

# Bethe-Salpeter Wave Functions of Hybrid Charmonia

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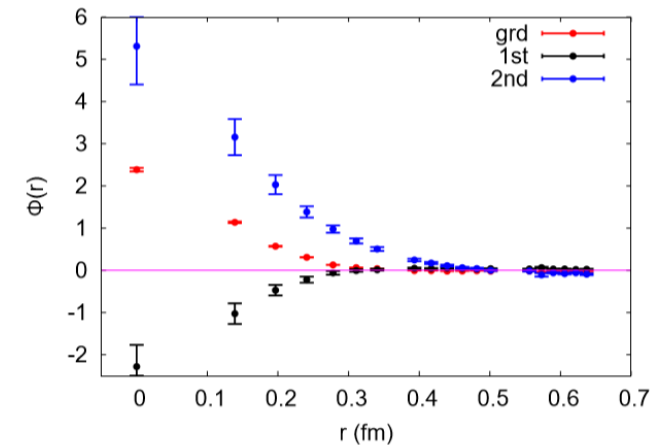
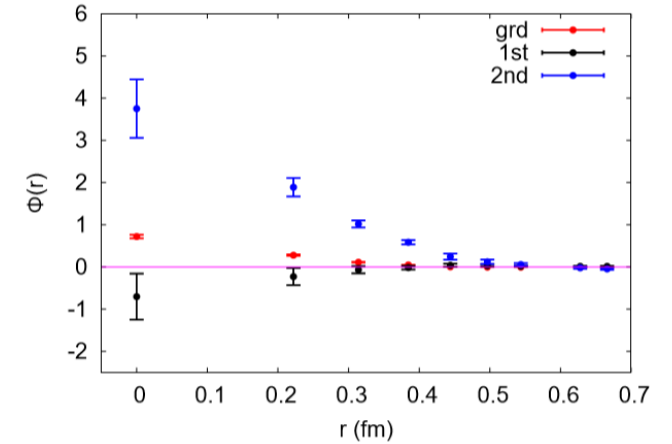
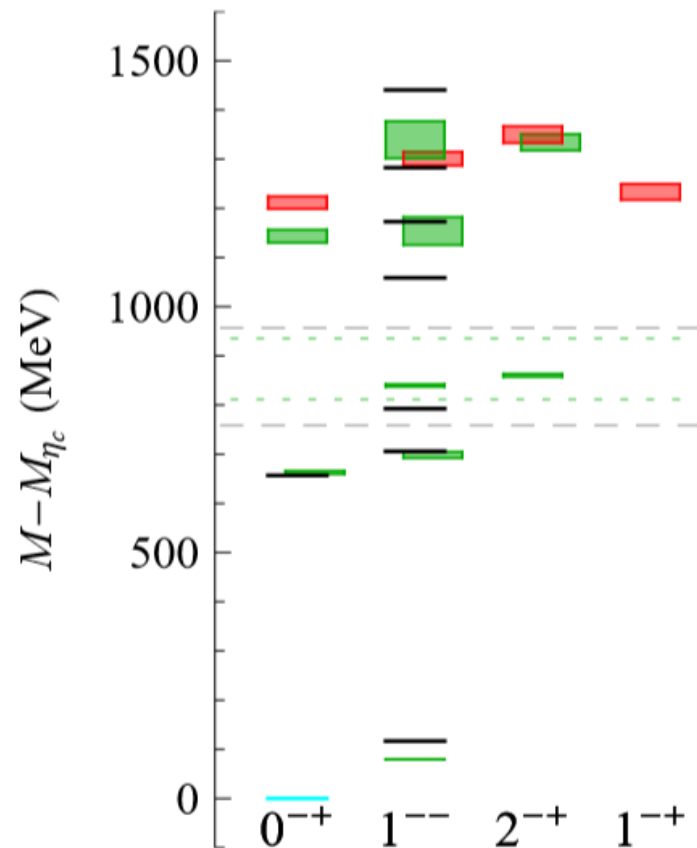
# Implication of Hybrid Charmonia

*“Excited and exotic charmonium spectroscopy from lattice QCD”*, Liuming Liu et al. [arXiv:1204.5425v2[hep-ph]

--Four lightest hybrids as a supermultiplet in channel  $1^{--}, (0,1,2)^{-+}$ .

*“Exotic vector charmonium and its leptonic decay width”*, Ying Chen et al. [arXiv:1604.03401v1[hep-lat]

--Bethe-Salpeter Amplitude as a tool to recognize hybrid state out.



# Basic Point of View

- A hybrid is a  $\bar{c}c$  with a **gluonic** component
- Spin of  $\bar{c}c$  in these four could be **spin singlet and triplet**, respectively
- Their masses are in near degenerate
- Hybrids are well-defined in quenched approximation
- Quenched gauge configurations used

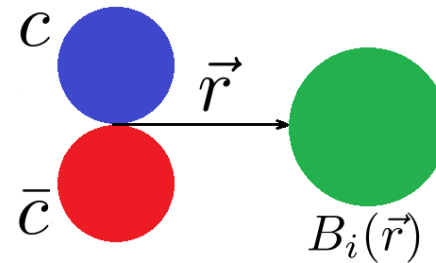
$\beta$	$\xi$	$a_s$	$La_s(fm)$	$L^3 \times T$	$N_{conf}$
2.4	5	0.222(2)(11)	3.55	$16^3 \times 160$	500
2.8	5	0.138(1)(7)	3.31	$24^3 \times 192$	200

- The configurations have been fixed into **Coulomb gauge**

# Hybrid-like Operators & Bethe-Salpeter(BS) Wave Functions

- We construct a group of hybrid-like interpolating operators as

$$\begin{aligned}
 O_i^{(H1)}(\mathbf{x}, t; \mathbf{r}) &= \bar{c}^a(\mathbf{x}, t) \gamma_5 B_i^{ab}(\mathbf{x} + \mathbf{r}, t) c^b(\mathbf{x}, t), & 1^{--} \\
 O_i^{(H2)}(\mathbf{x}, t; \mathbf{r}) &= \bar{c}^a(\mathbf{x}, t) \gamma_i B_i^{ab}(\mathbf{x} + \mathbf{r}, t) c^b(\mathbf{x}, t), & 0^{-+} \\
 O_i^{(H3)}(\mathbf{x}, t; \mathbf{r}) &= \bar{c}^a(\mathbf{x}, t) \varepsilon_{ijk} \gamma_j B_k^{ab}(\mathbf{x} + \mathbf{r}, t) c^b(\mathbf{x}, t), & 1^{-+} \\
 O_i^{(H4)}(\mathbf{x}, t; \mathbf{r}) &= \bar{c}^a(\mathbf{x}, t) |\varepsilon_{ijk}| \gamma_j B_k^{ab}(\mathbf{x} + \mathbf{r}, t) c^b(\mathbf{x}, t). & 2^{-+}
 \end{aligned}$$



$\bar{c}c$  in spin singlet  
 $1^{--}$   
 $\bar{c}c$  in spin triplet  
 $(0,1,2)^{-+}$

$\mathbf{r}$  is the displacement between  $\bar{c}c$  and gluon (represented by chromomagnetic field  $B_i^{ab}$ )

- The Bethe-Salpeter amplitude is defined as  $\langle 0 | O^H(r) | H \rangle$

# Data Analysis Strategy

- The real two-point correlation functions we calculated

$$C(r, t) = \langle 0 | O^H(r, t) O^W(\tau) | 0 \rangle$$

- Simultaneous fitting with many  $C(r, t)$  under multi-exponential model,  $r$  corresponds to different separation displacement

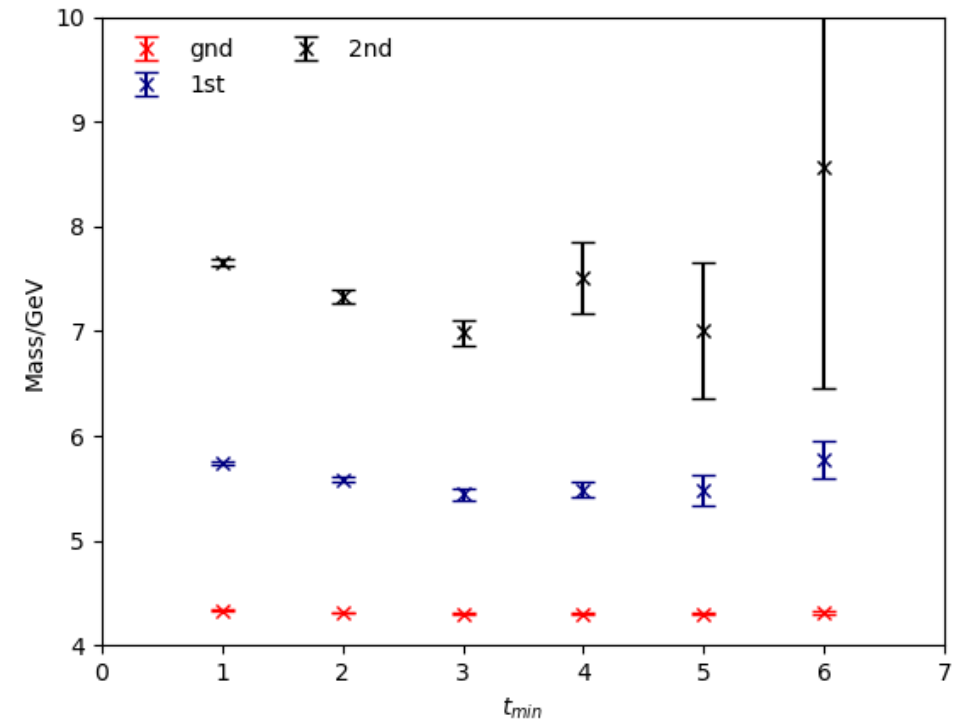
$$C(r, t) = \sum_i \Phi_i(r) \exp\{-m_i t\}$$

$\Phi_i(r)$  is proportional to BS wave functions.

- Fit window is

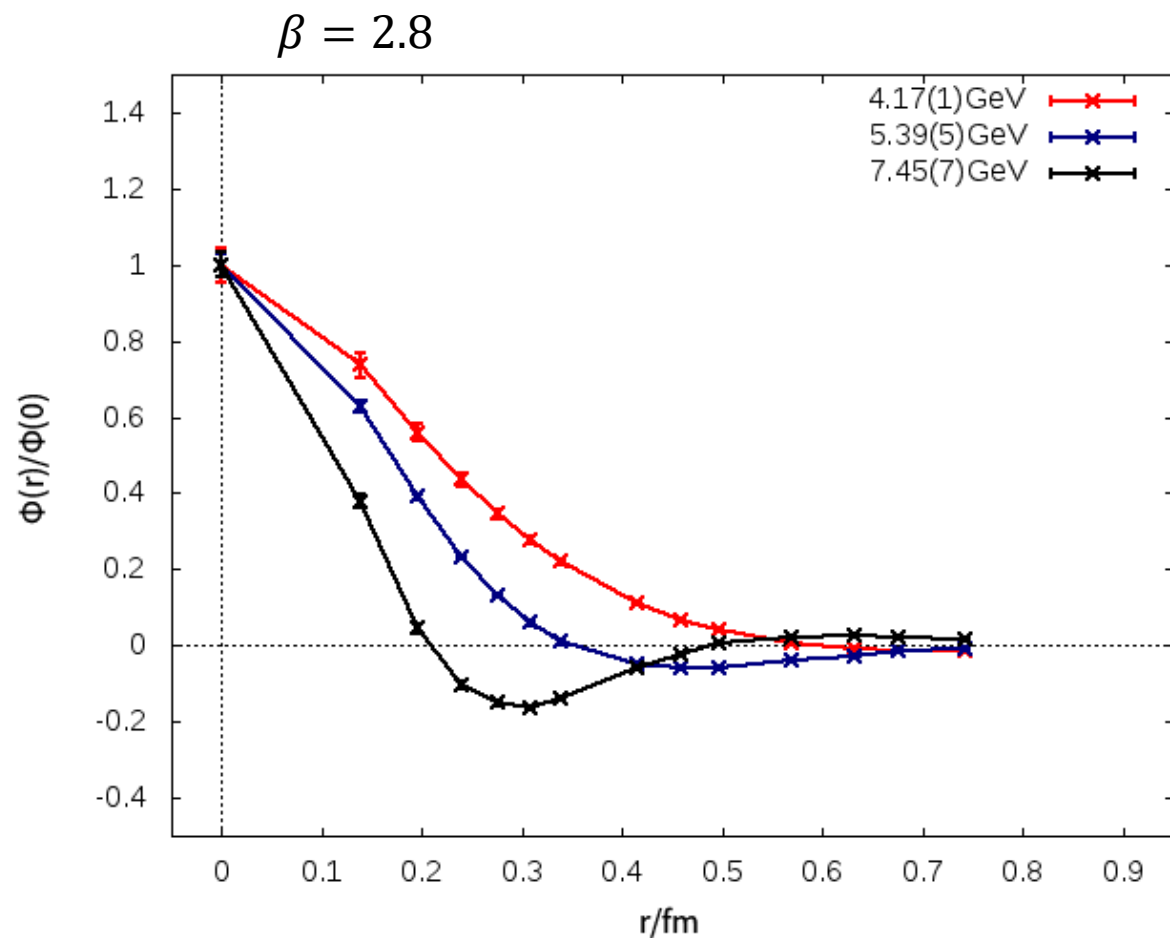
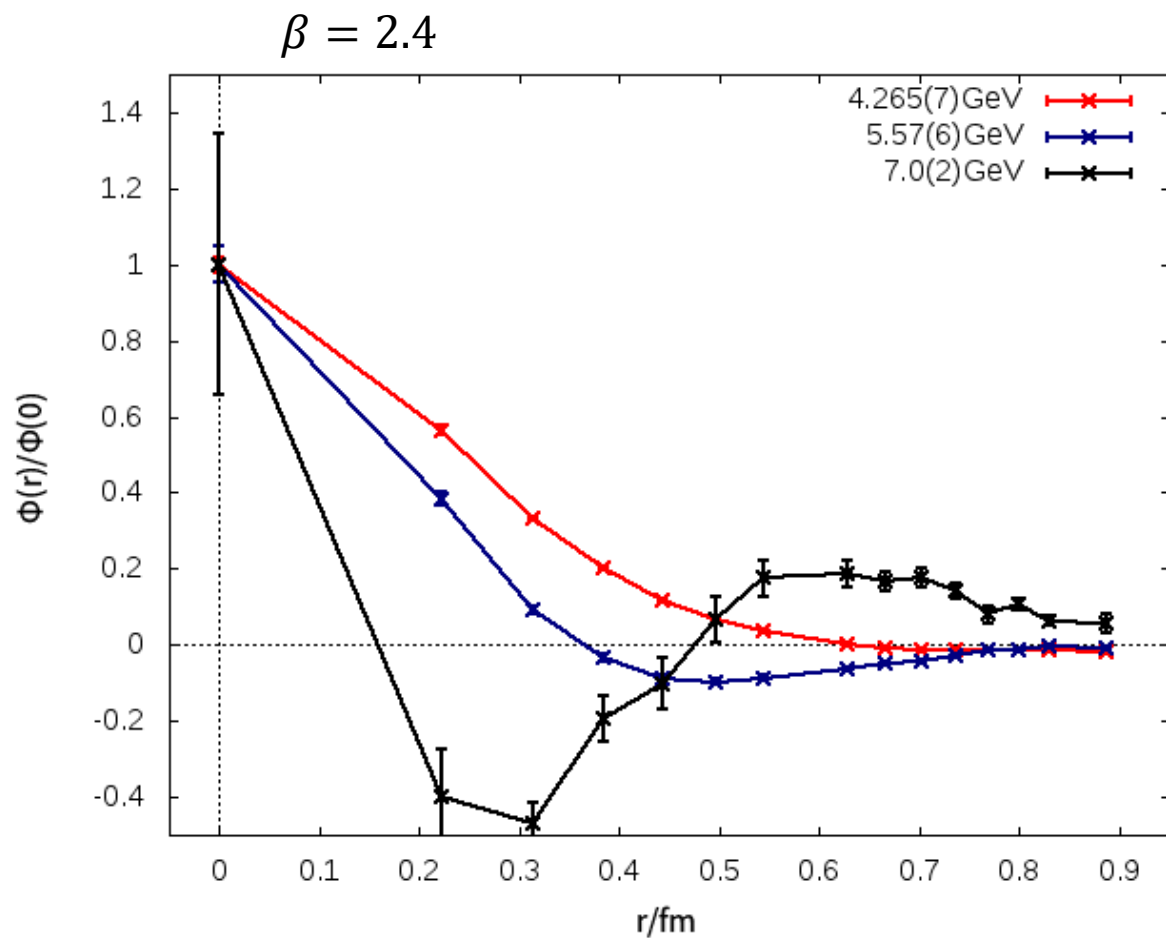
$$t \in [t_{min}, t_{max}]$$

Shift  $t_{min}$  with  $t_{max}$  fixed to find a stable region.



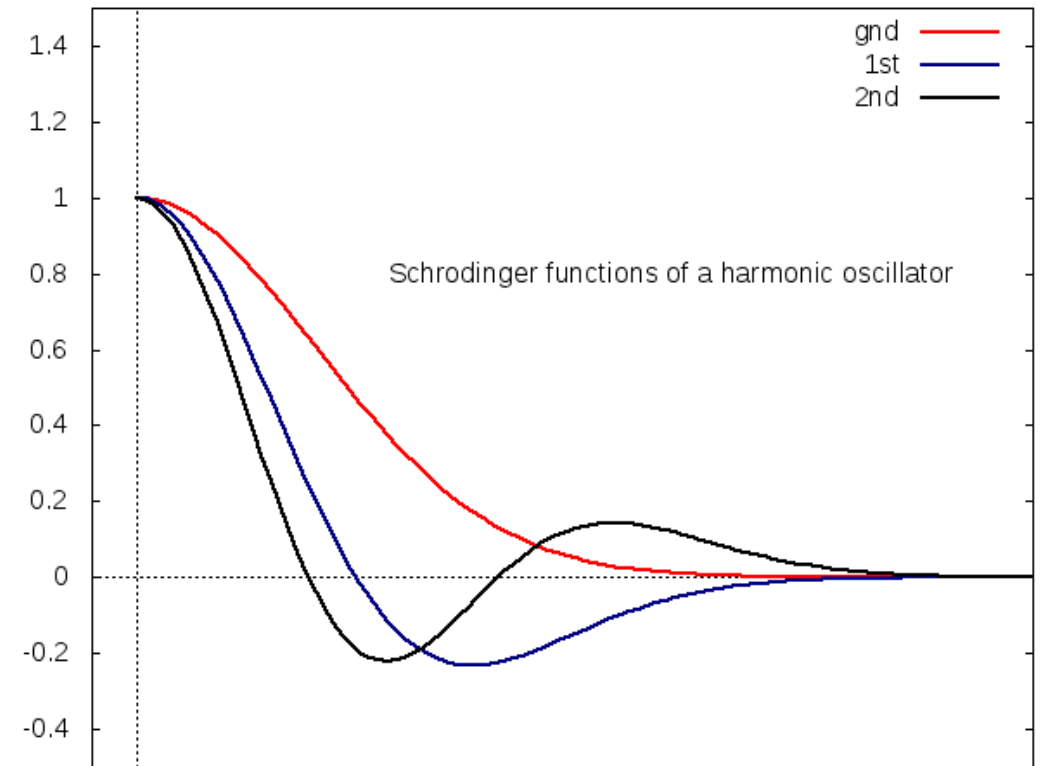
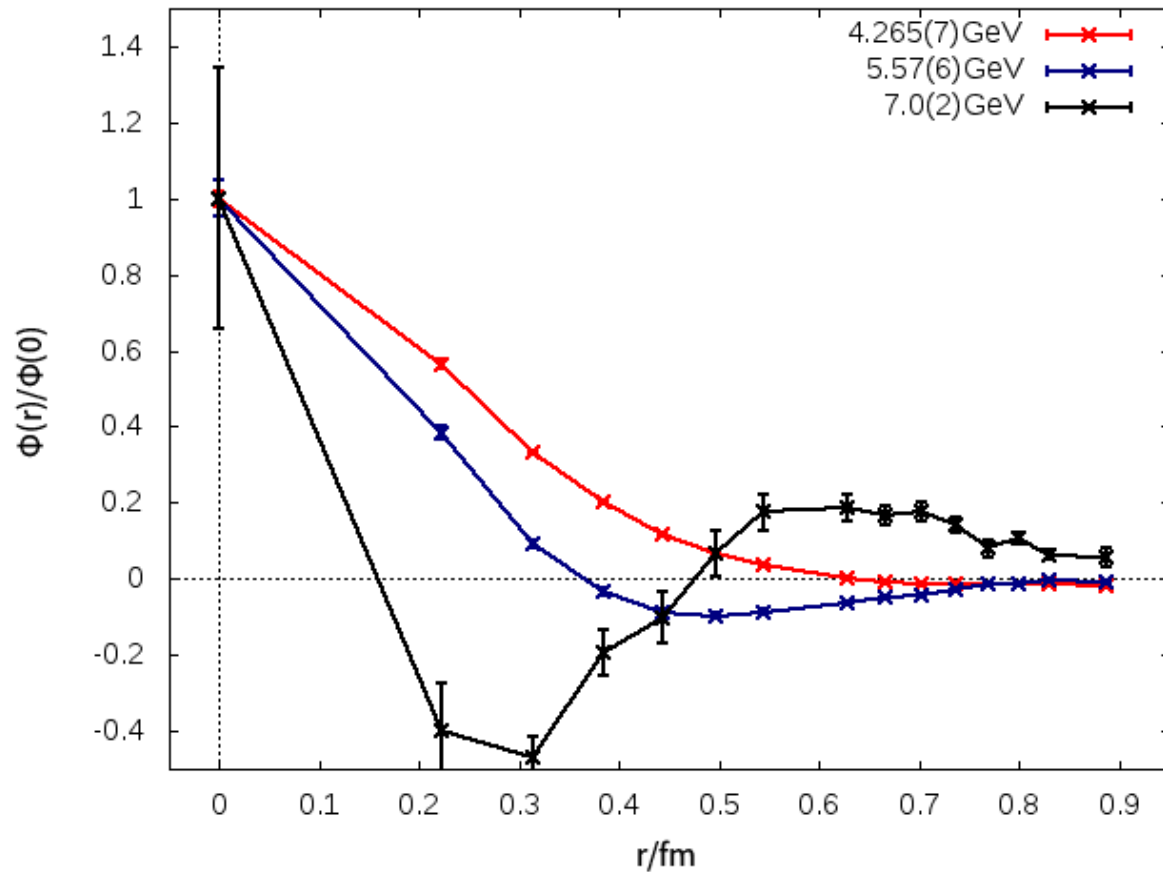
Exam.  $1^{-+}, \beta = 2.4$

# Exotic Channel $1^{-+}$



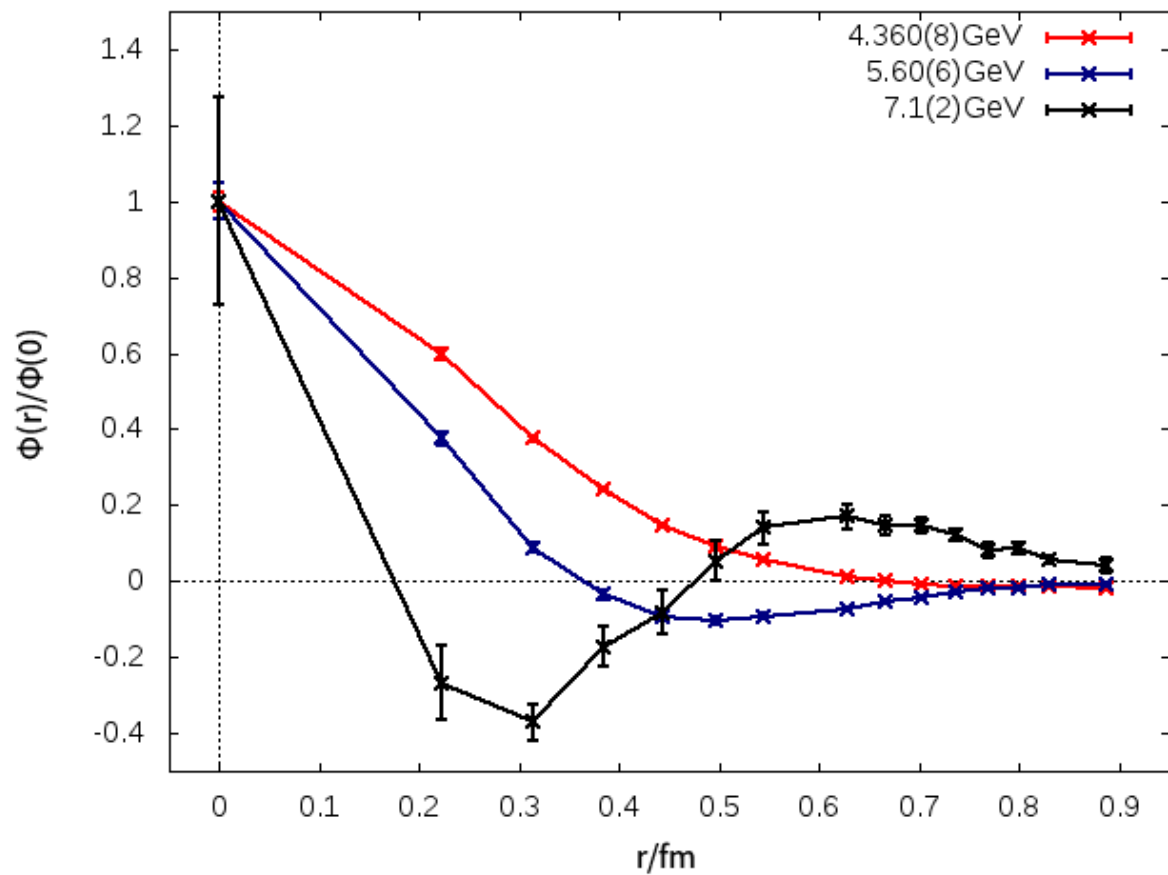
# Compare to Shrodinger Functions of A Harmonic Oscillator

$\beta = 2.4$

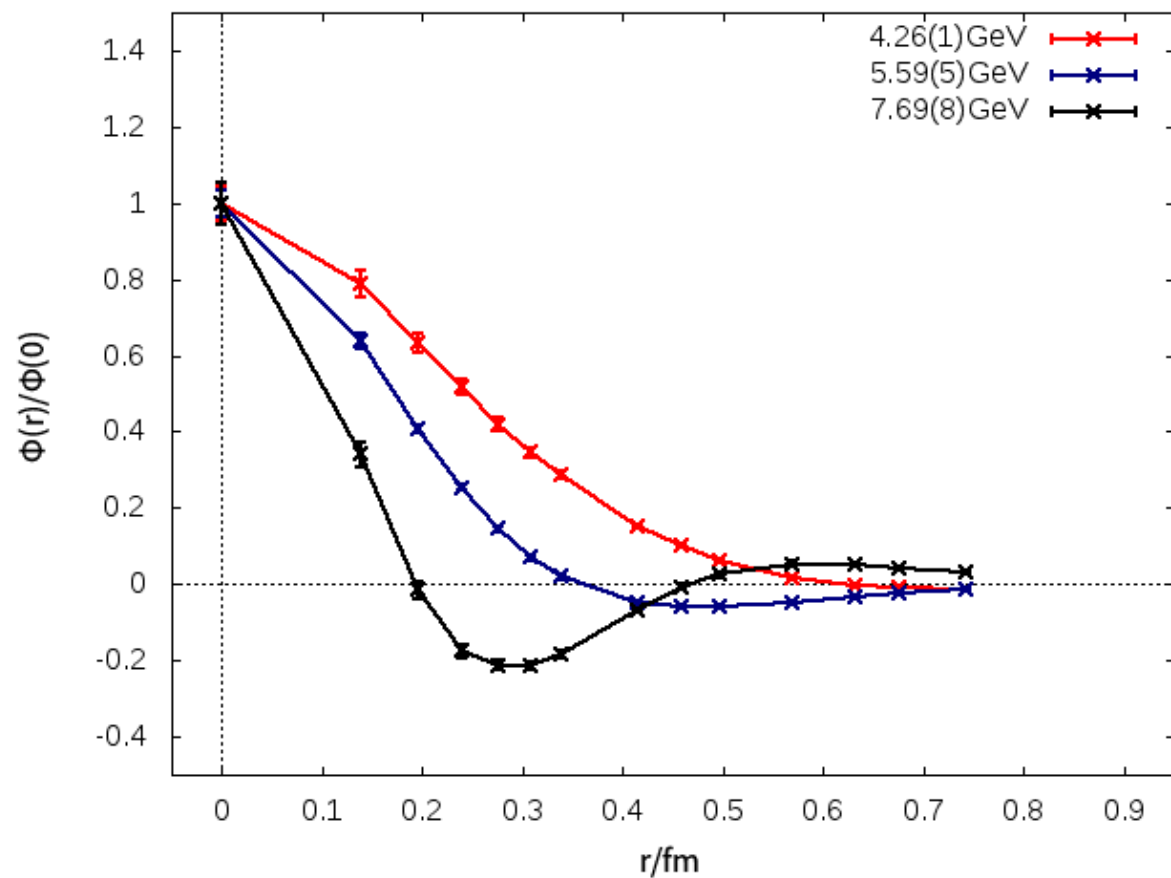


$2^{-+}$

$\beta = 2.4$



$\beta = 2.8$



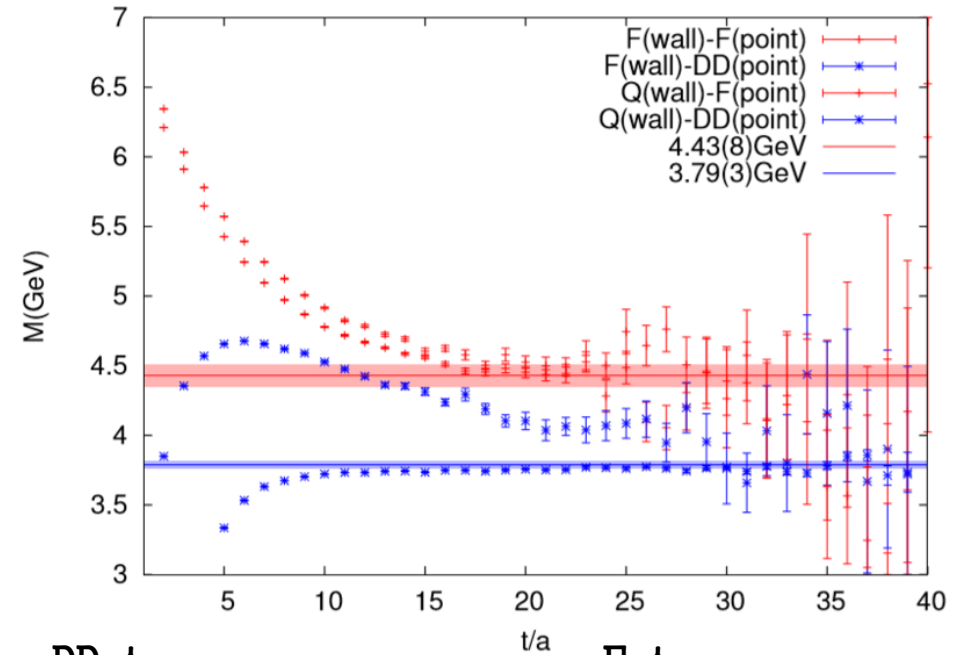


# Why no conventional $2^{-+}$ states found ?

According to

“Lattice study on  $\eta_{c2}$  and  $X(3872)$ ” ( Y.B.Yang et al. PhysRevD.87.014501[arXiv:1206.2086 [hep-lat]]),

a  $q\bar{q}B$  type(F-type, red dots in figure) operator hardly couples to conventional states like  $\eta_{c2}(2^{-+})$ , instead, it couples to a state around 4.43GeV mostly (which we treat as a ground state of hybrids here).



DD-type

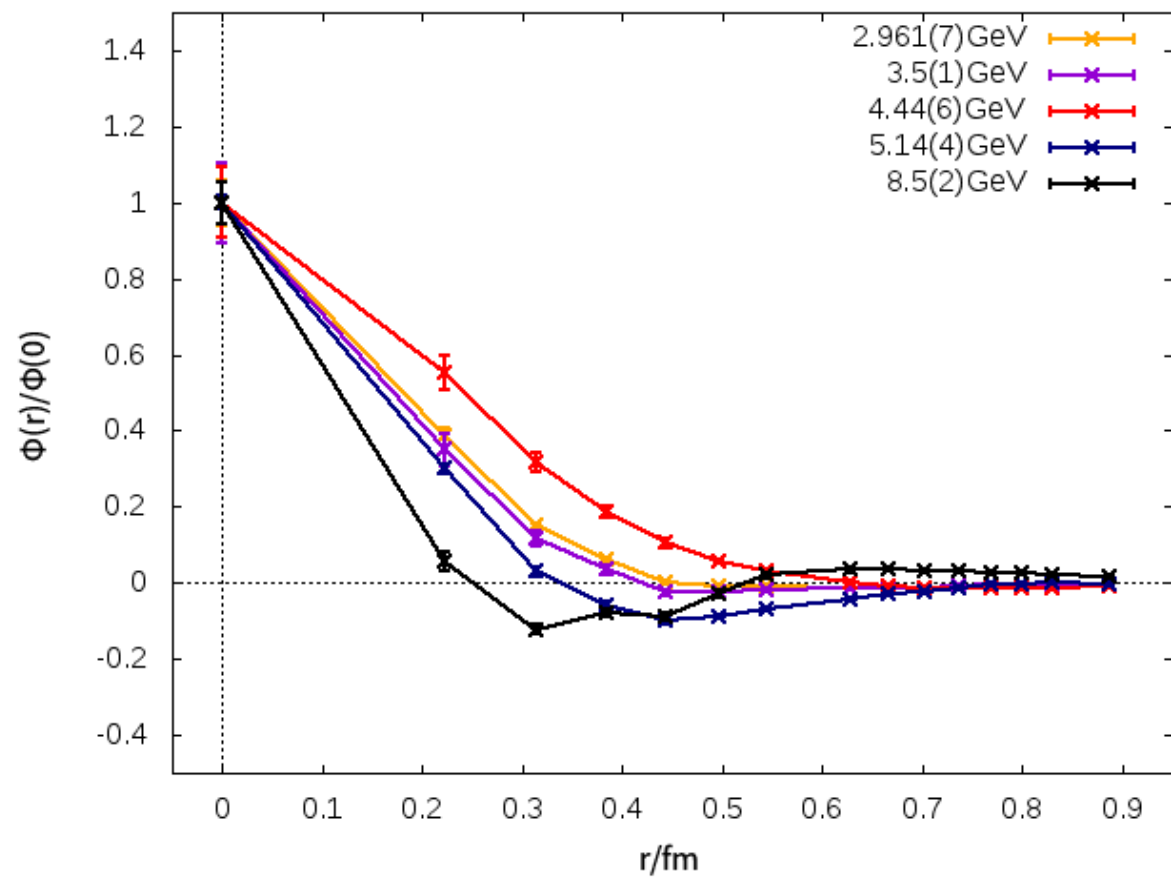
$$|\epsilon_{ijk} \bar{q} \gamma_5 \vec{D}_i \vec{D}_j q$$

F-type

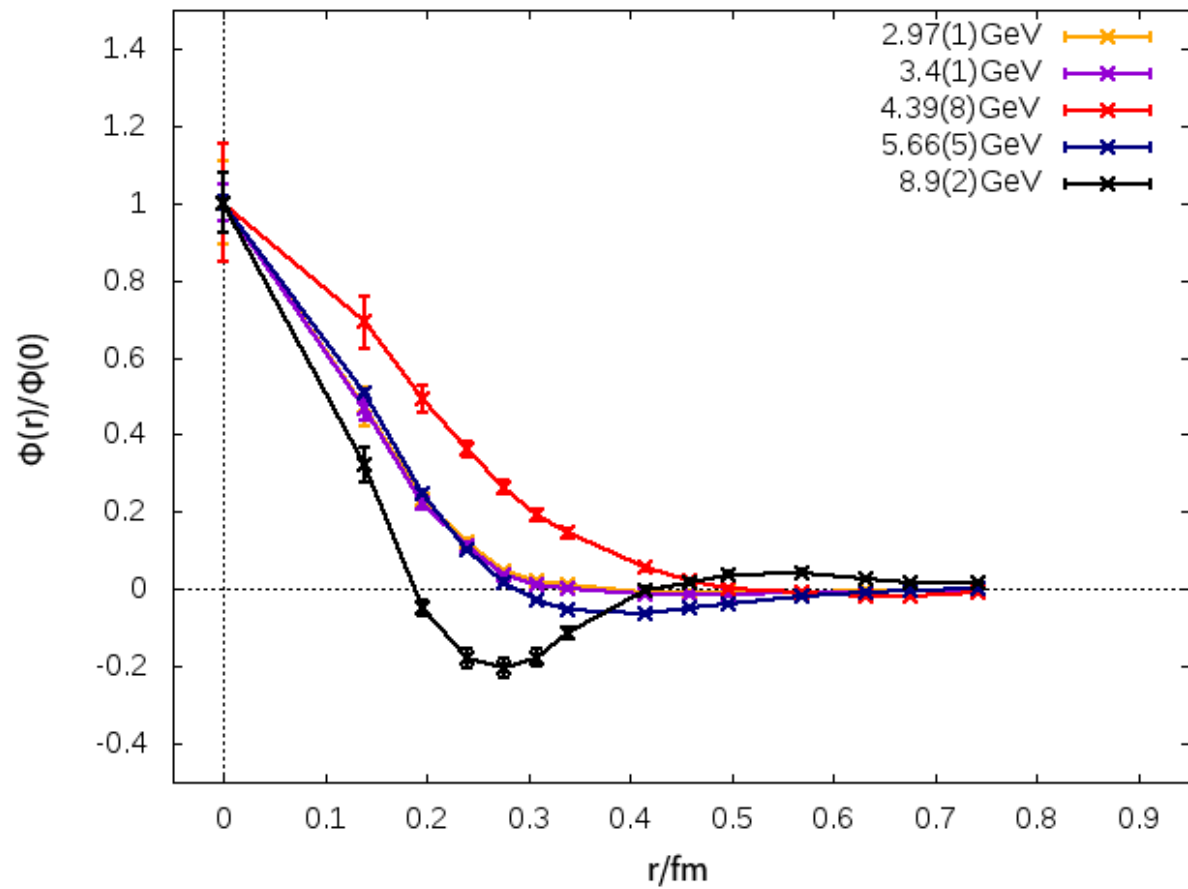
$$|\epsilon_{ijk} \bar{q}^a \gamma_i q^b B_j^{ab}$$

$0^{-+}$

$\beta = 2.4$

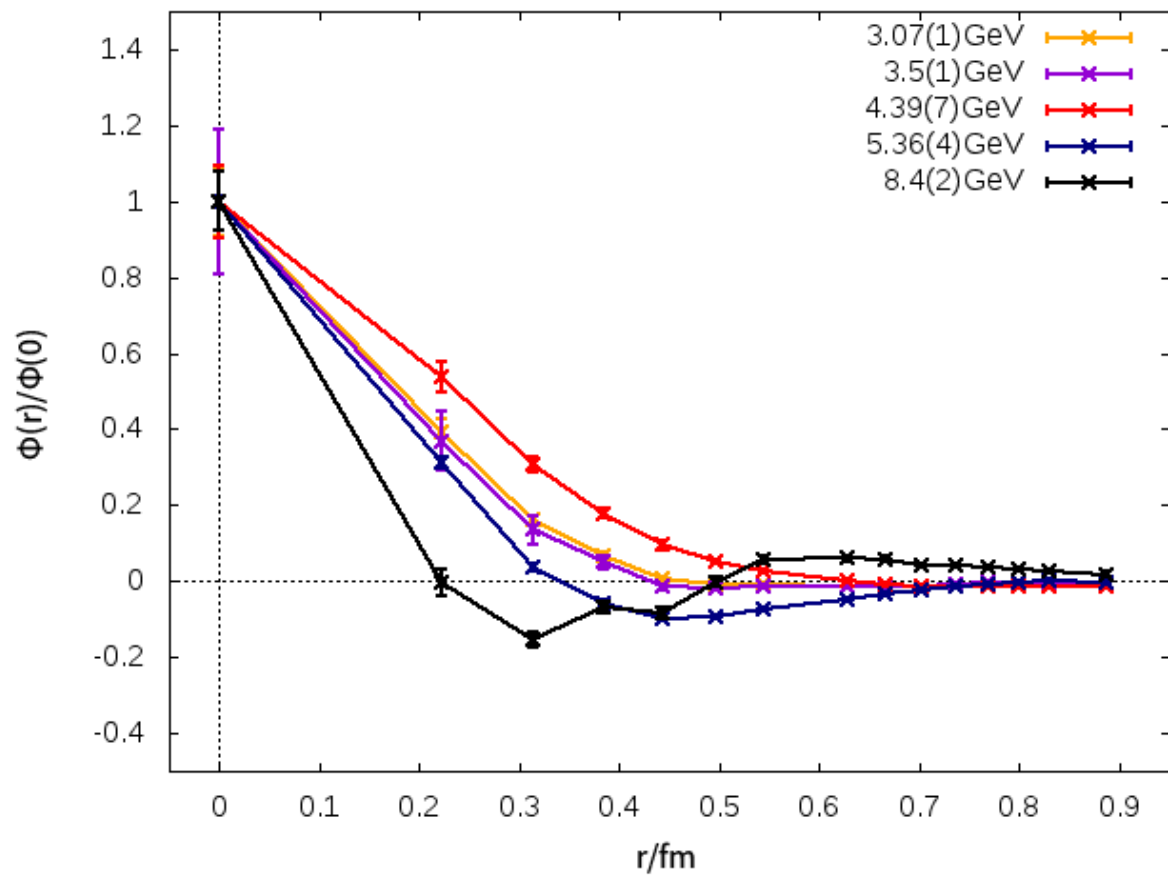


$\beta = 2.8$

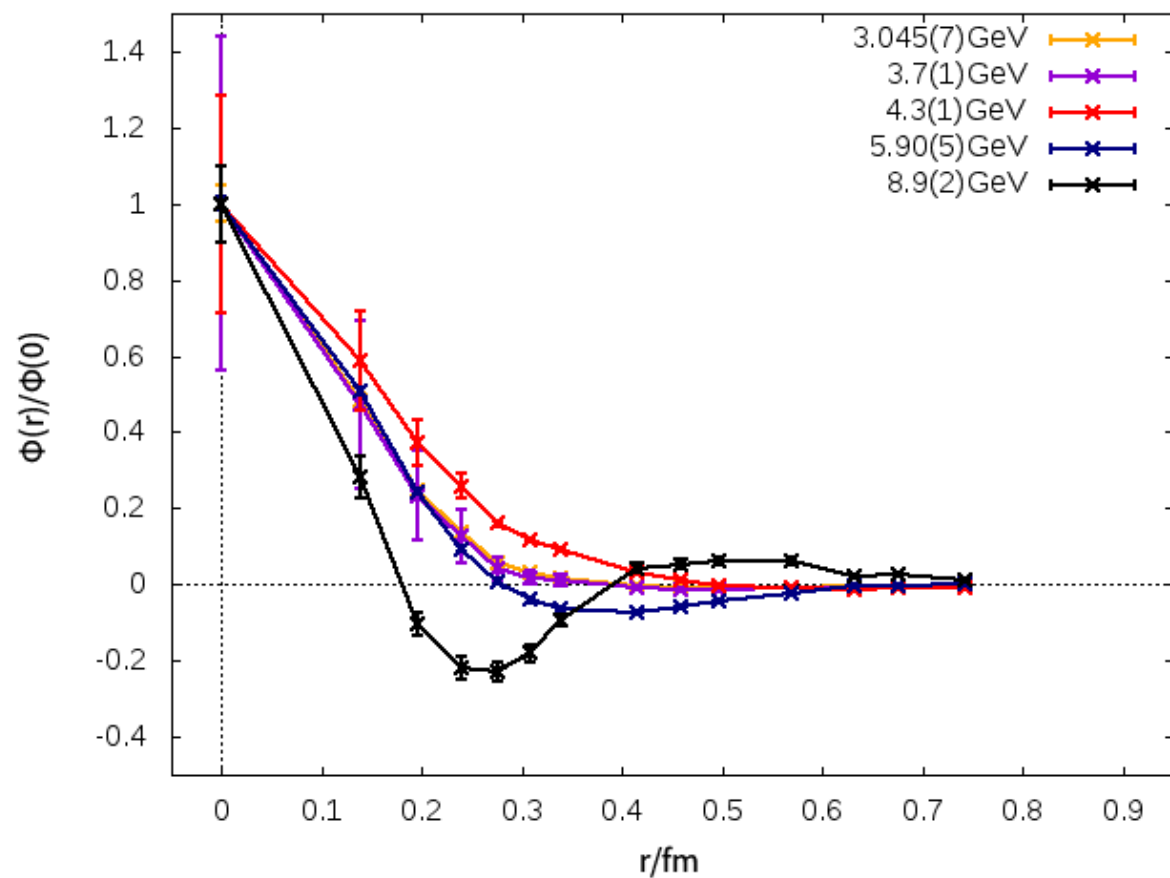


1--

$\beta = 2.4$



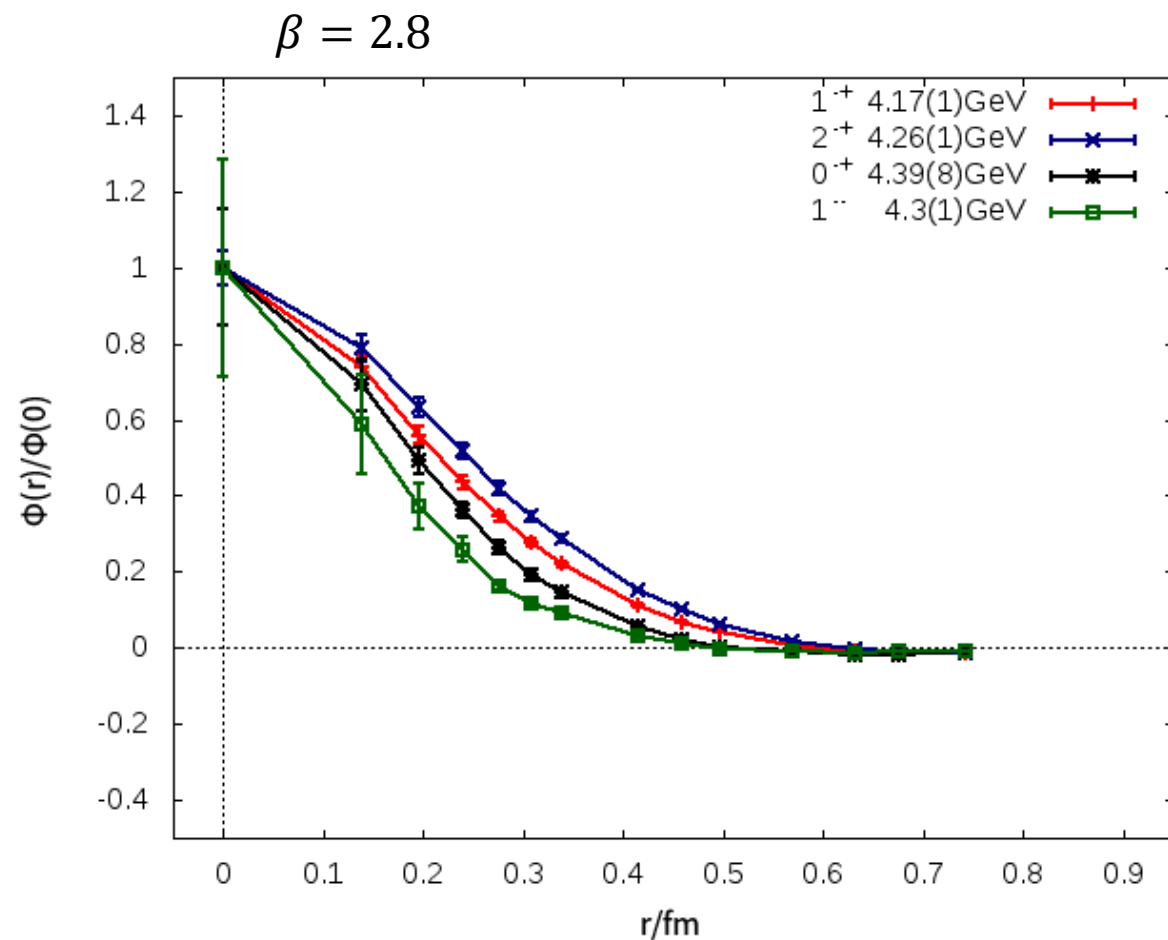
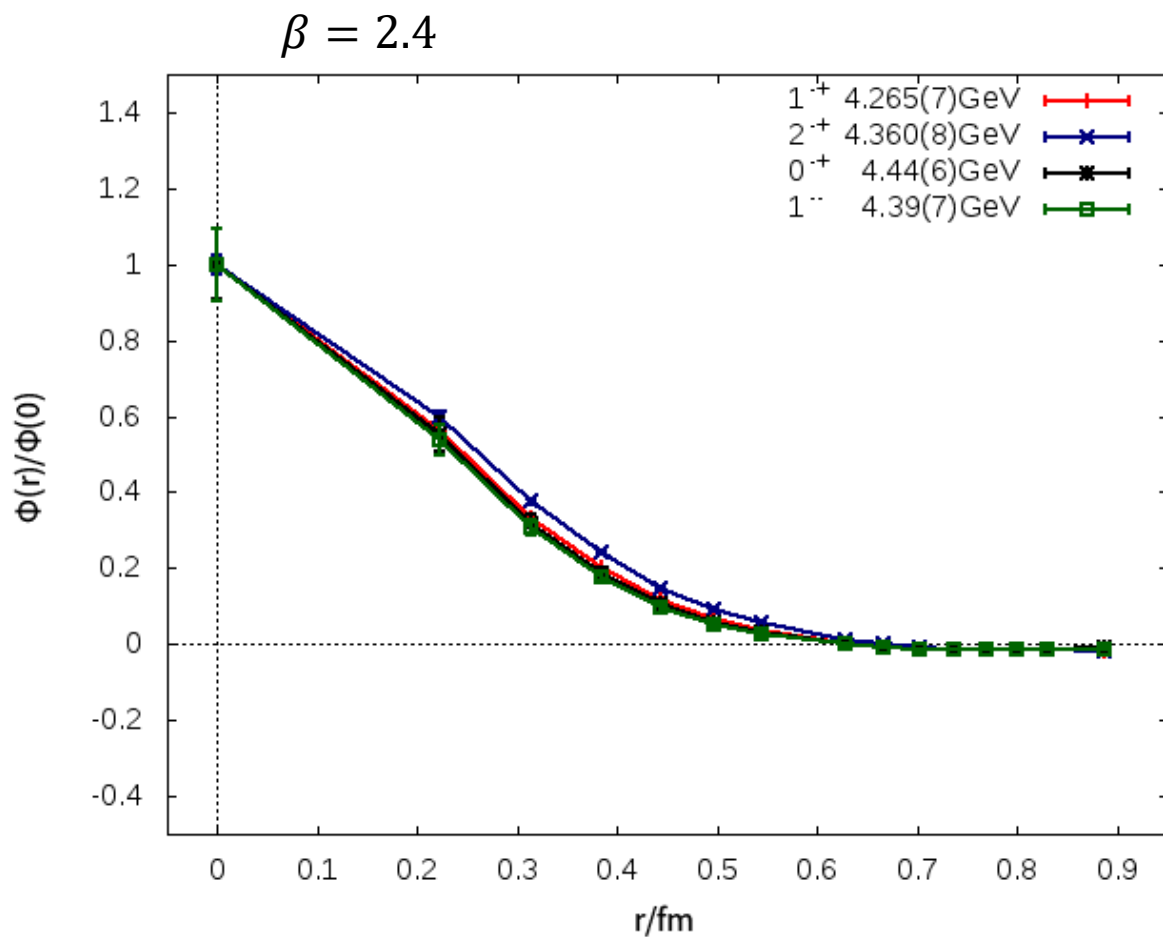
$\beta = 2.8$



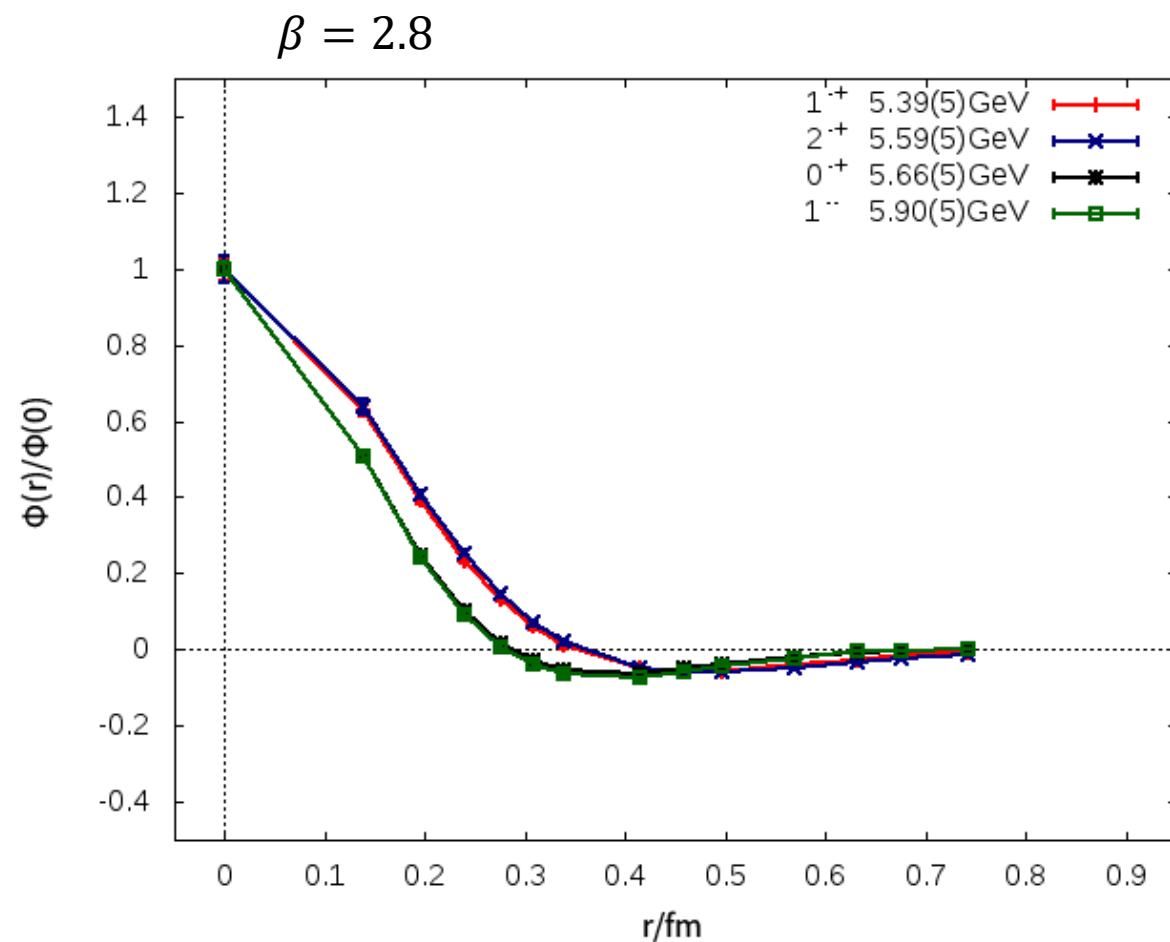
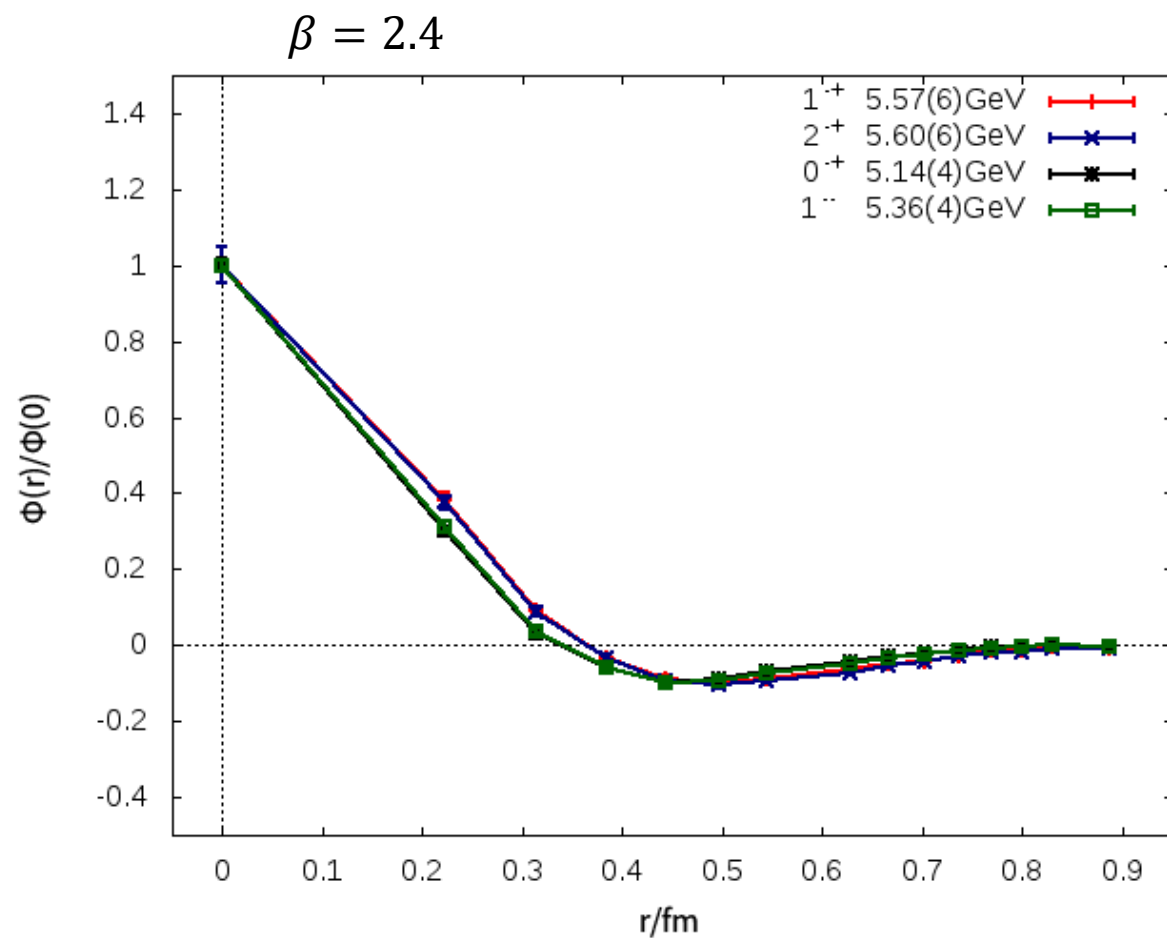
# Masses From Fitting (Unit: GeV)

1 <sup>--</sup>		0 <sup>-+</sup>		1 <sup>-+</sup>		2 <sup>-+</sup>		node
$\beta = 2.4$	$\beta = 2.8$	$\beta = 2.4$	$\beta = 2.8$	$\beta = 2.4$	$\beta = 2.8$	$\beta = 2.4$	$\beta = 2.8$	
3.07(1)	3.045(7)	2.961(7)	2.97(1)	W	W	W	W	0
3.5(1)	3.7(1)	3.5(1)	3.4(1)	W	W	W	W	0
4.39(7)	4.3(1)	4.44(6)	4.39(8)	4.265(7)	4.17(1)	4.360(8)	4.26(1)	0
5.36(4)	5.90(5)	5.14(4)	5.66(5)	5.57(6)	5.39(5)	5.60(6)	5.59(5)	1
8.4(2)	8.9(2)	8.5(2)	8.9(2)	7.0(2)	7.45(7)	7.1(2)	7.69(8)	2

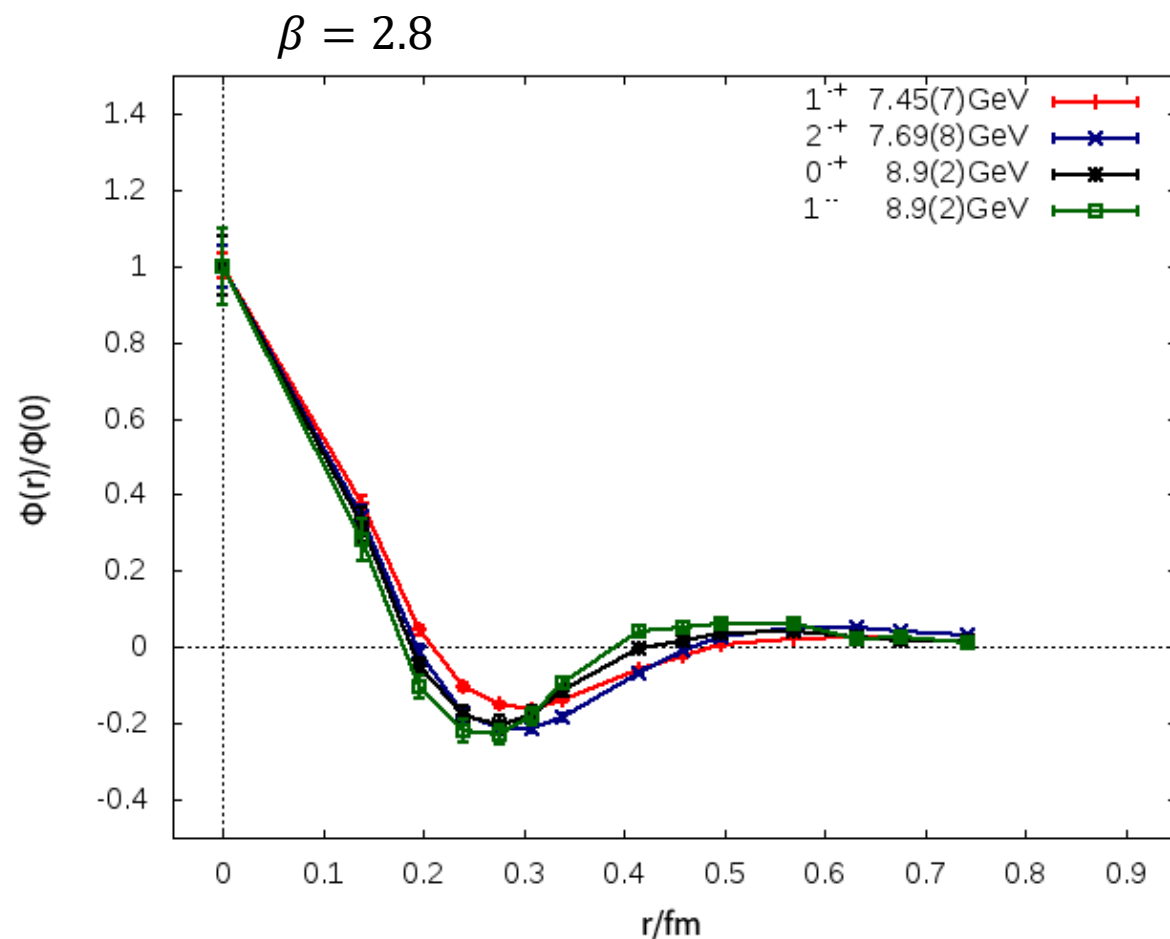
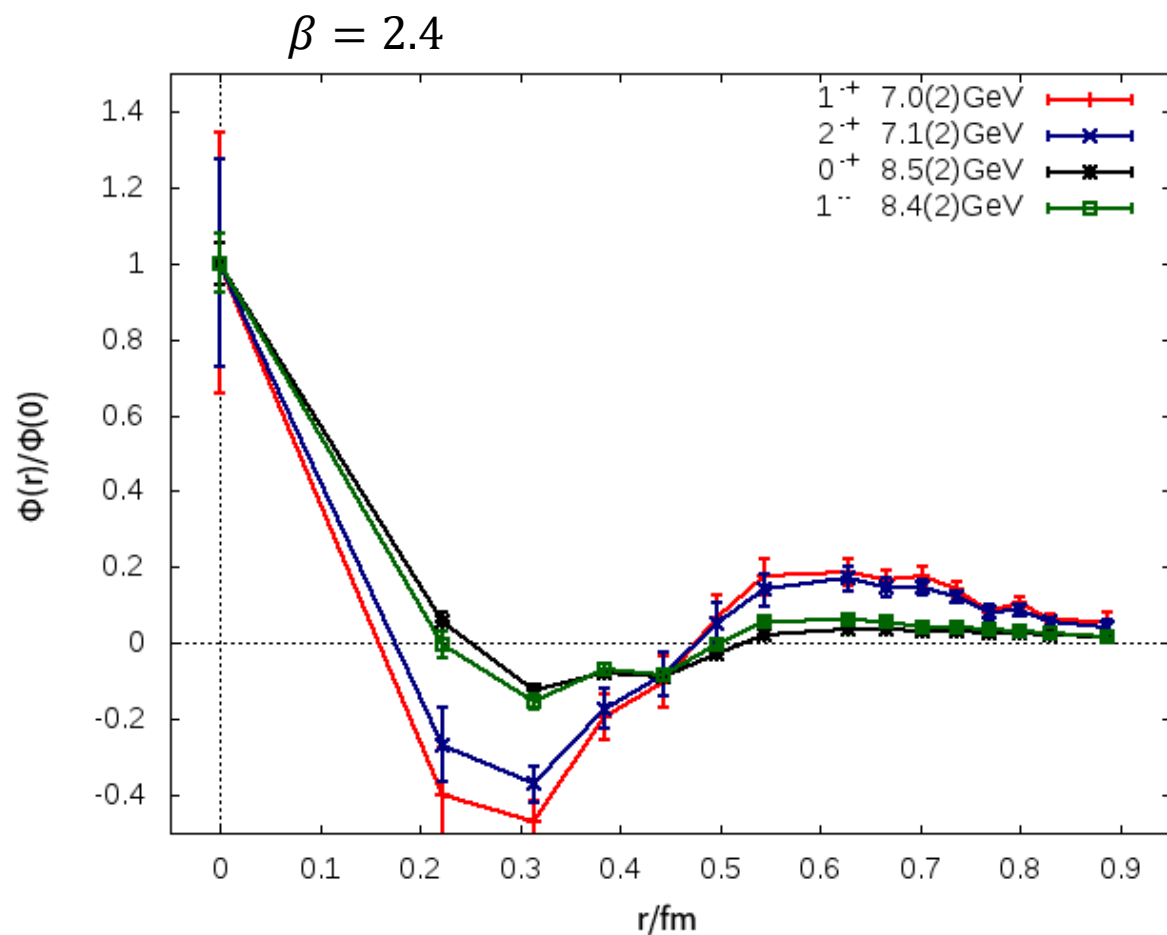
# Hybrid 1S (states around 4.3GeV)



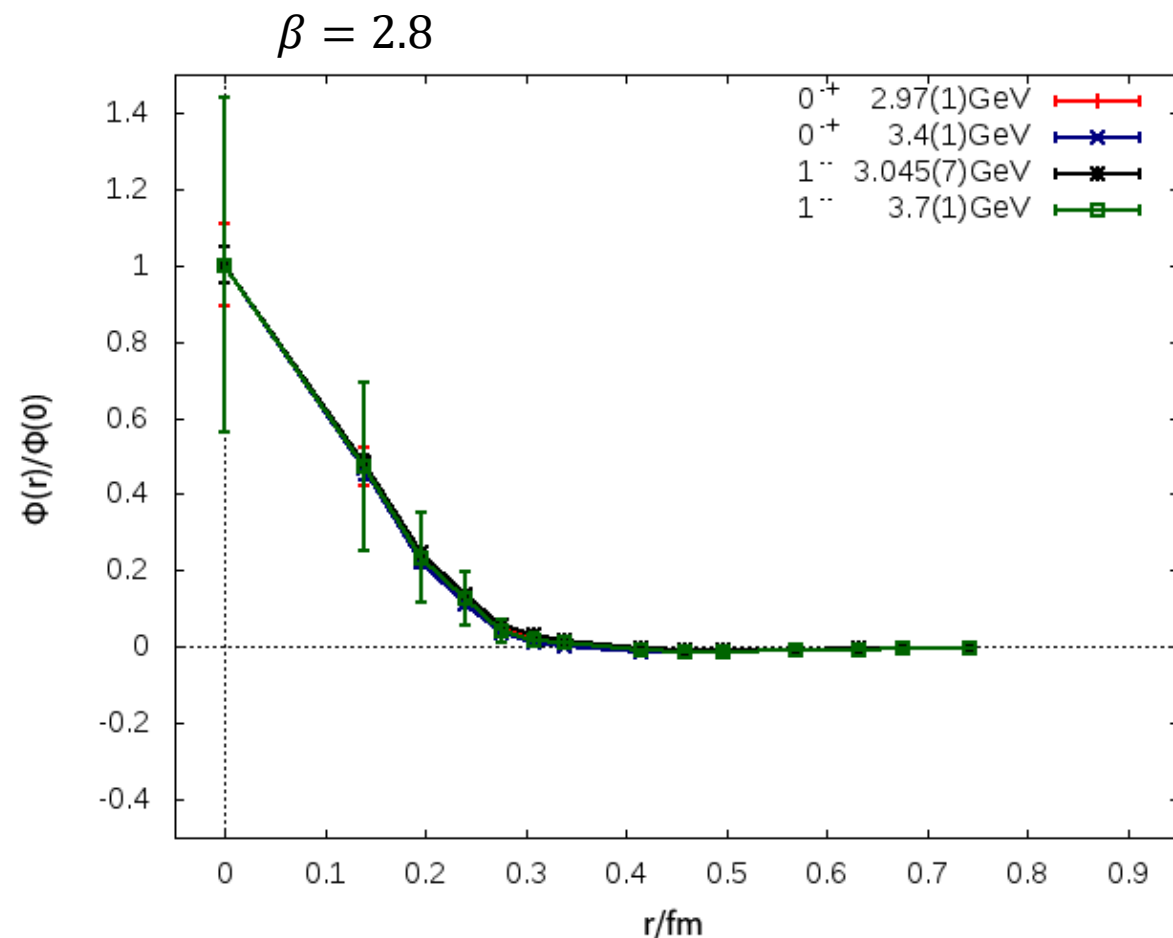
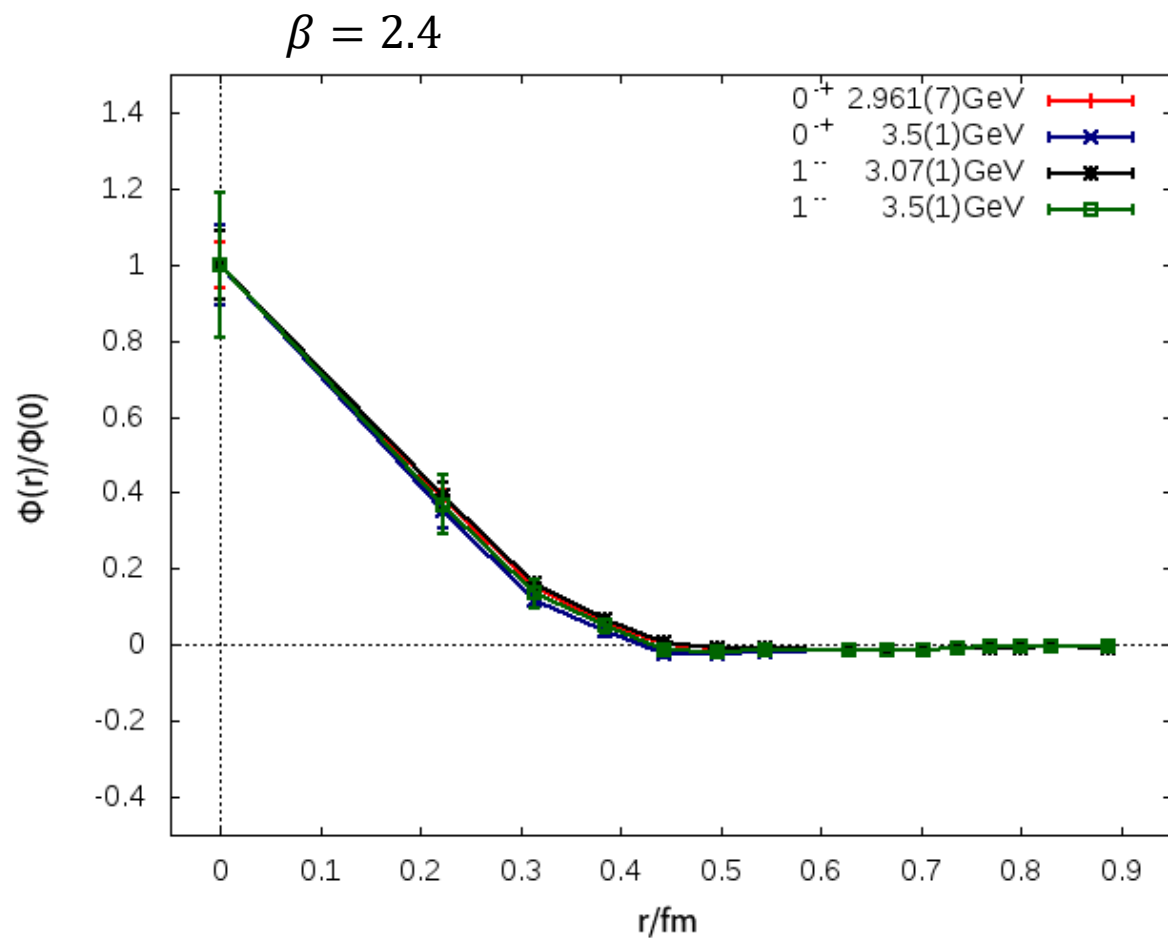
# Hybrid 2S (states around 5.5GeV)



# Hybrid 3S(?) (states above 7GeV)



# Conventional States ( $J/\psi$ , $\psi'$ , $\eta_c$ , $\eta_c'$ , maybe)





# Conclusion

- Clear nodal behavior of BS functions with respect to the spatial displacement reflects  $r$  in the operators is a meaningful dynamical variable.
- It implies that the inner structure of hybrid charmonia is a localized  $\bar{c}c$  kernel surrounded by a gluonic component, just like a halo.
- The hybrid states distribute across  $1^{--}$  and  $(0, 1, 2)^{-+}$  have similar structure, and their masses are almost in degenerate around 4.3GeV(hybrid 1S) and 5.5GeV(hybrid 2S), respectively.

End of Story