The hadronic contribution to the running of the electroweak mixing angle

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introduction – the electroweak mixing angle

the electroweak mixing (Weinberg) angle θ_W parametrizes the mixing between the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ sectors of the Standard Model. At tree level,

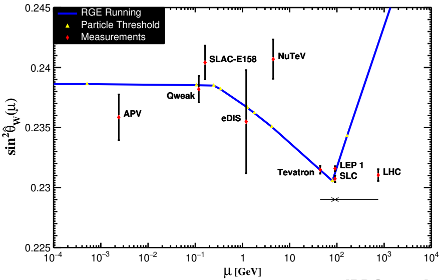
$$\sin^2\theta_\mathrm{W} = \frac{g'^2}{g^2 + g'^2},$$

where g and g' are the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ coupling respectively

- it is a free parameter of the Standard Model
- $\sin^2\theta_\mathrm{W} = 1 - \frac{M_W^2}{M_Z^2}$
- Z vector coupling $v_f = T_f - 2Q_f \sin^2\theta_f^\text{eff}$
- weak charge of the proton $Q_W(p) \sim 1 - 4\sin^2\theta_\mathrm{W}$

the precise numerical value of $\sin^2\theta_\mathrm{W}$
depends on the renormalization scheme and on the energy scale

the running – a precision test of the SM



[PDG 2018]

$$\sin^2\theta_W(Q^2) = \sin^2\theta_0\left[1 + \Delta\sin^2\theta_W(Q^2)\right]$$

experiments: at high Q^2

- measurements at colliders

at low Q^2, upcoming

- MOLLER @ JLab
- P2 @ MESA, Mainz [Becker *et al.* 2018]

⇒ non-perturbative QCD effects

[talk by J. Wilhelm, Had. Struct., Fri. 15:00]

theory: running, in the $\overline{\text{MS}}$ scheme

- $\sin^2\theta_0 = 0.238\,68(5)$ in the Thomson limit [Erler, Ferro-Hernández 2017]
- the running at scales $\lesssim \Lambda_{\text{QCD}}$ is affected by non-perturbative QCD physics

and global fits to EW precision data

the hadronic contribution

the leading hadronic contribution to the running of $\sin^2\theta_\mathrm{W}$ is [Jegerlehner 1986; 2011]

$$\Delta_\text{had}\sin^2\theta_\mathrm{W}(Q^2) = -\frac{4\pi\alpha}{\sin^2\theta_\mathrm{W}}\Pi_R^{Z\gamma}(Q^2), \qquad \Pi_R^{Z\gamma}(Q^2) = \Pi^{Z\gamma}(Q^2) - \Pi^{Z\gamma}(0),$$

proportional to the subtracted hadronic vacuum polarization

$$(Q_\mu Q_\nu - \delta_{\mu\nu}Q^2)\Pi^{Z\gamma}(Q^2) = \Pi_{\mu\nu}^{Z\gamma}(Q^2) = \int \mathrm{d}^4x\, e^{\mathrm{i}Qx}\left\langle j_\mu^Z(x) j_\nu^\gamma(0)\right\rangle$$

of the e.m. current and the vector part of the Z current

$$j_\mu^\gamma = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c,$$

$$j_\mu^{T_3} = \frac{1}{4}\bar{u}\gamma_\mu u - \frac{1}{4}\bar{d}\gamma_\mu d - \frac{1}{4}\bar{s}\gamma_\mu s + \frac{1}{4}\bar{c}\gamma_\mu c,$$

$$j_\mu^Z = j_\mu^{T_3} - \sin^2\theta_\mathrm{W} j_\mu^\gamma,$$

- can be extracted from phenomenology using dispersion relations
- or can be computed *ab initio* on the lattice [Burger *et al.* 2015; Gülpers *et al.* 2015]
- similarly, the hadronic contribution to the running of α_QED is given by $\Pi_R^{\gamma\gamma}(Q^2)$

[next talk by M. T. San José Pérez]

the time-momentum representation (TMR) method

introduced for the HVP contribution to $(g-2)_\mu$ [Bernecker, Meyer 2011; Francis *et al.* 2013]

$$\Pi_R^{Z\gamma}(Q^2) = \int_0^\infty \mathrm{d}x_0\, G^{Z\gamma}(x_0) \left[x_0^2 - \frac{4}{Q^2} \sin^2\left(\frac{Qx_0}{2} \right) \right],$$

$$G^{Z\gamma}(x_0) = -\frac{1}{3} \int \mathrm{d}^3x \sum_{k=1}^{3} \left\langle j_k^Z(x) j_k^\gamma(0) \right\rangle,$$

⇒ using correlators from $N_\mathrm{f} = 2+1$ Mainz effort in computing $(g-2)_\mu^{\mathrm{HVP}}$

[Gérardin *et al.* 2019; talk by A. Gérardin, Had. Struct., Tue. 14:40]

- non-perturbatively $\mathcal{O}(a)$-improved vector currents [Gérardin, Harris, Meyer 2018]
- two discretizations: local-local and local-conserved
- w.r.t. the $(g-2)_\mu^{\mathrm{HVP}}$ case, the kernel has a shorter range
- expect $\Delta_{\mathrm{had}} \sin^2\theta_\mathrm{W}$ to be more sensitive at cut-off effects, especially at high Q^2
- but much simpler large-distance systematic
  ⇒ no loss of signal in the tail of the connected correlator

lattice correlators

with $\mathrm{SU}(3)_F$ notation, in the isospin-symmetric limit (light quark ℓ: either u or d):

$$G^{33}_{\mu\nu}(x) = \frac{1}{2} C^{\ell,\ell}_{\mu\nu}(x),$$
$$G^{88}_{\mu\nu}(x) = \frac{1}{6}\Big[C^{\ell,\ell}_{\mu\nu}(x) + 2C^{s,s}_{\mu\nu}(x) + 2D^{\ell-s,\ell-s}_{\mu\nu}(x)\Big],$$
$$G^{08}_{\mu\nu}(x) = \frac{1}{2\sqrt{3}}\Big[C^{\ell,\ell}_{\mu\nu}(x) - C^{s,s}_{\mu\nu}(x) + D^{2\ell+s,\ell-s}_{\mu\nu}(x)\Big],$$

where the connected and disconnected Wick's contractions are

$$C^{f_1,f_2}_{\mu\nu}(x) = -\Big\langle \mathrm{Tr}\Big\{ D^{-1}_{f_1}(x,0)\gamma_\mu D^{-1}_{f_2}(0,x)\gamma_\nu \Big\}\Big\rangle,$$
$$D^{f_1,f_2}_{\mu\nu}(x) = \Big\langle \mathrm{Tr}\Big\{ D^{-1}_{f_1}(x,x)\gamma_\mu \Big\}\, \mathrm{Tr}\Big\{ D^{-1}_{f_2}(0,0)\gamma_\nu \Big\}\Big\rangle,$$

the $Z\gamma$ correlator is given by

$$G^{Z\gamma} = \Big(\frac{1}{2} - \sin^2\theta_{\mathrm{W}}\Big)\big(G^{\gamma\gamma}\big) - \frac{1}{6\sqrt{3}} G^{08}, \quad G^{\gamma\gamma} = G^{33} + \frac{1}{3} G^{88},$$

where $G^{\gamma\gamma}$ is the e.m. current correlator, relevant for e.g. $\Delta_{\text{had}}\alpha_{\text{QED}}(Q^2)$, a_μ^{HVP}, …

ensembles

from the CLS initiative

[Bruno *et al.* 2015, Bruno, Korzec, Schaefer 2017]

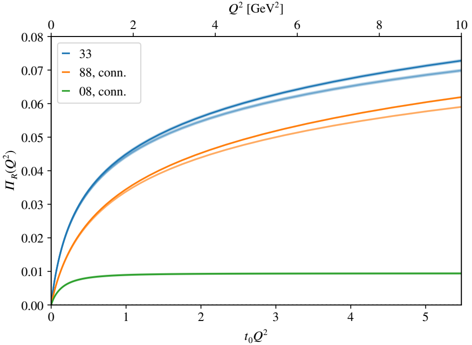
tree-level Lüscher-Weisz gauge action, non-perturbatively $\mathcal{O}(a)$-improved Wilson fermions, open BCs in time

	T/a	L/a	a [fm]	L [fm]	m_π [MeV]	m_K [MeV]	$m_\pi L$
H101	96	32	0.086	2.8	415	415	5.8
H102	96	32		2.8	355	440	5.0
H105*	96	32		2.8	280	460	3.9
N101	128	48		4.1	280	460	5.8
C101*	96	48		4.1	220	470	4.6
S400	128	32	0.076	2.4	350	440	4.3
N401*	128	48		3.7	285	460	5.3
H200	96	32	0.064	2.1	420	420	4.4
N202	128	48		3.1	410	410	6.4
N203*	128	48		3.1	345	440	5.4
N200*	128	48		3.1	285	465	4.4
D200*	128	64		4.1	200	480	4.2
E250§	192	96		6.2	130	490	4.1
N300	128	48	0.050	2.4	420	420	5.1
N302*	128	48		2.4	345	460	4.2
J303	192	64		3.2	260	475	4.2

* disconnected contribution available, § periodic BCs in time

preliminary results

E250: physical meson masses, $a = 0.064\,26(74)\,\mathrm{fm}$



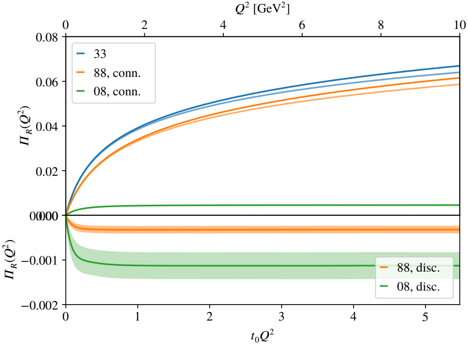
at $Q^2 = 1\,\mathrm{GeV}^2$

	l.c.	l.l.
33	0.035 70(36)	0.035 29(36)
88	0.026 00(12)	0.025 59(12)
08	0.008 26(21)	

$$\Delta_{\text{had}} \sin^2\theta_W(Q^2) = \begin{cases} -0.002\,484(39) & Q^2 = 0.24\,\mathrm{GeV}^2 \\ -0.005\,888(40) & Q^2 = 1\,\mathrm{GeV}^2 \\ -0.010\,329(41) & Q^2 = 4.22\,\mathrm{GeV}^2 \end{cases}$$

preliminary results – including disconnected

N200: $M_\pi \approx 285\,\mathrm{MeV}$, $a = 0.064\,26(74)\,\mathrm{fm}$



at $Q^2 = 1\,\mathrm{GeV}^2$

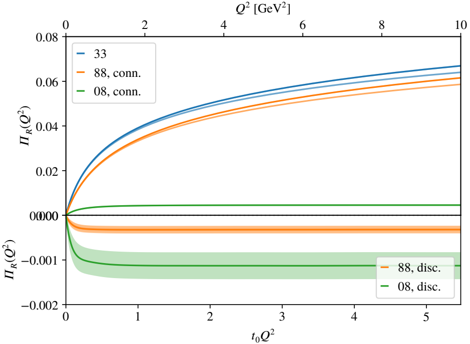
	l.c.	l.l.
33	0.030 11(11)	0.029 72(11)
88	0.025 40(5)	0.025 00(5)
08	0.003 93(6)	
88	−0.000 32(7)	−0.000 32(7)
08	−0.001 09(29)	

$$\Delta_{\mathrm{had}} \sin^2\theta_W(Q^2) = \begin{cases} -0.002\,115(10) & Q^2 = 0.24\,\mathrm{GeV}^2 \\ -0.005\,427(14) & Q^2 = 1\,\mathrm{GeV}^2 \\ -0.009\,874(15) & Q^2 = 4.22\,\mathrm{GeV}^2 \end{cases}$$

connected only!

preliminary results – including disconnected

N200: $M_\pi \approx 285\,\mathrm{MeV}$, $a = 0.064\,26(74)\,\mathrm{fm}$



at $Q^2 = 1\,\mathrm{GeV}^2$

	l.c.	l.l.
33	0.030 11(11)	0.029 72(11)
88	0.025 40(5)	0.025 00(5)
08	0.003 93(6)	
88	−0.000 32(7)	−0.000 32(7)
08	−0.001 09(29)	

$$\Delta_{\text{had}} \sin^2\theta_{\mathrm{W}}(Q^2) = \begin{cases} -0.002\,138(13) & Q^2 = 0.24\,\mathrm{GeV}^2 \\ -0.005\,457(16) & Q^2 = 1\,\mathrm{GeV}^2 \\ -0.009\,905(17) & Q^2 = 4.22\,\mathrm{GeV}^2 \end{cases}$$

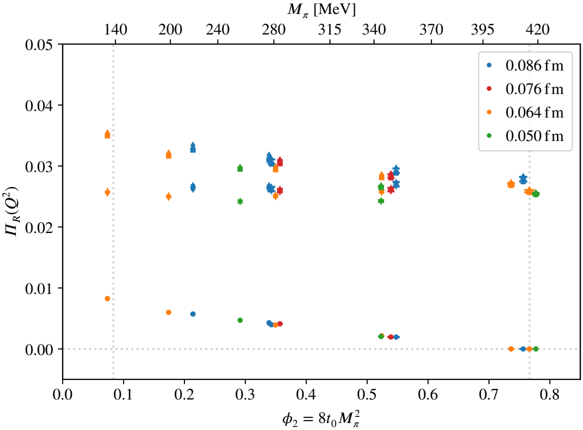
preliminary results – chiral and continuum extrapolation

at $Q^2 = 1\,\mathrm{GeV}^2$

■, ♦ : $\Pi_R^{33} \Rightarrow$

▼, ▲ : $\Pi_R^{88} \Rightarrow$

• : $\Pi_R^{08} \Rightarrow$

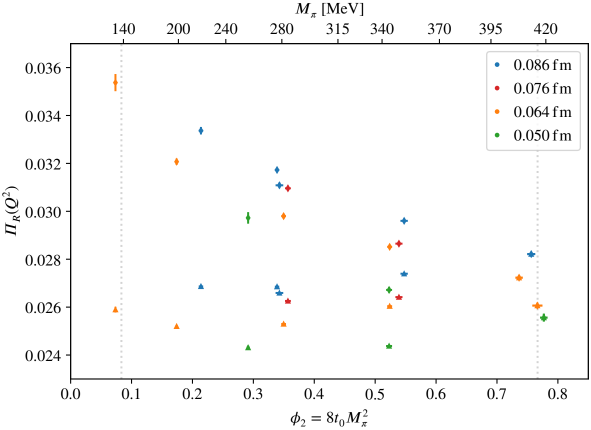


preliminary results – chiral and continuum extrapolation

at $Q^2 = 1\,\mathrm{GeV}^2$, local-conserved only

$\blacklozenge : \Pi_R^{33} \Rightarrow$

$\blacktriangle : \Pi_R^{88} \Rightarrow$



finite-size correction

added to the $I = 1$ correlator $G^{33}(t)$, with $t_i = (m_\pi L/4)^2/m_\pi$

[Gérardin *et al.* 2019; talk by A. Gérardin, Had. Struct., Tue. 14:40]

$t < t_i$: correction from scalar QED (a.k.a. NLO χPT)

[Francis *et al.* 2013; Della Morte *et al.* 2017]

$$G^{33}(t,L) - G^{33}(t,\infty) = \frac{1}{3}\left(\frac{1}{L^3}\sum_{\vec{k}} - \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3}\right)\frac{\vec{k}^2 + m_\pi^2}{\vec{k}^2}\mathrm{e}^{-2t\sqrt{\vec{k}^2+m_\pi^2}}$$

$t > t_i$: correction from GS model of $F_\pi(\omega)$

[Gounaris, Sakurai 1968]

$$G^{33}(t,\infty) = \int_0^\infty \mathrm{d}\omega\, \omega^2 \rho(\omega^2) \mathrm{e}^{-\omega t} \qquad \rho(\omega^2) = \frac{1}{48\pi^2}\left(1 - \frac{4m_\pi^2}{\omega^2}\right)^{\frac{3}{2}} \left|F_\pi(\omega)\right|^2$$

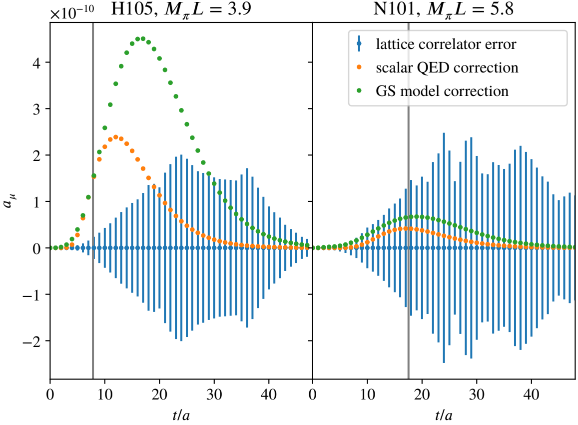
and the corresponding finite-volume correlator

[Lüscher 1991; Lellouch, Lüscher 2000; Meyer 2011]

$$G^{33}(t,L) = \sum_n \left|A_n\right|^2 \mathrm{e}^{-\omega_n t}$$ with Lüscher's ω_n and LL's A_n

finite-size correction – an example

using the TMR kernel to compute $(g-2)_\mu$

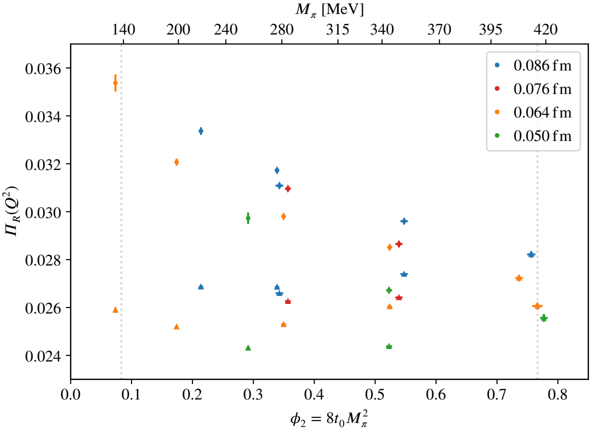


preliminary results – chiral and continuum extrapolation

at $Q^2 = 1\,\mathrm{GeV}^2$, local-conserved only, without finite-size correction

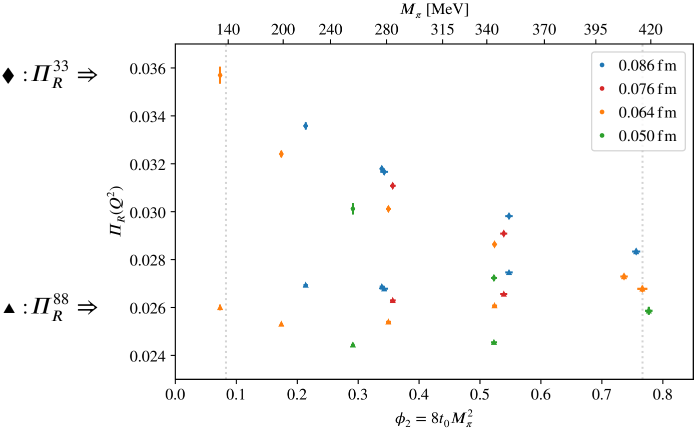
♦ : $\Pi_R^{33} \Rightarrow$

▲ : $\Pi_R^{88} \Rightarrow$



preliminary results – chiral and continuum extrapolation

at $Q^2 = 1\,\mathrm{GeV}^2$, local-conserved only, with finite-size correction

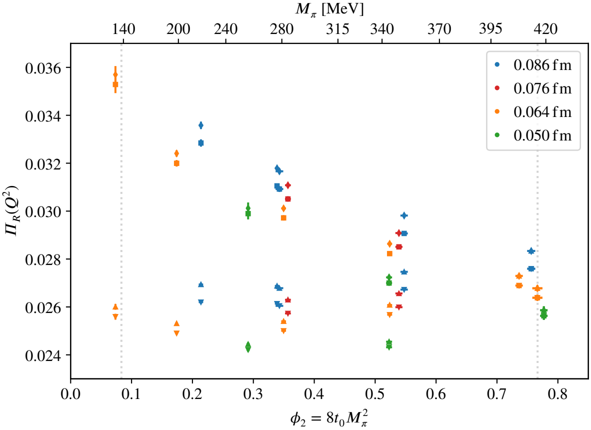


preliminary results – chiral and continuum extrapolation

at $Q^2 = 1\,\mathrm{GeV}^2$, local-conserved and local-local, with finite-size correction

$\blacksquare, \blacklozenge : \Pi_R^{33} \Rightarrow$

$\blacktriangledown, \blacktriangle : \Pi_R^{88} \Rightarrow$

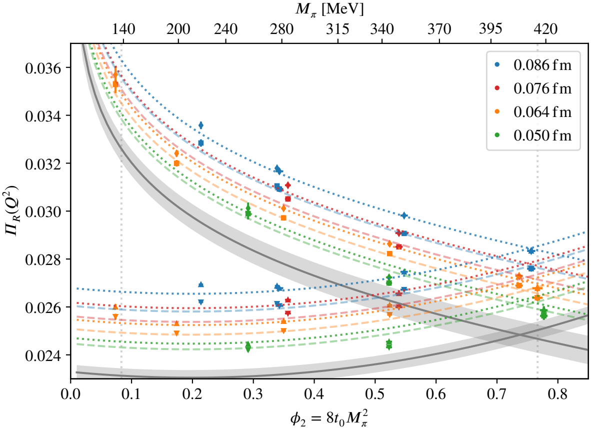


preliminary results – chiral and continuum extrapolation

at $Q^2 = 1\,\mathrm{GeV}^2$, local-conserved and local-local, with finite-size correction

■, ♦ : $\Pi_R^{33} \Rightarrow$

▼, ▲ : $\Pi_R^{88} \Rightarrow$



fit ansatz and preliminary results in the appendix

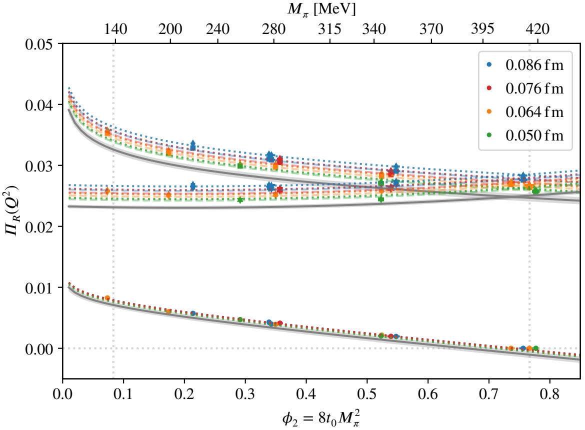
preliminary results – chiral and continuum extrapolation

at $Q^2 = 1\,\mathrm{GeV}^2$, local-conserved and local-local, with finite-size correction

■, ♦ : $\Pi_R^{33} \Rightarrow$

▼, ▲ : $\Pi_R^{88} \Rightarrow$

• : $\Pi_R^{08} \Rightarrow$



fit ansatz and preliminary results in the appendix

conclusions & outlook

the leading hadronic contribution to the running $\sin^2\theta_\mathrm{W}$ can be computed on the lattice

- with $\approx 1\,\%$ errors, competitive with phenomenology
- including the disconnected contribution, with sub-percent determination
- lattice provides flavour separation ⇒ input for the dispersive approach
- correction for finite-size effects is essential

- include the valence charm contribution (correlators already available)
- investigate the systematics of the chiral continuum extrapolation
- isospin breaking effects [talk by A. Risch, Had. Struct., Tue. 14:20]

thanks
for your attention!

questions?

backup slides

preliminary fits

input: a^2/t_0, $\phi_2 = 8t_0 M_\pi^2$, $\phi_4 = 8t_0(M_K^2 + M_\pi^2/2)$

- t_0 from CLS scale-setting paper [Bruno, Korzec, Schaefer 2017]
- aM_π from Mainz $N_\mathrm{f} = 2+1$ $(g-2)_\mu$ paper [Gérardin *et al.* 2019]
- aM_K form CLS scale-setting paper and dedicated (preliminary!) measurements

fit ansatz, at fixed Q^2:

$$f\left(a^2/t_0, \phi_2, \phi_4; p_0, \delta_1, \delta_2, \gamma_1, \gamma_2, \gamma_4, \gamma_9\right) = p_0 + \delta_1 a^2/t_0 + \delta_2 (a^2/t_0)^{3/2} + \gamma_1(\phi_2 - \phi_2^0) + \gamma_2(\log\phi_2 - \log\phi_2^0) + \gamma_4(\phi_2 - \phi_2^0)^2 + \gamma_9(\phi_4 - \phi_4^0)$$

- combined fit of the local-local and local-conserved discretizations
- no error on a^2/t_0, $\phi_2 = 8t_0 M_\pi^2$, $\phi_4 = 8t_0(M_K^2 + M_\pi^2/2)$ contributing to the χ^2 (yet)

preliminary results:

- fit of Π_R^{33}, with $\gamma_1 = 0$: $\chi^2/\#\mathrm{dof} = 26.99/24 = 1.12$, p-value $= 0.30$
- fit of Π_R^{88}, with $\gamma_2 = 0$: $\chi^2/\#\mathrm{dof} = 73.40/24 = 3.06$, p-value < 0.01
- fit of $\Pi_R^{\gamma\gamma}$, with $\gamma_1 = 0$: $\chi^2/\#\mathrm{dof} = 32.28/24 = 1.35$, p-value $= 0.12$

renormalization and $\mathcal{O}(a)$ improvement

for the local current [Bhattacharya et al. 2006, […], Gérardin, Harris, Meyer 2018]

$$V^3_{\mu,R} = Z_V(1 + 3\bar{b}_V a m_q^{\mathrm{av}} + b_V a m_{q,\ell}) V_\mu^{3,I},$$

$$V^8_{\mu,R} = Z_V\left[\left(1 + 3\bar{b}_V a m_q^{\mathrm{av}} + b_V \frac{a(m_{q,\ell}+2m_{q,s})}{3}\right) V_\mu^{8,I} + \left(\frac{b_V}{3} + f_V\right)\frac{2a(m_{q,\ell}-m_{q,s})}{\sqrt{3}} V_\mu^{0,I}\right],$$

$$V^0_{\mu,R} = Z_V r_V\left[\left(1 + (3\bar{d}_V + d_V) a m_q^{\mathrm{av}}\right) V_\mu^{0,I} + d_V \frac{a(m_{q,\ell}-m_{q,s})}{\sqrt{3}} V_\mu^{8,I}\right]$$

where

$$V_\mu^{a,I} = V_\mu^a + a c_V \partial_0 T^a_{0\mu}, \qquad V_\mu^{0,I} = V_\mu^0 + a\bar{c}_V \partial_0 T^0_{0\mu}.$$

while for the conserved current

$$V^a_{\mu,R} = V^a_\mu + a c_V^{cs} \partial_0 T^a_{0\mu}, \qquad V^0_{\mu,R} = V^0_\mu + a\bar{c}_V^{cs} \partial_0 T^0_{0\mu}.$$

⇒ we use only the conserved vector current for the flavour-singlet component, and we set

$$f_V = 0, \qquad \bar{c}_V^{cs} = c_V^{cs}.$$

finite-size correction – an example

using the TMR kernel to compute $(g-2)_\mu$