Recent progress of two-baryon problem and ΩΩ interaction on the lattice

Shinya Gongyo (RIKEN)

HAL(Hadrons to Atomic nuclei from Lattice) QCD Collaboration

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T. Hatsuda (RIKEN), Y. Ikeda (RCNP), T. Inoue (Nihon U.),
T. Iritani (RIKEN), N. Ishii (RCNP), T. Miyamoto (YITP),
H. Nemura (RCNP), K.Sasaki(YITP), T. Sugiura(RIKEN)

Lattice 19

Wuhan·China | June 16-22

Outline

Part I: Recent progress of two-baryon system

- Pseudo plateaux problem in direct plateaux fitting method
- Good convergence of derivative expansion in HAL method

T. Iritani+ [HAL Coll.] JHEP1610 (2016) 101 ← “src dep. of pseudo plateaux”
PRD96 (2017) 034521 ← “normality check of phase shift”
PRD99 (2019) 014514 ← “convergence of der. expansion”
JHEP1903 (2019) 007 ← “consistency b/w Lüscher & HAL”

Part II: ΩΩ interaction near the physical point

SG+ [HAL Coll.] PRL 120 (2018) 212001

Conventional method for Hadron Interaction

Lüscher's method

Input: $E_n(L)$

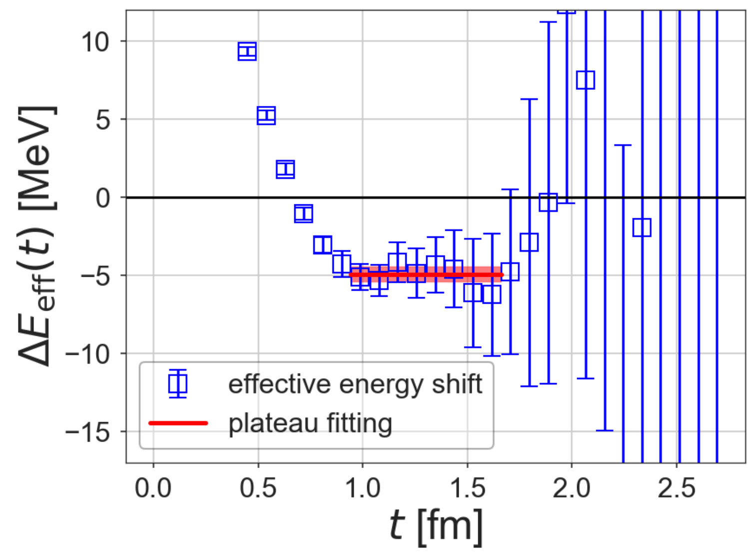
$$C_{NN}(t) \equiv \langle 0 \big| N(t)N(t)\mathcal{J}^{\dagger}_{\rm src}(t=0) \big| 0 \rangle = \sum_n A_n \psi_n(\vec{r}) \, \exp\left[-E_n \, t\right]$$

Lüscher's formula

$$k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbf{Z}} \frac{1}{\vec{n}^2 - (kL/2\pi)^2}$$

Phase shifts & Binding energy@*L*=∞

Direct method= Plateaux fitting + Lüscher's formula



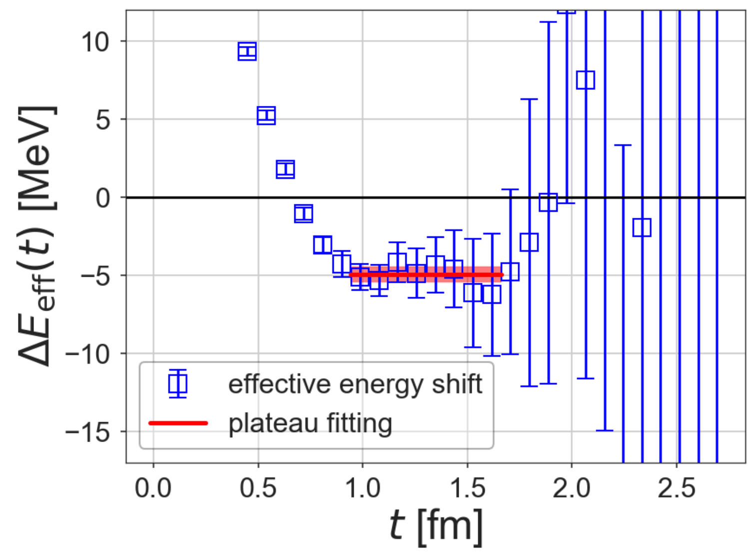
Inconsistent results in Direct method

NN(di-neutron)@ heavy quark masses

Direct method with **Smeared Source** [YIKU 2011/2012/2015, NPL 2012/2013/2015/2017, Cal Lat 2017]	**Bound**
Direct method with **Wall Source** [Iritani+ 2016] Direct method with **Improved Source (LapH)**[Mainz 2018]	**Unbound**

$m_\pi = 0.30 - 0.96[\mathrm{GeV}]$

Direct method= <u>Plateaux fitting</u> + Lüscher's formula



Inconsistent results in Direct method

What is the origin of inconsistency?

Direct method with **Wall Source** [Iritani+ 2016] Direct method with **Improved Source (LapH)**[Mainz 2018]	**Unbound**

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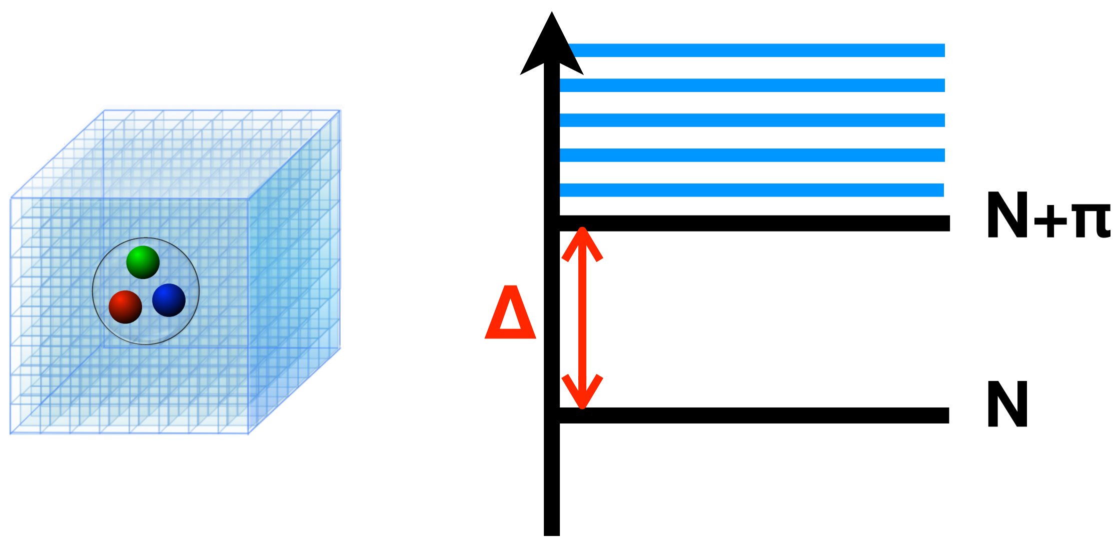
Pseudo plateaux in direct method

S/N in two-baryon system

Single Nucleon

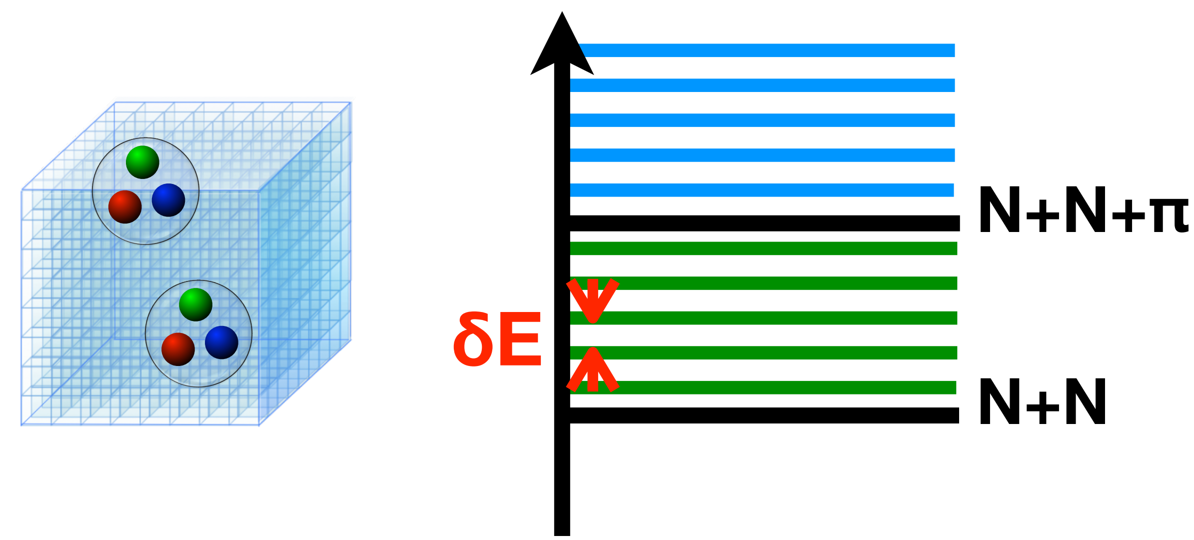


$$C_N(t) = a_0 e^{-m_N t} + a_1 e^{-(m_N + m_\pi)t} + \cdots$$
$$\to a_0 e^{-m_N t} \quad (t > t^*)$$

t* ~ Δ^{-1} ~ 1 fm

$$\mathcal{S}/\mathcal{N} \sim \sqrt{N_{\rm conf.}} \exp\left[-(m_{\rm N} - 3/2 m_\pi)\, t^*\right]$$
$$\sim \sqrt{N_{\rm conf.}} \times 10^{-2}$$

Two Nucleons



$$C_{NN}(t) = b_0 e^{-W_0 t} + b_1 e^{-W_1 t} + \cdots$$
$$\to b_0 e^{-W_0 t} \quad (t > t^*)$$

t* ~ δE^{-1} = m_N (L/2π)^2 ~ 10 fm

$$\mathcal{S}/\mathcal{N} \sim \sqrt{N_{\rm conf.}} \exp\left[-2\,(m_{\rm N} - 3/2 m_\pi)\, t^*\right]$$
$$\sim \sqrt{N_{\rm conf.}} \times 10^{-32}$$

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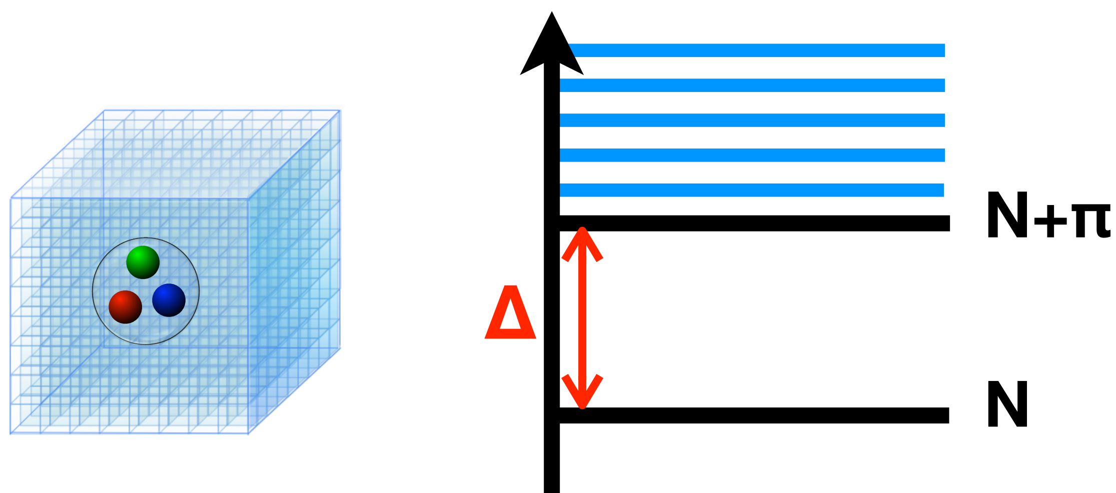

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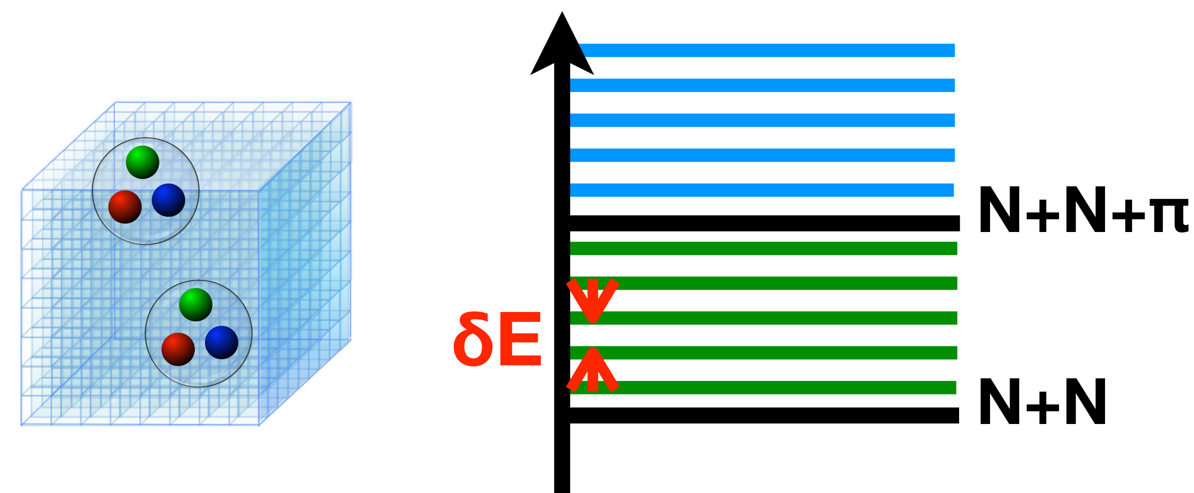

How did they practically calculate so far?

$t^* \sim \Delta^{-1} \sim 1$ fm

$$\mathcal{S}/\mathcal{N} \sim \sqrt{N_{\text{conf.}}} \exp\left[-(m_N - 3/2 m_\pi)\, t^*\right] \sim \sqrt{N_{\text{conf.}}} \times 10^{-2}$$

$t^* \sim \delta E^{-1} = m_N (L/2\pi)^2 \sim$ 10 fm

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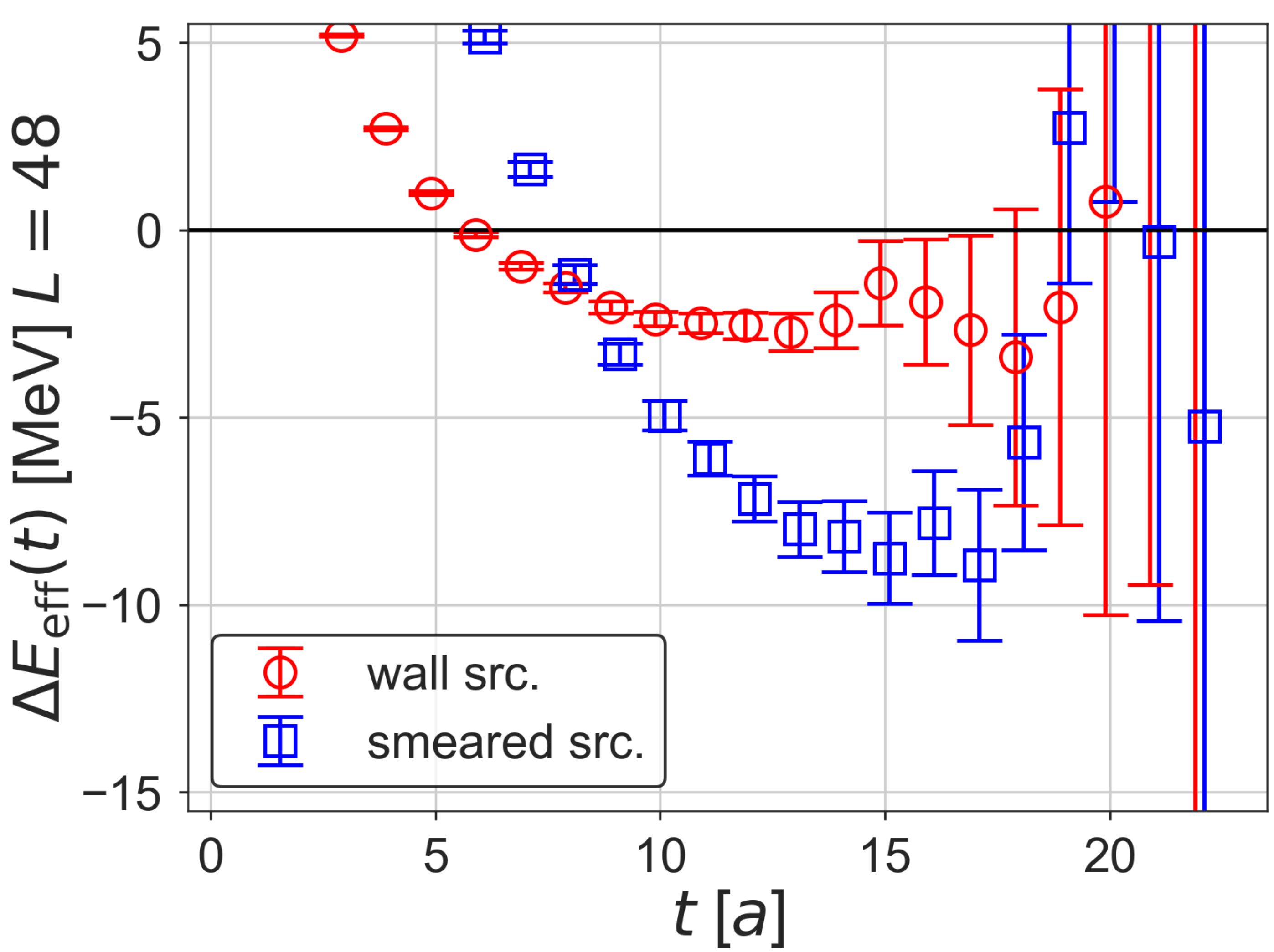
Pseudo Plateaux in the direct method

$$R(t) = \sum_{\vec{x},\vec{y}} \langle 0 | \, B_1(\vec{x},t) B_2(\vec{y},t) \mathcal{J}^{\dagger}_{\rm src}(t=0) \, | 0 \rangle / C_B(t)^2$$

wall src or **smeared src**



$\Xi\Xi(^1S_0)$

$=NN(^1S_0)$ in SU(3) limit

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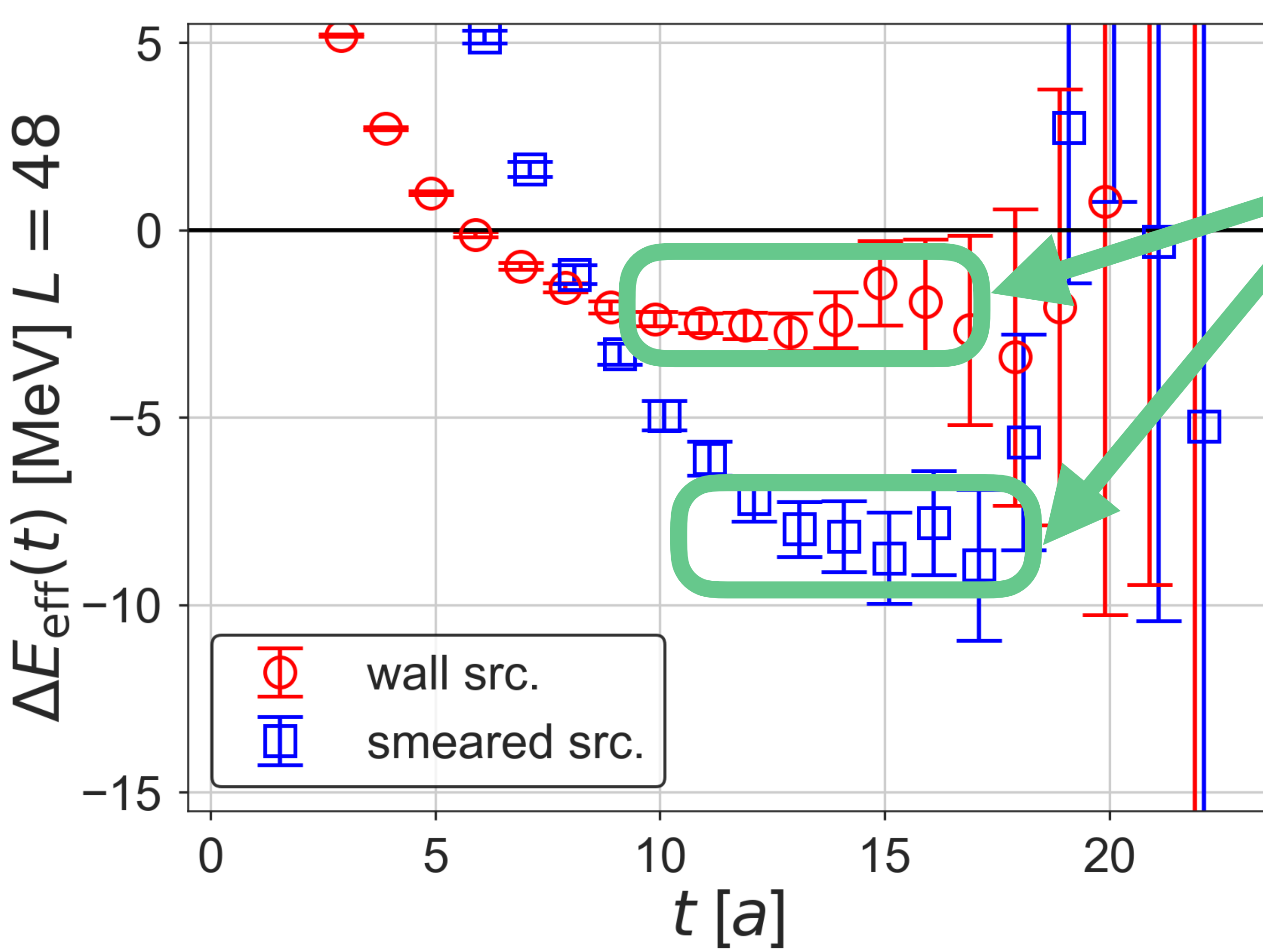

Inconsistent plateaux at t~1 fm (< t*~10 fm)

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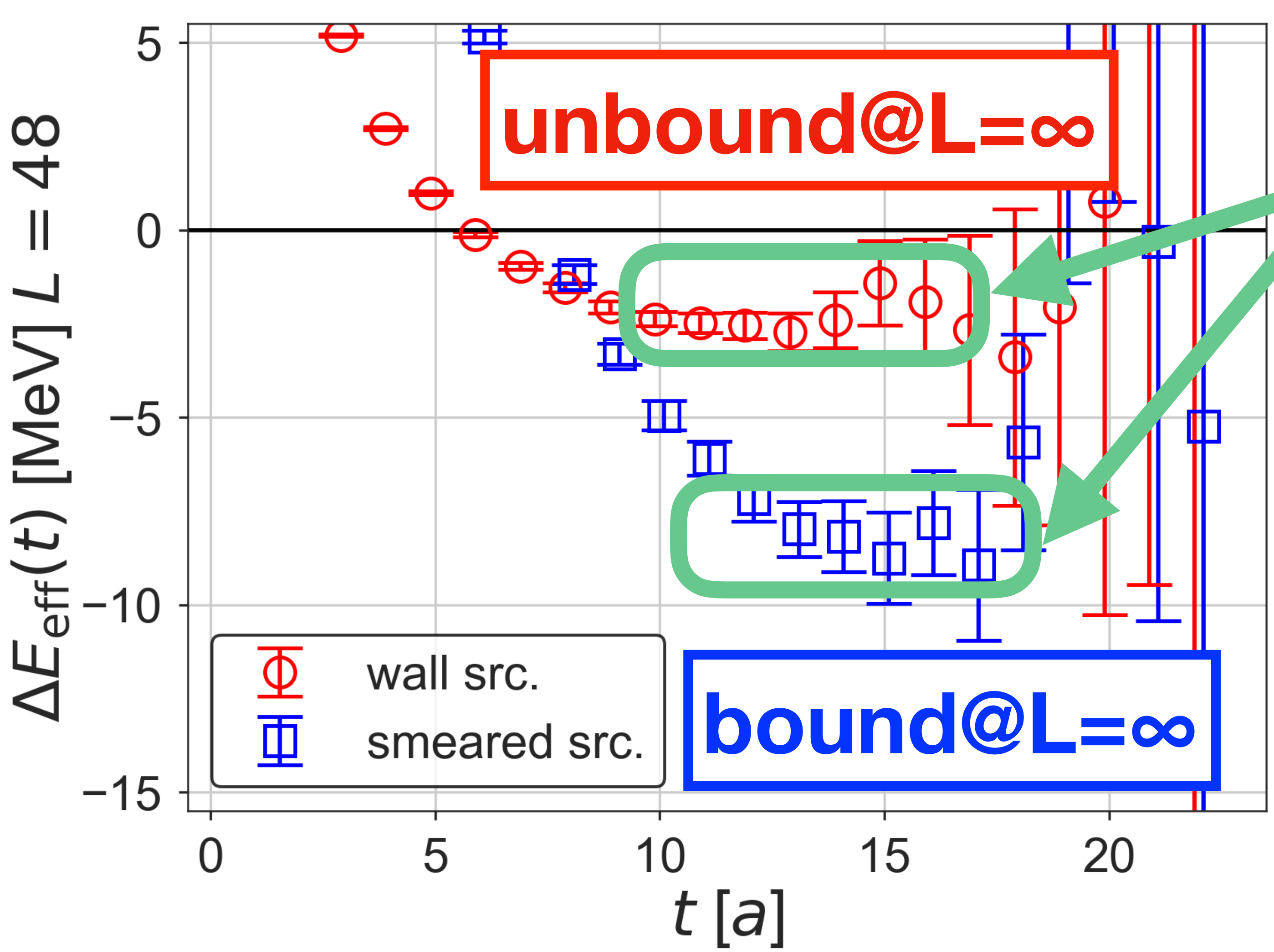

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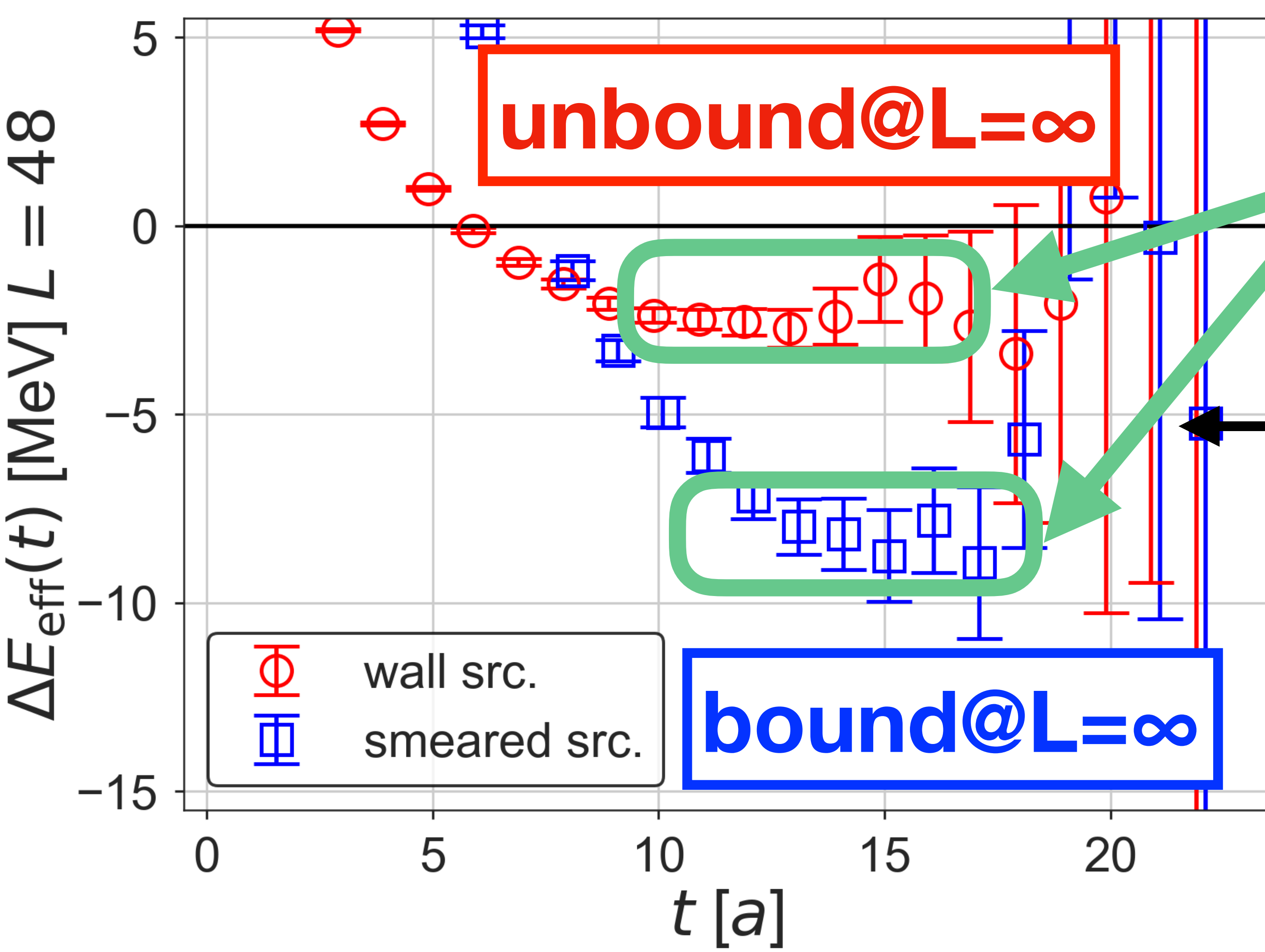

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$$\mathcal{S}/\mathcal{N} \sim \sqrt{N_{\mathrm{conf.}}} \exp\left[-2\left(m_{\mathrm{N}} - 3/2 m_{\pi}\right) t\right]$$

Exponentially divergent noise

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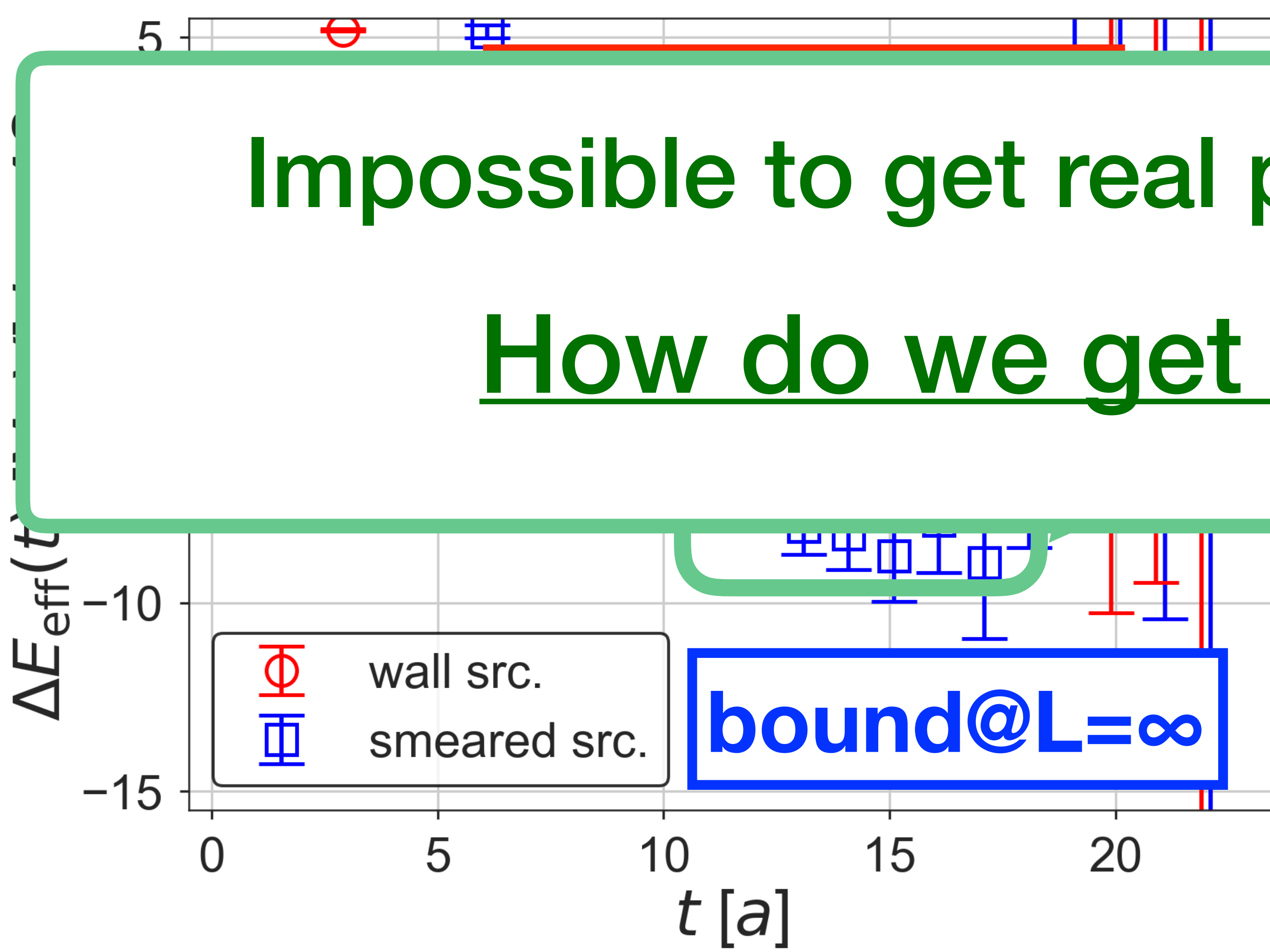

Impossible to get real plateaux at t* ~ 10 fm

How do we get correct result?

Exponentially divergent noise

ΞΞ($^1S_0$)

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1. Variational method

M. Lüscher and U. Wolff, NPB 339, 222 (1990)

2. HAL QCD method

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$$C_{ij}(t) = \langle 0 \Big| B_i(t) B_i(t) \mathcal{J}^{\dagger}_{j\ \mathrm{src}}(t=0) \Big| 0 \rangle$$

$$\boxed{B_1(t), \ldots, B_N(t)} \xrightarrow{\text{diagonalization}} \boxed{\Delta E_1, \ldots, \Delta E_N}$$

successfully used in two-meson system

R. Briceno, J. Dudek and R. Young, Rev. Mod. Phys. 90, no. 2, 025001 (2018)

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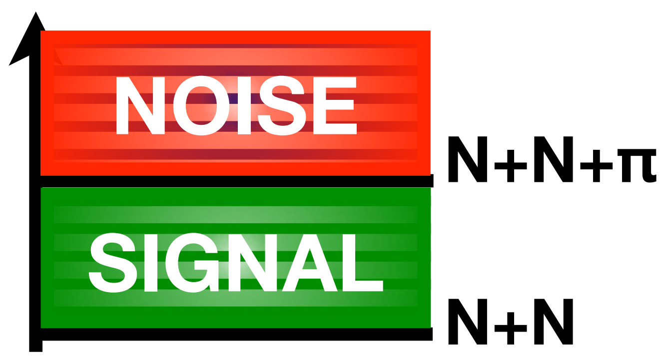
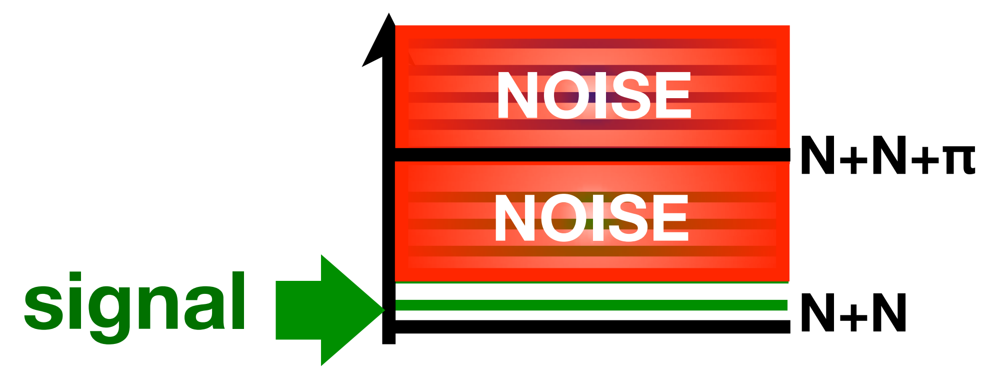


all elastic scattering states gives signals of the interaction

Direct method



ground state extraction is mandatory

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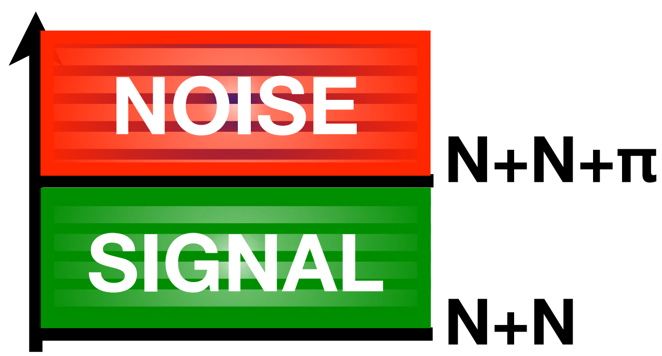

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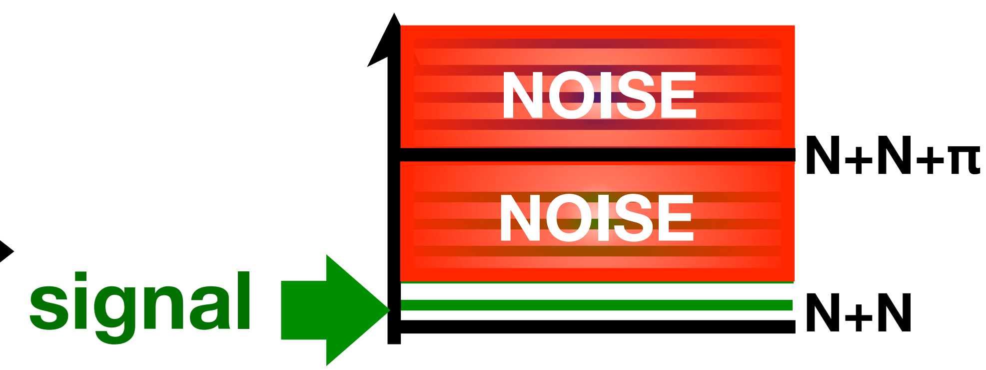

ground state extraction is mandatory

HAL QCD method → check contamination of the excited scattering states in the direct method.

HAL QCD method

Aoki, Hatsuda, Ishii, PTP123, 89 (2010)
N. Ishii+ [HAL QCD Coll.], PLB712, 437 (2012)

Space-time BB correlator (R-correlator)

$$R(\mathbf{r},t) = \langle 0 \big| B(\mathbf{r},t)B(\mathbf{0},t)\mathcal{J}^{\dagger}_{\mathrm{src}}(t=0) \big| 0 \rangle / \left\{ C_B(t) \right\}^2$$
$$= \sum_n A_n \underbrace{\psi_n(\mathbf{r})}_{\textbf{NBS w.f.}} e^{-\Delta E_n t}$$

NBS w.f. satisfies

$$\left[E_n - H_0 \right] \psi_n(\mathbf{r}) = \int d\vec{r}' \boldsymbol{U}(\mathbf{r},\mathbf{r}')\psi_n(\mathbf{r}')$$

non-local potential
independent of energy

t-dependent HAL method

$$\left[-\partial_t + \frac{1}{4m_B}\partial_t^2 - H_0 \right] R(\mathbf{r},t) = \int d\mathbf{r}' U(\mathbf{r},\mathbf{r}') R(\mathbf{r}',t)$$

$\Rightarrow U(\mathbf{r},\mathbf{r}') \Rightarrow$ **phase shifts**
binding energy

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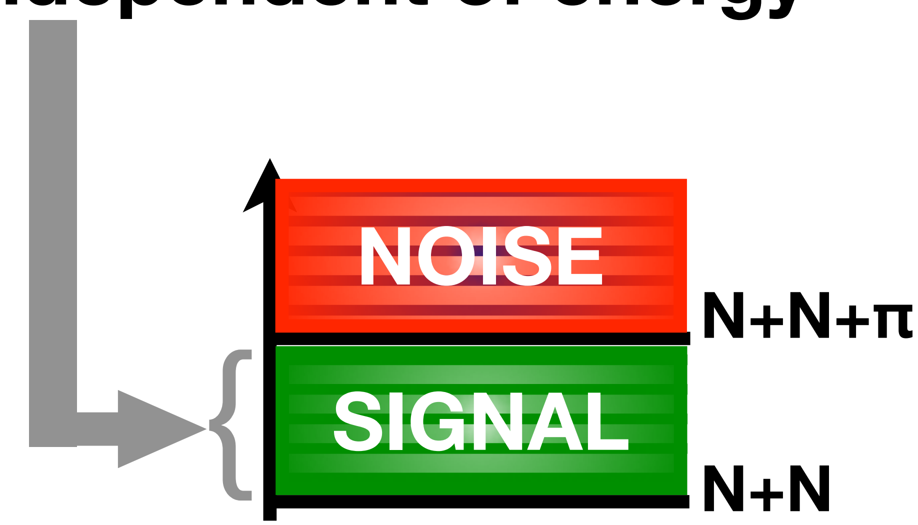


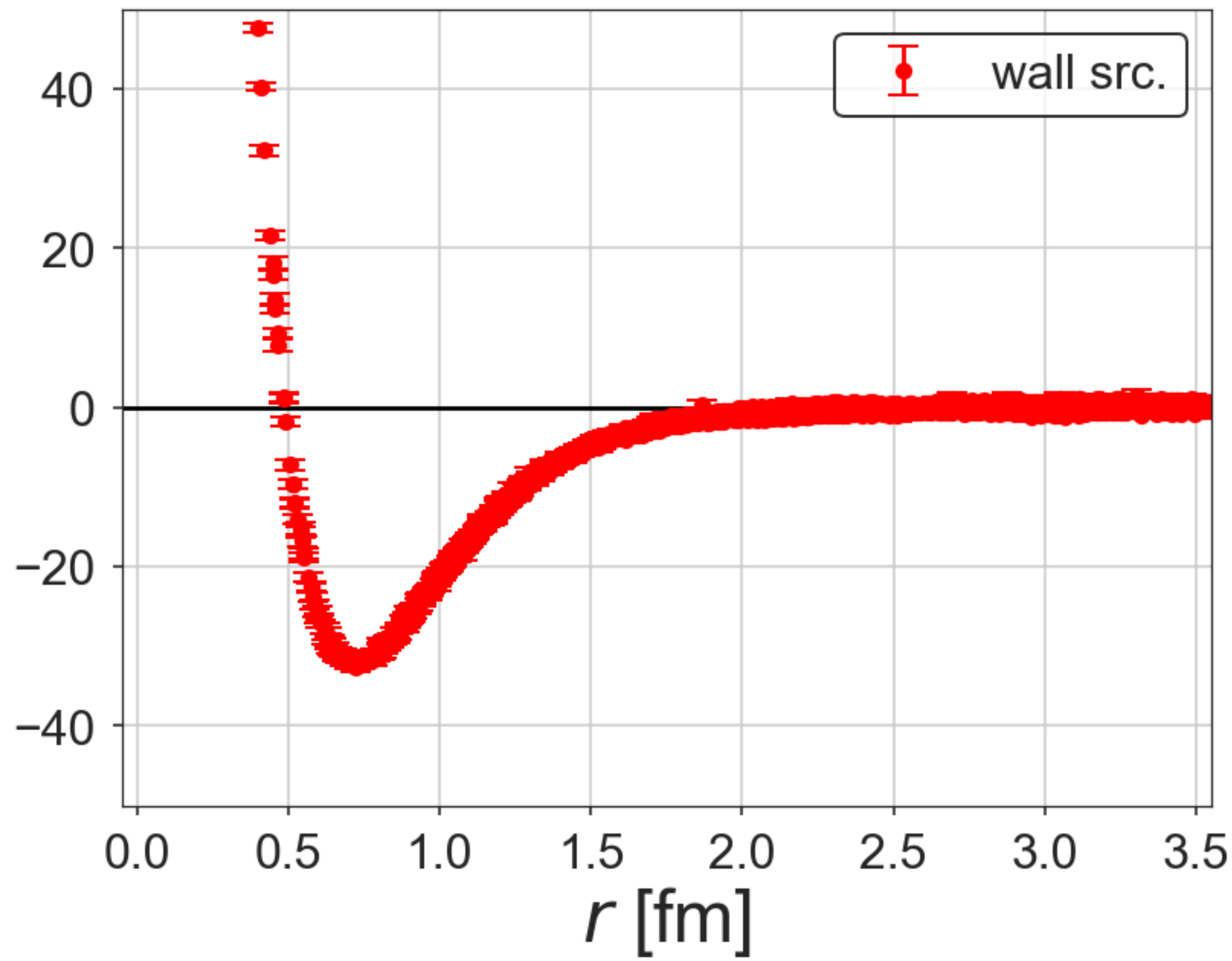
t-dependent HAL method

$$\left[-\partial_t + \frac{1}{4m_B} \partial_t^2 - H_0 \right] R(\mathbf{r}, t) =$$

$\Rightarrow U(\mathbf{r}, \mathbf{r}') \Rightarrow$ p

binding energy

HAL potential

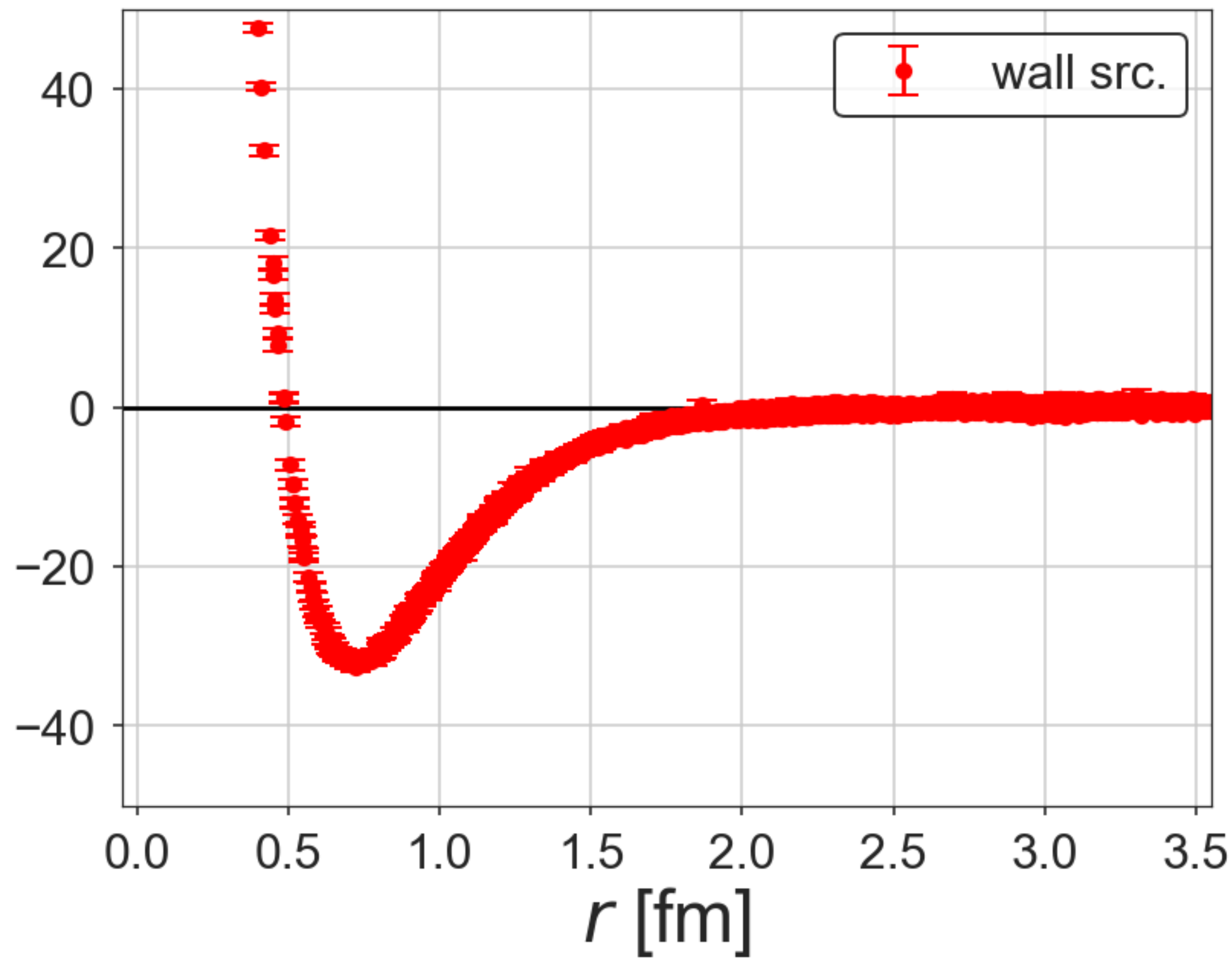


Derivative expansion + truncation of higher-order derivatives

$$U(\mathbf{r}, \mathbf{r}') = V_0(r)\delta(\mathbf{r} - \mathbf{r}') + \sum_{n=1} V_{2n}(r)\,\nabla^{2n}\delta(\mathbf{r} - \mathbf{r}')$$

LO approx.

HAL potential



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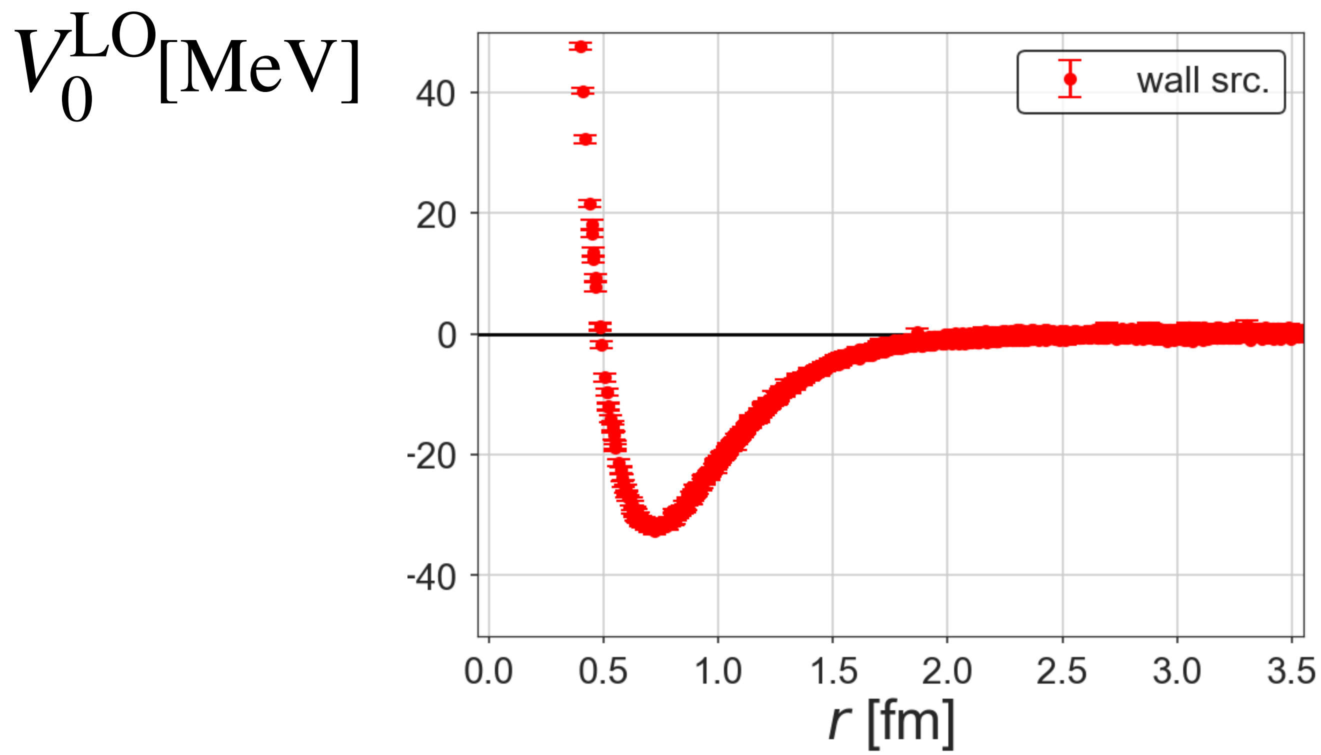
LO approx.

Higher order contribution will be shown to be small later.

Strategy to extract contribution from each state in space-time BB correlator

Use HAL pot and solve Schrödinger eq. at finite volume



$$\left[H_0 + V_0^{\mathrm{LO}}\right] \Psi_n(\mathbf{r}) = E_n \Psi_n(\mathbf{r})$$

$$R^{\mathrm{wall/smear}}(\mathbf{r}, t) = \sum_n a_n^{\mathrm{wall/smear}} \Psi_n(\mathbf{r}) e^{-E_n t}$$

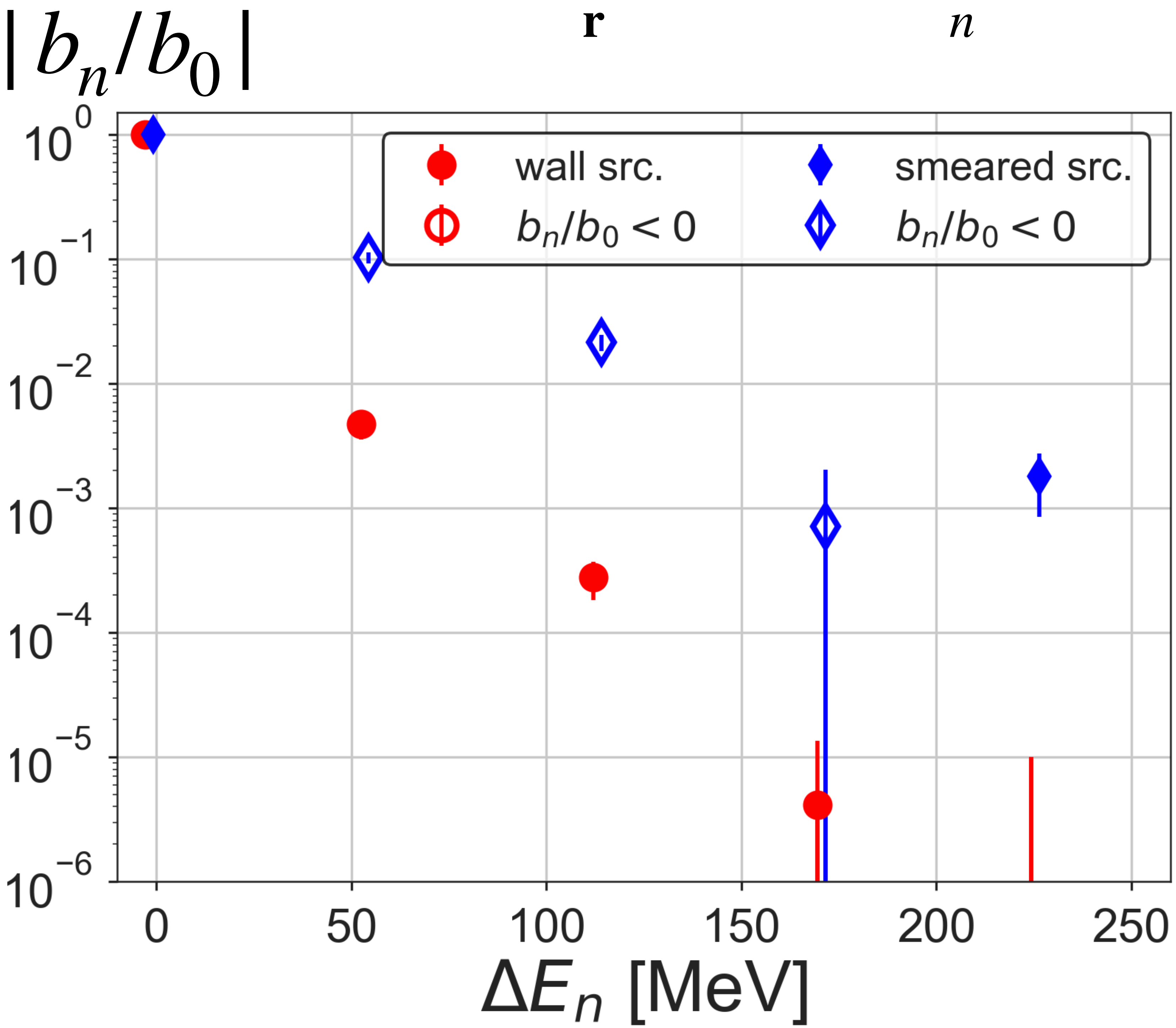
Use $\Psi_n(\mathbf{r})$ & E_n and derive contribution from each state

$$a_n^{\mathrm{wall/smear}} = \sum_{\mathbf{r}} R^{\mathrm{wall/smear}}(\mathbf{r}, t) \Psi_n(\mathbf{r}) e^{E_n t}$$

Excited state contamination in direct method

$$R^{\text{wall/smear}}(t) = \sum_{\mathbf{r}} R(\mathbf{r}, t) = \sum_n b_n^{\text{wall/smear}} e^{-\Delta E_n t}$$

$$b_n^{\text{wall/smear}} = \sum_{\mathbf{r}} a_n^{\text{wall/smear}} \Psi_n(\mathbf{r})$$



Wall source

$$|b_1/b_0| \ll 1\,\%$$

Smeared source

$$|b_1/b_0| \simeq 10\,\%$$

Excited state contamination in direct method

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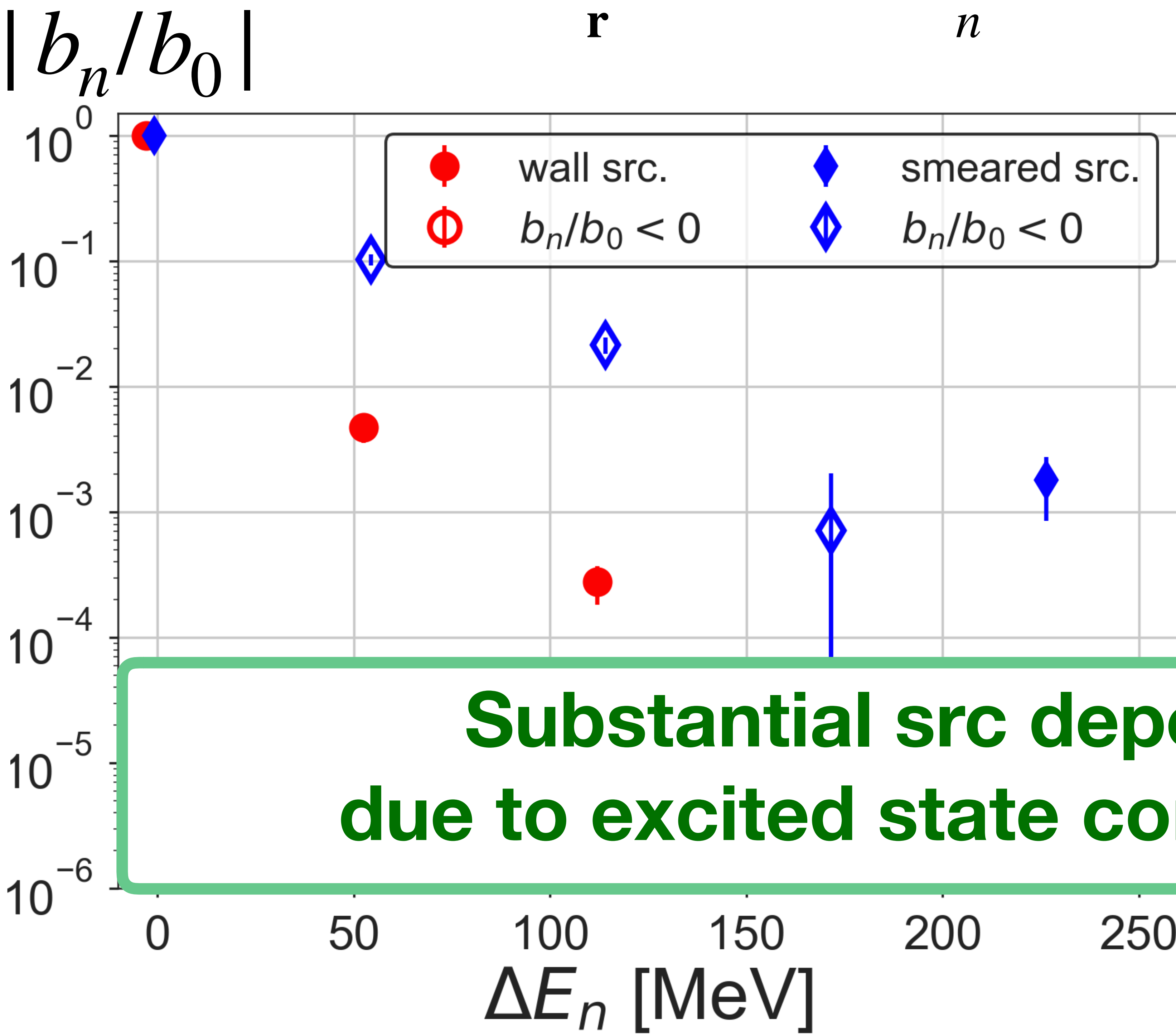

Wall source

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Smeared source

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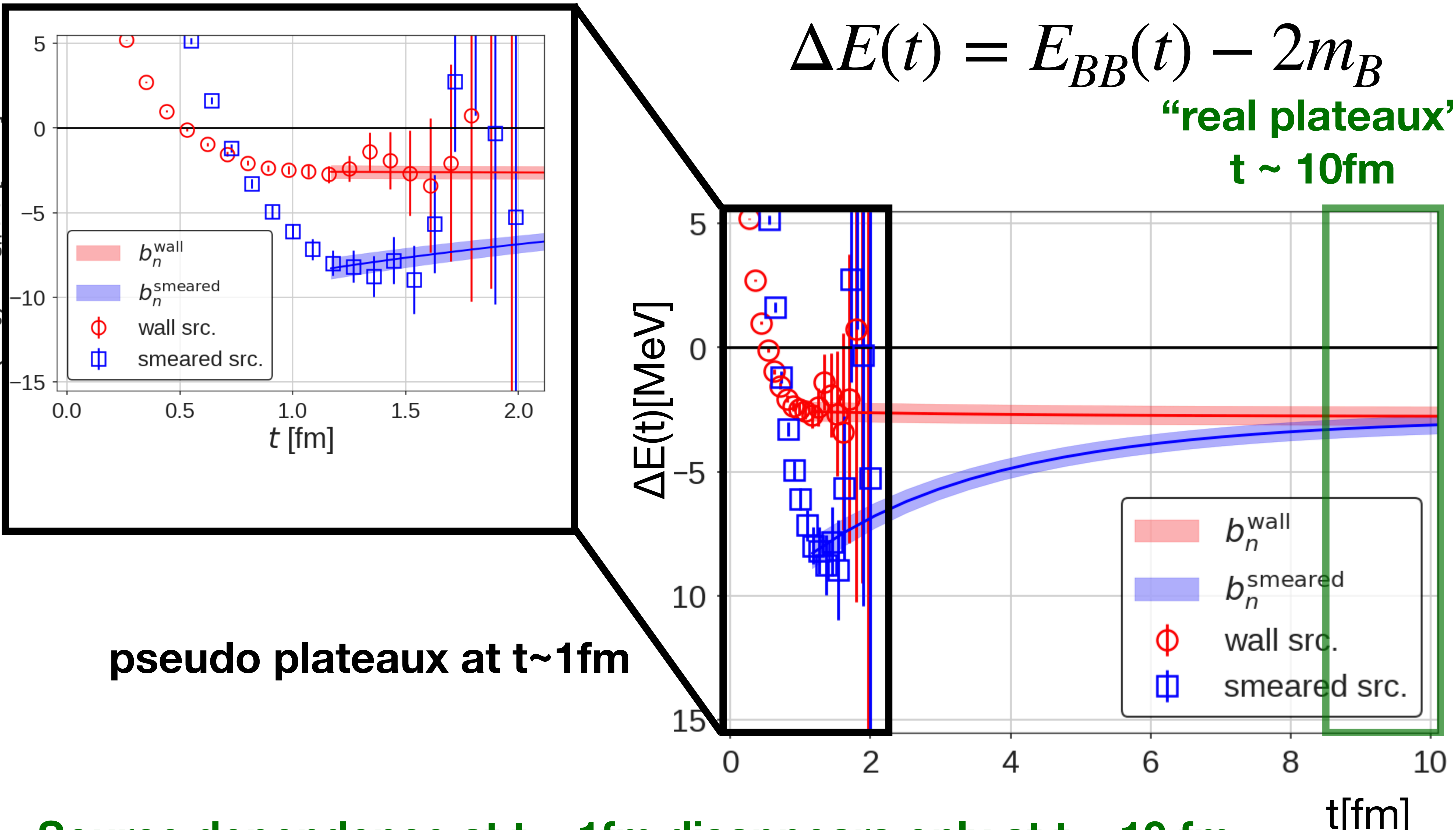
Substantial src dependence
due to excited state contamination

Effective Energy shift of two baryons

Reconstruction of energy shift from HAL QCD method

$$\Delta E(t) = E_{BB}(t) - 2m_B$$



Source dependence at t ~ 1fm disappears only at t ~ 10 fm

Effective Energy shift of two baryons

Reconstruction of energy shift from HAL QCD method

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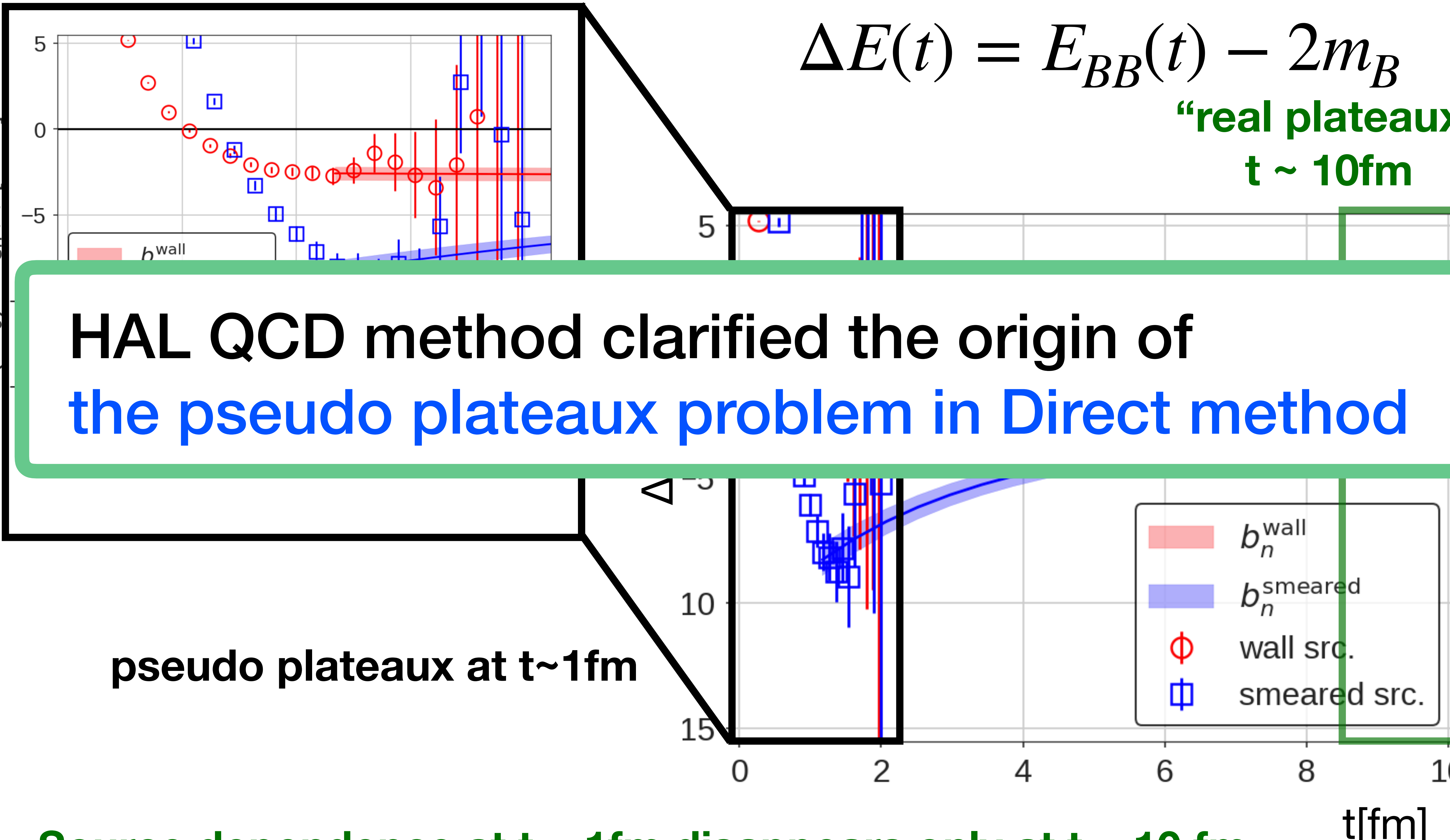
"real plateaux"
t ~ 10fm



HAL QCD method clarified the origin of
the pseudo plateaux problem in Direct method

pseudo plateaux at t~1fm

Source dependence at t ~ 1fm disappears only at t ~ 10 fm

Short summary

NN(di-neutron)@ heavy quark masses

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1. Pseudo plateaux problem is resolved by HAL QCD Method

2. NN(di-neutron) is found to be unbound

3. Further confirmation by the variational method is called for.

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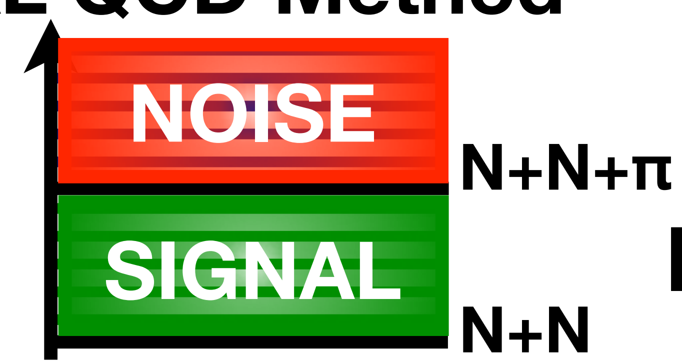
HAL QCD Method



$\Rightarrow U(\mathbf{r}, \mathbf{r}') \Rightarrow$

phase shifts
binding energy

elastic scattering states at t~1fm→signal

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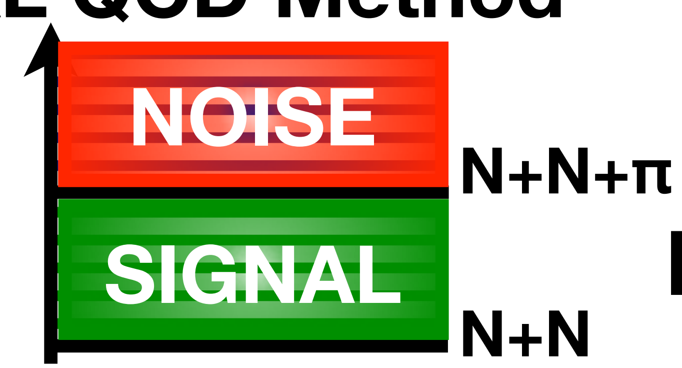
HAL QCD Method

LO aprox.
in derivative expansion



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Convergence of derivative expansion in HAL method

$$U(\mathbf{r},\mathbf{r}') = V_0(r)\delta(\mathbf{r}-\mathbf{r}') + \underbrace{V_2(r)\,\nabla^2\delta(\mathbf{r}-\mathbf{r}')}_{\text{N}^2\text{LO approx.}} + \sum_{n=2} V_{2n}(r)\,\nabla^{2n}\delta(\mathbf{r}-\mathbf{r}')$$

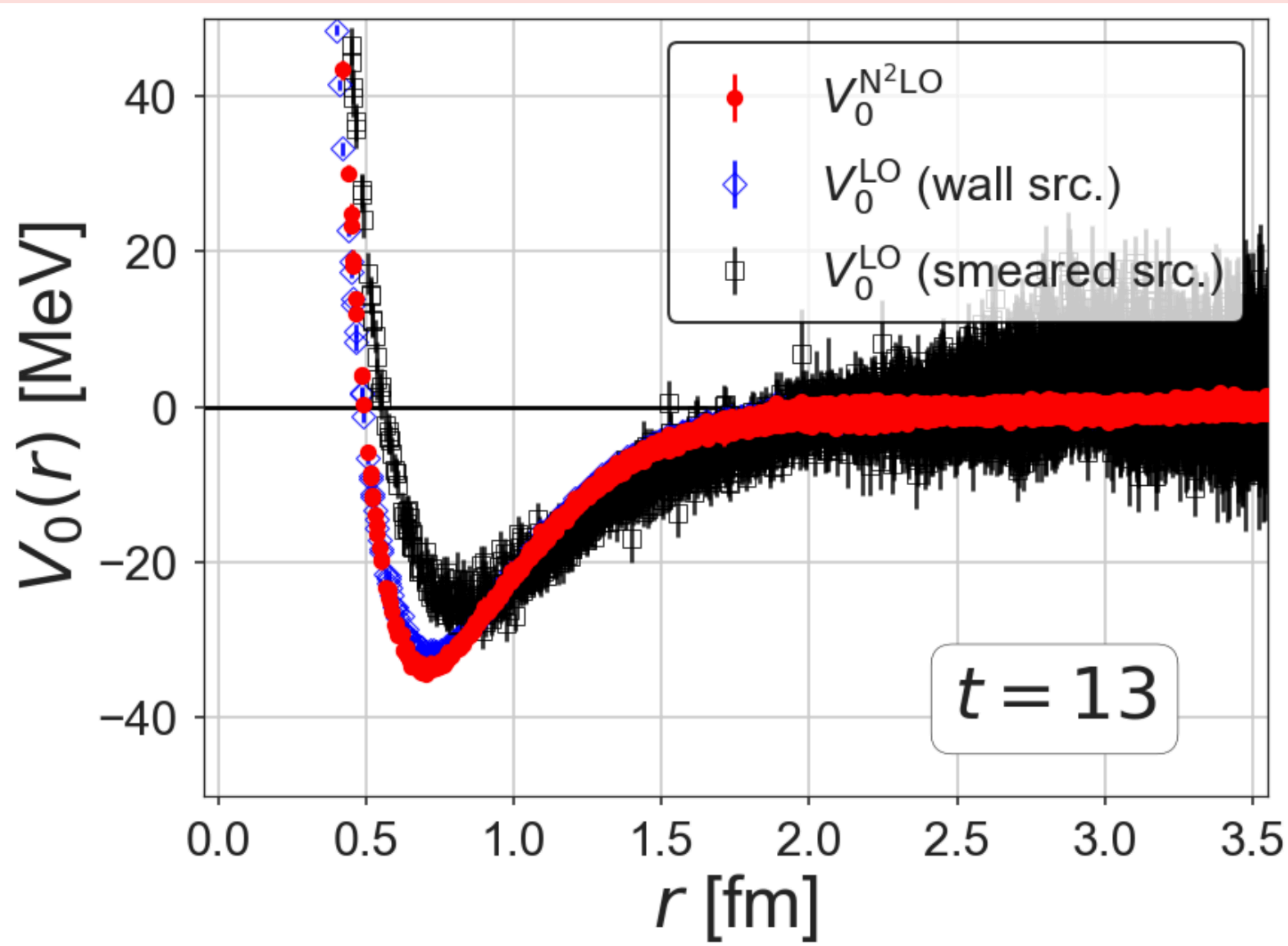
$$\rightarrow \quad V_0^{\mathrm{N^2LO}}(r),\, V_2^{\mathrm{N^2LO}}(r)$$

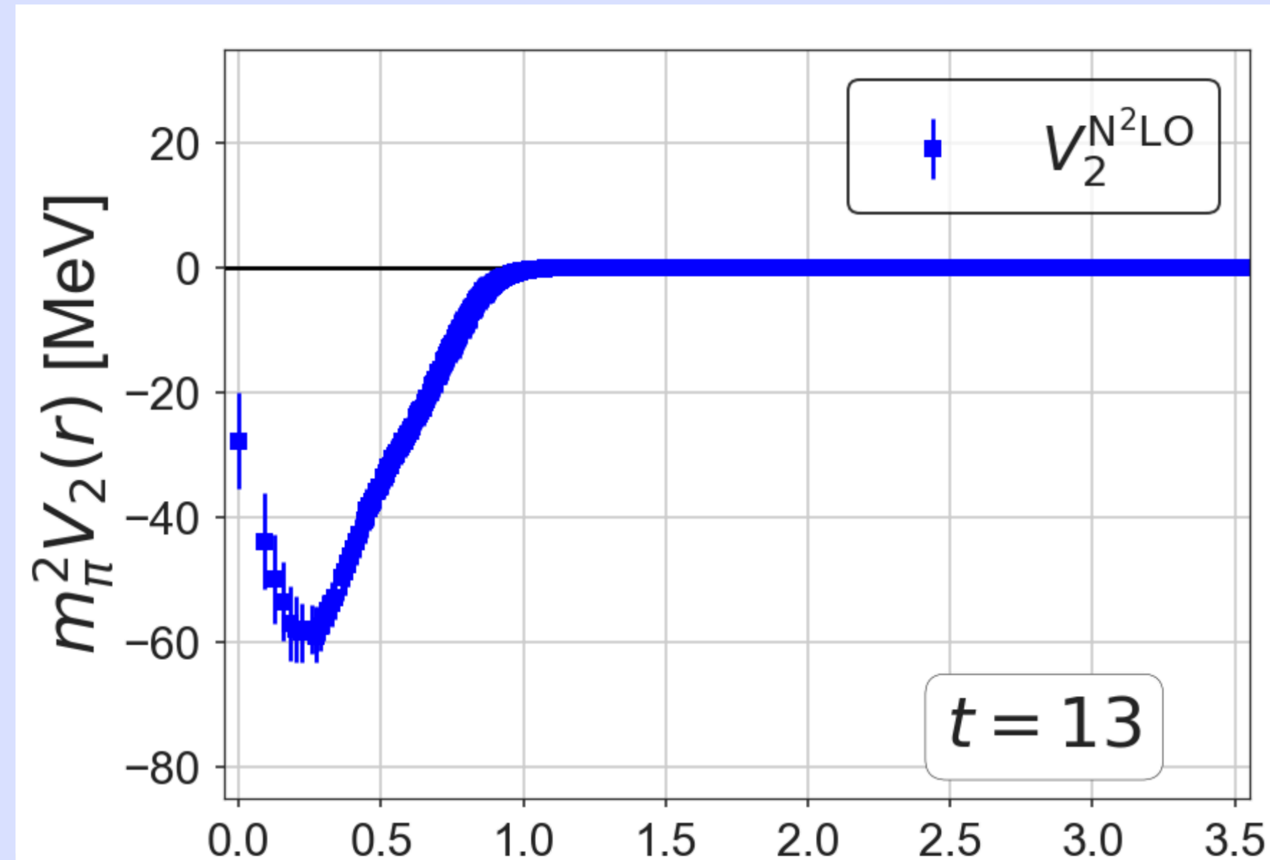
Convergence of derivative expansion in HAL method:

LO & N²LO potentials in HAL method I

smeared src & wall src $\longrightarrow V_0^{\mathrm{N^2LO}}(r), V_2^{\mathrm{N^2LO}}(r)$

$\Xi\Xi(^1S_0)$@m_π=0.51GeV





wall src $V_0^{\mathrm{LO\ wall}} \simeq V_0^{\mathrm{N^2LO}}(r)$

smeared src $V_0^{\mathrm{LO\ \ smeared}} \neq V_0^{\mathrm{N^2LO}}(r)$

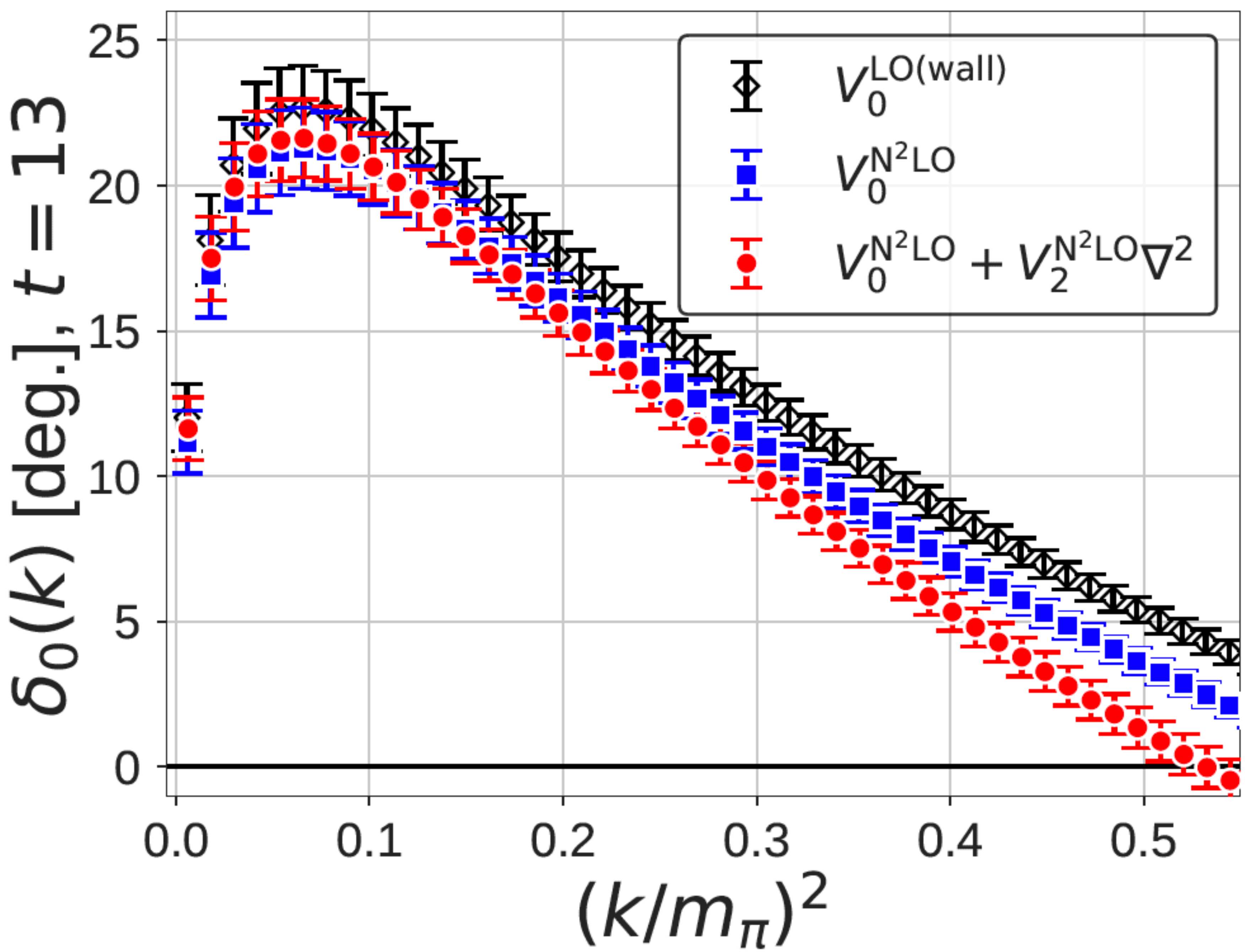
$V_2^{\mathrm{N^2LO}}(r)$

→**short range contribution**

$$V_0^{\mathrm{LO}}(r) = V_0^{\mathrm{N^2LO}}(r) + V_2^{\mathrm{N^2LO}}(r)\frac{\nabla^2 R(r,t)}{R(r,t)}$$

Convergence of derivative expansion in HAL method:

LO & N^{2}LO potentials in HAL method II



Convergence of derivative expansion in HAL method:

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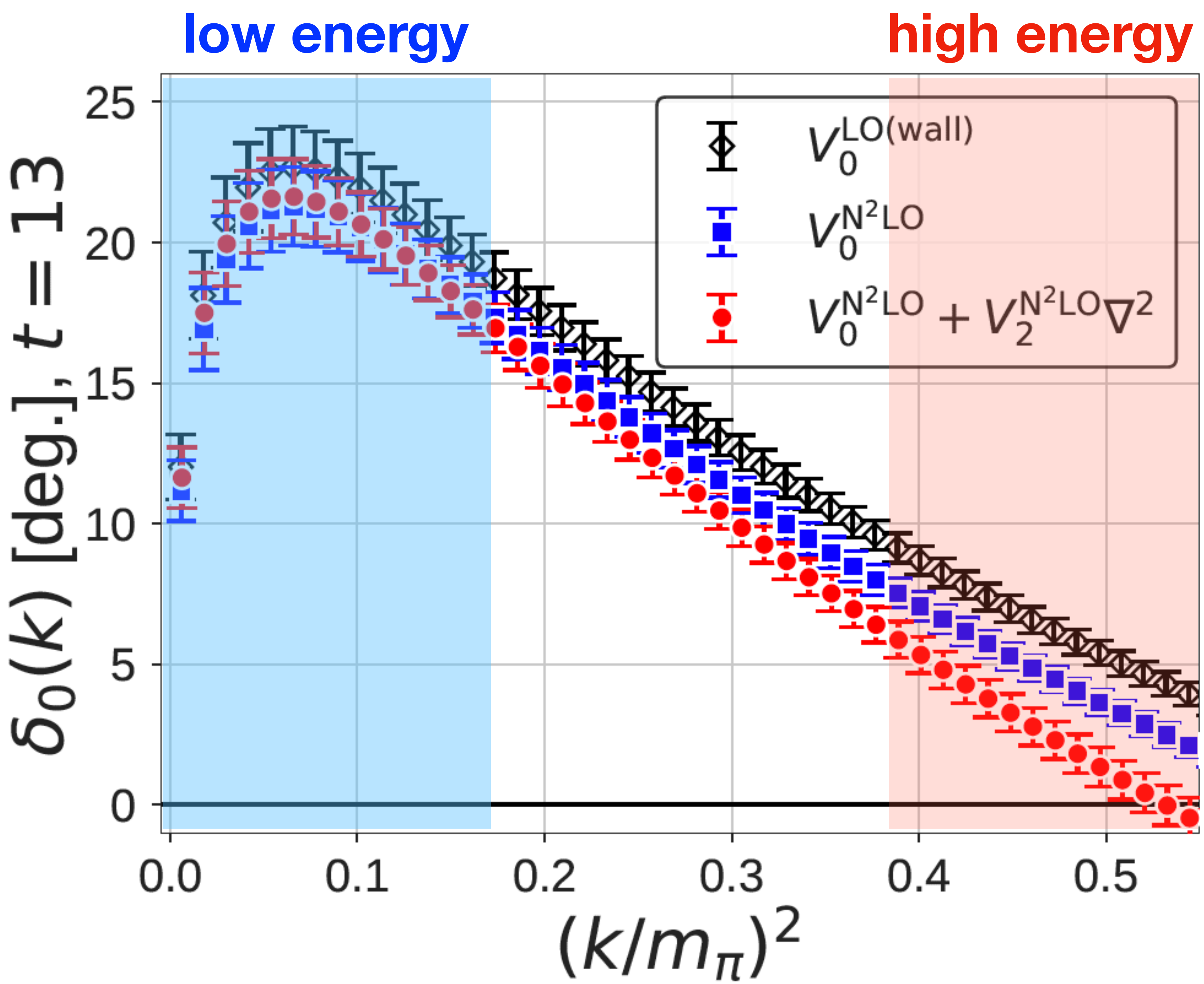
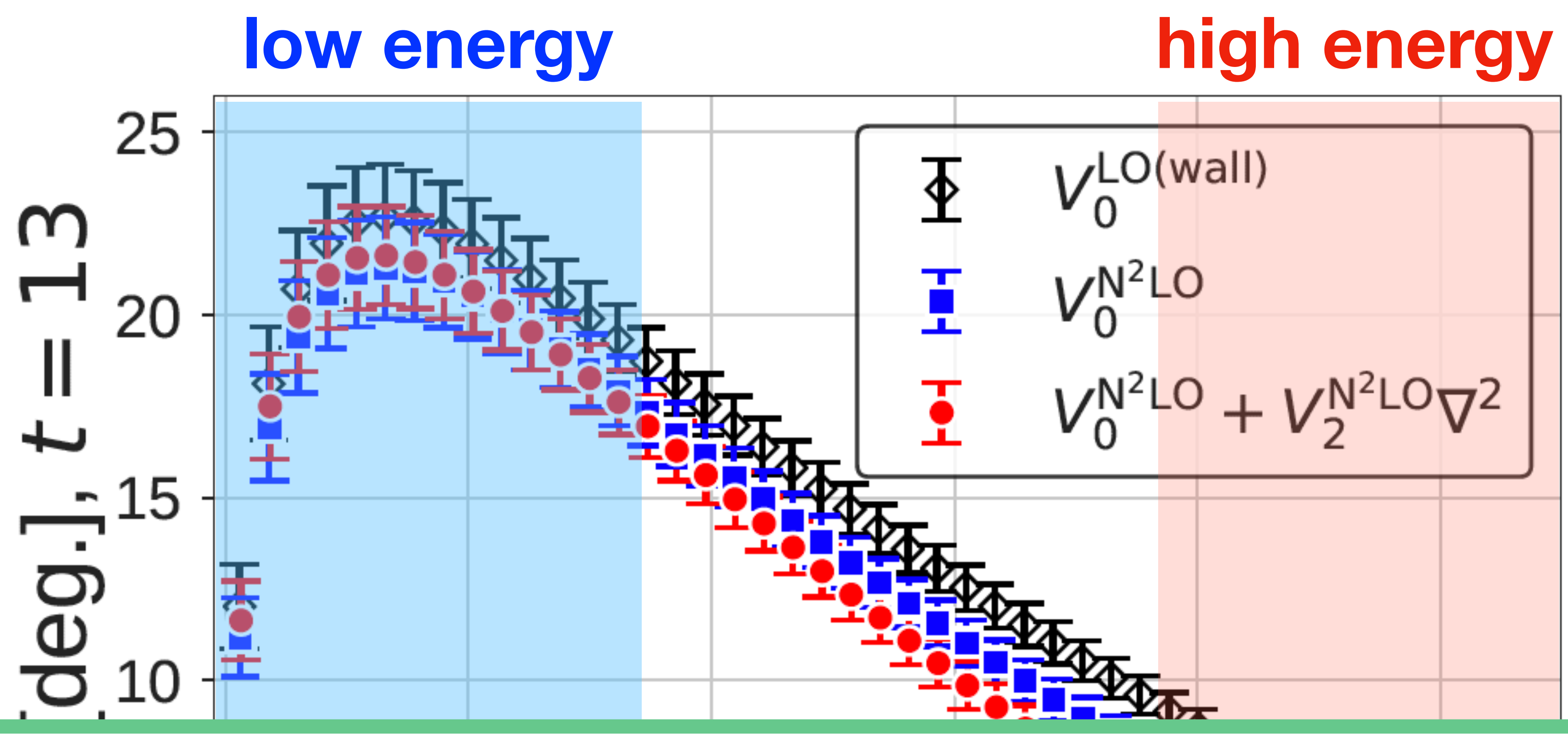


At low energy LO. result from wall src. ≒ N²LO results

At high energy small $V_2^{\mathrm{N^2LO}}$ correction

LO & N²LO potentials in HAL method II



LO pot in wall src works well at low energy.

Derivative expansion converges well.

At low energy LO. result from wall src. ≒ N²LO results

At high energy small $V_2^{N^2LO}$ correction

HAL method

Good convergence of derivative expansion

Phase shifts & Binding energy@$L=\infty$

16

HAL method

↓

Good convergence of derivative expansion

↓

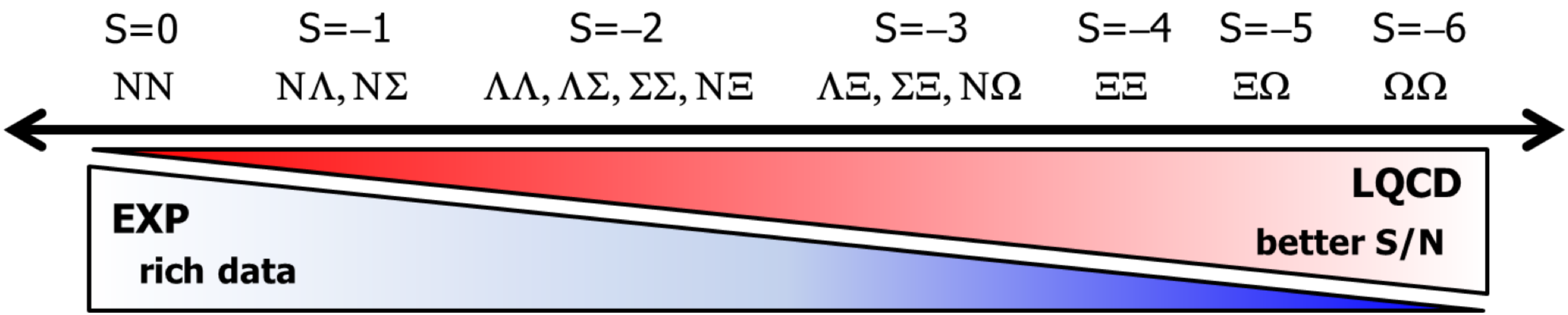
Phase shifts & Binding energy@L=∞

BB interaction near the physical point

$m_\pi = 146[\mathrm{MeV}]$

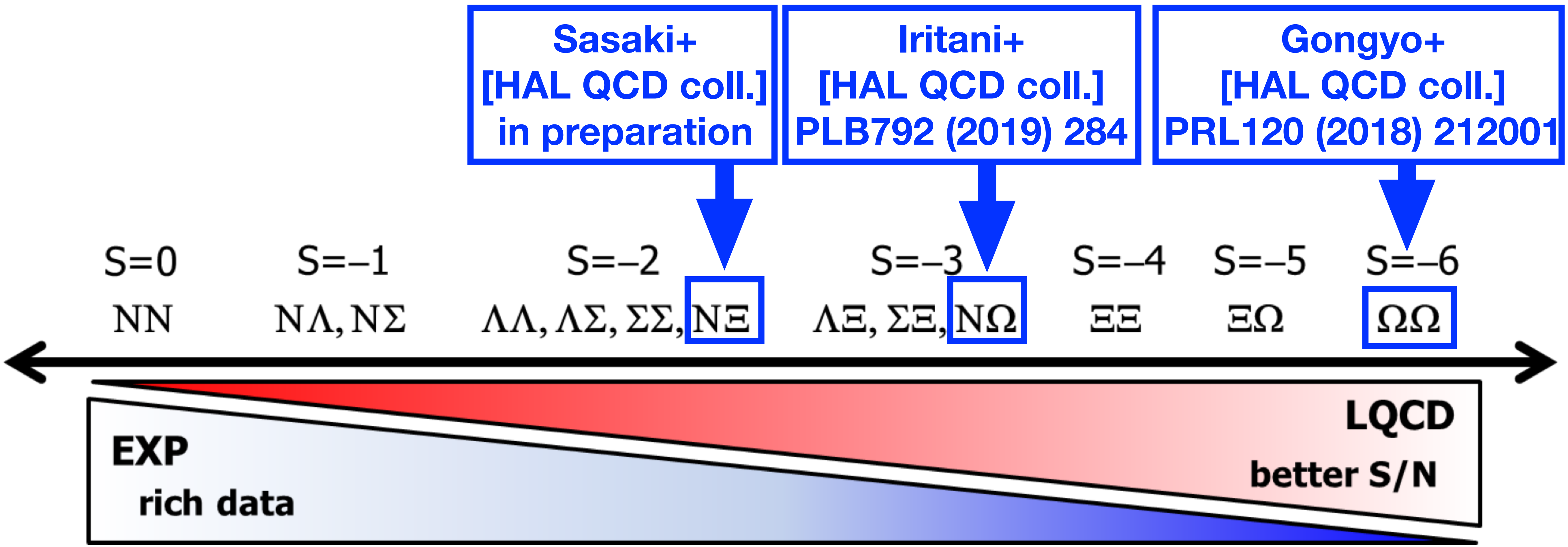
16

BB interaction near the physical point from HAL method



HAL QCD Coll. have studied from S= 0 to S= -6.

BB interaction near the physical point from HAL method



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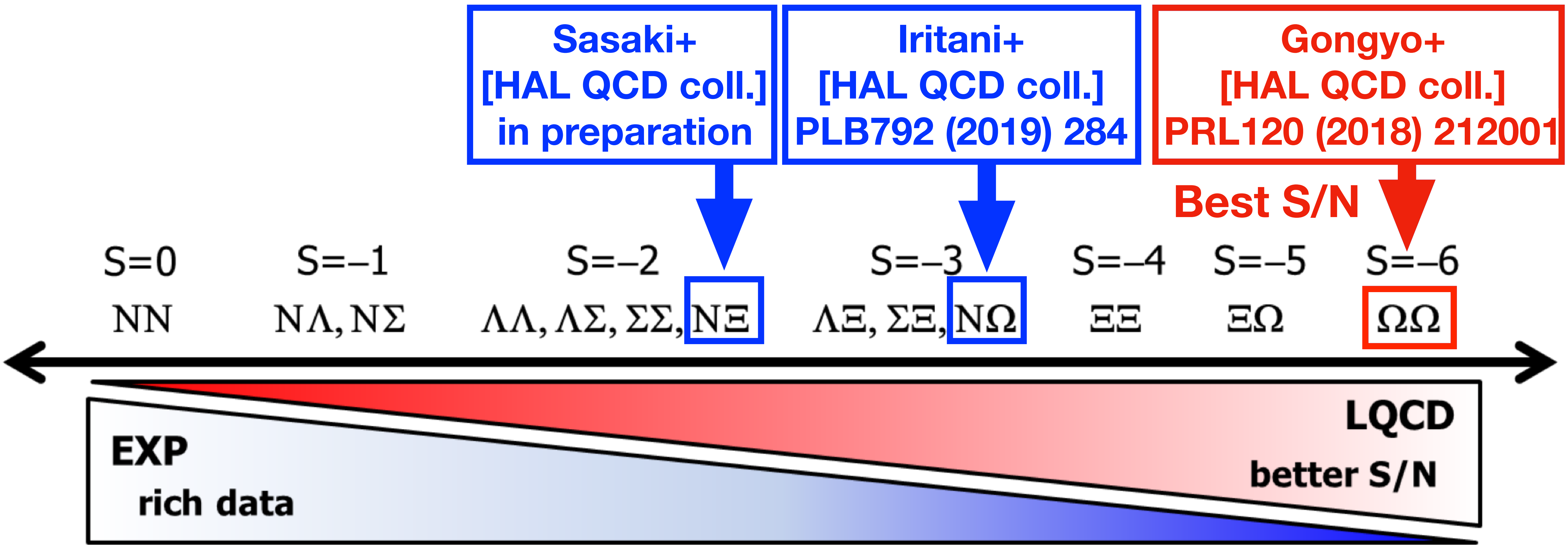


HAL QCD Coll. have studied from S= 0 to S= -6.

ΩΩ interaction near the physical point

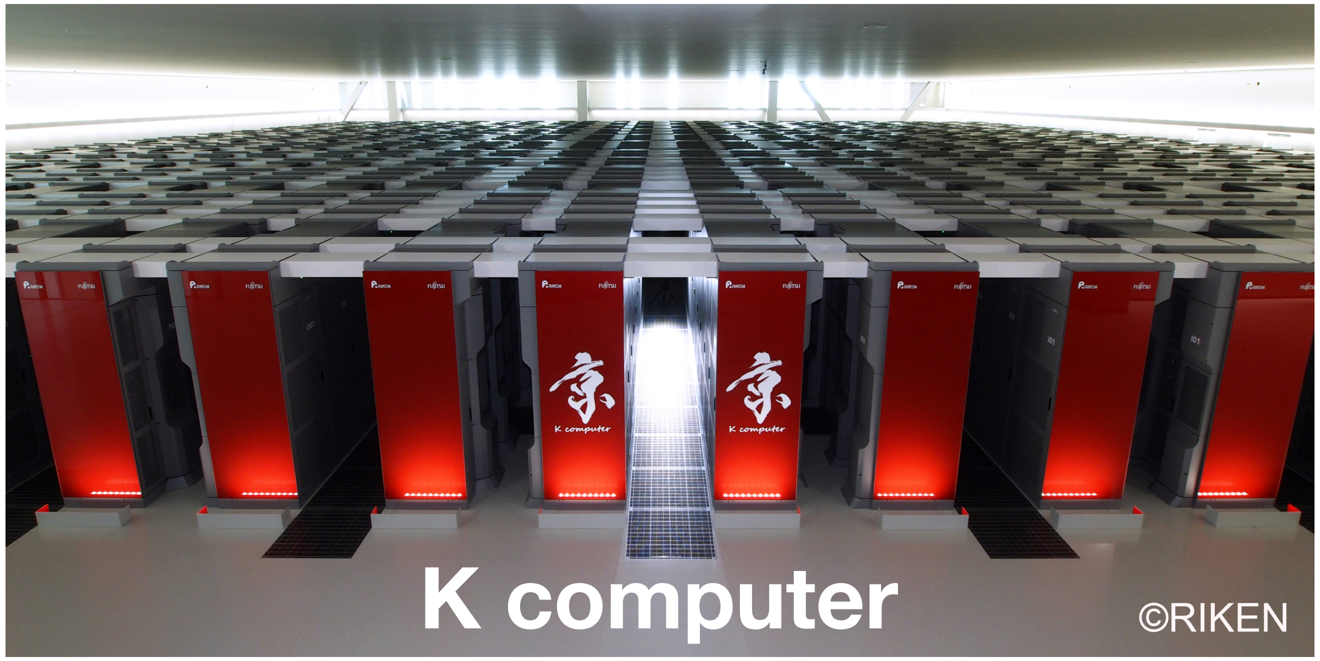
Numerical Setup near the physical point

2+1 flavor gauge configurations

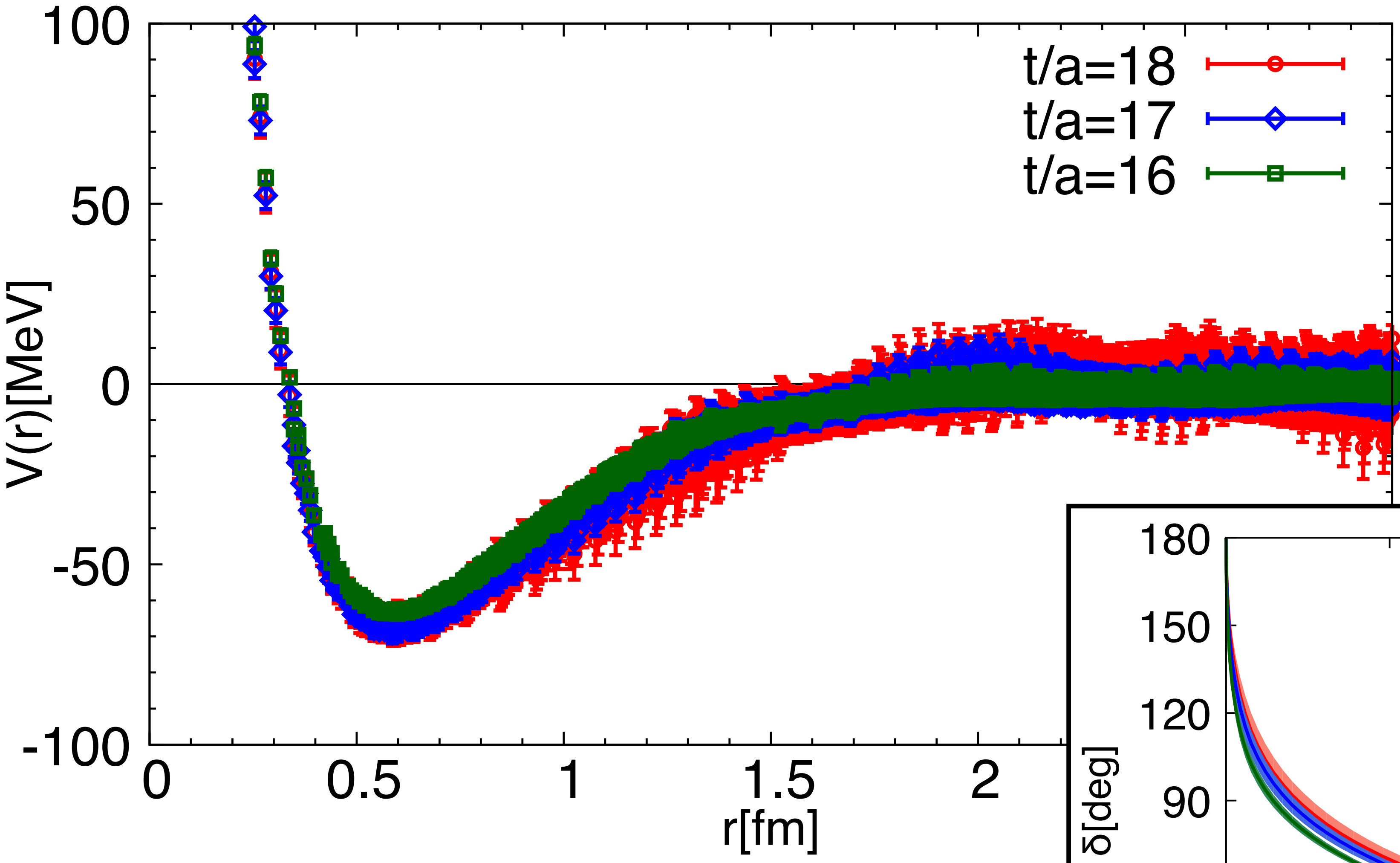
- Iwasaki gauge action & O(a) improved Wilson quark action
- a= 0.0846 [fm], a^{-1} = 2333 [MeV]
- 96^3x96 lattice, L = 8.1[fm]
- 400 confs x 48 source positions x 4 rotations

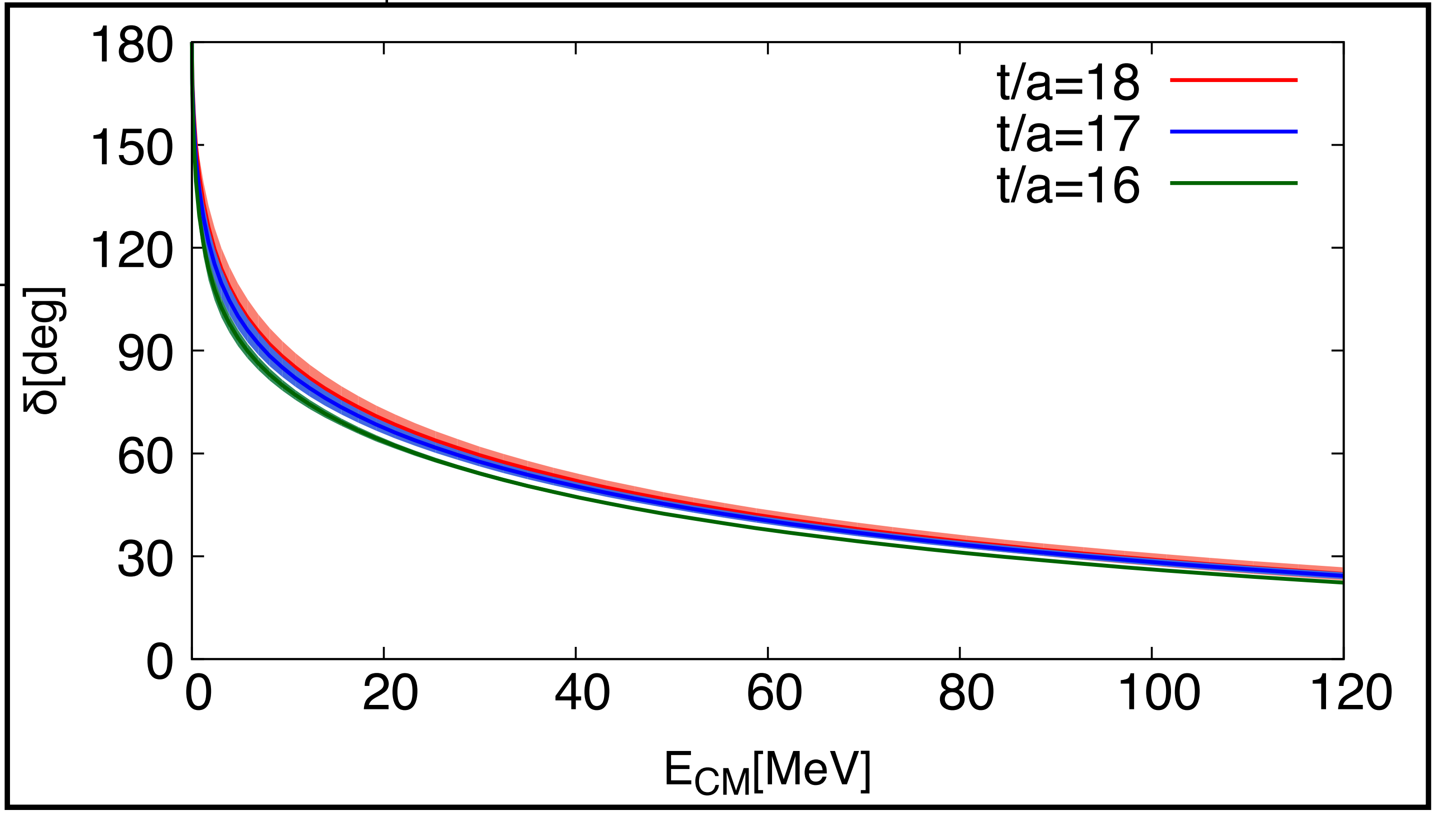
Wall source is employed.

	[MeV]	phys.
π	146	8%
K	525	6%
N	964	3%
Ω	1712	2%



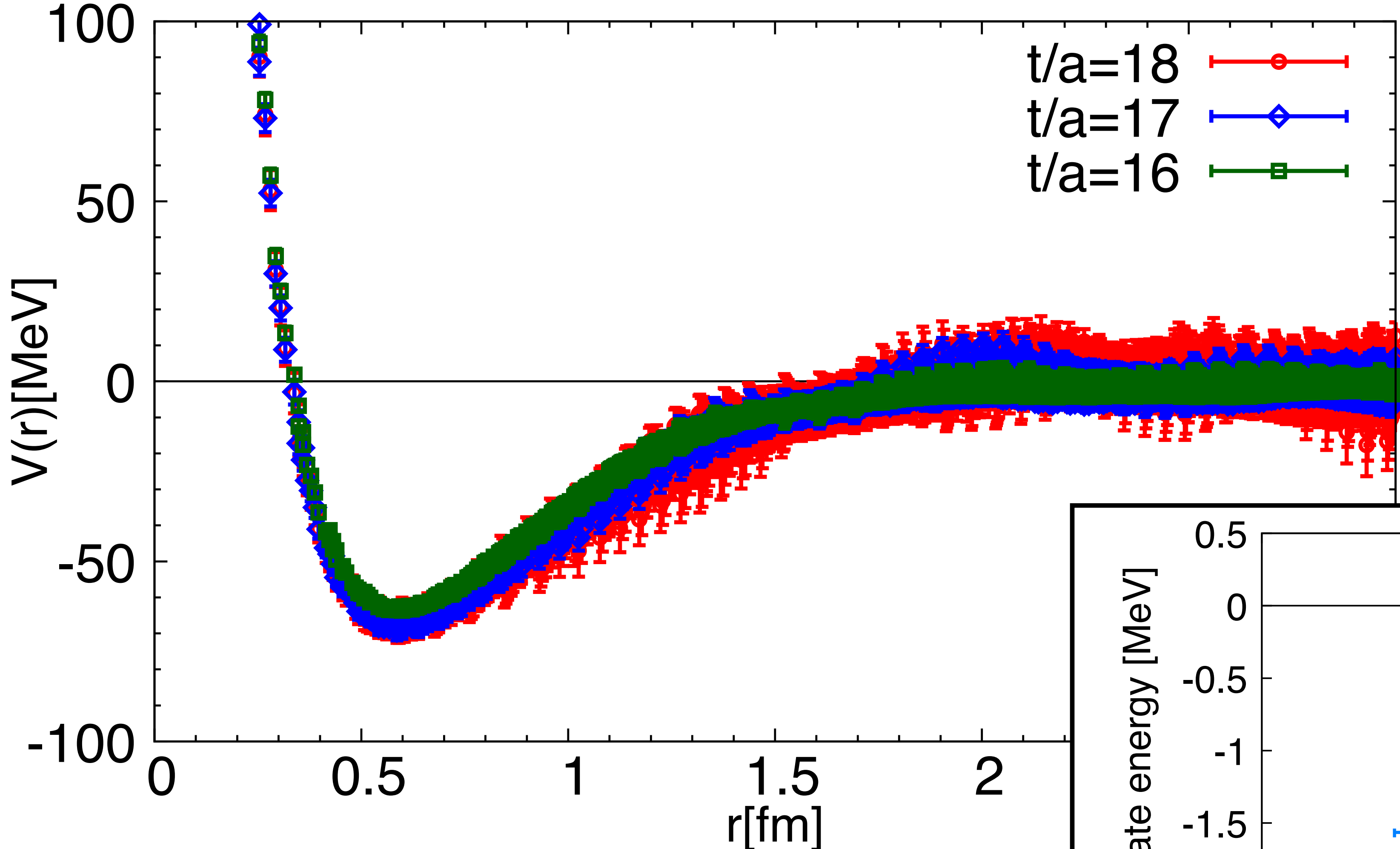
di-Omega: $\Omega\Omega(^1S_0)$

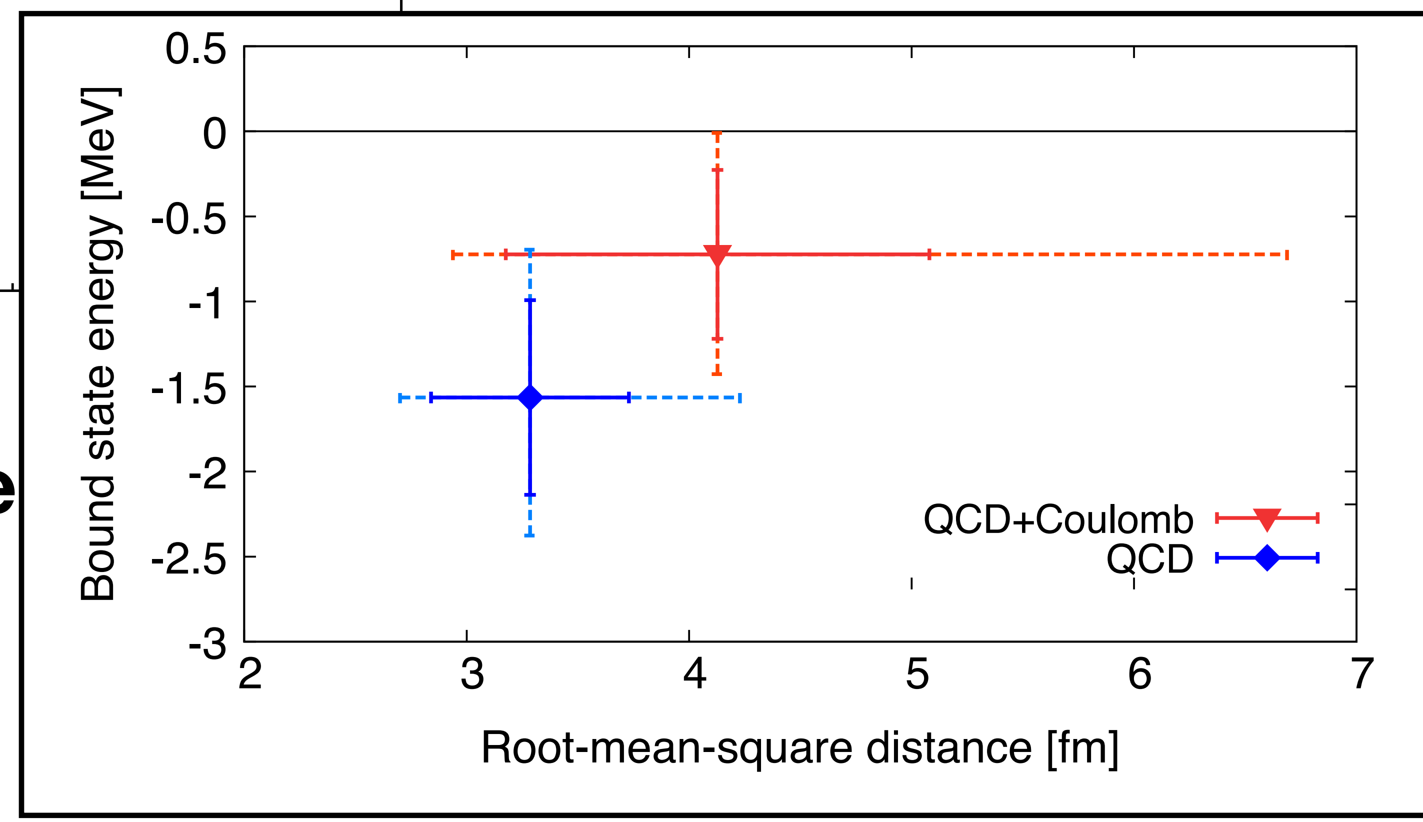




attractive region→ bound state

di-Omega: ΩΩ($^1S_0$)



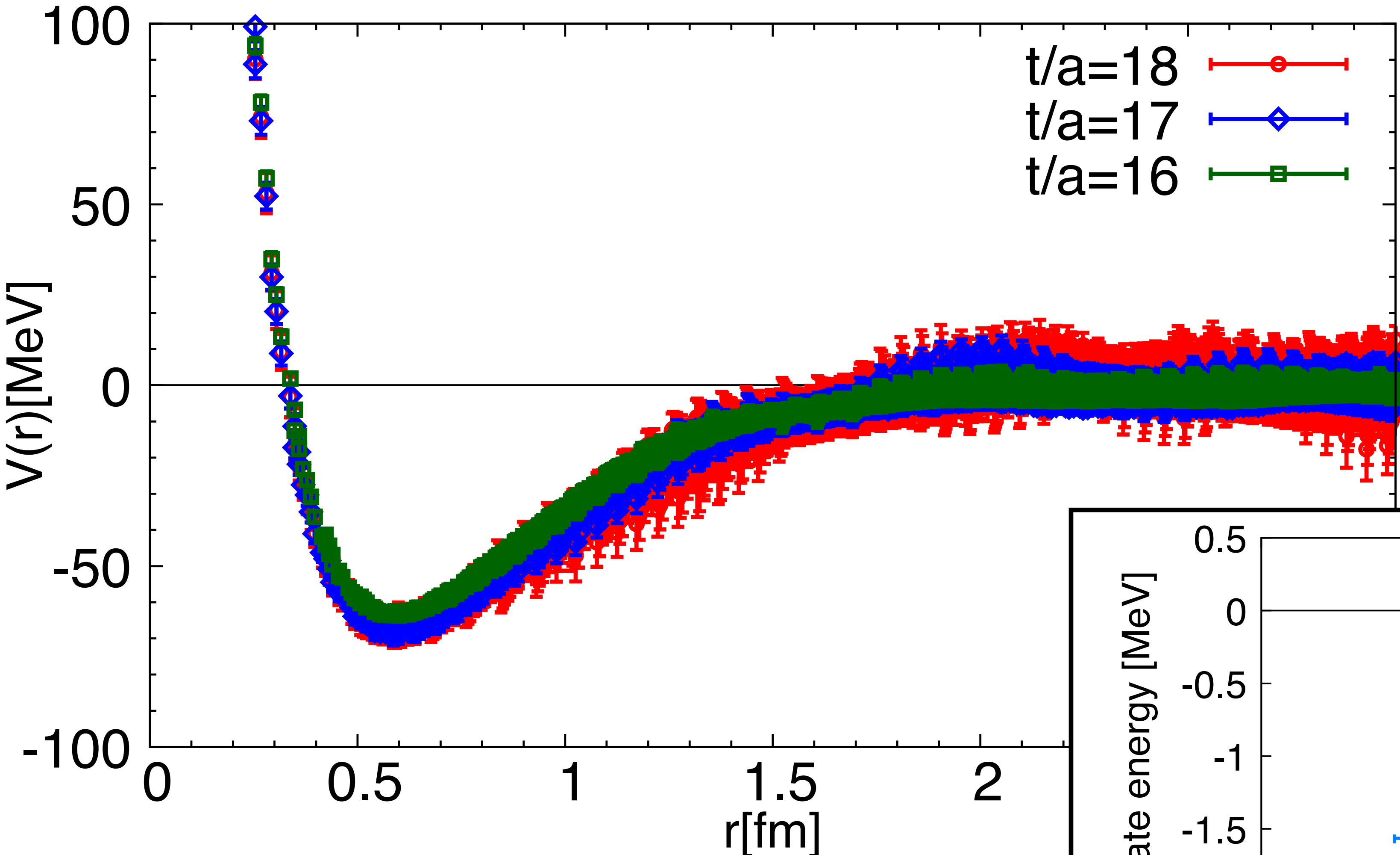


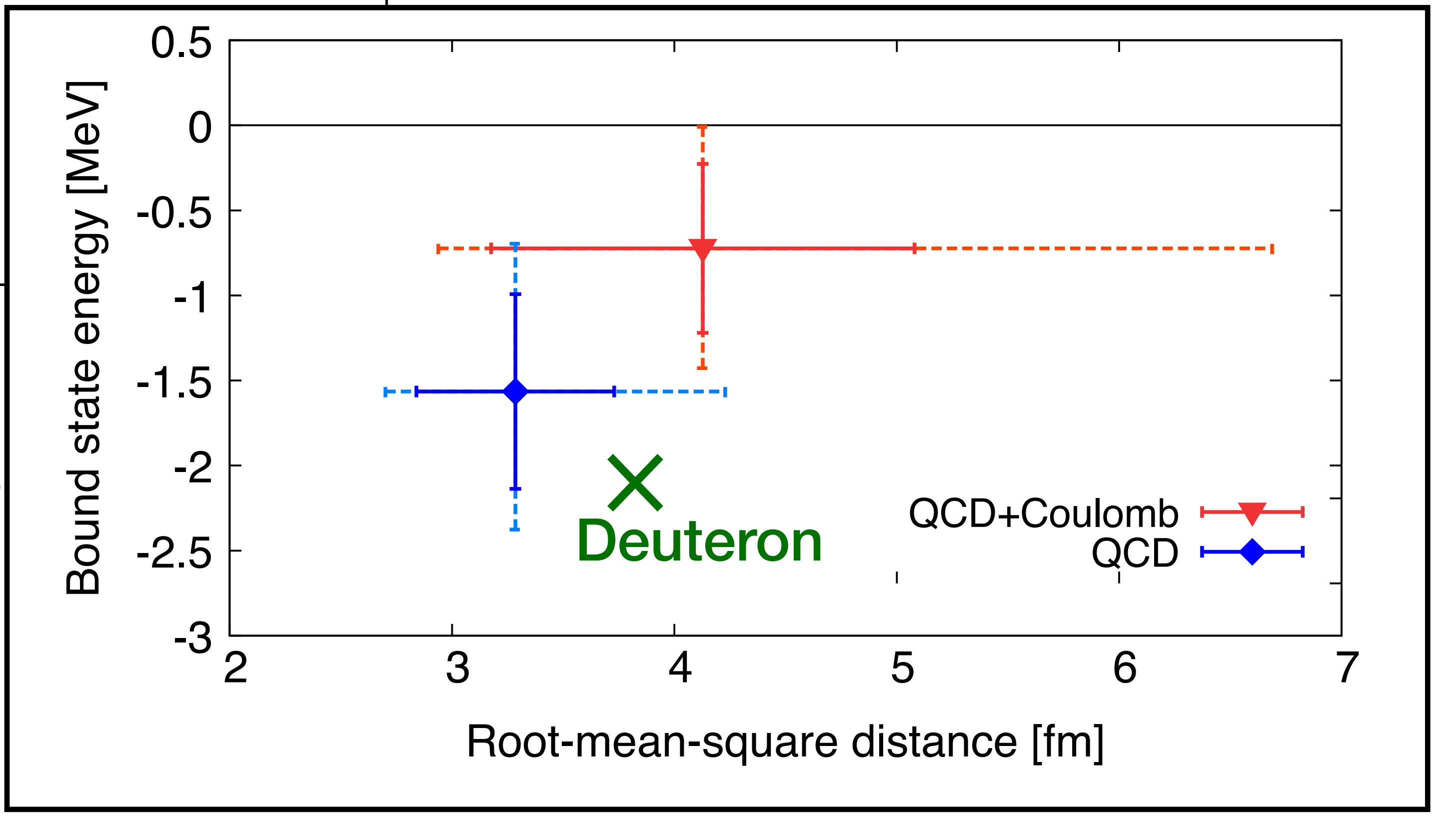
attractive region + repulsive core

→ loosely bound state

(~unitary limit)

di-Omega: $\Omega\Omega(^1S_0)$





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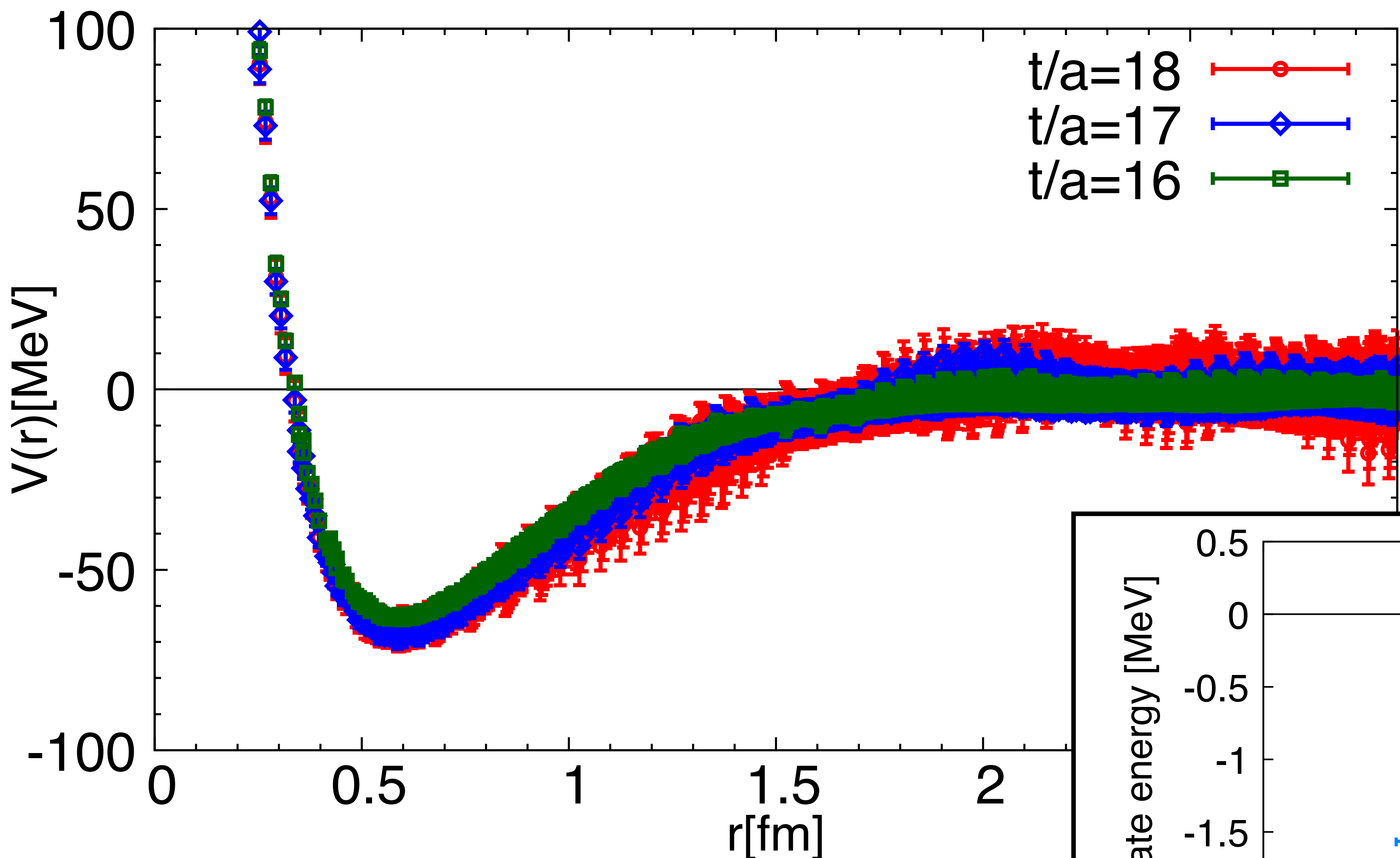
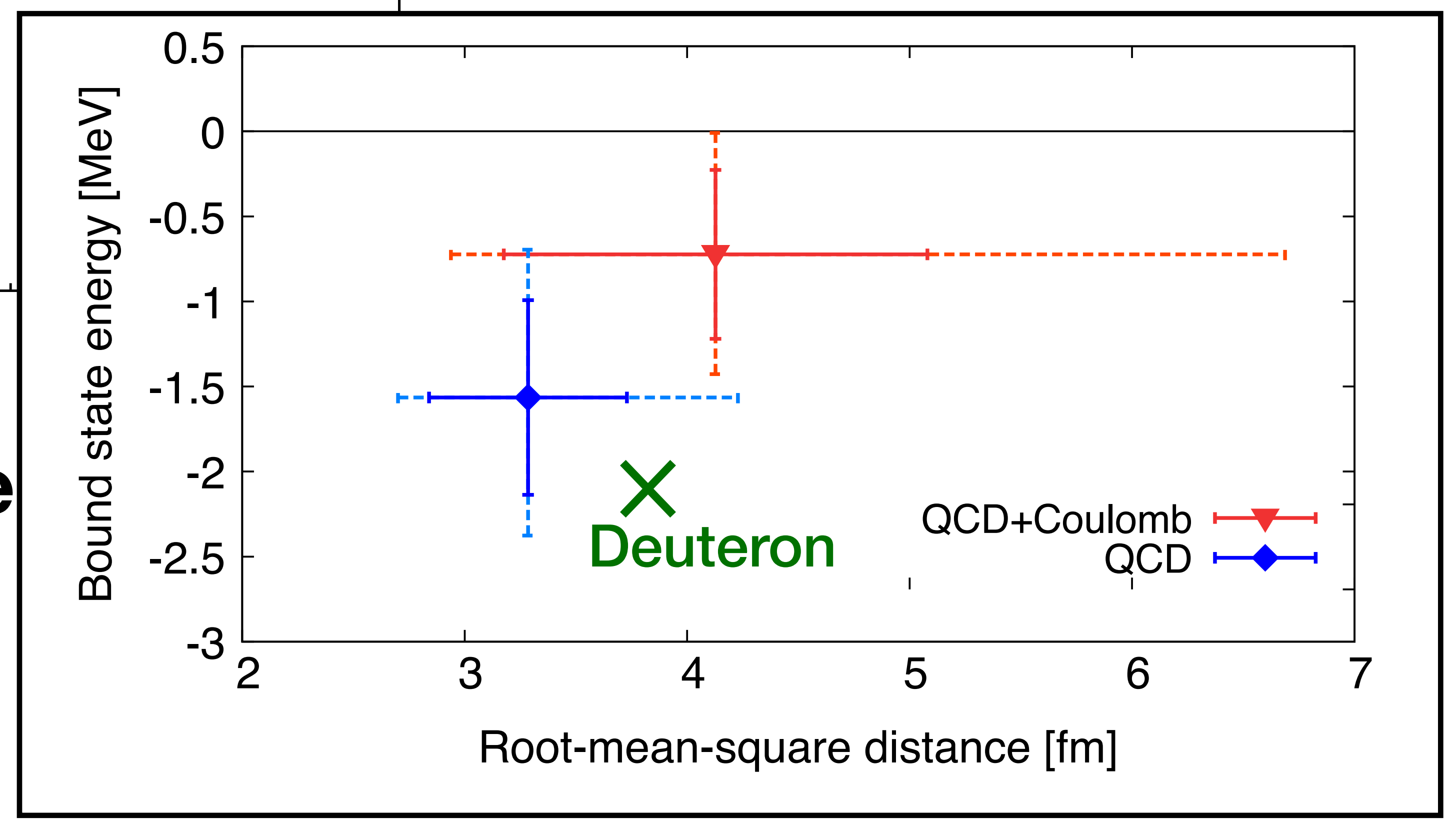
similar to deuteron

di-Omega: $\Omega\Omega(^1S_0)$





attractive region + repulsive core
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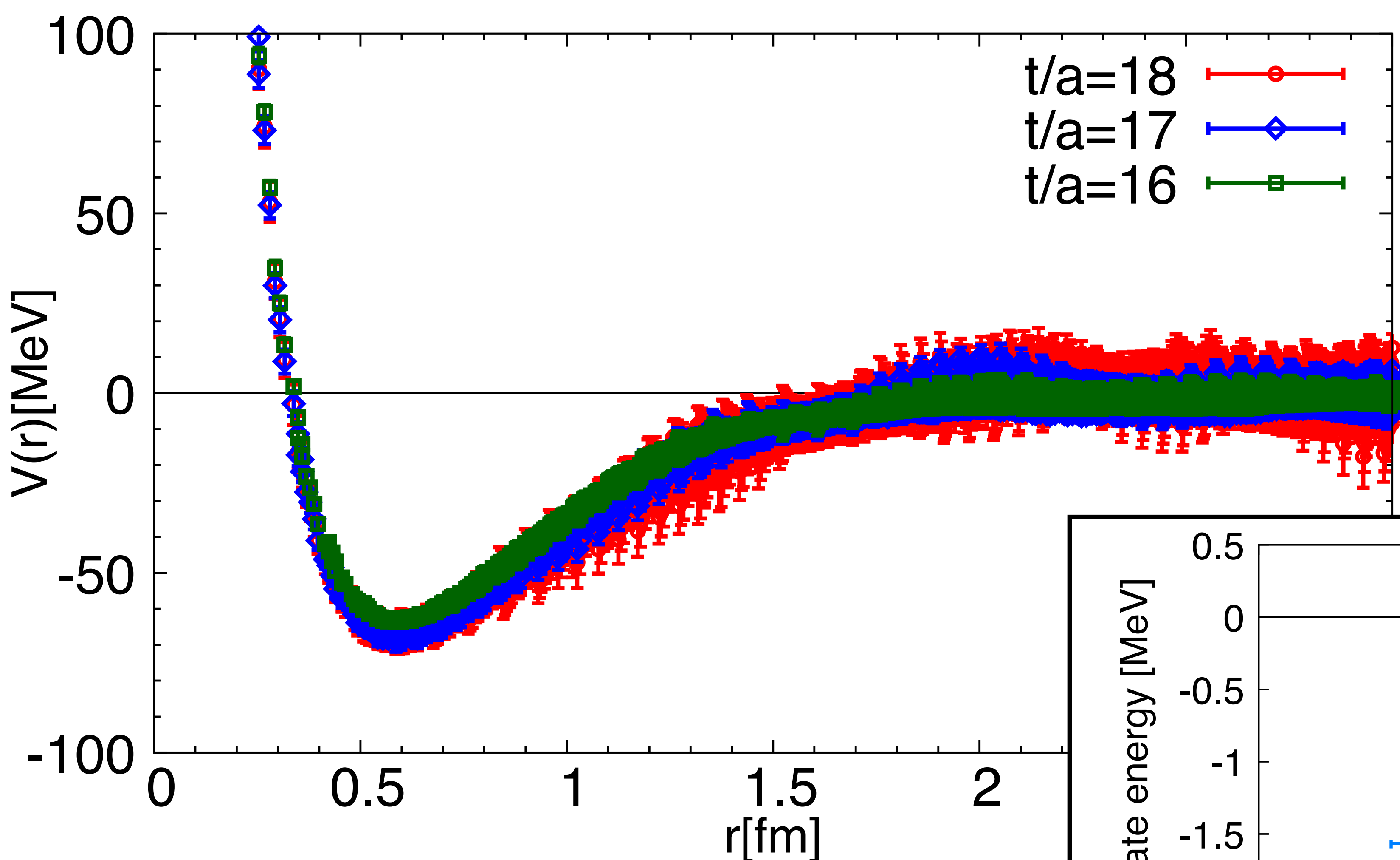
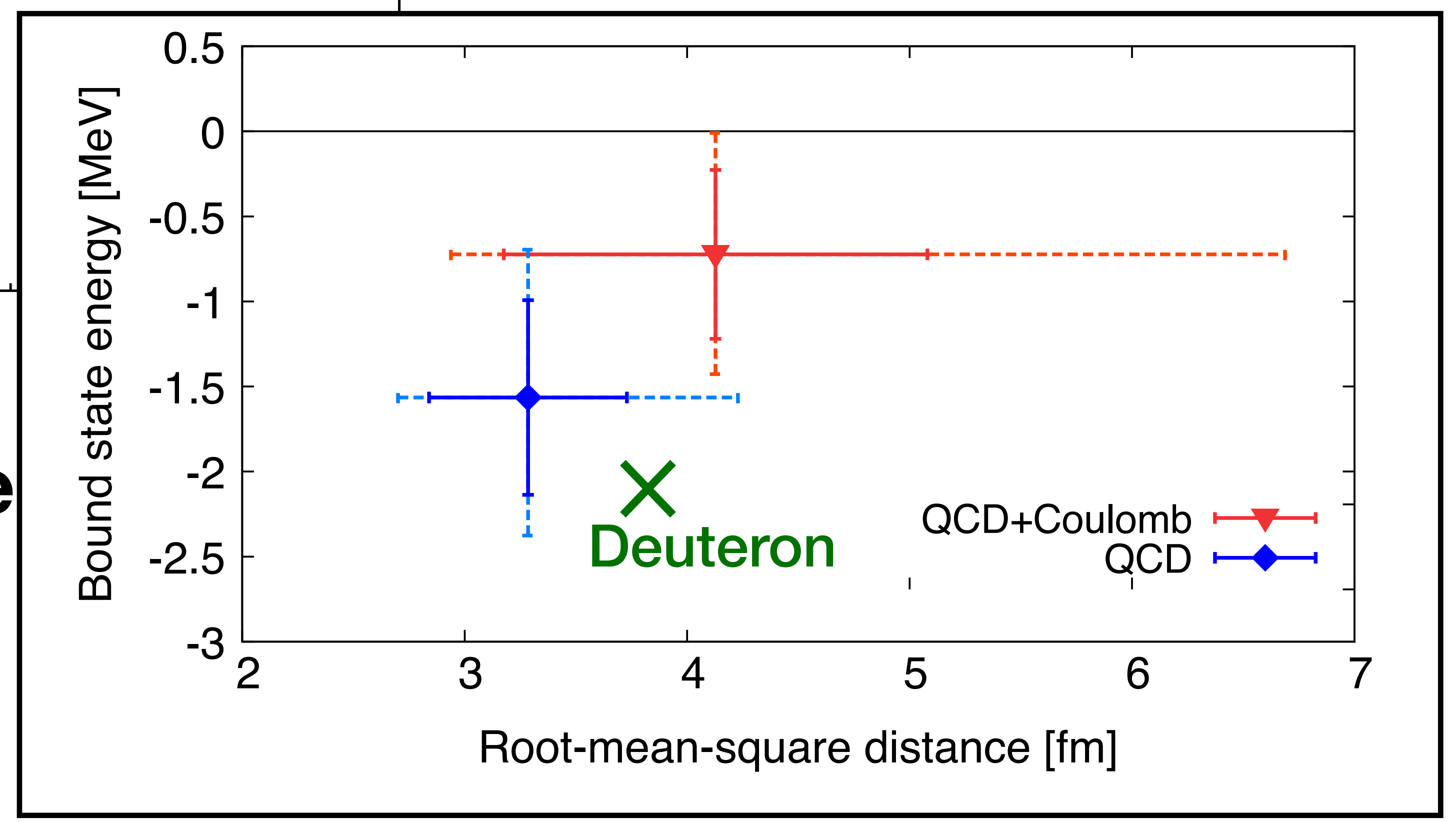
(~unitary limit)
similar to deuteron

- **Allowed by Pauli statistics for quarks**
- **gluon exchange at short distance (Oka &Yazaki (1980))→ repulsive core**

di-Omega: $\Omega\Omega(^1S_0)$





attractive region + repulsive core
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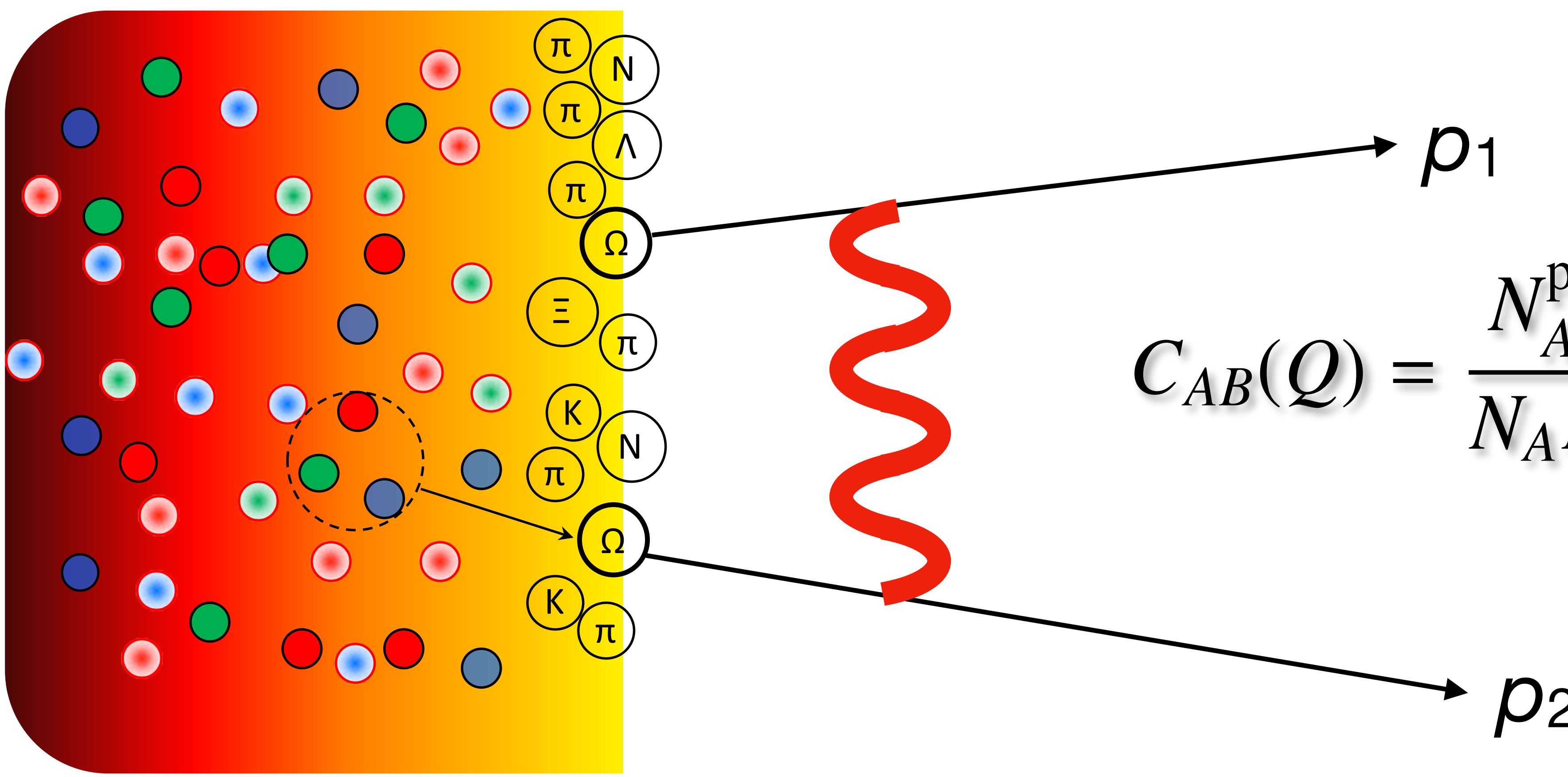
- **Allowed by Pauli statistics for quarks**
- **gluon exchange at short distance (Oka &Yazaki (1980))→ repulsive core**

- **Coulomb effect reduces B.E. by a factor of 2.**

Measurement of two-baryon correlation at RHIC & LHC

STAR Coll., Phys. Lett. B790 (2019) 490 **"NΩ correlation in Au+Au"**
ALICE Coll., arXiv:1905.07209 **"ΛΛ correlation in p+p, p+Pb"**
ALICE Coll., arXiv:1904.12198 **"NXi correlation in p+p, p+Pb"**

two-baryon interaction ⇔ two-baryon correlation



$$C_{AB}(Q) = \frac{N_{AB}^{\text{pair}}(Q)}{N_A N_B(Q)} = \begin{cases} 1 \\ \text{others} \end{cases}$$

$$Q = \sqrt{-\left(\frac{p_1 - p_2}{2} - \frac{(p_1 - p_2)\cdot P}{P^2}P\right)^2}$$

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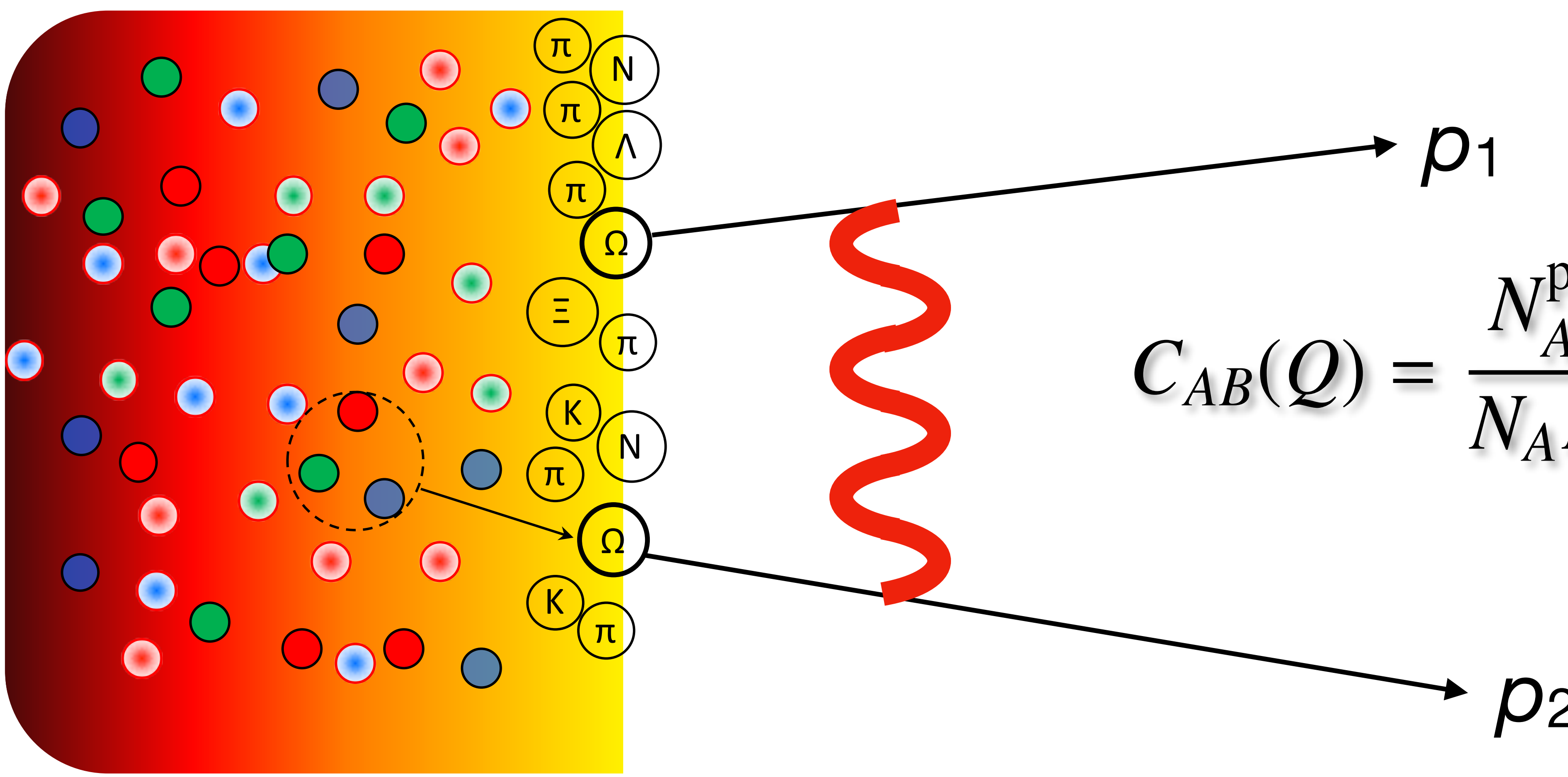

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K. Morita+ , PRC94 (2016) 031901 "NΩ correlation from HAL pot."
K. Morita+ , NPA967 (2017) 856 "NXi correlation from HAL pot."
K. Morita+ in preparation "NΩ & ΩΩ correlations from HAL pot."

Summary

Part I

HAL method

Good convergence of derivative expansion

Lüscher's method

direct method

pseudo plateaux problem

variational method

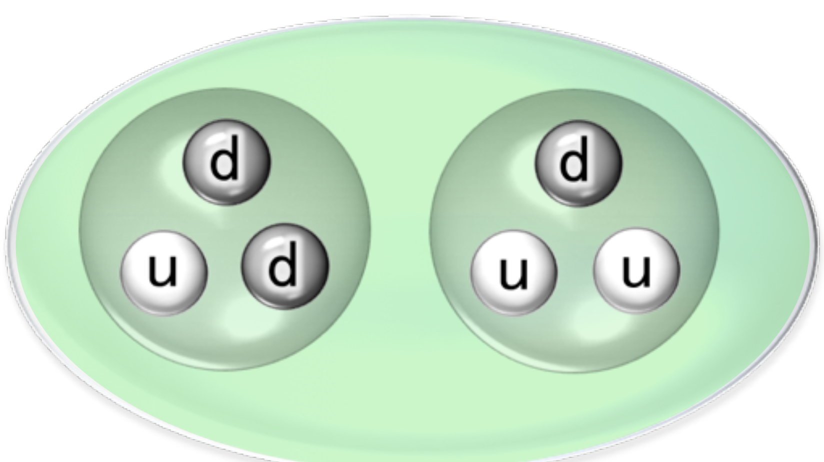
future study

Phase shifts & Binding energy@$L=\infty$

Part II

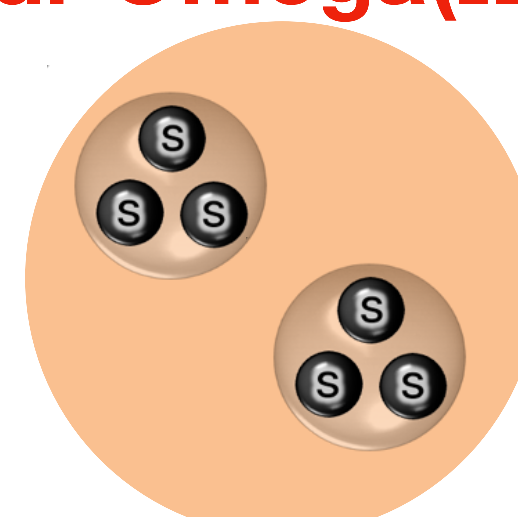
Stable dibaryon (B=2)

Deuteron(pn)



found in 1930s

di-Omega(ΩΩ)

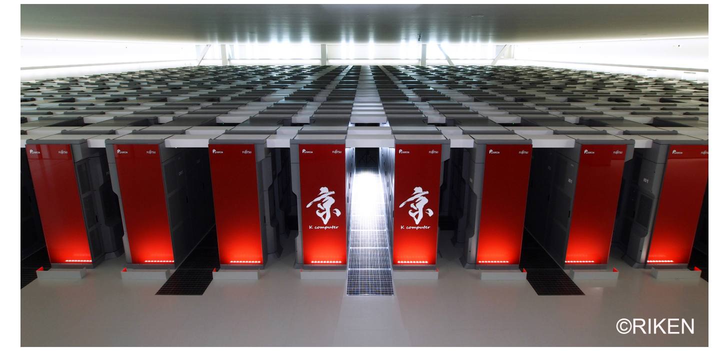


Future Expt.?

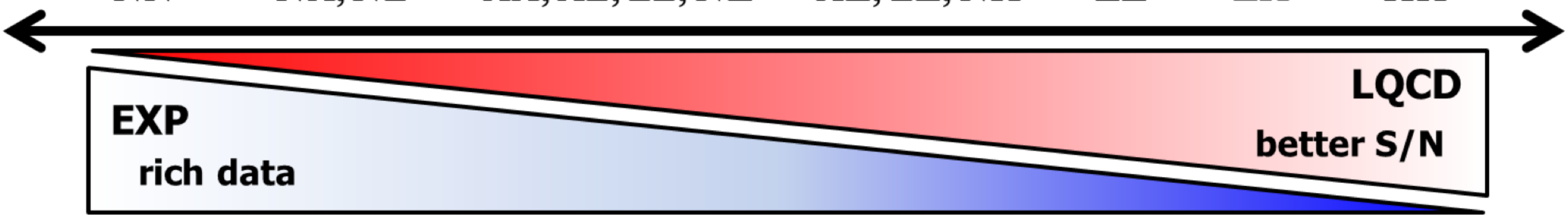
prediction by HAL

Future outlook

2011-2019 K (10PFLOPS)

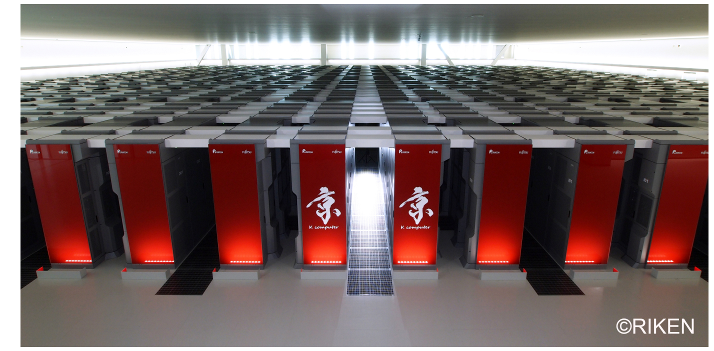


S=0	S=−1	S=−2	S=−3	S=−4	S=−5	S=−6
NN	NΛ, NΣ	ΛΛ, ΛΣ, ΣΣ, NΞ	ΛΞ, ΣΞ, NΩ	ΞΞ	ΞΩ	ΩΩ



Future outlook

2011-2019 K (10PFLOPS)



S=0	S=−1	S=−2	S=−3	S=−4	S=−5	S=−6
NN	$N\Lambda, N\Sigma$	$\Lambda\Lambda, \Lambda\Sigma, \Sigma\Sigma, N\Xi$	$\Lambda\Xi, \Sigma\Xi, N\Omega$	$\Xi\Xi$	$\Xi\Omega$	$\Omega\Omega$

EXP
rich data

LQCD
better S/N

2021- Fugaku (~1000PFLOPS)

S=0	S=−1	S=−2	S=−3	S=−4	S=−5	S=−6
NN	$N\Lambda, N\Sigma$	$\Lambda\Lambda, \Lambda\Sigma, \Sigma\Sigma, N\Xi$	$\Lambda\Xi, \Sigma\Xi, N\Omega$	$\Xi\Xi$	$\Xi\Omega$	$\Omega\Omega$

LQCD

Theoretical equivalence

$$R(\mathbf{r}, t) = \langle 0 \Big| \, B(\mathbf{r}, t) B(\mathbf{0}, t) \mathcal{J}^{\dagger}_{\mathrm{src}}(t=0) \Big| 0 \rangle / \left\{ C_B(t) \right\}^2$$

$$= \sum_n A_n \psi_n(\mathbf{r}) e^{-\Delta E_n t}$$

$$R(t) = \sum_{\mathbf{r}} R(\mathbf{r}, t)$$

HAL QCD method ⟷ **Lüscher's method**

$$R(\mathbf{r}, t) \qquad\qquad R(t)$$

Both relies on asymptotic behavior of NBS w.f.

$$\psi_n(r) \simeq A \frac{\sin\left(kr - l\pi/2 + \boldsymbol{\delta(k)}\right)}{kr}$$

<u>Both methods are equivalent</u>

Development of HAL QCD method

$$R(\vec{r},t) \equiv C_{BB}(\vec{r},t)/\{C_B(t)\}^2 \qquad C_{BB}(\vec{r},t) \equiv \langle 0 \big| B_1(\vec{r},t)B_2(\vec{0},t)\mathcal{J}^\dagger(t=0)\big| 0\rangle$$

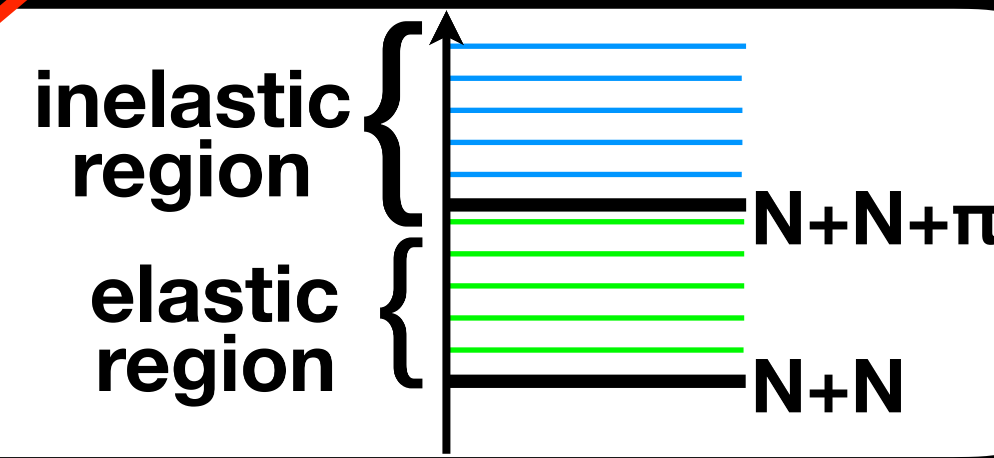
$$= \sum_n A_n \psi_n(\vec{r}) e^{-\Delta E_n t}$$

Independent of energy

$$\left[E_0 - H_0\right]\psi_0(\vec{r}) = \int d\vec{r}\,' U(\vec{r},\vec{r}\,')\psi_0(\vec{r}\,')$$

$$\left[E_1 - H_0\right]\psi_1(\vec{r}) = \int d\vec{r}\,' U(\vec{r},\vec{r}\,')\psi_1(\vec{r}\,')$$

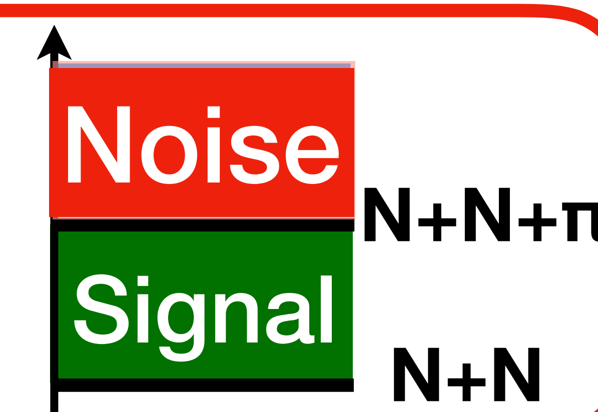
...



Elastic region

t-dependent HAL method

$$\left[-\frac{\partial}{\partial t} + \frac{1}{4m_B}\frac{\partial^2}{\partial t^2} - H_0\right] R(\vec{r},t) = \int d\vec{r}\,' U(\vec{r},\vec{r}\,') R(\vec{r}\,',t)$$



	original HAL QCD method	t-dependent HAL QCD
Inelastic states	noise	noise
elastic excited states	noise	signal
necessary t (> t*)	t*~10fm	t*~1fm

Truncation of derivative expansion

$$\left[-\partial_t + \frac{1}{4m_B}\partial_t^2 - H_0\right] R(\mathbf{r}, t) = \int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') R(\mathbf{r}', t)$$

$$U(\mathbf{r}, \mathbf{r}') = V_0(r)\delta(\mathbf{r} - \mathbf{r}') + V_2(r)\nabla^2\delta(\mathbf{r} - \mathbf{r}') + \sum_{n=2} V_{2n}(r)\nabla^{2n}\delta(\mathbf{r} - \mathbf{r}')$$

Non-local pot. & all terms does not depend on src

Truncation of derivative expansion

$$\left[-\partial_t + \frac{1}{4m_B}\partial_t^2 - H_0\right] R(\mathbf{r},t) = \int d\mathbf{r}' U(\mathbf{r},\mathbf{r}') R(\mathbf{r}',t)$$

$$U(\mathbf{r},\mathbf{r}') = V_0(r)\delta(\mathbf{r}-\mathbf{r}') + \cancel{V_2(r)\nabla^2\delta(\mathbf{r}-\mathbf{r}') + \sum_{n=2} V_{2n}(r)\nabla^{2n}\delta(\mathbf{r}-\mathbf{r}')}$$

Non-local pot. & all terms does not depend on src

LO truncation: LO pot from one src function

$$V^{\mathrm{LO}} = R^{-1}(\mathbf{r},t)\left[-\partial_t + \frac{1}{4m_B}\partial_t^2 - H_0\right] R(\mathbf{r},t)$$

$$= V_0(r) \boxed{+ R^{-1}(\mathbf{r},t)\sum_{n=1} V_{2n}\nabla^{2n} R(\mathbf{r},t)}$$

- **Systematics in V^{LO} depends on src**

Determination of N²LO potentials

$$\left[-\partial_t + \frac{1}{4m_B}\partial_t^2 - H_0\right] R(\mathbf{r}, t) = \int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') R(\mathbf{r}', t)$$

$$U(\mathbf{r}, \mathbf{r}') = V_0(r)\delta(\mathbf{r} - \mathbf{r}') + V_2(r)\nabla^2\delta(\mathbf{r} - \mathbf{r}') + \cancel{\sum_{n=2} V_{2n}(r)\nabla^{2n}\delta(\mathbf{r} - \mathbf{r}')}$$

N²LO truncation: N²LO pots from two src function R^A, R^B

$$\begin{cases} V_0^{\mathrm{N^2LO}} + V_2^{\mathrm{N^2LO}} \left(R^A\right)^{-1} \nabla^2 R^A = \left(R^A\right)^{-1} \left[-\partial_t + \frac{1}{4m_B}\partial_t^2 - H_0\right] R^A \\ V_0^{\mathrm{N^2LO}} + V_2^{\mathrm{N^2LO}} \left(R^B\right)^{-1} \nabla^2 R^B = \left(R^B\right)^{-1} \left[-\partial_t + \frac{1}{4m_B}\partial_t^2 - H_0\right] R^B \end{cases}$$

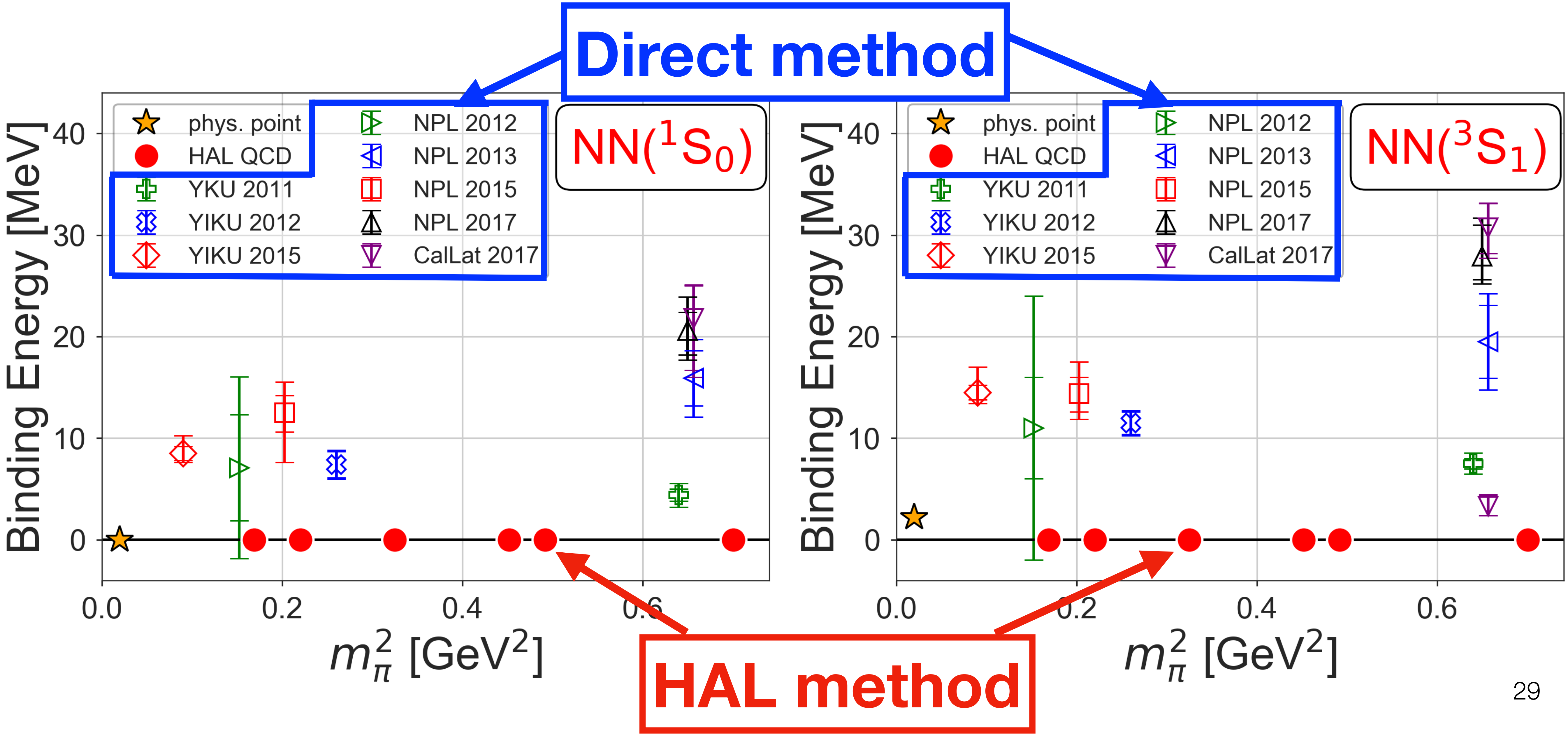
$$\Rightarrow \quad V_0^{\mathrm{N^2LO}}(r),\ V_2^{\mathrm{N^2LO}}(r)$$

Direct method vs HAL method

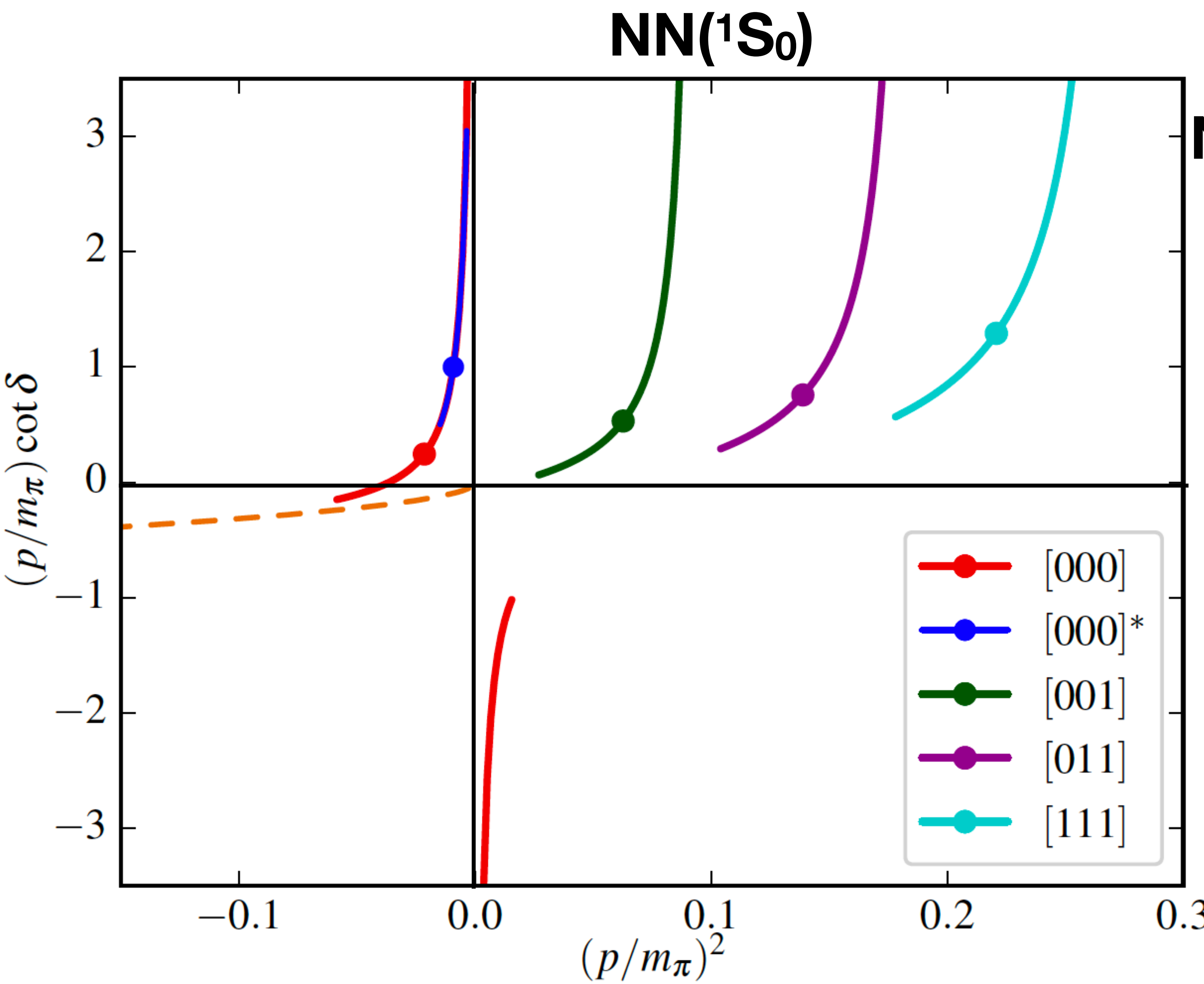
NN@ heavy quark masses

Direct method @PACS-CS/NPL/CalLat: bound

(t-dep.) HAL method: unbound

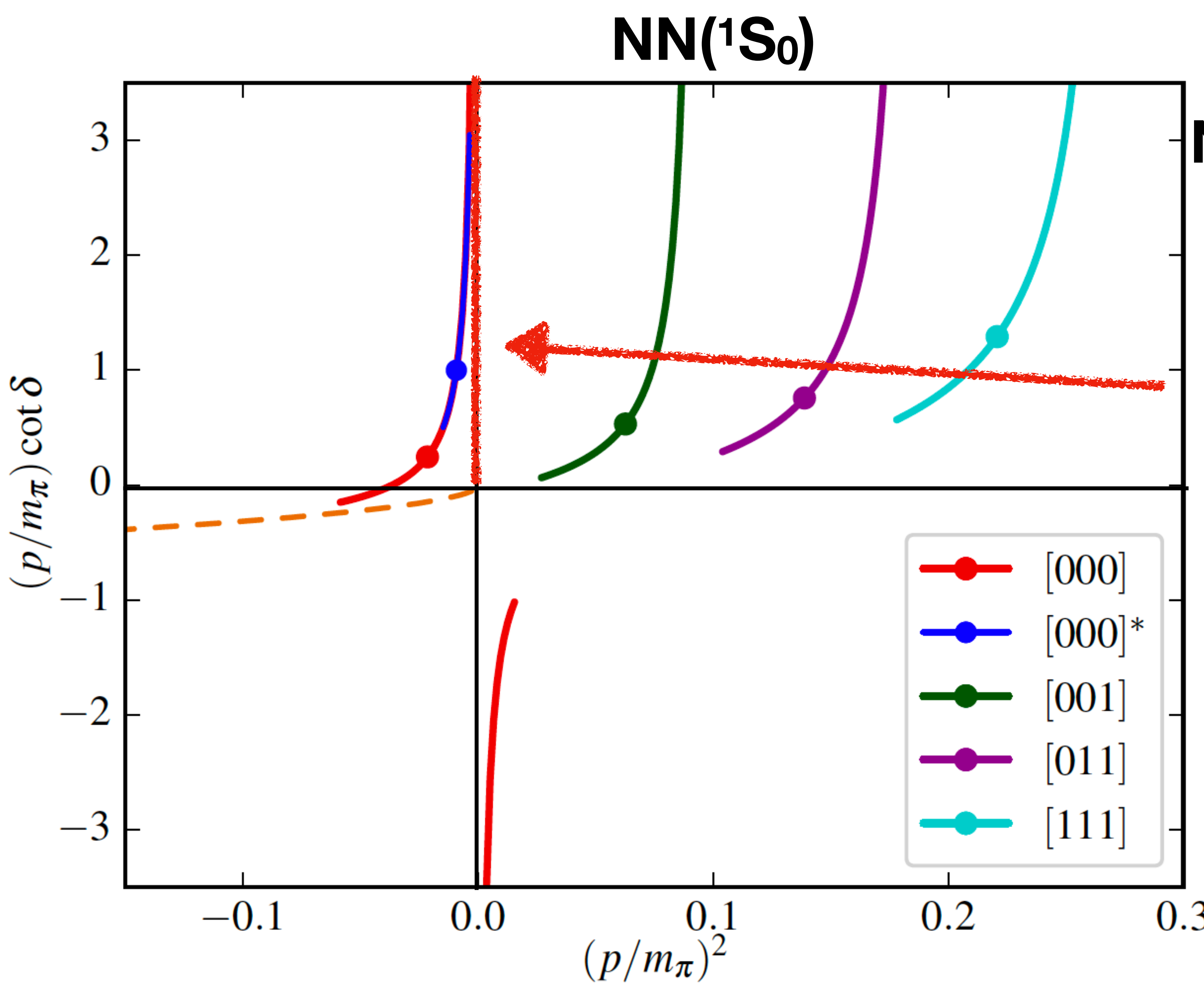


Mainz group, PRD99 (2019) 074505



No fail of normality condition (still large error)

Mainz group, PRD99 (2019) 074505



No fail of normality condition
(still large error)

ERE crosses positive at p=0

→ (likely) Unbound

Same result as HAL method

Normality check for results from direct method

T. Iritani et al. (HAL Coll.) PRD96(2017)034521

Effective Range Expansion (ERE)

$$k\cot\delta(k) = \frac{1}{a_s} + \frac{r_e}{2}k^2 + \cdots$$

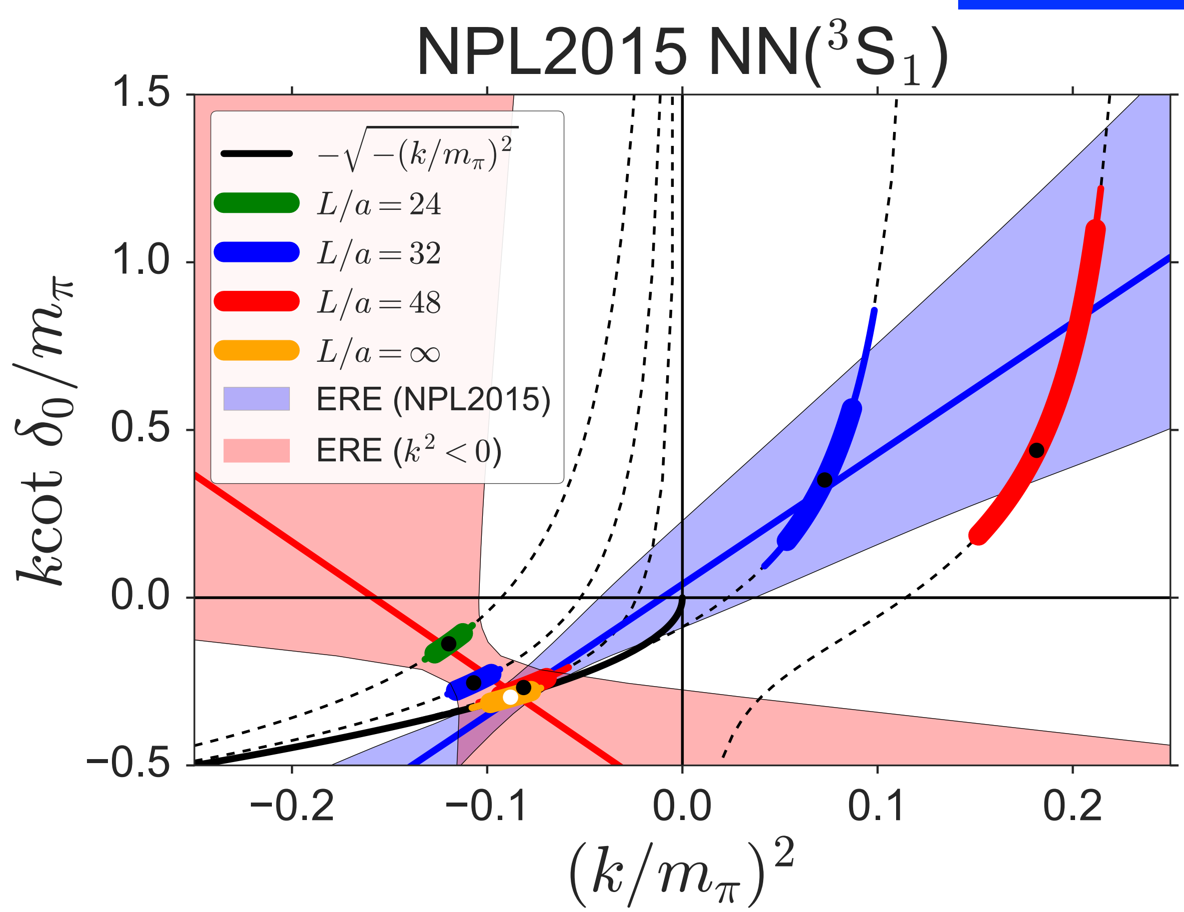
$\boxed{k\cot\delta(k)\ \text{vs}\ k^2}$

Necessary condition

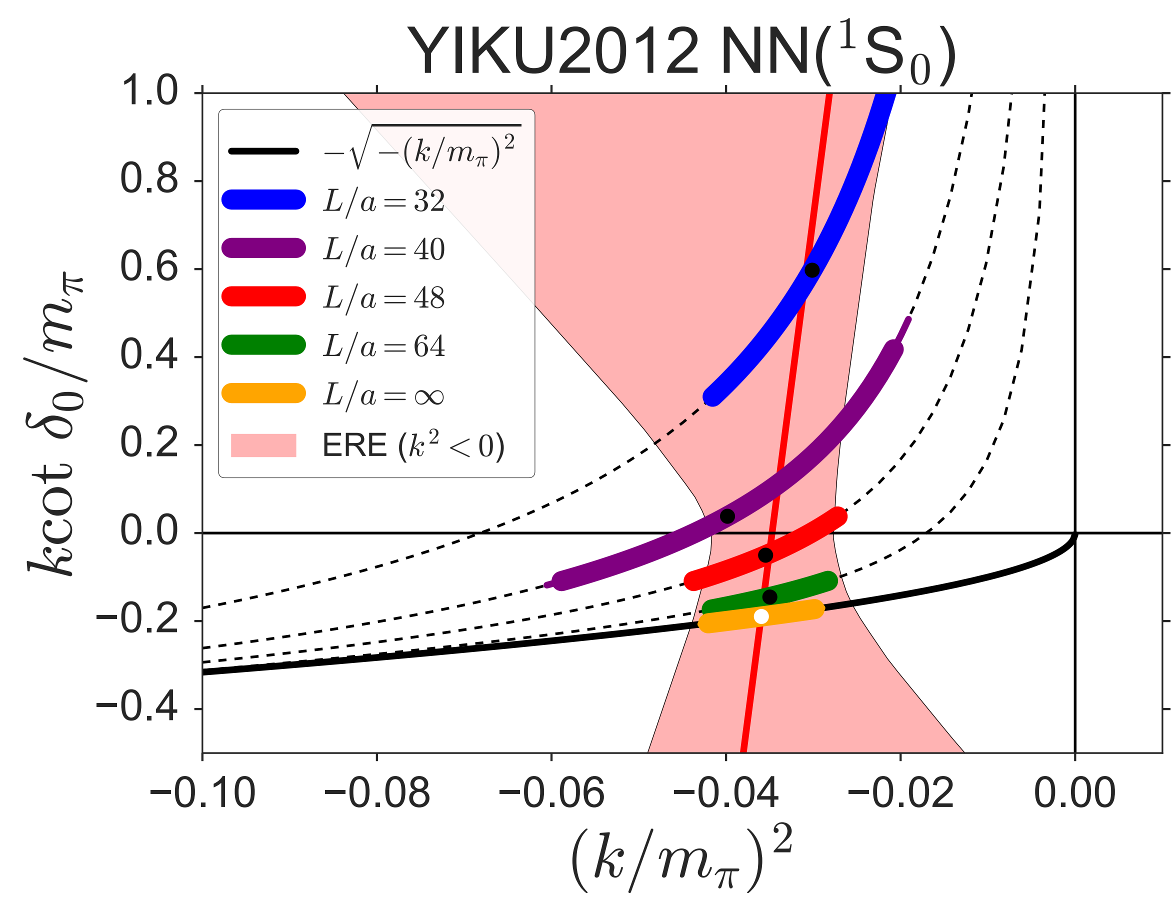
(i) consistent ERE line for $k^2<0$ and for $k^2>0$

(ii) reasonable parameterizations a_s, r_e

(iii) physical residue at pole position



(i) Inconsistent ERE line
(iii) Unphysical pole residue

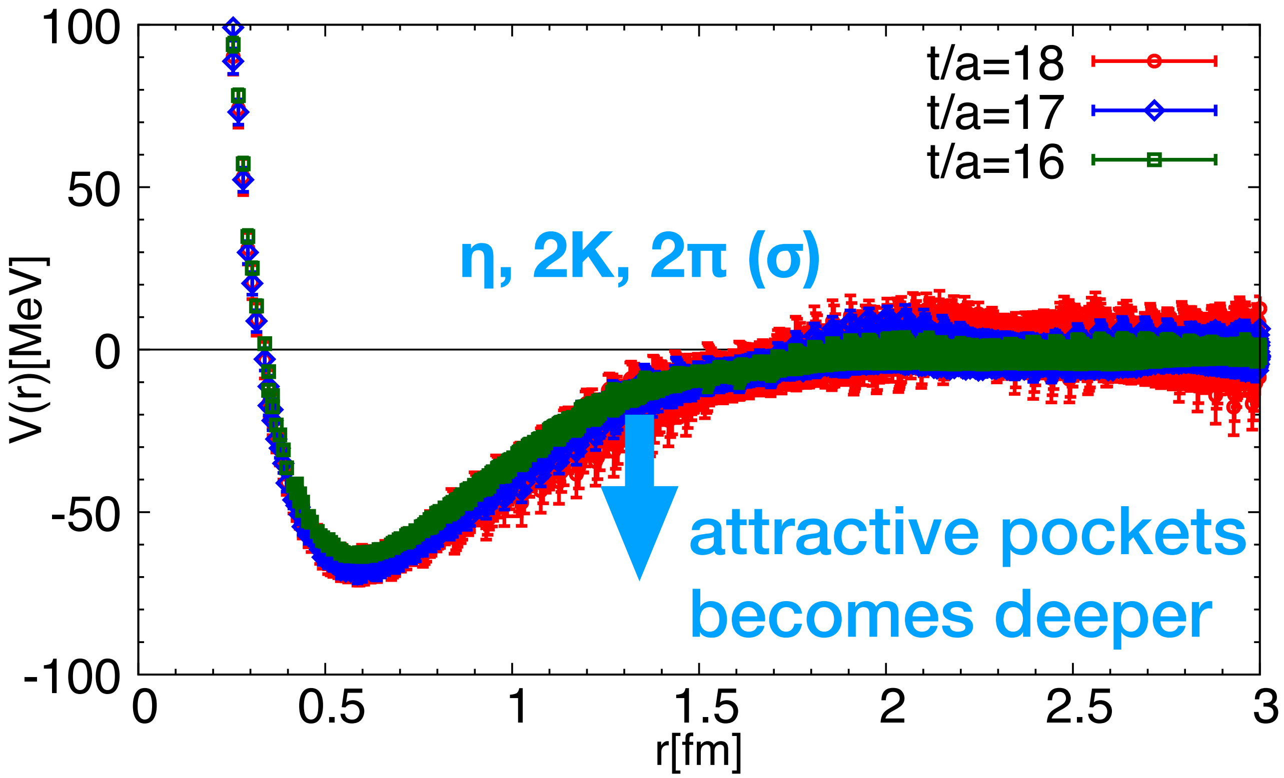


(ii) $r_e \simeq \infty$

Conservative estimate at exact phys. pt.

m_{π}=146 MeV -> 135 MeV, m_{Ω}= 1712MeV -> 1672 MeV

B.E. ↑



V.S.

B.E. ↓

$$\mathcal{H} = \boxed{-\frac{\nabla^2}{m_\Omega}} + V_{\Omega\Omega}^{\mathrm{LQCD}}(r)$$

kinetic energy is increasing
-> B.E. is reduced

conservative estimate:

only change the mass of kinetic term

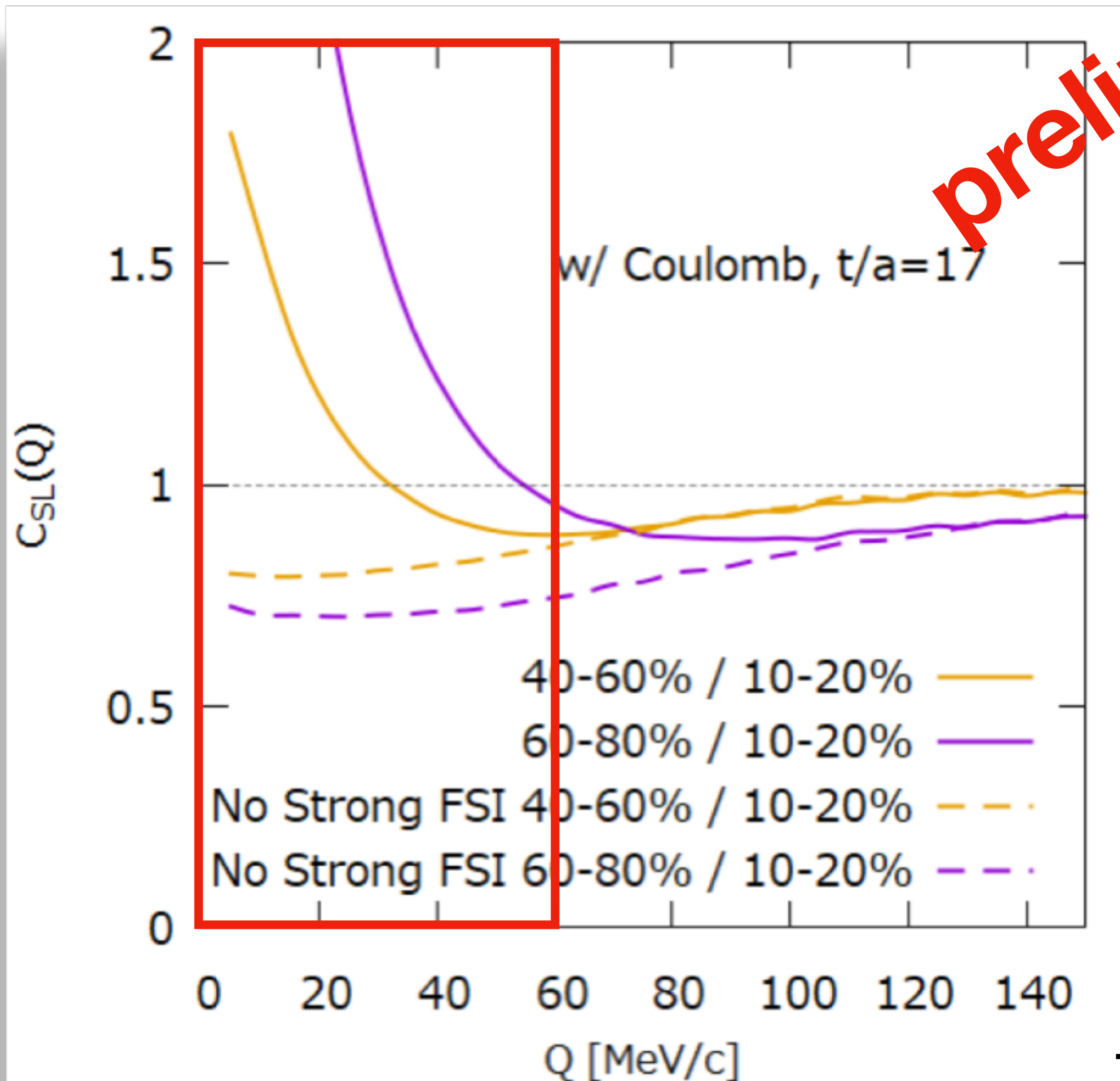
$$(B_{\Omega\Omega}^{(\mathrm{QCD})}, B_{\Omega\Omega}^{(\mathrm{QCD+Coulomb})}) = (1.6(6)\mathrm{MeV}, 0.7(5)\mathrm{MeV})$$
$$\rightarrow (1.3(5)\mathrm{MeV}, 0.5(5)\mathrm{MeV})$$

These changes are within errors

Kenji Morita (QST/Wroclaw/RIKEN)

ΩΩ Correlation@LHC

The Small-Large Ratio $C_{SL}(Q)$



Response to system size change

$$C_{SL}(Q) = C_R(Q)/C_{R'}(Q)$$

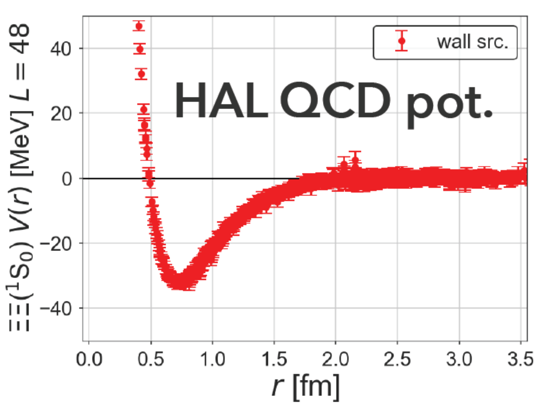
QS (HBT) Correlation suppresses the ratio

Nevertheless FSI dominates at low Q

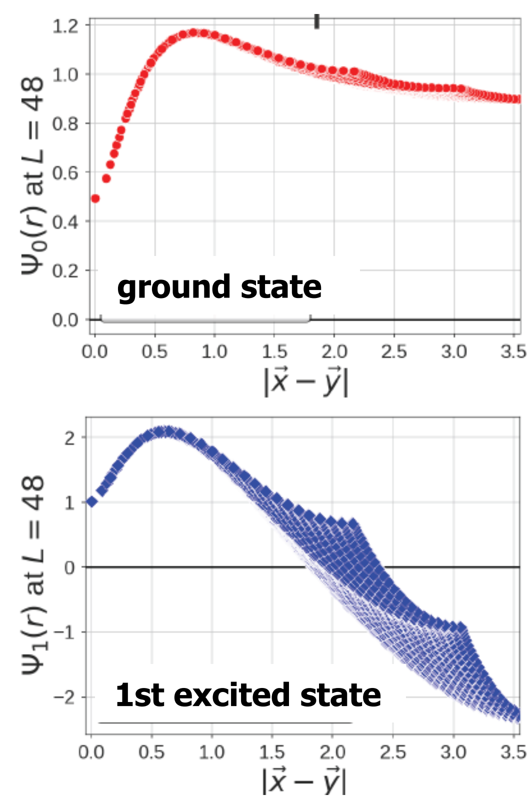
found in future HIC experiment?

Operator optimized for 2-body system by HAL

- HAL method ➔ HAL pot ➔ 2-body wave func. @ finite V
- 2-body wave func. ➔ optimized operator
  - Applicable for sink and/or src op : Here we apply for sink op
- While utilizing info by HAL, formulation is Luscher's method

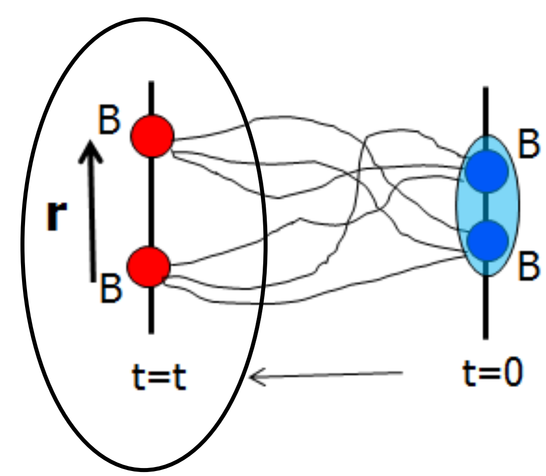


wave func. ψ(r)



HAL-optimized sink op

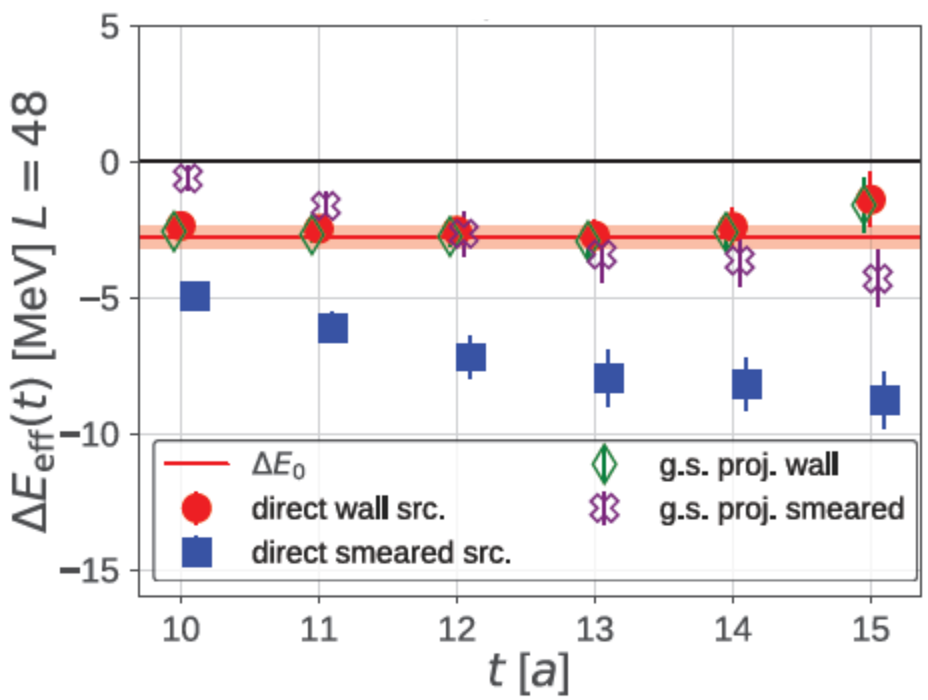
$$\mathcal{J}^{2B}_{\rm sink} = \sum_{\vec{r}} \psi^\dagger(r) \sum_{\vec{x}} B(\vec{r}+\vec{x})B(\vec{x})$$



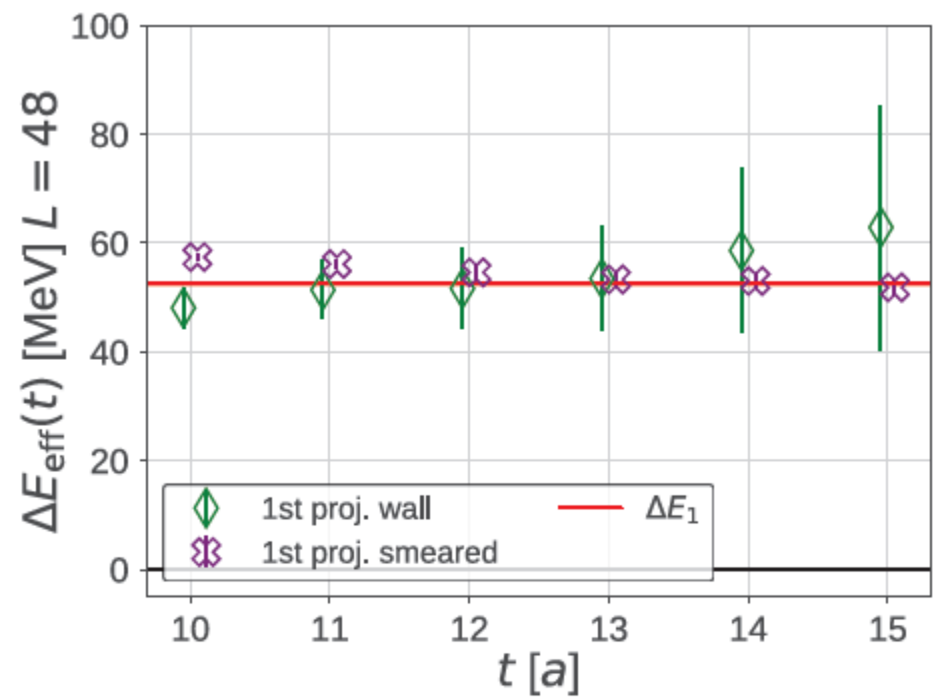
Effective energy shift ΔE from “HAL-optimized op”

HAL-optimized sink op ➔ projected to each state ➔ “True” plateaux

Ground State



1st excited state



HAL QCD pot = Lushcer’s method w/ proper projection

≠ Direct method w/ naïve plateau fitting