_V$ | 0.999(23) | 0.000(23) | 0.000489(84) |
| | $\pi_V^0$ | 0.000(23) | 0.999(23) | 0.00112(20) |
| | $\eta_V'$ | 0.00046(23) | 0.00114(28) | 0.9984(27) |

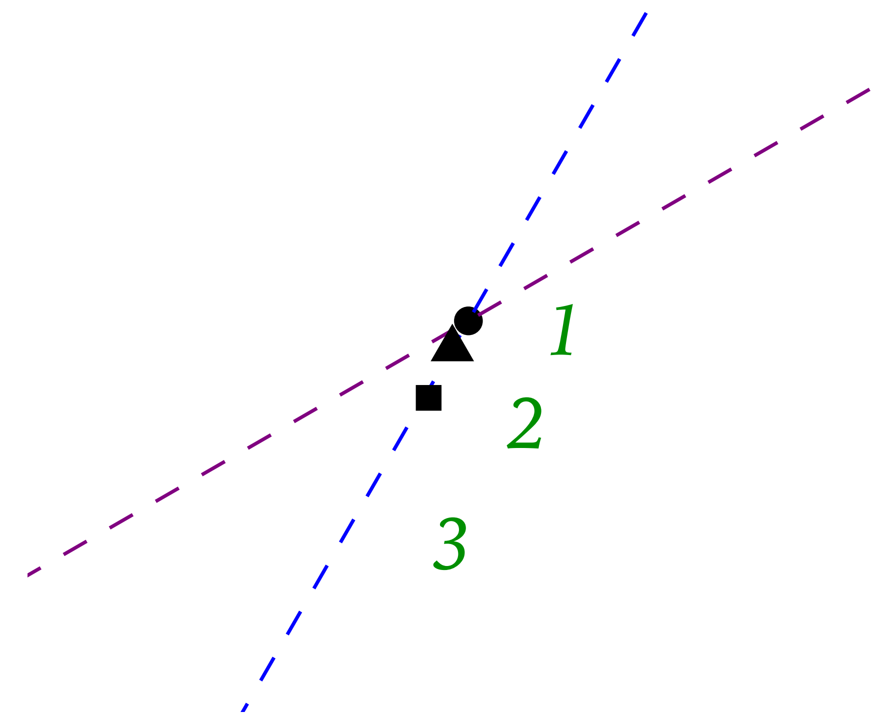
*Diagonalised state (using t=4,5): Effective mass*

*Ensemble 3 ($m_d < m_s$)*

| $m_{u\bar{u}}$ | $m_{d\bar{d}}$ | $m_{s\bar{s}}$ |
|---|---|---|
| 415 | 425 | 450 |



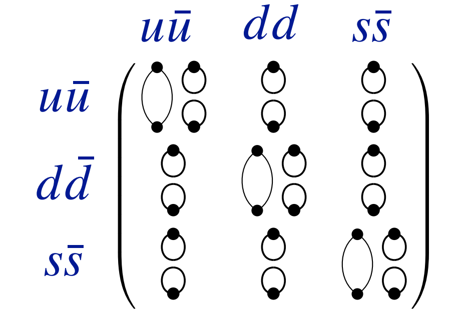

*Enhanced signal for $\pi^0$-$\eta$ splitting*

25(2) MeV

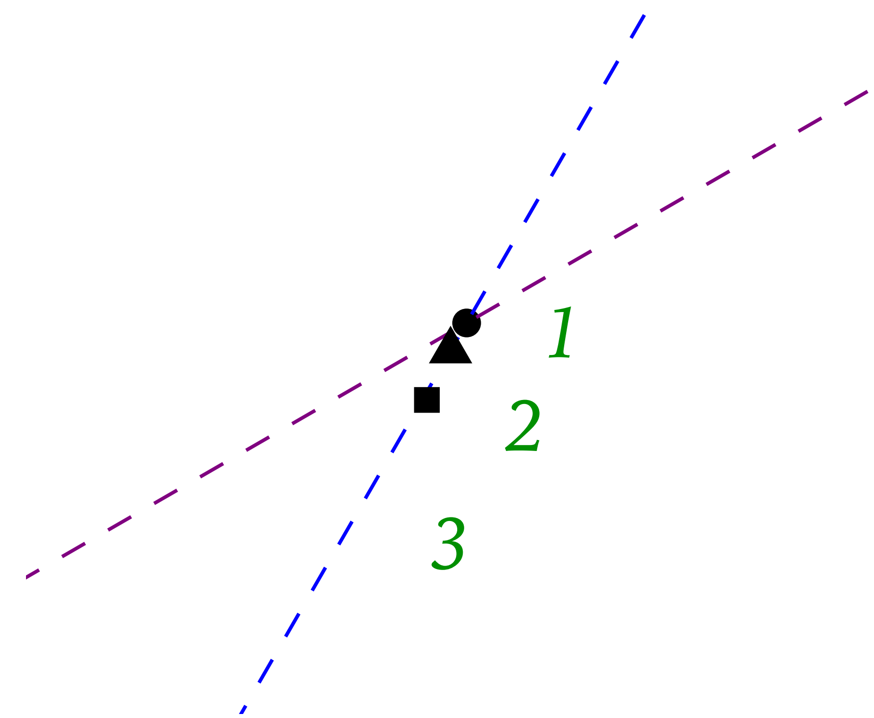
*Diagonalised state: Ensemble 3 ($m_d < m_s$)*

| $m_{u\bar{u}}$ | $m_{d\bar{d}}$ | $m_{s\bar{s}}$ |
|---|---|---|
| 415 | 425 | 450 |

$$C^{\alpha} = v_q^{\alpha\dagger} C_{qq'} v_{q'}^{\alpha}$$




$$v^1 = \begin{pmatrix} 0.774(6) \\ -0.628(1) \\ 0.074(18) \end{pmatrix} \quad v^2 = \begin{pmatrix} 0.338(12) \\ 0.510(1) \\ -0.791(3) \end{pmatrix} \quad v^3 = \begin{pmatrix} 0.535(5) \\ 0.587(2) \\ 0.607(4) \end{pmatrix}$$

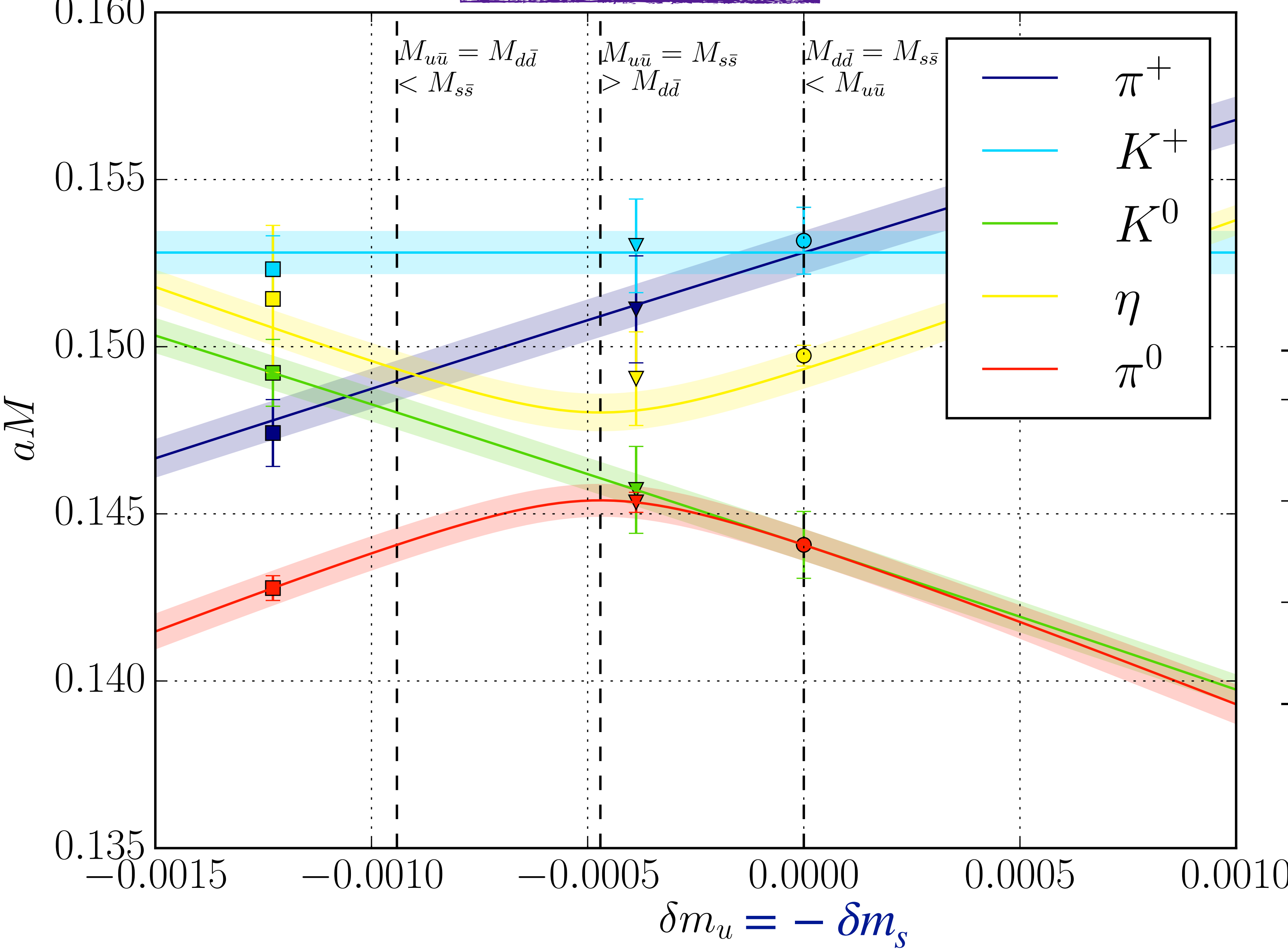



$$v^{\pi^0} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad v^{\eta} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \quad v^{\eta'} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$v^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1.09(23) \\ -0.89(3) \\ 0.10(18) \end{pmatrix} \quad v^2 = \frac{1}{\sqrt{6}} \begin{pmatrix} 0.83(3) \\ 1.25(1) \\ -1.94(1) \end{pmatrix} \quad v^3 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0.927(8) \\ 1.017(3) \\ 1.052(7) \end{pmatrix}$$

| | | *state 1* | *state 2* | *state 3* |
|---|---|---|---|---|
| | Basis, $\phi$ | $\langle \phi \mid \pi^0 \rangle^2$ | $\langle \phi \mid \eta \rangle^2$ | $\langle \phi \mid \eta' \rangle^2$ |
| Ensemble 3 | $\pi_T^0$ | 0.9838(53) | 0.0148(53) | 0.00136(32) |
| | $\eta_T$ | 0.0145(52) | 0.9841(53) | 0.00142(42) |
| | $\eta'_T$ | 0.00169(42) | 0.00108(31) | 0.99722(69) |

$m_d = constant$




| # | $m_{u\bar{u}}$ | $m_{d\bar{d}}$ | $m_{s\bar{s}}$ |
|---|---|---|---|
| 1 | 440 | 415 | 415 |
| 2 | 425 | 410 | 420 |
| 3 | 415 | 425 | 450 |

| | | state 1 | state 2 | state 3 |
|---|---|---|---|---|
| | Basis, $\phi$ | $\langle\phi\,\vert\pi^0\rangle^2$ | $\langle\phi\,\vert\eta\rangle^2$ | $\langle\phi\,\vert\eta'\rangle^2$ |
| Ensemble 1 | $\pi^0_U$ | 1.0000(1) | 0.0000(1) | 0.0000(1) |
| | $\eta_U$ | 0.0000(1) | 0.999623(83) | 0.000377(83) |
| | $\eta'_U$ | 0.0000(1) | 0.000377(83) | 0.999623(83) |
| Ensemble 2 | $\eta_V$ | 0.999(23) | 0.000(23) | 0.000489(84) |
| | $\pi^0_V$ | 0.000(23) | 0.999(23) | 0.00112(20) |
| | $\eta'_V$ | 0.00046(23) | 0.00114(28) | 0.9984(27) |
| Ensemble 3 | $\pi^0_T$ | 0.9838(53) | 0.0148(53) | 0.00136(32) |
| | $\eta_T$ | 0.0145(52) | 0.9841(53) | 0.00142(42) |
| | $\eta'_T$ | 0.00169(42) | 0.00108(31) | 0.99722(69) |

## SU(3) flavour-breaking expansion

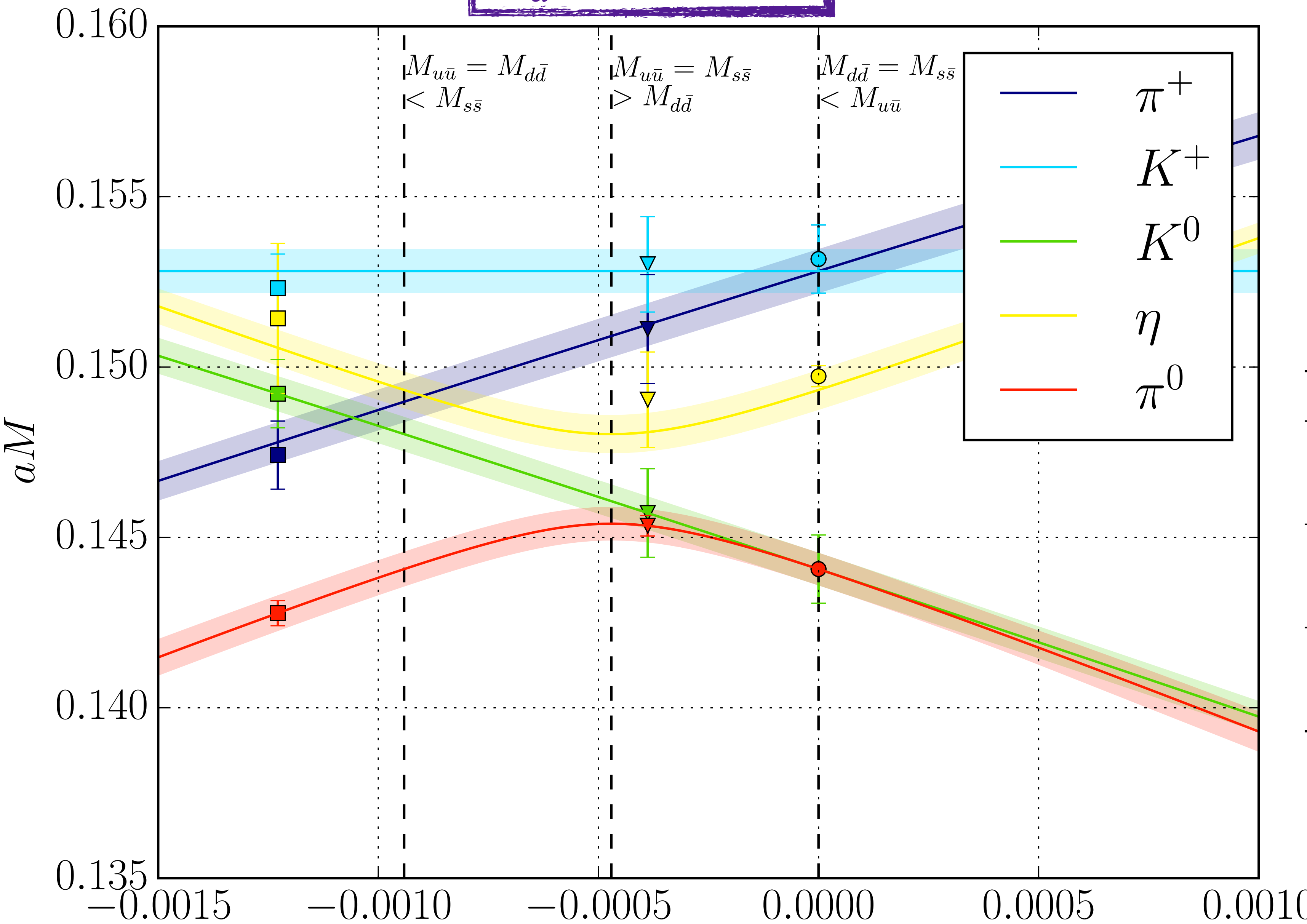
$$M^2_{\pi^+} = M^2_0 + b_1(\delta m_u + \delta\mu_u) + c_4^{EM}$$

$$M^2_{K^+} = M^2_0 + b_1\delta\mu_u + c_4^{EM}$$

$$M^2_{K^0} = M^2_0 - b_1(\delta m_u + 2\delta\mu_u)$$

$$M^2_{\eta\pm} = M^2_0 + \frac{2}{3}c_6^{EM} \pm \frac{2}{3}\sqrt{3b_1^2\delta m_u^2 + 3b_1c_6^{EM}\delta m_u + (c_6^{EM})^2 + \delta\mu_u(9b_1^2\delta\mu_u + 9b_1^2\delta m_u + 6b_1c_6^{EM})}$$

$m_d = constant$




$\delta m_u = -\delta m_s$

| # | $m_{u\bar{u}}$ | $m_{d\bar{d}}$ | $m_{s\bar{s}}$ |
|---|---|---|---|
| 1 | 440 | 415 | 415 |
| 2 | 425 | 410 | 420 |
| 3 | 415 | 425 | 450 |

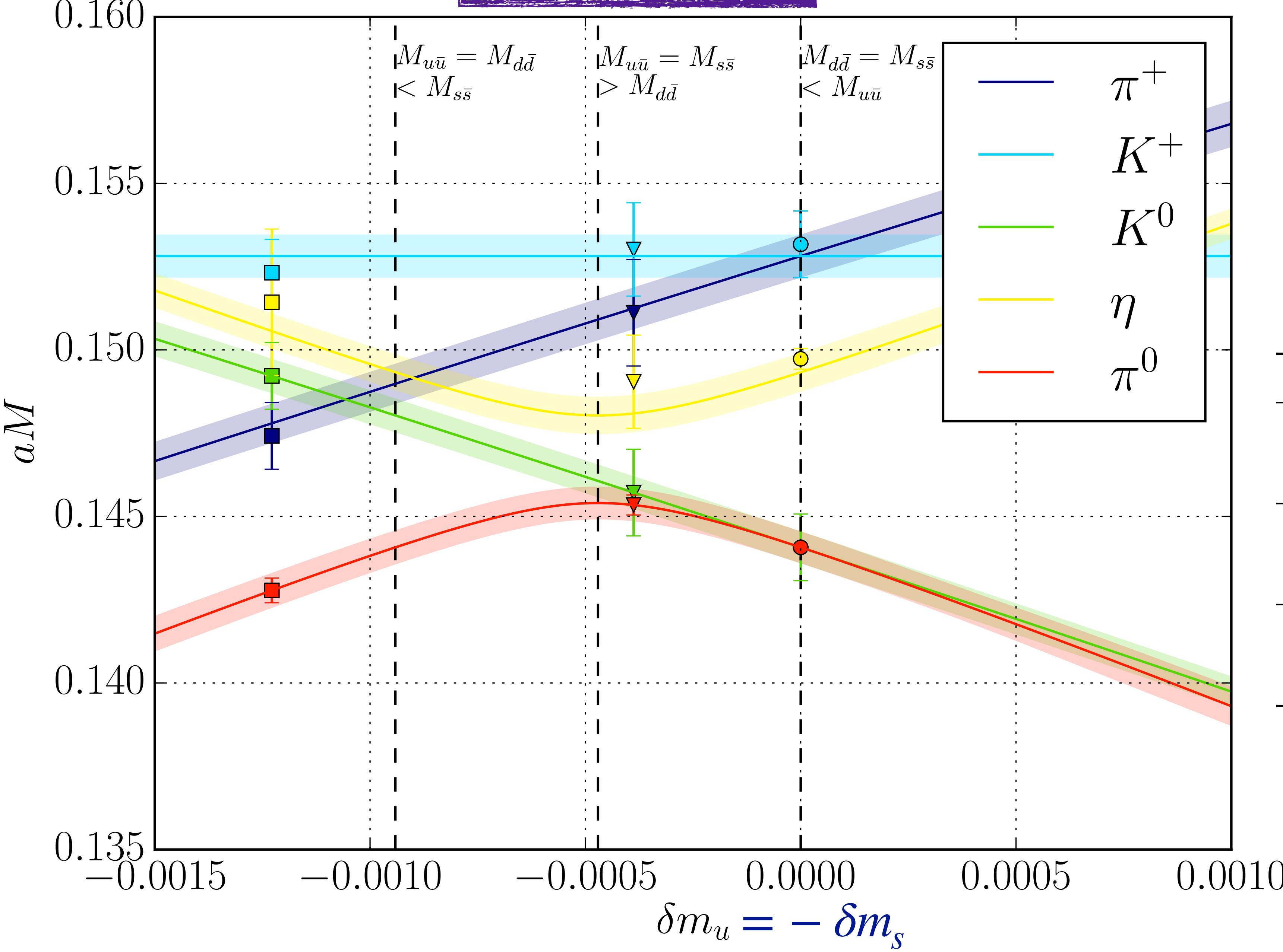
| | | state 1 | state 2 | state 3 |
|---|---|---|---|---|
| | Basis, $\phi$ | $\langle\phi\,\vert\pi^0\rangle^2$ | $\langle\phi\,\vert\eta\rangle^2$ | $\langle\phi\,\vert\eta'\rangle^2$ |
| Ensemble 1 | $\pi^0_U$ | 1.0000(1) | 0.0000(1) | 0.0000(1) |
| | $\eta_U$ | 0.0000(1) | 0.999623(83) | 0.000377(83) |
| | $\eta'_U$ | 0.0000(1) | 0.000377(83) | 0.999623(83) |
| Ensemble 2 | $\eta_V$ | 0.999(23) | 0.000(23) | 0.000489(84) |
| | $\pi^0_V$ | 0.000(23) | 0.999(23) | 0.00112(20) |
| | $\eta'_V$ | 0.00046(23) | 0.00114(28) | 0.9984(27) |
| Ensemble 3 | $\pi^0_T$ | 0.9838(53) | 0.0148(53) | 0.00136(32) |
| | $\eta_T$ | 0.0145(52) | 0.9841(53) | 0.00142(42) |
| | $\eta'_T$ | 0.00169(42) | 0.00108(31) | 0.99722(69) |

*Constrained from fits to flavour-off-diagonal ("outer ring") states*

## *SU(3) flavour-breaking expansion*

$$M^2_{\pi^+} = M^2_0 + b_1(\delta m_u + \delta\mu_u) + c^{EM}_4$$
$$M^2_{K^+} = M^2_0 + b_1\delta\mu_u + c^{EM}_4$$
$$M^2_{K^0} = M^2_0 - b_1(\delta m_u + 2\delta\mu_u)$$
$$M^2_{\eta_\pm} = M^2_0 + \frac{2}{3}c^{EM}_6 \pm \frac{2}{3}\sqrt{3b_1^2\delta m_u^2 + 3b_1 c^{EM}_6\delta m_u + (c^{EM}_6)^2 + \delta\mu_u(9b_1^2\delta\mu_u + 9b_1^2\delta m_u + 6b_1 c^{EM}_6)}$$

## $m_d$ = constant




| # | $m_{u\bar{u}}$ | $m_{d\bar{d}}$ | $m_{s\bar{s}}$ |
|---|---|---|---|
| 1 | 440 | 415 | 415 |
| 2 | 425 | 410 | 420 |
| 3 | 415 | 425 | 450 |

| | | state 1 | state 2 | state 3 |
|---|---|---|---|---|
| | Basis, $\phi$ | $\langle\phi\,\lvert\pi^0\rangle^2$ | $\langle\phi\,\lvert\eta\rangle^2$ | $\langle\phi\,\lvert\eta'\rangle^2$ |
| Ensemble 1 | $\pi^0_U$ | 1.0000(1) | 0.0000(1) | 0.0000(1) |
| | $\eta_U$ | 0.0000(1) | 0.999623(83) | 0.000377(83) |
| | $\eta'_U$ | 0.0000(1) | 0.000377(83) | 0.999623(83) |
| Ensemble 2 | $\eta_V$ | 0.999(23) | 0.000(23) | 0.000489(84) |
| | $\pi^0_V$ | 0.000(23) | 0.999(23) | 0.00112(20) |
| | $\eta'_V$ | 0.00046(23) | 0.00114(28) | 0.9984(27) |
| Ensemble 3 | $\pi^0_T$ | 0.9838(53) | 0.0148(53) | 0.00136(32) |
| | $\eta_T$ | 0.0145(52) | 0.9841(53) | 0.00142(42) |
| | $\eta'_T$ | 0.00169(42) | 0.00108(31) | 0.99722(69) |

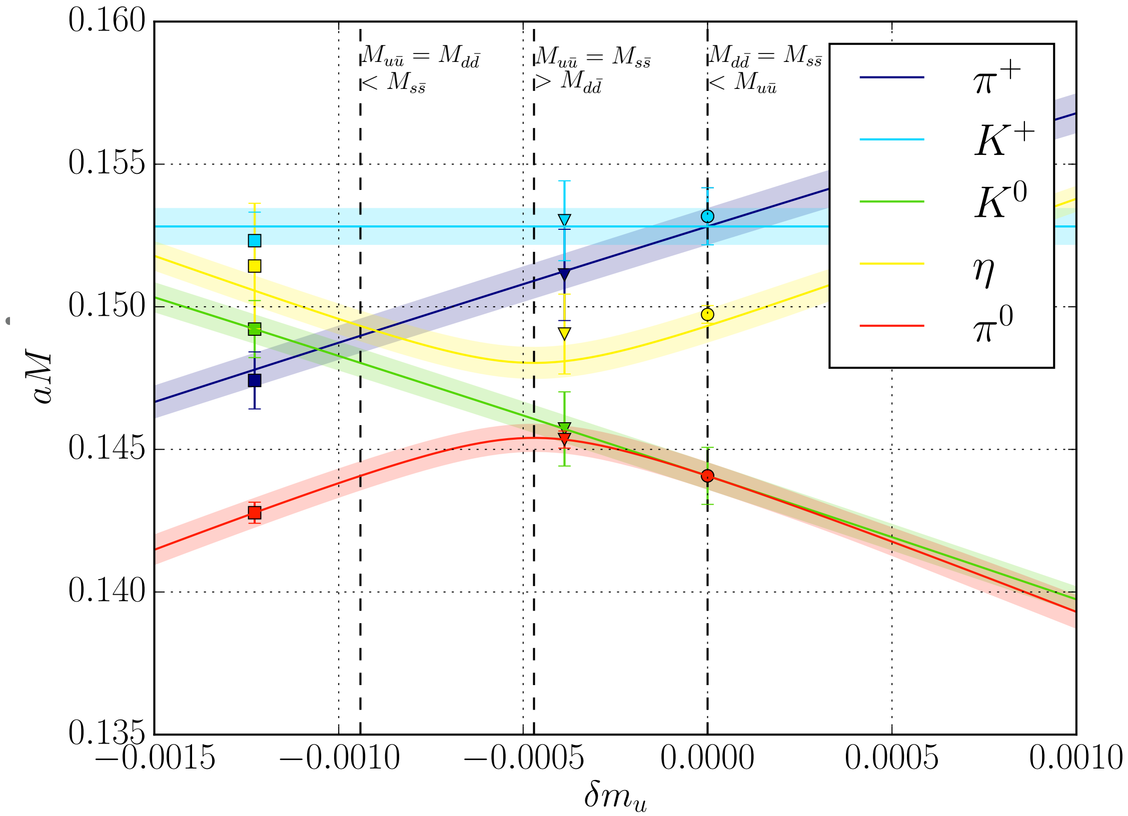
### SU(3) flavour-breaking expansion ($\delta m_d=0$)

$$M^2_{\pi^+} = M_0^2 + b_1(\delta m_u + \delta\mu_u) + c_4^{EM}$$
$$M^2_{K^+} = M_0^2 + b_1\delta\mu_u + c_4^{EM}$$
$$M^2_{K^0} = M_0^2 - b_1(\delta m_u + 2\delta\mu_u)$$
$$M^2_{\eta_\pm} = M_0^2 + \frac{2}{3}c_6^{EM} \pm \frac{2}{3}\sqrt{3b_1^2\delta m_u^2 + 3b_1c_6^{EM}\delta m_u + (c_6^{EM})^2 + \delta\mu_u(9b_1^2\delta\mu_u + 9b_1^2\delta m_u + 6b_1c_6^{EM})}$$

Constrained from fits to flavour-off-diagonal ("outer ring") states

Single parameter describes $\pi^0$-$\eta$ splitting

# Summary




- ➤ Observe mass eigenstates rotating between U, V and T states
- ➤ Clear QED effect in flavour-symmetry breaking, e.g.

$$\delta m_u = \delta \mu_u = 0 \quad \overset{\alpha_{QED}=1/137}{\Longrightarrow} \quad M_\eta - M_{\pi^0} = \frac{4}{3}\frac{c_6^{EM}}{M_\eta + M_{\pi^0}} = 0.55(8)\,\text{MeV}$$

*Note: not full QED effect!*

- ➤ Similar observations made in $\Sigma^0$-$\Lambda$ system

- ➤ Current work:
  - ➤ more physical masses (splittings)
  - ➤ improved method for A2A to resolve *η'*