Status of the muon g-2 hadronic vacuum polarization calculation by RBC/UKQCD

Christoph Lehner (UR & BNL)

June 17, 2019 – Lattice 2019, Wuhan

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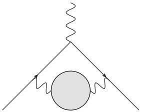
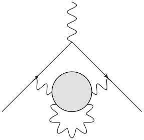
There is a tension of 3.7σ for the muon $a_\mu = (g_\mu - 2)/2$:

$$a_\mu^{\rm EXP} - a_\mu^{\rm SM} = 27.4\,\underbrace{(2.7)}_{\rm HVP}\,\underbrace{(2.6)}_{\rm HLbL}\,\underbrace{(0.1)}_{\rm other}\,\underbrace{(6.3)}_{\rm EXP}\times 10^{-10}$$

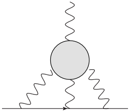
HVP





This talk

HLbL

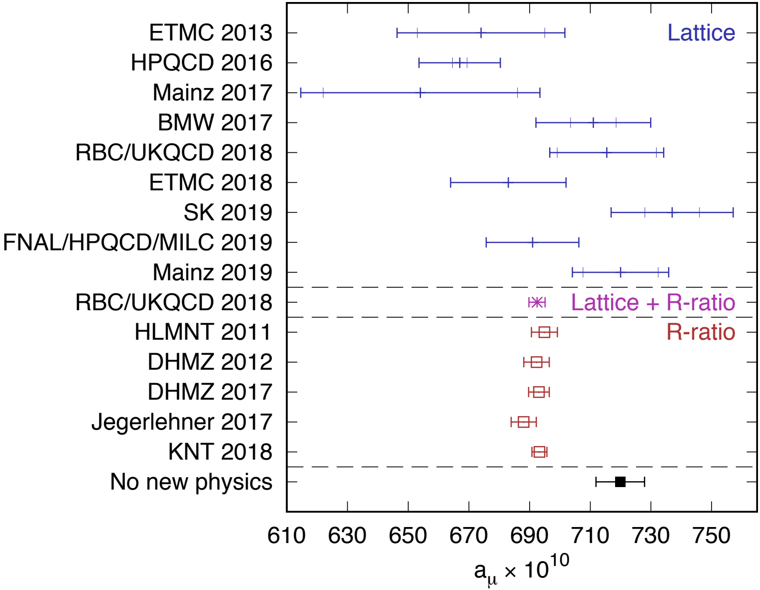


2019: $\delta a_\mu^{\rm EXP} \to 4.5 \times 10^{-10}$ (avg. of BNL/estimate of 2019 Fermilab result)

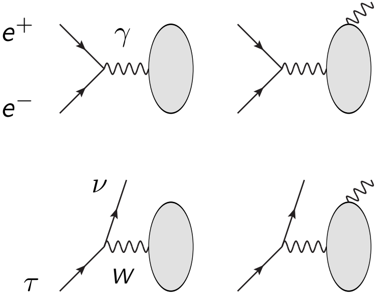
Targeted final uncertainty of Fermilab E989: $\delta a_\mu^{\rm EXP} \to 1.6 \times 10^{-10}$

$\Rightarrow$ by 2019 consolidate HVP/HLbL, over the next years uncertainties to $O(1 \times 10^{-10})$

Status of HVP determinations



Dispersive method - Overview



$$e^+e^- \to \mathrm{hadrons}(\gamma)$$

$$J_\mu = V_\mu^{I=1,I_3=0} + V_\mu^{I=0,I_3=0}$$

$$\tau \to \nu\mathrm{hadrons}(\gamma)$$

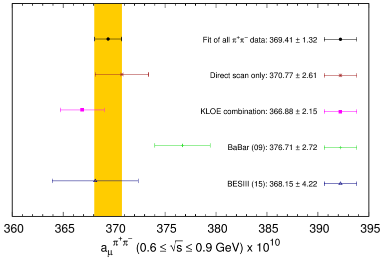
$$J_\mu = V_\mu^{I=1,I_3=\pm1} - A_\mu^{I=1,I_3=\pm1}$$

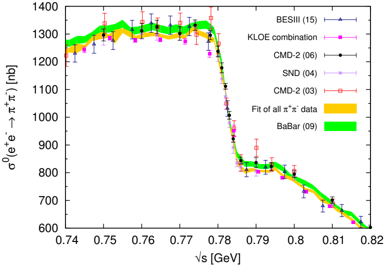
Knowledge of isospin-breaking corrections and separation of vector and axial-vector components needed to use τ decay data. Can do this from LQCD+QED (Bruno, Izubuchi, CL, Meyer, 1811.00508)!

Can have both energy-scan and ISR setup.

Dispersive method - e^+e^- status

Tension in 2π experimental input. BaBar and KLOE central values differ by $\delta a_\mu = 9.8(3.5) \times 10^{-10}$, compare to quoted total uncertainties of dispersive results of order $\delta a_\mu = 3 \times 10^{-10}$.





Conflicting input limits the precision and reliability of the dispersive results. Can we replace some of this data with LQCD+QED?

Looking for more data and insight: energy-scans update from CMD-3 in Novosibirsk and ISR updates from KLOE2, BaBar, Belle, BESIII and BelleII.

RBC/UKQCD status 2018

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Editors' Suggestion

Calculation of the Hadronic Vacuum Polarization Contribution to the Muon Anomalous Magnetic Moment

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We present a first-principles lattice QCD + QED calculation at physical pion mass of the leading-order hadronic vacuum polarization contribution to the muon anomalous magnetic moment. The total contribution of up, down, strange, and charm quarks including QED and strong isospin breaking effects is $a_\mu^{\mathrm{HVP\ LO}} = 715.4(18.7) \times 10^{-10}$. By supplementing lattice data for very short and long distances with R-ratio data, we significantly improve the precision to $a_\mu^{\mathrm{HVP\ LO}} = 692.5(2.7) \times 10^{-10}$. This is the currently most precise determination of $a_\mu^{\mathrm{HVP\ LO}}$.

Pure lattice result and dispersive result with reduced $\pi\pi$ dependence (window method)

Lattice QCD – Time-Moment Representation

Starting from the vector current $J_\mu(x) = i \sum_f Q_f \overline{\Psi}_f(x)\gamma_\mu \Psi_f(x)$ we may write

$$a_\mu^{\mathrm{HVP\ LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

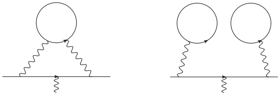
$$C(t) = \frac{1}{3} \sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x}, t) J_j(0) \rangle$$

and w_t capturing the photon and muon part of the HVP diagrams (Bernecker-Meyer 2011).

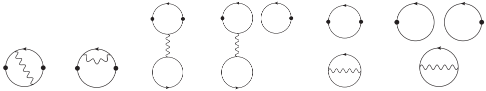
The correlator $C(t)$ is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, and charm quark contributions. The missing bottom quark contributions are computed in pQCD.

Diagrams

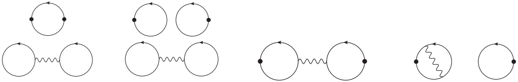
Isospin limit



QED corrections

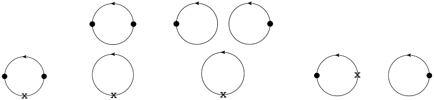


(a) V (b) S (c) T (d) T_d (e) D1 (f) $D1_d$



(g) D2 (h) $D2_d$ (i) F (j) D3

Strong isospin breaking



(a) M (b) R (c) R_d (d) O

Window method (implemented in RBC/UKQCD 2018)

We therefore also consider a window method. Following Meyer-Bernecker 2011 and smearing over t to define the continuum limit we write

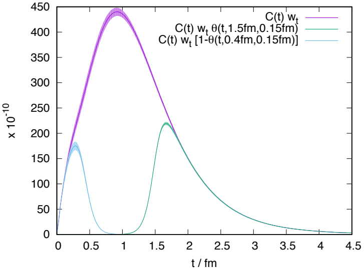
$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

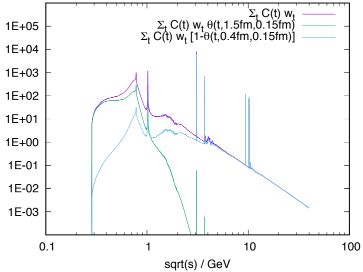
with

$$\begin{aligned}
a_\mu^{\mathrm{SD}} &= \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)]\,, \\
a_\mu^{\mathrm{W}} &= \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]\,, \\
a_\mu^{\mathrm{LD}} &= \sum_t C(t) w_t \Theta(t, t_1, \Delta)\,, \\
\Theta(t, t', \Delta) &= [1 + \tanh[(t - t')/\Delta]]/2\,.
\end{aligned}$$

In this version of the calculation, we use
$C(t) = \frac{1}{12\pi^2} \int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to \mathrm{had})$
to compute a_μ^{SD} and a_μ^{LD}.

How does this translate to the time-like region?





Most of $\pi\pi$ peak is captured by window from $t_0 = 0.4$ fm to $t_1 = 1.5$ fm, so replacing this region with lattice data reduces the dependence on BaBar versus KLOE data sets.

The pure lattice calculation of RBC/UKQCD 2018:

$$10^{10} \times a_\mu^{\mathrm{HVP\ LO}} = 715.4(18.7)$$
$$= 715.4(16.3)_{\mathrm{S}}(7.8)_{\mathrm{V}}(3.0)_{\mathrm{C}}(1.9)_{\mathrm{A}}(3.2)_{\mathrm{other}}$$

(S) statistics, (V) finite-volume errors, (C) the continuum limit extrapolation, (A) scale setting uncertainty;

other $\supset$ neglected diagrams for QED and SIB, estimate of bottom quark contribution

Statistical noise mostly from isospin symmetric light quark connected (14.2) and disconnected (3.3), QED (5.7), SIB (4.3)

RBC/UKQCD 2019 update (in preparation):

- Improved methodology
- A lot of new data

Improved methodology

Improved statistics and systematics – Bounding Method

BMW/RBC/UKQCD 2016

The correlator in finite volume

$$C(t) = \sum_n |\langle 0|V|n\rangle|^2 e^{-E_n t}\,.$$

We can bound this correlator at each t from above and below by the correlators

$$\tilde{C}(t;T,\tilde{E}) = \begin{cases} C(t) & t < T\,, \\ C(T)e^{-(t-T)\tilde{E}} & t \geq T \end{cases}$$

for proper choice of $\tilde{E}$. We can chose $\tilde{E} = E_0$ (assuming $E_0 < E_1 < \ldots$) to create a strict upper bound and any $\tilde{E}$ larger than the local effective mass to define a strict lower bound.

Improved Bounding Method

RBC/UKQCD 2018, A. Meyer's talk today at 5:30pm

Therefore if we had precise knowledge of the lowest $n = 0, \ldots, N$ values of $|\langle 0|V|n\rangle|$ and E_n, we could define a new correlator

$$C^N(t) = C(t) - \sum_{n=0}^{N} |\langle 0|V|n\rangle|^2 e^{-E_n t}$$

which we could bound much more strongly through the larger lowest energy $E_{N+1} \gg E_0$. New method: do a GEVP study of FV spectrum to perform this subtraction.

Reduces statistical error of light quark contribution by more than a factor of 3.

Improved systematics – compute finite-volume effects from first-principles

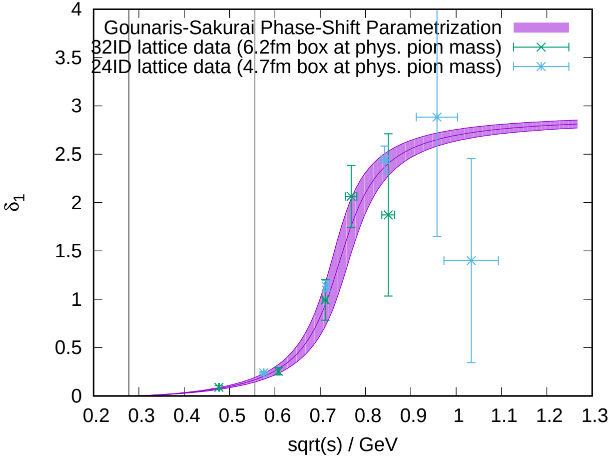
RBC/UKQCD study of QCD at **physical pion mass** at three different volumes:

$\boxed{L = 4.66 \text{ fm}, L = 5.47 \text{ fm}, L = 6.22 \text{ fm}}$

Results for light-quark isospin-symmetric connected contribution:

- $a_\mu(L = 6.22 \text{ fm}) - a_\mu(L = 4.66 \text{ fm}) = 12.2 \times 10^{-10}$ (sQED), $21.6(6.3) \times 10^{-10}$ (lattice QCD)
- Need to do better than sQED in finite-volume

First constrain the p-wave phase shift from our $L = 6.22$ fm physical pion mass lattice:



$E_\rho = 0.766(21)$ GeV (PDG 0.77549(34) GeV)
$\Gamma_\rho = 0.139(18)$ GeV (PDG 0.1462(7) GeV)

GSL^2 finite-volume results compared to sQED and lattice

GSL^2 method of Meyer 2012

Results for light-quark isospin-symmetric connected contribution:

- FV difference between $a_\mu(L = 6.22\text{ fm}) - a_\mu(L = 4.66\text{ fm}) = 12.2 \times 10^{-10}$ (sQED), $21.6(6.3) \times 10^{-10}$ (lattice QCD), $20(3) \times 10^{-10}$ (GSL^2)
- GSL^2 prediction agrees with actual FV effect measured on the lattice, sQED is in slight tension, two-loop FV ChPT to be compared next Bijnens and Relefors 2017
- Use GSL^2 to update FV correction of Phys. Rev. Lett. 121, 022003 (2018): $a_\mu(L \to \infty) - a_\mu(L = 5.47\text{ fm}) = 16(4) \times 10^{-10}$ (sQED), $22(1) \times 10^{-10}$ (GSL^2); sQED error estimate based on Bijnens and Relefors 2017, table 1.
- Compare also to Hansen-Patella 2019 1904.10010: $a_\mu(L \to \infty) - a_\mu(L = 5.47\text{ fm}) \approx 14 \times 10^{-10}$, effect of neglected $e^{-\sqrt{2} m_\pi L}$ likely significant

Other improvements:

- HVP QED from re-analysis of HLbL point-source data (see also τ project, 1811.00508) reduces statistical noise by $\approx 10\times$ for V and S
- Infinte-volume and continuum limit also for diagram V, S, and F
- First results for T, D1, and R; other sub-leading in preparation
- Global fit combined with calculation of mass derivatives gives much reduced uncertainty for diagrams M and O (connected and disconnected SIB)

New data set

Ensembles at physical pion mass:

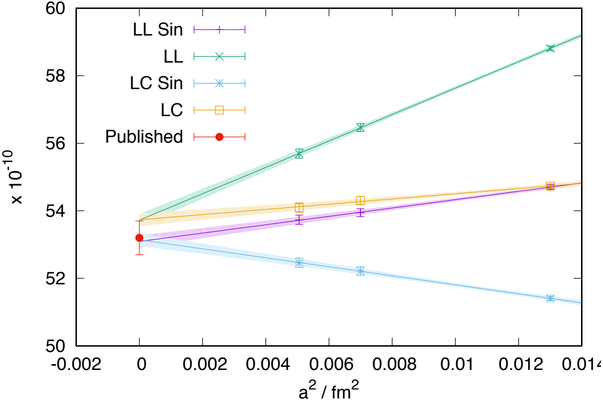
48I (1.73 GeV, 5.5fm), 64I (2.359 GeV, 5.4fm), 24ID (1 GeV, 4.7fm), 32ID (1 GeV, 6.2fm), 48ID (1 GeV, 9.3fm), 32IDf (1.37 GeV, 4.6fm)

RBC/UKQCD 2019 (data for light quarks, changes from 2018):

- A2A data for connected isospin symmetric: 48I (127 conf $\rightarrow$ 400 conf), 64I (160 conf $\rightarrow$ 250 conf), 24ID (new 130 conf, multi mass), 32ID (new 88 conf, multi mass)
- A2A data (tadpole fields) for disconnected: 48I (33 conf), 24ID (new 260 conf, multi mass), 32IDf (new 103 conf)
- QED and SIB corrections to meson and Ω masses, Z_V: 48I (30 conf) and 64I (new 30 conf)
- QED and SIB from HLbL point sources on 48I, 24ID, 32ID, 32IDf (on order of 20 conf each, 2000 points per config)
- Distillation data on 48I (33 conf), 64I (in progr.), 24ID (33 conf), 32ID (11 conf, multi-mass)
- New Ω mass operators (excited states control): 48I (130 conf)

Add $a^{-1} = 2.77$ GeV lattice spacing

- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):



- For light quark need new ensemble at physical pion mass. Started run on Summit Machine at Oak Ridge this year ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV).

Conclusions and Outlook

- Target precision for HVP is of $O(1 \times 10^{-10})$ in a few years; for now consolidate error at $O(3 \times 10^{-10})$
- Dispersive result from $e^+e^- \to$ hadrons right now is at 3×10^{-10} but limited by experimental tensions
- Two-pion channel from DHMZ17, KNT18 (e^+e^-) and DHMYZ13 (τ) are scattered by 12.5×10^{-10}

  Experimental updates and first-principles calculation of isospin-breaking corrections desirable. Combination of dispersive and lattice results can in short term lessen dependence on contested experimental data.
- New methods to reduce statistical and systematic errors and a lot of additional data.
- By end of this year, first-principles lattice result could have error of $O(5 \times 10^{-10})$

Backup

Dispersive method - e^+e^- status

Recent results by Keshavarzi et al. 2018, Davier et al. 2017:

Channel	This work (KNT18)	DHMZ17 [78]	Difference
Data based channels ($\sqrt{s} \leq 1.8$ GeV)			
$\pi^0\gamma$ (data + ChPT)	4.58 ± 0.10	4.29 ± 0.10	0.29
$\pi^+\pi^-$ (data + ChPT)	503.74 ± 1.96	507.14 ± 2.58	-3.40
$\pi^+\pi^-\pi^0$ (data + ChPT)	47.70 ± 0.89	46.20 ± 1.45	1.50
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
...			
Total	693.3 ± 2.5	693.1 ± 3.4	0.2

Good agreement for total, individual channels disagree to some degree. **Muon g-2 Theory Initiative workshops** recently held at Fermilab, KEK, UConn, and Mainz, intend to facilitate discussions and further understanding of these tensions.

One difference: treatment of correlations, impactful in particular in case when not all experimental data agrees

Gounaris-Sakurai-Lüscher method [H. Meyer 2012, Mainz 2017]

- Produce FV spectrum and matrix elements from phase-shift study (Lüscher method for spectrum and amplitudes, GS for phase-shift parametrization)
- This allows for a prediction of FV effects beyond chiral perturbation theory given that the phase-shift parametrization captures all relevant effects (can be checked against lattice data)
- This method is now being employed by ETMC, Mainz, and RBC/UKQCD.

Dispersive method - τ status

Experiment	$a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau]$ (10^{-10}) $2m_{\pi^\pm} - 0.36$ GeV	$0.36 - 1.8$ GeV
ALEPH	$9.80 \pm 0.40 \pm 0.05 \pm 0.07$	$501.2 \pm 4.5 \pm 2.7 \pm 1.9$
CLEO	$9.65 \pm 0.42 \pm 0.17 \pm 0.07$	$504.5 \pm 5.4 \pm 8.8 \pm 1.9$
OPAL	$11.31 \pm 0.76 \pm 0.15 \pm 0.07$	$515.6 \pm 9.9 \pm 6.9 \pm 1.9$
Belle	$9.74 \pm 0.28 \pm 0.15 \pm 0.07$	$503.9 \pm 1.9 \pm 7.8 \pm 1.9$
Combined	$9.82 \pm 0.13 \pm 0.04 \pm 0.07$	$506.4 \pm 1.9 \pm 2.2 \pm 1.9$

Davier et al. 2013: $a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau] = 516.2(3.5) \times 10^{-10}$ $(2m_\pi^\pm - 1.8$ GeV)

Compare to e^+e^-:

- $a_\mu^{\mathrm{had,LO}}[\pi\pi, e^+e^-] = 507.1(2.6) \times 10^{-10}$ (DHMZ17, $2m_\pi^\pm - 1.8$ GeV)
- $a_\mu^{\mathrm{had,LO}}[\pi\pi, e^+e^-] = 503.7(2.0) \times 10^{-10}$ (KNT18, $2m_\pi^\pm - 1.937$ GeV)

Here treatment of isospin-breaking to relate matrix elements of $V_\mu^{I=1,I_3=1}$ to $V_\mu^{I=1,I_3=0}$ crucial. Progress towards a first-principles calculation from LQCD+QED, see 1811.00508.

Regions of precision (R-ratio data here is from **Fred Jegerlehner** 2017)

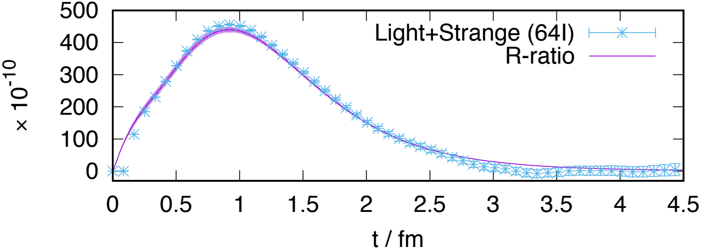


FIG. 4. Comparison of $w_t C(t)$ obtained using R-ratio data [1] and lattice data on our 64I ensemble.

The precision of lattice data deteriorates exponentially as we go to large t, however, is precise at intermediate distances. The R-ratio is very precise at long distances.

Note: in this plot a direct comparison of R-ratio and lattice data is not appropriate. Continuum limit, infinite-volume corrections, charm contributions, and IB corrections are missing from lattice data shown here.

We perform the calculation as a perturbation around an isospin-symmetric lattice QCD computation with two degenerate light quarks with mass $m_{\rm light}$ and a heavy quark with mass $m_{\rm heavy}$ tuned to produce a pion mass of 135.0 MeV and a kaon mass of 495.7 MeV.

The correlator is expanded in the fine-structure constant α as well as $\Delta m_{\rm up,\ down} = m_{\rm up,\ down} - m_{\rm light}$, and $\Delta m_{\rm strange} = m_{\rm strange} - m_{\rm heavy}$. We write

$$C(t) = C^{(0)}(t) + \alpha C^{(1)}_{\rm QED}(t) + \sum_f \Delta m_f C^{(1)}_{\Delta \rm m_f}(t) + \mathcal{O}(\alpha^2, \alpha\Delta m, \Delta m^2)\,.$$

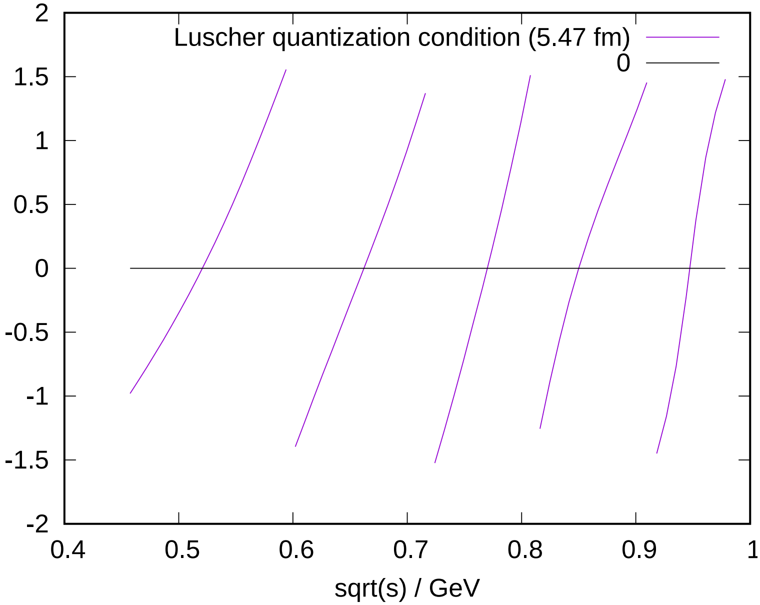
The correlators of this expansion are computed in lattice QCD with dynamical up, down, and strange quarks. We compute the missing contributions to a_μ from charm sea quarks in perturbative QCD (RHAD) by integrating the time-like region above 2 GeV and find them to be smaller than 0.3×10^{-10}.

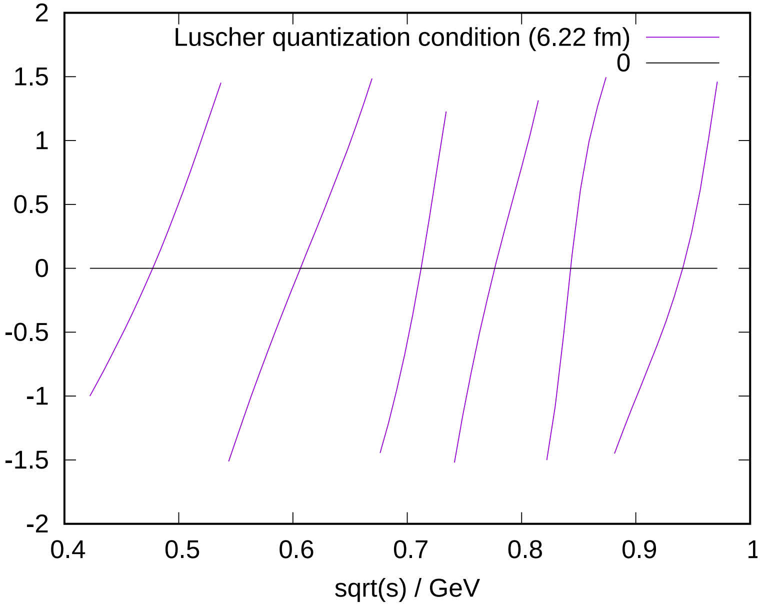
We tune the bare up, down, and strange quark masses m_{up}, m_{down}, and m_{strange} such that the π^0, π^+, K^0, and K^+ meson masses computed in our calculation agree with the respective experimental measurements. The lattice spacing is determined by setting the Ω^- mass to its experimental value.

We perform the lattice calculations for the light quark contributions using RBC/UKQCD's 48I and 64I lattice configurations with lattice cutoffs $a^{-1} = 1.730(4)$ GeV and $a^{-1} = 2.359(7)$ GeV and a larger set of ensembles with up to $a^{-1} = 2.774(10)$ GeV for the charm contribution.

From the parameter tuning procedure on the 48I we find $\Delta m_{\mathrm{up}} = -0.00050(1)$, $\Delta m_{\mathrm{down}} = 0.00050(1)$, and $\Delta m_{\mathrm{strange}} = -0.0002(2)$.

The shift of the Ω^- mass due to the QED correction is significantly smaller than the lattice spacing uncertainty and its effect on $C(t)$ is therefore not included separately.


Luscher quantization condition (5.47 fm)
0
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.4
0.5
0.6
0.7
0.8
0.9
1
sqrt(s) / GeV

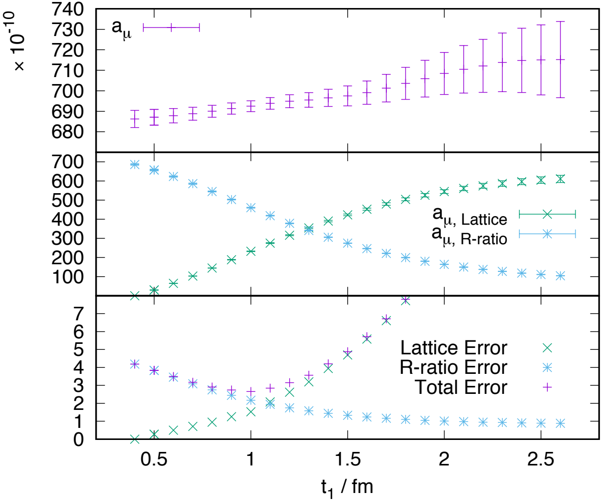


Luscher quantization condition (6.22 fm)
0
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.4
0.5
0.6
0.7
0.8
0.9
1
sqrt(s) / GeV

Consolidate continuum limit

Adding a finer lattice

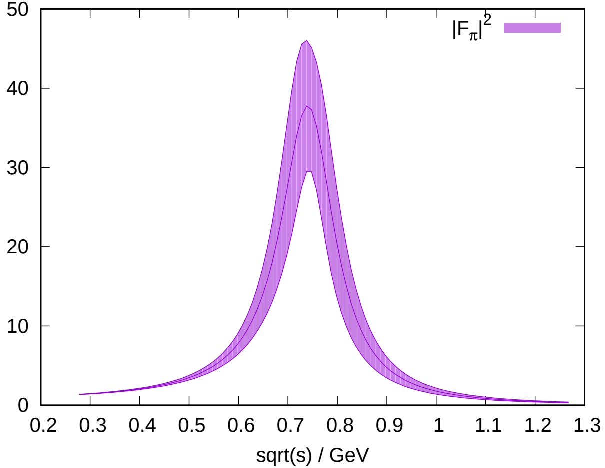
Window method with fixed $t_0 = 0.4$ fm



For $t = 1$ fm approximately 50% of uncertainty comes from lattice and 50% of uncertainty comes from the R-ratio. Is there a small slope? More in a few slides!

Can use this to check experimental data sets; see my KEK talk for more details

Predicts $|F_\pi(s)|^2$:



We can then also predict matrix elements and energies for our other lattices; successfully checked!