Status of the muon g-2 hadronic vacuum polarization calculation by RBC/UKQCD

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June 17, 2019 – Lattice 2019, Wuhan
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There is a tension of $3.7\sigma$ for the muon $a_\mu = (g_\mu - 2)/2$:

$$a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = 27.4 (2.7) (2.6) (0.1) (6.3) \times 10^{-10}$$

HVP HLbL other EXP

2019: $\delta a_\mu^{\text{EXP}} \rightarrow 4.5 \times 10^{-10}$ (avg. of BNL/estimate of 2019 Fermilab result)

Targeted final uncertainty of Fermilab E989: $\delta a_\mu^{\text{EXP}} \rightarrow 1.6 \times 10^{-10}$

⇒ by 2019 consolidate HVP/HLbL, over the next years uncertainties to $O(1 \times 10^{-10})$
Status of HVP determinations

- ETMC 2013
- HPQCD 2016
- Mainz 2017
- BMW 2017
- RBC/UKQCD 2018
- ETMC 2018
- SK 2019
- FNAL/HPQCD/MILC 2019
- Mainz 2019
- RBC/UKQCD 2018
- HLMNT 2011
- DHMZ 2012
- DHMZ 2017
- Jegerlehner 2017
- KNT 2018
- No new physics

\[ a_\mu \times 10^{10} \]
Dispersive method - Overview

$e^+ e^- \rightarrow \text{hadrons}(\gamma)$

$J_{\mu} = V^{l=1,l_3=0} + V^{l=0,l_3=0}$

$\tau \rightarrow \nu \text{hadrons}(\gamma)$

$J_{\mu} = V^{l=1,l_3=\pm 1} - A^{l=1,l_3=\pm 1}$

Knowledge of isospin-breaking corrections and separation of vector and axial-vector components needed to use $\tau$ decay data. Can do this from LQCD+QED (Bruno, Izubuchi, CL, Meyer, 1811.00508)!

Can have both energy-scan and ISR setup.
Dispersive method - $e^+e^-$ status

Tension in $2\pi$ experimental input. BaBar and KLOE central values differ by $\delta a_\mu = 9.8(3.5) \times 10^{-10}$, compare to quoted total uncertainties of dispersive results of order $\delta a_\mu = 3 \times 10^{-10}$.

Conflicting input limits the precision and reliability of the dispersive results. Can we replace some of this data with LQCD+QED?

Looking for more data and insight: energy-scans update from CMD-3 in Novosibirsk and ISR updates from KLOE2, BaBar, Belle, BESIII and BelleII.
Calculation of the Hadronic Vacuum Polarization Contribution to the Muon Anomalous Magnetic Moment

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(Received 25 January 2018; published 12 July 2018)

We present a first-principles lattice QCD + QED calculation at physical pion mass of the leading-order hadronic vacuum polarization contribution to the muon anomalous magnetic moment. The total contribution of up, down, strange, and charm quarks including QED and strong isospin breaking effects is \( a_{\mu}^{\text{HVP LO}} = 715.4(18.7) \times 10^{-10} \). By supplementing lattice data for very short and long distances with \( R \)-ratio data, we significantly improve the precision to \( a_{\mu}^{\text{HVP LO}} = 692.5(2.7) \times 10^{-10} \). This is the currently most precise determination of \( a_{\mu}^{\text{HVP LO}} \).

Pure lattice result and dispersive result with reduced \( \pi \pi \) dependence (window method)
Lattice QCD – Time-Moment Representation

Starting from the vector current \( J_\mu(x) = i \sum_f Q_f \Psi_f(x) \gamma_\mu \Psi_f(x) \) we may write

\[
a_{HVP \, LO}^{\mu} = \sum_{t=0}^{\infty} w_t C(t)
\]

with

\[
C(t) = \frac{1}{3} \sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x}, t) J_j(0) \rangle
\]

and \( w_t \) capturing the photon and muon part of the HVP diagrams (Bernecker-Meyer 2011).

The correlator \( C(t) \) is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, and charm quark contributions. The missing bottom quark contributions are computed in pQCD.
Diagrams

Isospin limit

QED corrections

Strong isospin breaking

(a) V  (b) S  (c) T  (d) T_d  (e) D1  (f) D1_d  
(g) D2  (h) D2_d  (i) F  (j) D3

(a) M  (b) R  (c) R_d  (d) O
Window method (implemented in RBC/UKQCD 2018)

We therefore also consider a window method. Following Meyer-Bernecker 2011 and smearing over $t$ to define the continuum limit we write

$$a_{\mu} = a_{\mu}^{SD} + a_{\mu}^{W} + a_{\mu}^{LD}$$

with

$$a_{\mu}^{SD} = \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)],$$

$$a_{\mu}^{W} = \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)],$$

$$a_{\mu}^{LD} = \sum_t C(t) w_t \Theta(t, t_1, \Delta),$$

$$\Theta(t, t', \Delta) = \left[1 + \tanh \left[\frac{(t - t')}{\Delta}\right]\right]/2.$$

In this version of the calculation, we use

$$C(t) = \frac{1}{12\pi^2} \int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$$

with $R(s) = \frac{3s}{4\pi\alpha^2} \sigma(s, e^+ e^- \to \text{had})$ to compute $a_{\mu}^{SD}$ and $a_{\mu}^{LD}$. 
How does this translate to the time-like region?

Most of $\pi\pi$ peak is captured by window from $t_0 = 0.4$ fm to $t_1 = 1.5$ fm, so replacing this region with lattice data reduces the dependence on BaBar versus KLOE data sets.
The pure lattice calculation of RBC/UKQCD 2018:

\[ 10^{10} \times a_\mu^{\text{HVP LO}} = 715.4(18.7) \]

\[ = 715.4(16.3)_{\text{S}}(7.8)_{\text{V}}(3.0)_{\text{C}}(1.9)_{\text{A}}(3.2)_{\text{other}} \]

(S) statistics, (V) finite-volume errors, (C) the continuum limit extrapolation, (A) scale setting uncertainty;
other \( \supset \) neglected diagrams for QED and SIB, estimate of bottom quark contribution

Statistical noise mostly from isospin symmetric light quark connected (14.2) and disconnected (3.3), QED (5.7), SIB (4.3)

RBC/UKQCD 2019 update (in preparation):

- Improved methodology
- A lot of new data
Improved methodology
The correlator in finite volume

\[ C(t) = \sum_n |\langle 0 | V | n \rangle|^2 e^{-E_n t}. \]

We can bound this correlator at each \( t \) from above and below by the correlators

\[ \tilde{C}(t; T, \tilde{E}) = \begin{cases} C(t) & t < T, \\ C(T)e^{-(t-T)\tilde{E}} & t \geq T \end{cases} \]

for proper choice of \( \tilde{E} \). We can choose \( \tilde{E} = E_0 \) (assuming \( E_0 < E_1 < \ldots \)) to create a strict upper bound and any \( \tilde{E} \) larger than the local effective mass to define a strict lower bound.
Therefore if we had precise knowledge of the lowest \( n = 0, \ldots, N \) values of \( |\langle 0 \mid V \mid n \rangle| \) and \( E_n \), we could define a new correlator

\[
C^N(t) = C(t) - \sum_{n=0}^{N} |\langle 0 \mid V \mid n \rangle|^2 e^{-E_n t}
\]

which we could bound much more strongly through the larger lowest energy \( E_{N+1} \gg E_0 \). New method: do a GEVP study of FV spectrum to perform this subtraction.

Reduces statistical error of light quark contribution by more than a factor of 3.
Improved systematics – compute finite-volume effects from first-principles

RBC/UKQCD study of QCD at **physical pion mass** at three different volumes:

\[ L = 4.66 \, \text{fm}, \quad L = 5.47 \, \text{fm}, \quad L = 6.22 \, \text{fm} \]

Results for light-quark isospin-symmetric connected contribution:

\[ a_\mu(L = 6.22 \, \text{fm}) - a_\mu(L = 4.66 \, \text{fm}) = 12.2 \times 10^{-10} \, \text{(sQED)}, \]
\[ 21.6(6.3) \times 10^{-10} \, \text{(lattice QCD)} \]

- Need to do better than sQED in finite-volume
First constrain the p-wave phase shift from our $L = 6.22$ fm physical pion mass lattice:

\[
\begin{align*}
&\delta_1 \sqrt{s} \text{ GeV} \\
&\text{Gounaris-Sakurai Phase-Shift Parametrization} \\
&\text{32ID lattice data (6.2fm box at phys. pion mass)} \\
&\text{24ID lattice data (4.7fm box at phys. pion mass)} \\
\end{align*}
\]

\[
\begin{align*}
E_\rho &= 0.766(21) \text{ GeV (PDG 0.77549(34) GeV)} \\
\Gamma_\rho &= 0.139(18) \text{ GeV (PDG 0.1462(7) GeV)}
\end{align*}
\]
GSL$^2$ finite-volume results compared to sQED and lattice

GSL$^2$ method of Meyer 2012

Results for light-quark isospin-symmetric connected contribution:

- FV difference between $a_\mu(L = 6.22 \text{ fm}) - a_\mu(L = 4.66 \text{ fm}) = 12.2 \times 10^{-10}$ (sQED), $21.6(6.3) \times 10^{-10}$ (lattice QCD), $20(3) \times 10^{-10}$ (GSL$^2$)

- GSL$^2$ prediction agrees with actual FV effect measured on the lattice, sQED is in slight tension, two-loop FV ChPT to be compared next Bijnens and Relefors 2017

- Use GSL$^2$ to update FV correction of Phys. Rev. Lett. 121, 022003 (2018): $a_\mu(L \to \infty) - a_\mu(L = 5.47 \text{ fm}) = 16(4) \times 10^{-10}$ (sQED), $22(1) \times 10^{-10}$ (GSL$^2$); sQED error estimate based on Bijnens and Relefors 2017, table 1.

- Compare also to Hansen-Patella 2019 1904.10010: $a_\mu(L \to \infty) - a_\mu(L = 5.47 \text{ fm}) \approx 14 \times 10^{-10}$, effect of neglected $e^{-\sqrt{2}m_\pi L}$ likely significant
Other improvements:

▶ HVP QED from re-analysis of HLbL point-source data (see also $\tau$ project, 1811.00508) reduces statistical noise by $\approx 10\times$ for V and S

▶ Infinite-volume and continuum limit also for diagram V, S, and F

▶ First results for T, D1, and R; other sub-leading in preparation

▶ Global fit combined with calculation of mass derivatives gives much reduced uncertainty for diagrams M and O (connected and disconnected SIB)
New data set
Ensembles at physical pion mass:

48I (1.73 GeV, 5.5fm), 64I (2.359 GeV, 5.4fm), 24ID (1 GeV, 4.7fm), 32ID (1 GeV, 6.2fm), 48ID (1 GeV, 9.3fm), 32IDf (1.37 GeV, 4.6fm)

RBC/UKQCD 2019 (data for light quarks, changes from 2018):

- A2A data for connected isospin symmetric: 48I (127 conf → 400 conf), 64I (160 conf → 250 conf), 24ID (new 130 conf, multi mass), 32ID (new 88 conf, multi mass)

- A2A data (tadpole fields) for disconnected: 48I (33 conf), 24ID (new 260 conf, multi mass), 32IDf (new 103 conf)

- QED and SIB corrections to meson and Ω masses, $Z_V$: 48I (30 conf) and 64I (new 30 conf)

- QED and SIB from HLbL point sources on 48I, 24ID, 32ID, 32IDf (on order of 20 conf each, 2000 points per config)

- Distillation data on 48I (33 conf), 64I (in progr.), 24ID (33 conf), 32ID (11 conf, multi-mass)

- New Ω mass operators (excited states control): 48I (130 conf)
Add $a^{-1} = 2.77$ GeV lattice spacing

- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):

- For light quark need new ensemble at physical pion mass. Started run on Summit Machine at Oak Ridge this year ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV).
**Conclusions and Outlook**

- Target precision for HVP is of \(O(1 \times 10^{-10})\) in a few years; for now consolidate error at \(O(3 \times 10^{-10})\)

- Dispersive result from \(e^+e^- \rightarrow \text{hadrons}\) right now is at \(3 \times 10^{-10}\) but limited by experimental tensions

- Two-pion channel from DHMZ17, KNT18 \((e^+e^-)\) and DHMYZ13 \((\tau)\) are scattered by \(12.5 \times 10^{-10}\)

  Experimental updates and first-principles calculation of isospin-breaking corrections desirable. **Combination of dispersive and lattice results can in short term lessen dependence on contested experimental data.**

- New methods to reduce statistical and systematic errors and a lot of additional data.

- By end of this year, first-principles lattice result could have error of \(O(5 \times 10^{-10})\)
Backup
Dispersive method - $e^+e^-$ status

Recent results by Keshavarzi et al. 2018, Davier et al. 2017:

<table>
<thead>
<tr>
<th>Channel</th>
<th>This work (KNT18)</th>
<th>DHMZ17 [78]</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data based channels ($\sqrt{s} \leq 1.8$ GeV)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^0\gamma$ (data + ChPT)</td>
<td>4.58 ± 0.10</td>
<td>4.29 ± 0.10</td>
<td>0.29</td>
</tr>
<tr>
<td>$\pi^+\pi^-$ (data + ChPT)</td>
<td>503.74 ± 1.96</td>
<td>507.14 ± 2.58</td>
<td>−3.40</td>
</tr>
<tr>
<td>$\pi^+\pi^-\pi^0$ (data + ChPT)</td>
<td>47.70 ± 0.89</td>
<td>46.20 ± 1.45</td>
<td>1.50</td>
</tr>
<tr>
<td>$\pi^+\pi^-\pi^+\pi^-$</td>
<td>13.99 ± 0.19</td>
<td>13.68 ± 0.31</td>
<td>0.31</td>
</tr>
</tbody>
</table>

... 

| Total | 693.3 ± 2.5 | 693.1 ± 3.4 | 0.2 |

Good agreement for total, individual channels disagree to some degree. **Muon g-2 Theory Initiative workshops** recently held at Fermilab, KEK, UConn, and Mainz, intend to facilitate discussions and further understanding of these tensions.

One difference: treatment of correlations, impactful in particular in case when not all experimental data agrees
Gounaris-Sakurai-Lüscher method [H. Meyer 2012, Mainz 2017]

- Produce FV spectrum and matrix elements from phase-shift study (Lüscher method for spectrum and amplitudes, GS for phase-shift parametrization)

- This allows for a prediction of FV effects beyond chiral perturbation theory given that the phase-shift parametrization captures all relevant effects (can be checked against lattice data)

- This method is now being employed by ETMC, Mainz, and RBC/UKQCD.
Dispersive method - $\tau$ status

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$2m_\pi^\pm - 0.36$ GeV</th>
<th>$a^\text{had,LO}[\pi\pi, \tau] (10^{-10})$</th>
<th>$0.36 - 1.8$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>9.80 ± 0.40 ± 0.05 ± 0.07</td>
<td>501.2 ± 4.5 ± 2.7 ± 1.9</td>
<td></td>
</tr>
<tr>
<td>CLEO</td>
<td>9.65 ± 0.42 ± 0.17 ± 0.07</td>
<td>504.5 ± 5.4 ± 8.8 ± 1.9</td>
<td></td>
</tr>
<tr>
<td>OPAL</td>
<td>11.31 ± 0.76 ± 0.15 ± 0.07</td>
<td>515.6 ± 9.9 ± 6.9 ± 1.9</td>
<td></td>
</tr>
<tr>
<td>Belle</td>
<td>9.74 ± 0.28 ± 0.15 ± 0.07</td>
<td>503.9 ± 1.9 ± 7.8 ± 1.9</td>
<td></td>
</tr>
<tr>
<td>Combined</td>
<td>9.82 ± 0.13 ± 0.04 ± 0.07</td>
<td>506.4 ± 1.9 ± 2.2 ± 1.9</td>
<td></td>
</tr>
</tbody>
</table>

Davier et al. 2013: $a^\text{had,LO}_{\mu}[\pi\pi, \tau] = 516.2(3.5) \times 10^{-10} \ (2m_\pi^\pm - 1.8 \text{ GeV})$

Compare to $e^+e^-$:

$\implies a^\text{had,LO}_{\mu}[\pi\pi, e^+ e^-] = 507.1(2.6) \times 10^{-10} \ (\text{DHMZ17, } 2m_\pi^\pm - 1.8 \text{ GeV})$

$\implies a^\text{had,LO}_{\mu}[\pi\pi, e^+ e^-] = 503.7(2.0) \times 10^{-10} \ (\text{KNT18, } 2m_\pi^\pm - 1.937 \text{ GeV})$

Here treatment of isospin-breaking to relate matrix elements of $V_{\mu}^{l=1, l_3=1}$ to $V_{\mu}^{l=1, l_3=0}$ crucial. Progress towards a first-principles calculation from LQCD+QED, see 1811.00508.
Regions of precision (R-ratio data here is from Fred Jegerlehner 2017)

FIG. 4. Comparison of $w_t C(t)$ obtained using R-ratio data [1] and lattice data on our 64I ensemble.

The precision of lattice data deteriorates exponentially as we go to large $t$, however, is precise at intermediate distances. The R-ratio is very precise at long distances.

Note: in this plot a direct comparison of R-ratio and lattice data is not appropriate. Continuum limit, infinite-volume corrections, charm contributions, and IB corrections are missing from lattice data shown here.
We perform the calculation as a perturbation around an isospin-symmetric lattice QCD computation with two degenerate light quarks with mass $m_{\text{light}}$ and a heavy quark with mass $m_{\text{heavy}}$ tuned to produce a pion mass of 135.0 MeV and a kaon mass of 495.7 MeV.

The correlator is expanded in the fine-structure constant $\alpha$ as well as $\Delta m_{\text{up, down}} = m_{\text{up, down}} - m_{\text{light}}$, and $\Delta m_{\text{strange}} = m_{\text{strange}} - m_{\text{heavy}}$. We write

$$C(t) = C^{(0)}(t) + \alpha C^{(1)}_{\text{QED}}(t) + \sum_{f} \Delta m_{f} C^{(1)}_{\Delta m_{f}}(t) + \mathcal{O}(\alpha^2, \alpha \Delta m, \Delta m^2).$$

The correlators of this expansion are computed in lattice QCD with dynamical up, down, and strange quarks. We compute the missing contributions to $a_\mu$ from charm sea quarks in perturbative QCD (RHAD) by integrating the time-like region above 2 GeV and find them to be smaller than $0.3 \times 10^{-10}$. 
We tune the bare up, down, and strange quark masses $m_{\text{up}}$, $m_{\text{down}}$, and $m_{\text{strange}}$ such that the $\pi^0$, $\pi^+$, $K^0$, and $K^+$ meson masses computed in our calculation agree with the respective experimental measurements. The lattice spacing is determined by setting the $\Omega^-$ mass to its experimental value.

We perform the lattice calculations for the light quark contributions using RBC/UKQCD’s 48I and 64I lattice configurations with lattice cutoffs $a^{-1} = 1.730(4)$ GeV and $a^{-1} = 2.359(7)$ GeV and a larger set of ensembles with up to $a^{-1} = 2.774(10)$ GeV for the charm contribution.

From the parameter tuning procedure on the 48I we find $\Delta m_{\text{up}} = -0.00050(1)$, $\Delta m_{\text{down}} = 0.00050(1)$, and $\Delta m_{\text{strange}} = -0.0002(2)$.

The shift of the $\Omega^-$ mass due to the QED correction is significantly smaller than the lattice spacing uncertainty and its effect on $C(t)$ is therefore not included separately.
Luscher quantization condition (5.47 fm)
Luscher quantization condition (6.22 fm)
Consolidate continuum limit

Adding a finer lattice
Window method with fixed $t_0 = 0.4$ fm

For $t = 1$ fm approximately 50% of uncertainty comes from lattice and 50% of uncertainty comes from the R-ratio. Is there a small slope? More in a few slides!

Can use this to check experimental data sets; see my KEK talk for more details
Predicts $|F_\pi(s)|^2$:

We can then also predict matrix elements and energies for our other lattices; successfully checked!