Bethe-Salpeter Wave Functions of Hybrid Charmonia

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Implication of Hybrid Charmonia

“Excited and exotic charmonium spectroscopy from lattice QCD”, Liuming Liu et al. [arXiv:1204.5425v2[hep-ph]]

— Four lightest hybrids as a supermultiplet in channel $1^{-+}, (0,1,2)^{-+}$.

“Exotic vector charmonium and its leptonic decay width”, Ying Chen et al. [arXiv:1604.03401v1[hep-lat]]

— Bethe–Salpeter Amplitude as a tool to recognize hybrid state out.
Basic Point of View

• A hybrid is a $\bar{c}c$ with a **gluonic** component
• Spin of $\bar{c}c$ in these four could be **spin singlet and triplet**, respectively
• Their masses are in near degenerate
• Hybrids are well-defined in quenched approximation
• Quenched gauge configurations used

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\xi$</th>
<th>$a_s$</th>
<th>$La_s(fm)$</th>
<th>$L^3 \times T$</th>
<th>$N_{conf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>5</td>
<td>0.222(2)(11)</td>
<td>3.55</td>
<td>$16^3 \times 160$</td>
<td>500</td>
</tr>
<tr>
<td>2.8</td>
<td>5</td>
<td>0.138(1)(7)</td>
<td>3.31</td>
<td>$24^3 \times 192$</td>
<td>200</td>
</tr>
</tbody>
</table>

• The configurations have been fixed into **Coulomb gauge**
Hybrid-like Operators & Bethe-Salpeter (BS) Wave Functions

- We construct a group of hybrid-like interpolating operators as

\[
\begin{align*}
O^{(H1)}_i(x, t; r) &= \bar{c}^a(x, t) \gamma_5 B^{ab}_i(x + r, t) c^b(x, t), & 1^- \\
O^{(H2)}_i(x, t; r) &= \bar{c}^a(x, t) \gamma_i B^{ab}_i(x + r, t) c^b(x, t), & 0^- \\
O^{(H3)}_i(x, t; r) &= \bar{c}^a(x, t) \varepsilon_{ijk} \gamma_j B^{ab}_i(x + r, t) c^b(x, t), & 1^- \\
O^{(H4)}_i(x, t; r) &= \bar{c}^a(x, t) \varepsilon_{ijk} \gamma_j B^{ab}_i(x + r, t) c^b(x, t). & 2^- \\
\end{align*}
\]

\( r \) is the displacement between \( \bar{c}c \) and gluon (represented by chromomagnetic field \( B^{ab}_i \))

- The Bethe-Salpeter amplitude is defined as \( \langle 0 | O^H(r) | H \rangle \)
Data Analysis Strategy

- The real two-point correlation functions we calculated

\[ C(r, t) = \langle 0 | O^H(r, t) O^W(\tau) | 0 \rangle \]

- Simultaneous fitting with many \( C(r, t) \) under multi-exponential model. \( r \) corresponds to different separation displacement

\[ C(r, t) = \sum_i \Phi_i(r) \exp\{-m_i t\} \]

\( \Phi_i(r) \) is proportional to BS wave functions.

- Fit window is

\[ t \in [t_{\text{min}}, t_{\text{max}}] \]

Shift \( t_{\text{min}} \) with \( t_{\text{max}} \) fixed to find a stable region.

Exam. \( 1^{-+}, \beta = 2.4 \)
Exotic Channel $1^{--}$

$\beta = 2.4$

$\beta = 2.8$
Compare to Shroedinger Functions of A Harmonic Oscillator

$\beta = 2.4$
$2^{-+}$

$\beta = 2.4$

$\beta = 2.8$
Why no conventional $2^{-+}$ states found?

According to

"Lattice study on $\eta_{c2}$ and $X(3872)$" (Y.B.Yang et al. PhysRevD.87.014501 [arXiv:1206.2086 [hep-lat]]),

a $q\bar{q}B$ type (F-type, red dots in figure) operator hardly couples to conventional states like $\eta_{c2}(2^{-+})$. Instead, it couples to a state around 4.43 GeV mostly (which we treat as a ground state of hybrids here).

\[
\begin{align*}
D\bar{D}-\text{type} & : |\epsilon_{ijk}| \bar{q}_5 B_i D_j q \\
F\text{-type} & : |\epsilon_{ijk}| \bar{q}^a \gamma_i q^b B^{ab}
\end{align*}
\]
\[ \beta = 2.4 \]

\[ \beta = 2.8 \]

The graphs show the function \( \phi(r)/\phi(0) \) as a function of \( r/\text{fm} \) for different energy levels at \( \beta = 2.4 \) and \( \beta = 2.8 \). The energy levels are indicated as follows:

- \( 2.961(7) \text{GeV} \)
- \( 3.51 \text{GeV} \)
- \( 4.44(6) \text{GeV} \)
- \( 5.14(4) \text{GeV} \)
- \( 8.5(2) \text{GeV} \)

For \( \beta = 2.4 \):

- \( 2.97(1) \text{GeV} \)
- \( 3.4(1) \text{GeV} \)
- \( 4.39(8) \text{GeV} \)
- \( 5.66(5) \text{GeV} \)
- \( 8.9(2) \text{GeV} \)
\[ \beta = 2.4 \]

\[ \beta = 2.8 \]
Masses From Fitting (Unit: GeV)

<table>
<thead>
<tr>
<th></th>
<th>1−</th>
<th>0+</th>
<th>1+</th>
<th>2+</th>
<th>node</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta = 2.4)</td>
<td>3.07(1)</td>
<td>3.045(7)</td>
<td>2.961(7)</td>
<td>2.97(1)</td>
<td>0</td>
</tr>
<tr>
<td>(\beta = 2.8)</td>
<td>3.5(1)</td>
<td>3.7(1)</td>
<td>3.5(1)</td>
<td>3.4(1)</td>
<td>0</td>
</tr>
<tr>
<td>(\beta = 2.4)</td>
<td>4.39(7)</td>
<td>4.3(1)</td>
<td>4.44(6)</td>
<td>4.39(8)</td>
<td>0</td>
</tr>
<tr>
<td>(\beta = 2.8)</td>
<td>5.36(4)</td>
<td>5.90(5)</td>
<td>5.14(4)</td>
<td>5.66(5)</td>
<td>1</td>
</tr>
<tr>
<td>(\beta = 2.4)</td>
<td>8.4(2)</td>
<td>8.9(2)</td>
<td>8.5(2)</td>
<td>8.9(2)</td>
<td>2</td>
</tr>
</tbody>
</table>
Hybrid 1S (states around 4.3GeV)

$\beta = 2.4$

$1^+ 4.265(7) \text{GeV}$
$2^+ 4.360(8) \text{GeV}$
$0^+ 4.44(6) \text{GeV}$
$1^- 4.39(7) \text{GeV}$

$\beta = 2.8$

$1^+ 4.17(1) \text{GeV}$
$2^+ 4.26(1) \text{GeV}$
$0^+ 4.39(8) \text{GeV}$
$1^- 4.3(1) \text{GeV}$
Hybrid 2S (states around 5.5GeV)

$\beta = 2.4$

$1^+\ 5.57(6)\text{GeV}$
$2^+\ 5.60(6)\text{GeV}$
$0^+\ 5.14(4)\text{GeV}$
$1^-\ 5.38(4)\text{GeV}$

$\beta = 2.8$

$1^+\ 5.38(5)\text{GeV}$
$2^+\ 5.59(5)\text{GeV}$
$0^+\ 5.66(5)\text{GeV}$
$1^-\ 5.90(5)\text{GeV}$
Hybrid $3S(?)$ (states above 7GeV)

$\beta = 2.4$

$\beta = 2.8$
Conventional States($J/\psi, \psi', \eta_c, \eta_c'$, maybe)

\[ \beta = 2.4 \]

\[ \beta = 2.8 \]
Conclusion

• Clear nodal behavior of BS functions with respect to the spatial displacement reflects $r$ in the operators is a meaningful dynamical variable.

• It implies that the inner structure of hybrid charmonia is a localized $\bar{c}c$ kernel surrounded by a gluonic component, just like a halo.

• The hybrid states distribute across $1^{--}$ and $(0,1,2)^{--}$ have similar structure, and their masses are almost in degenerate around $4.3\text{GeV}(\text{hybrid 1S})$ and $5.5\text{GeV}(\text{hybrid 2S})$, respectively.
End of Story