

Quasi-PDFs with twisted mass fermions

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in collaboration with:

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Kyriakos Hadjiyiannakou (Univ. of Cyprus)

Karl Jansen (DESY Zeuthen)

Aurora Scapellato (Univ. of Cyprus, Univ. of Wuppertal)

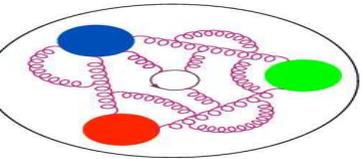
Fernanda Steffens (Univ. of Bonn)



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This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 642069



Outline of the talk



1. Quasi-PDFs and procedure
2. Investigation of systematic effects
3. Preliminary new results
4. Conclusions and prospects

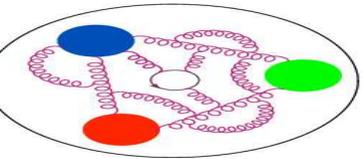


Based on:

- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato, F. Steffens, “Systematic uncertainties in parton distribution functions from lattice QCD simulations at the physical point”, Phys. Rev. D99 (2019) 114504
- C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, “Light-Cone Parton Distribution Functions from Lattice QCD”, Phys. Rev. Lett. 121 (2018) 112001
- C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, “Transversity parton distribution functions from lattice QCD”, Phys. Rev. D98 (2018) 091503 (Rapid Communications)
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, F. Steffens, “A complete non-perturbative renormalization prescription for quasi-PDFs”, Nucl. Phys. B923 (2017) 394-415 (invited Frontiers Article)

Review of the field:

- K. Cichy, M. Constantinou, “A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results”, invited review article for a special issue of Advances in High Energy Physics, Adv. High Energy Phys. 2019 (2019) 3036904, arXiv: 1811.07248 [hep-lat]



Quasi-PDFs

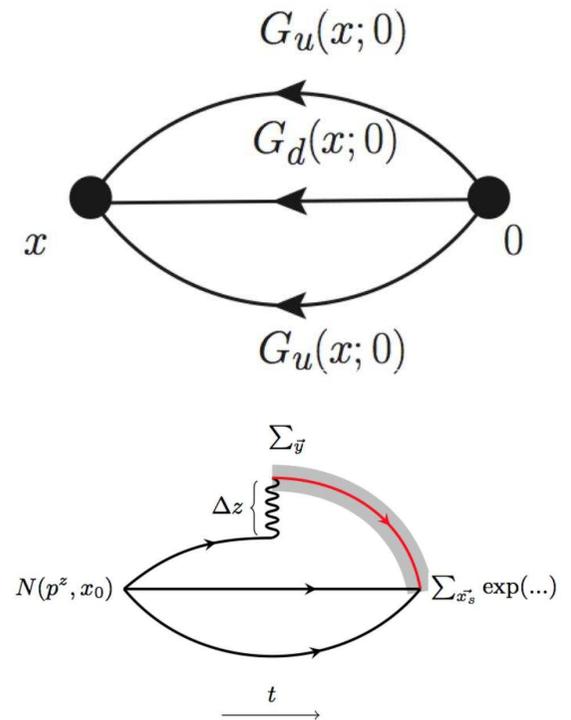
- Quasi-PDF approach:

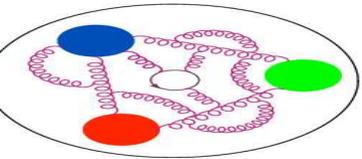
X. Ji, Parton Physics on a Euclidean Lattice, Phys. Rev. Lett. 110 (2013) 262002

- Compute a **quasi distribution** \tilde{q} , which is **purely spatial** and uses **nucleons with finite momentum**:

$$\tilde{q}(x, \mu^2, P_3) = \int \frac{dz}{4\pi} e^{ixP_3z} \langle N | \bar{\psi}(z) \Gamma \mathcal{A}(z, 0) \psi(0) | N \rangle.$$

- z – distance in any *spatial* direction z ,
- P_3 – momentum boost in this direction.
- e.g. $\Gamma = \gamma_0, \gamma_3$ – unpolarized, $\Gamma = \gamma_5 \gamma_3$ – helicity, $\Gamma = \sigma_{31}, \sigma_{32}$ – transversity
- Theoretically very appealing and intuitive!
- Differs from light-front PDFs by $\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{P_3^2}, \frac{m_N^2}{P_3^2}\right)$.
- The highly non-trivial aspect:
how to relate $\tilde{q}(x, \mu^2, P_3)$ to the light-front PDF $q(x, \mu^2)$ (infinite momentum frame)
 \Rightarrow **Large Momentum Effective Theory (LaMET)**





Summary of the procedure



The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

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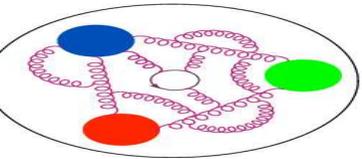
Quasi-PDFs

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Summary of the procedure



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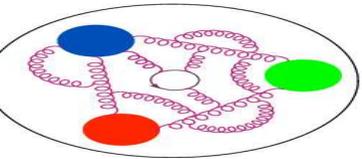
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1. Compute bare matrix elements: $\langle N | \bar{\psi}(z) \Gamma \mathcal{A}(z, 0) \psi(0) | N \rangle$.



Summary of the procedure



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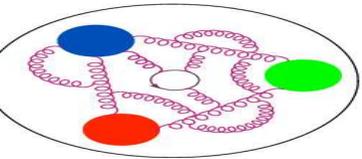
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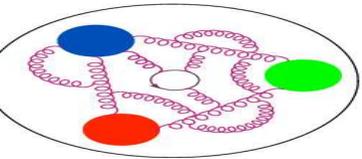
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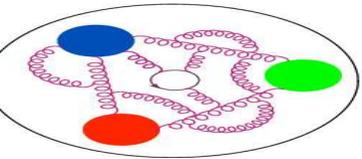
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$$\tilde{q}^{\overline{\text{MMS}}}(x, \bar{\mu}, P_3) = \int \frac{dz}{4\pi} e^{ixP_3z} \langle N | \bar{\psi}(z) \Gamma \mathcal{A}(z, 0) \psi(0) | N \rangle^{\overline{\text{MMS}}}.$$

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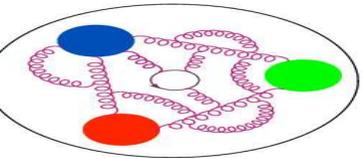
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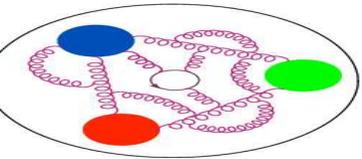
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7. Apply nucleon mass corr. to eliminate residual m_N^2/P_3^2 effects.

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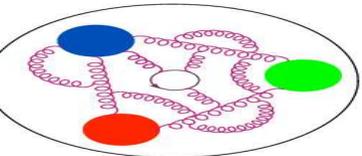
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Review of lattice partonic functions



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Review Article

A Guide to Light-Cone PDFs from Lattice QCD: An Overview of Approaches, Techniques, and Results

Krzysztof Cichy ¹ and **Martha Constantinou** ²

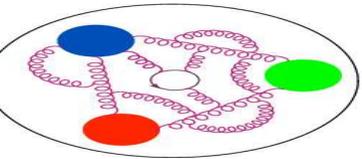
¹Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland

²Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA

Adv. High Energy Phys. 2019 (2019) 3036904, [arXiv:1811.07248](https://arxiv.org/abs/1811.07248)

Special issue *Transverse Momentum Dependent Observables from Low to High Energy: Factorization, Evolution, and Global Analyses*,

- discusses in detail quasi-distributions:
nucleon: **non-singlet quark qPDFs**, qGPDs, qTMDs, singlet qPDFs, gluon qPDFs; pion: qPDFs, qDAs
- reviews also other approaches:
hadronic tensor, auxiliary scalar quark, auxiliary heavy quark, auxiliary light quark, pseudo-distributions, “OPE without OPE”, lattice cross sections



Systematics



- Computation of quasi-PDFs subject to different systematic effects.

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Lattice setup

Excited states

Z-factors

Matching

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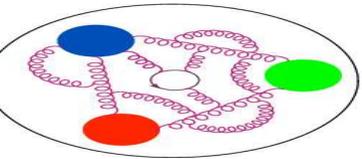
Final PDFs

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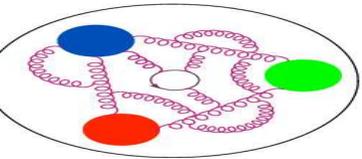
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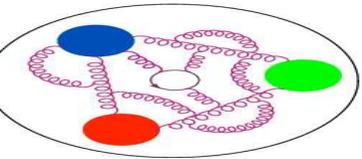
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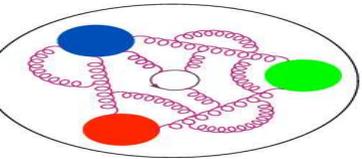
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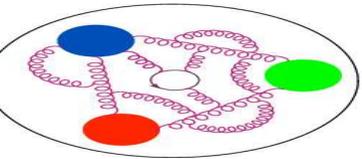
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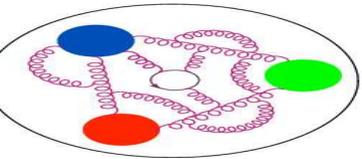
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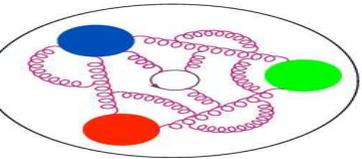
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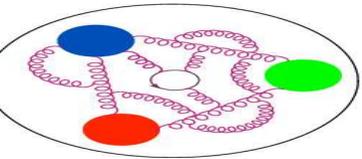
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 - ★ matching (vector current conservation, MS2MS vs. RI2MS, Fourier transform truncation),
 - ★ momentum dependence.
- Careful investigation of several systematic effects in:
[C. Alexandrou et al. \[ETM Collaboration\], “Systematic uncertainties in parton distribution functions from lattice QCD simulations at the physical point”, Phys. Rev. D99 \(2019\) 114504](#)



Lattice setup



- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 2.1$
- gauge field configurations generated by ETMC

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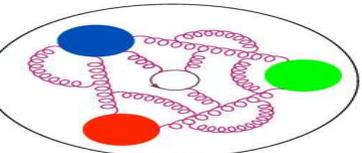
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$\beta=2.10,$	$c_{\text{SW}}=1.57751,$	$a=0.0938(3)(2)$ fm
$48^3 \times 96$	$a\mu = 0.0009$	$m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV	$m_\pi L = 2.98(1)$

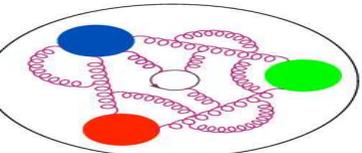
	$P_3 = \frac{6\pi}{L}$		$P_3 = \frac{8\pi}{L}$		$P_3 = \frac{10\pi}{L}$	
Insertion	N_{conf}	N_{meas}	N_{conf}	N_{meas}	N_{conf}	N_{meas}
γ^0	50	4800	425	38250	811	72990
$\gamma^5 \gamma^3$	65	6240	425	38250	811	72990
σ^{3j}	50	9600	425	38250	811	72990



Excited states effects



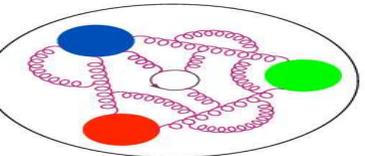
Very important aspect in lattice hadron structure: **excited states effects**.



Excited states effects



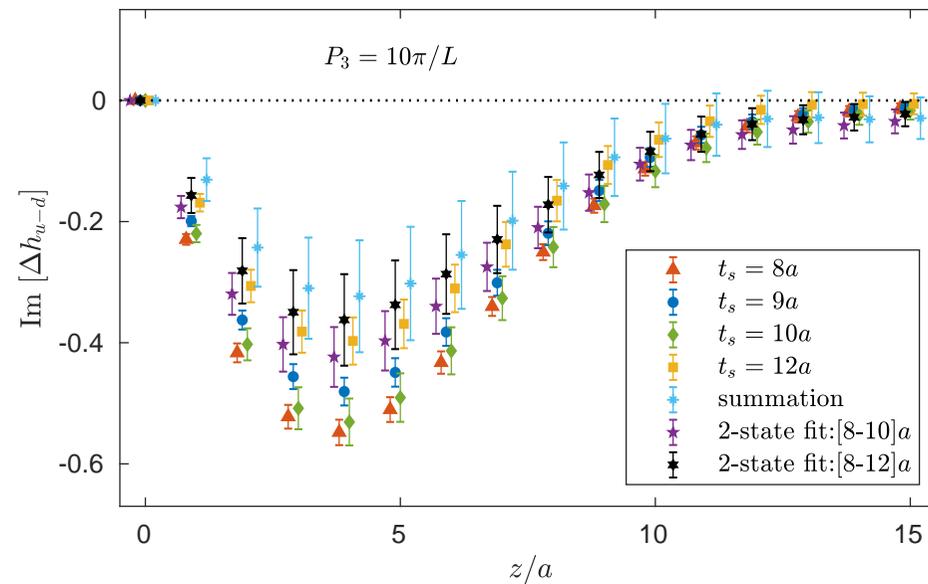
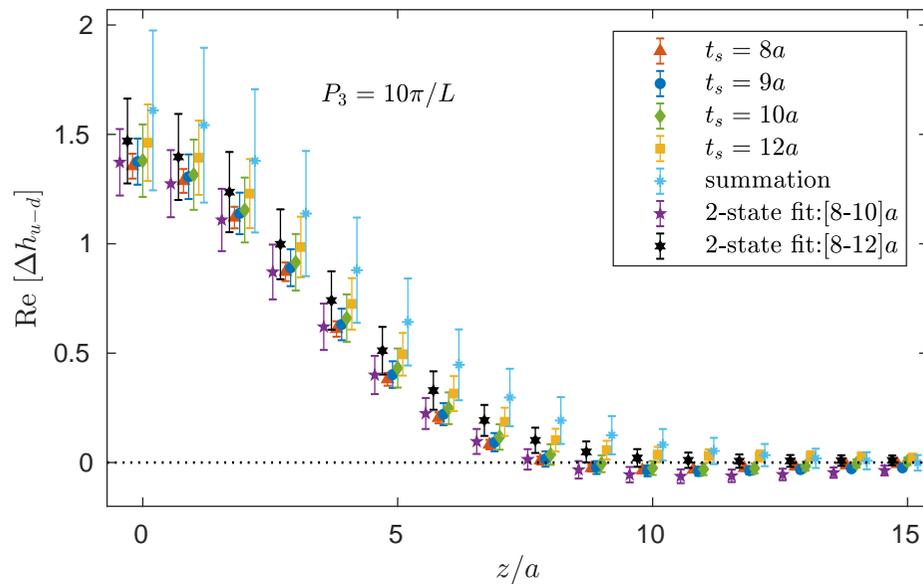
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One needs to go to large enough source-sink separation $t_s \Rightarrow$ **COSTLY!**

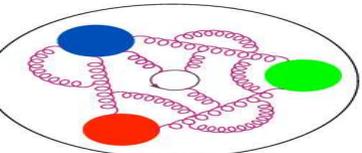


Excited states effects



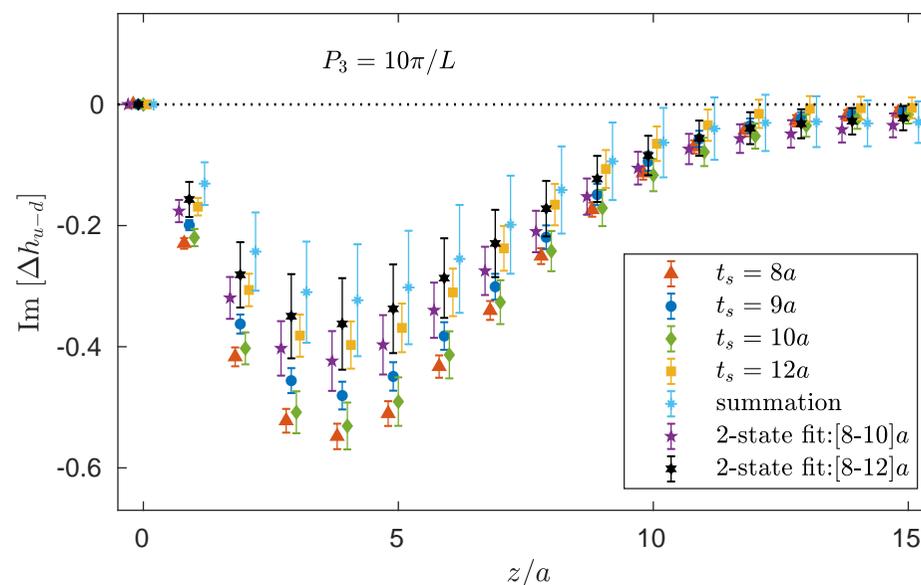
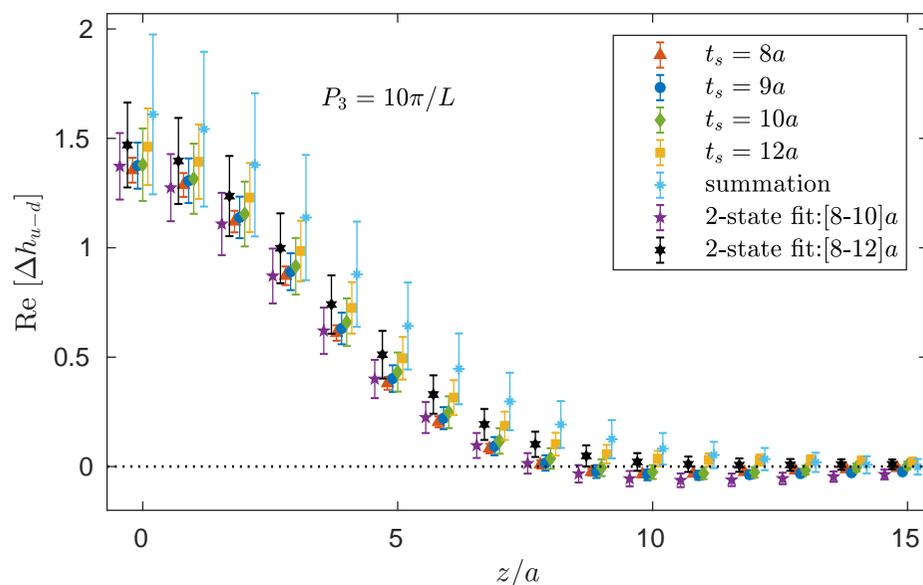
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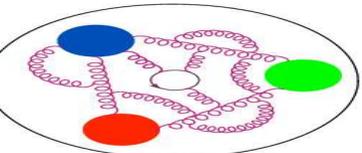


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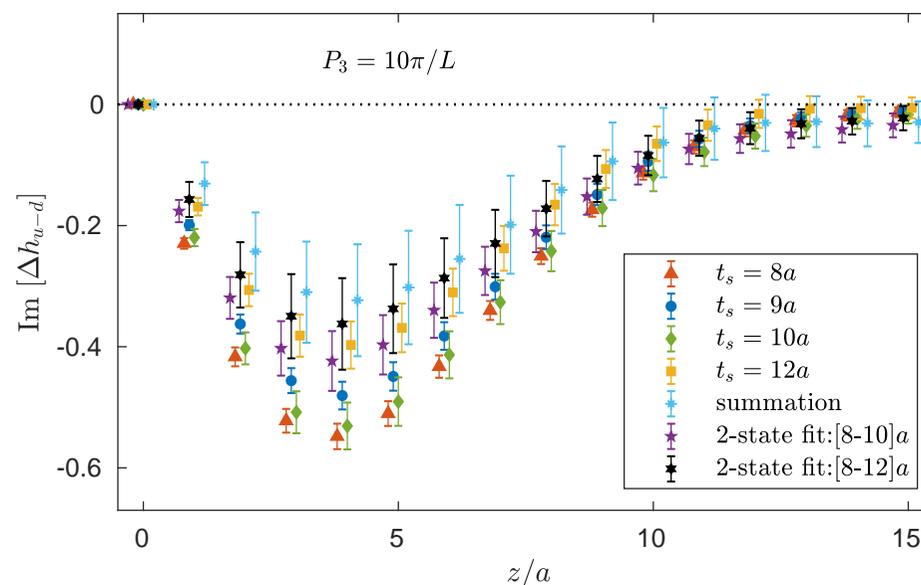
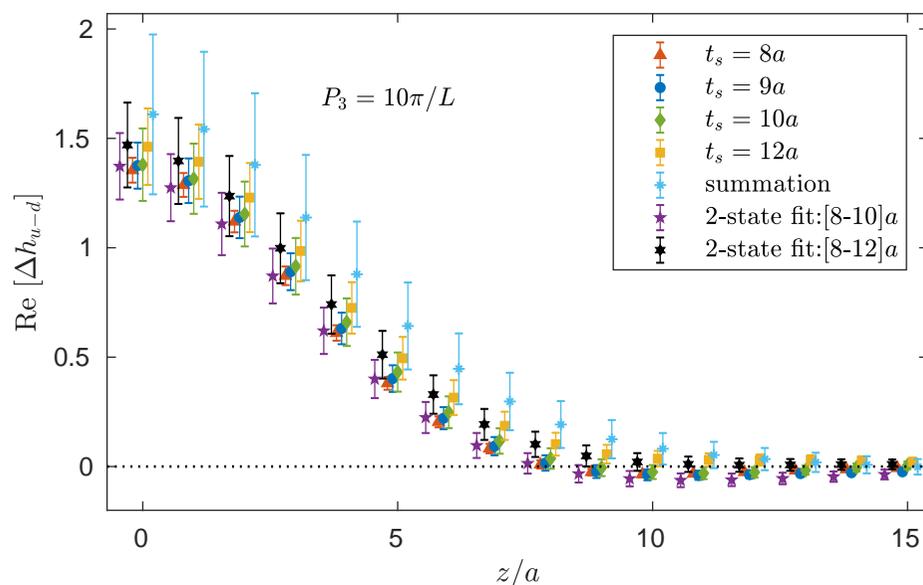


- Especially important for this kind of study with boosted nucleons – spectrum becomes denser for increasing boost and for decreasing pion mass.

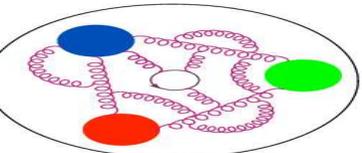


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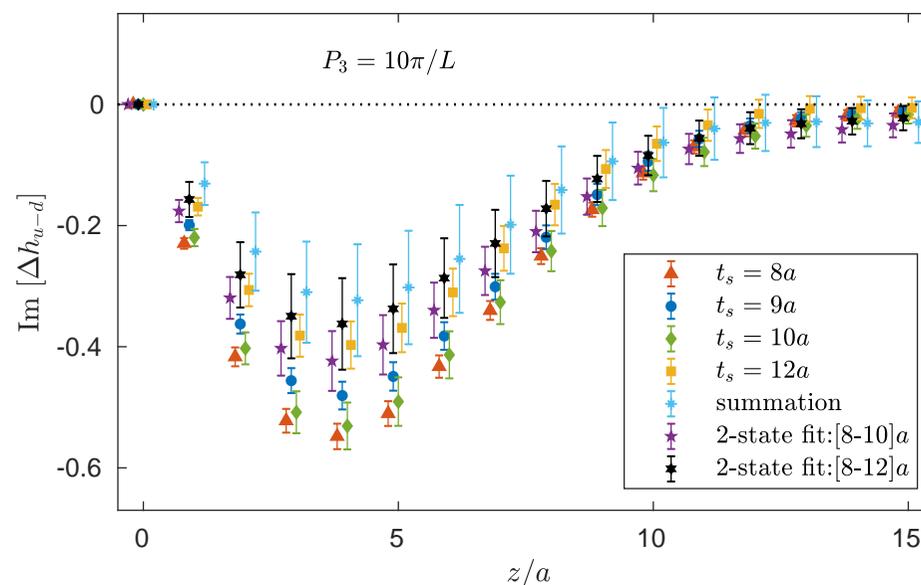
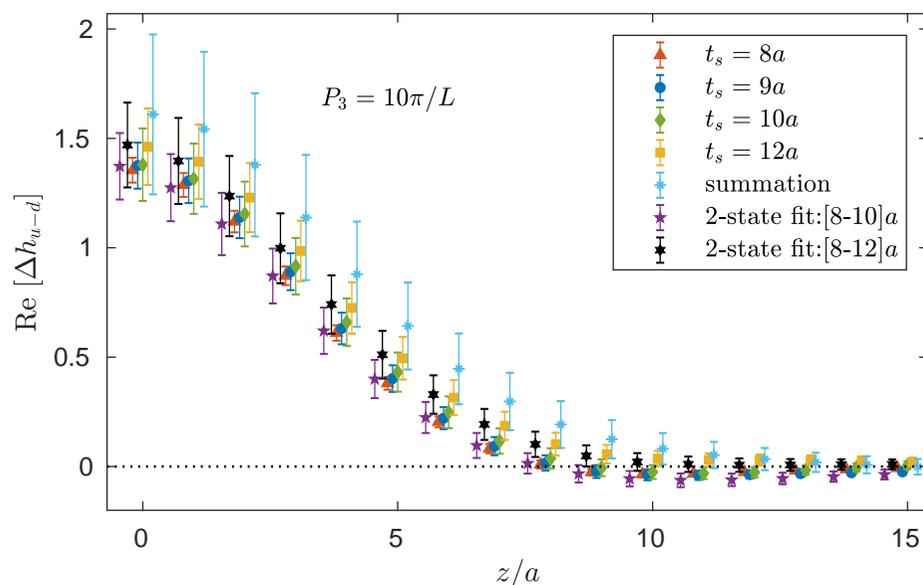


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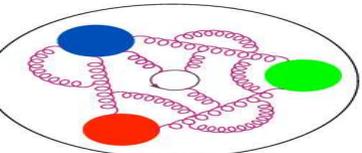


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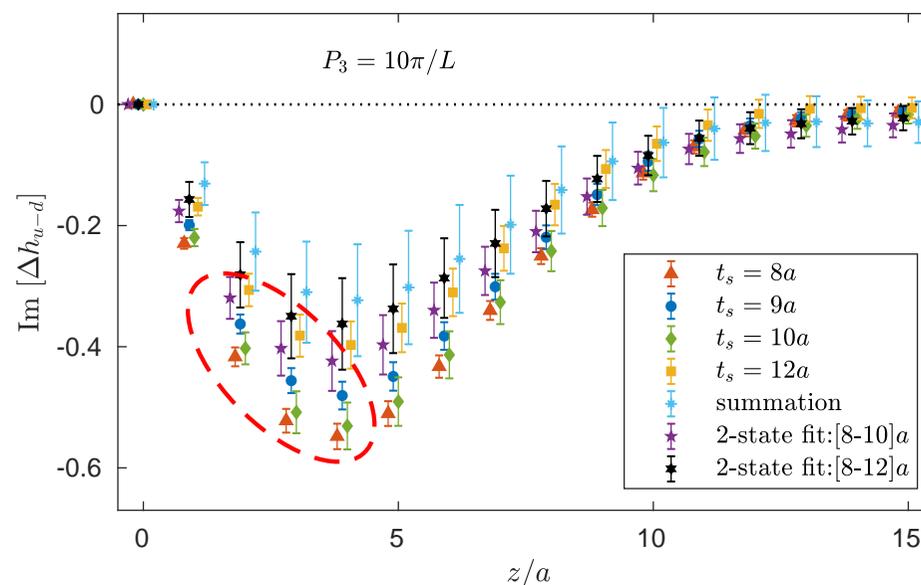
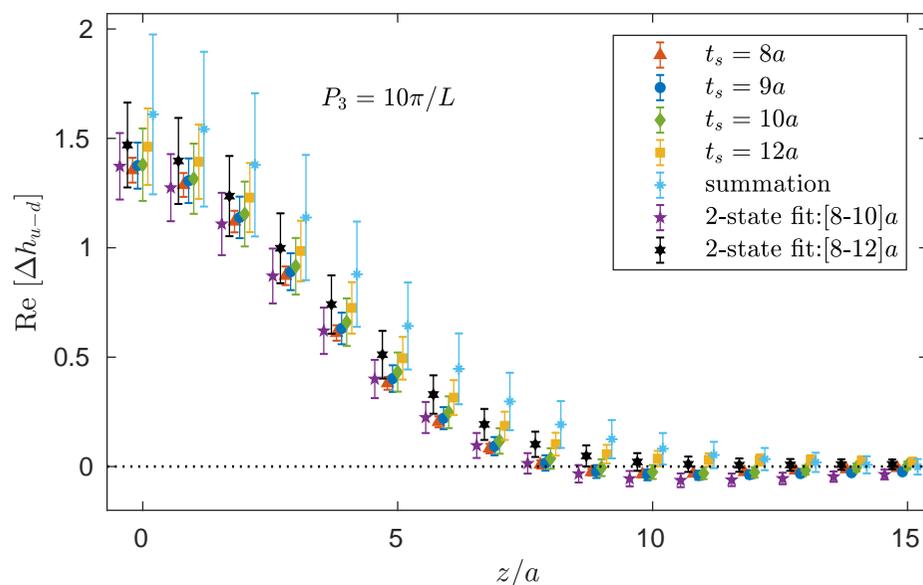


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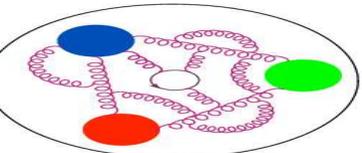


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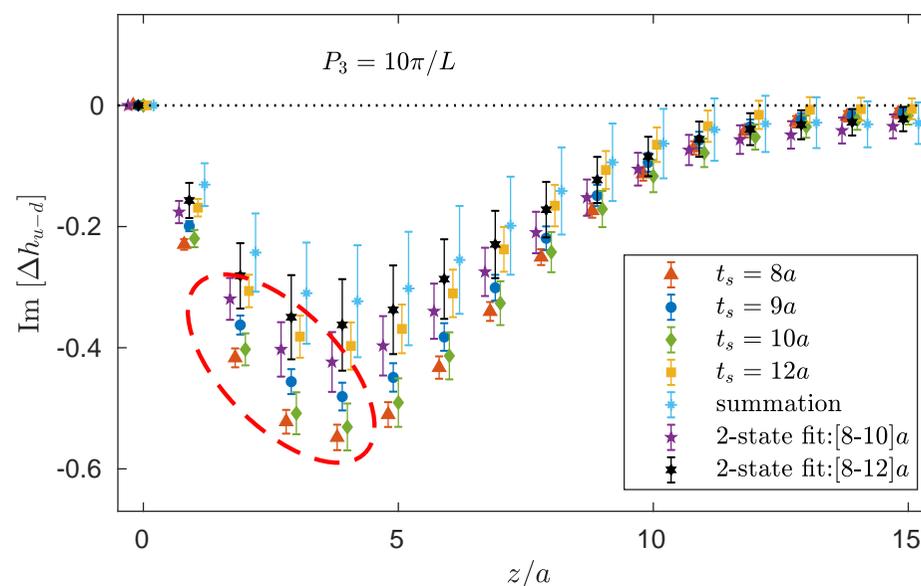
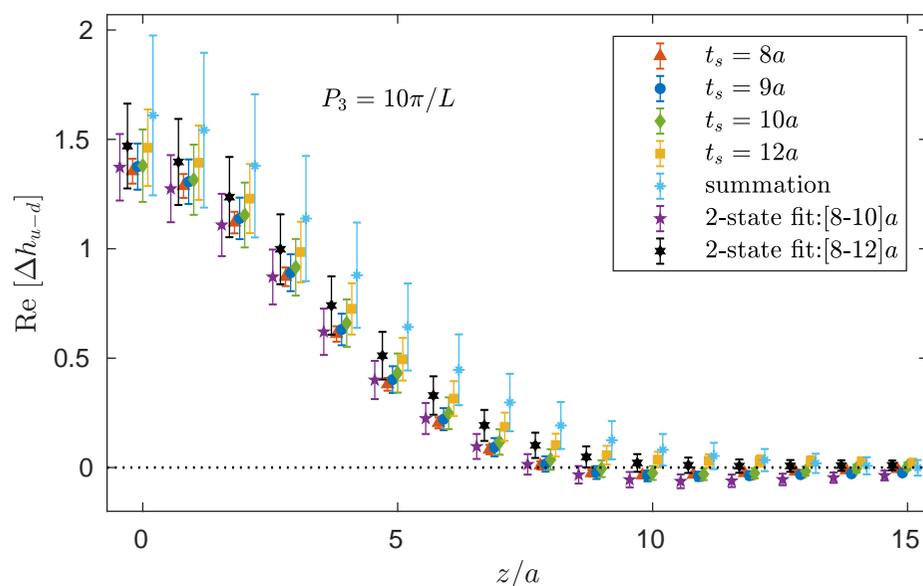
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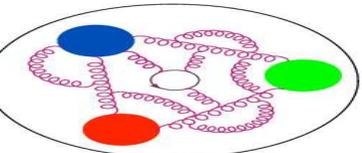
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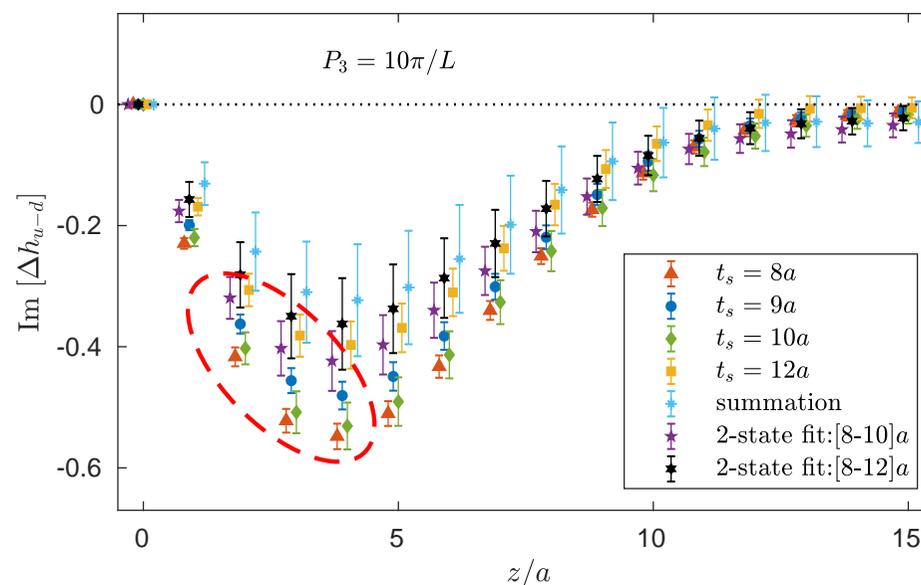
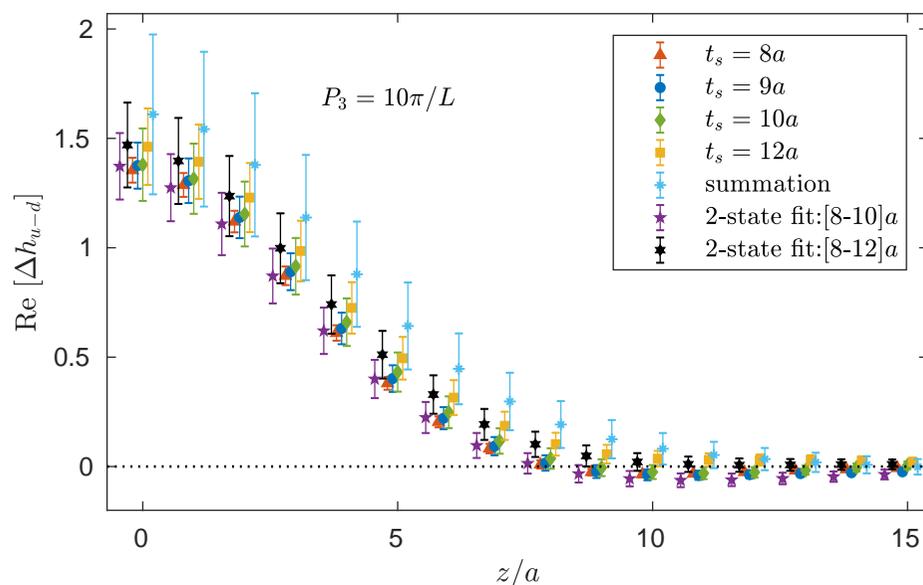


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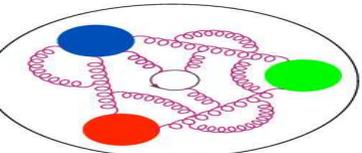


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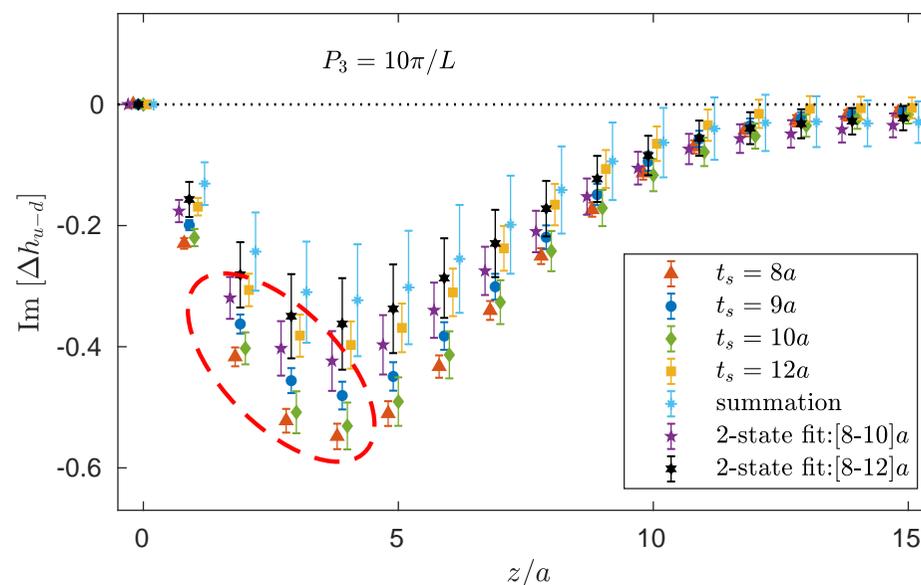
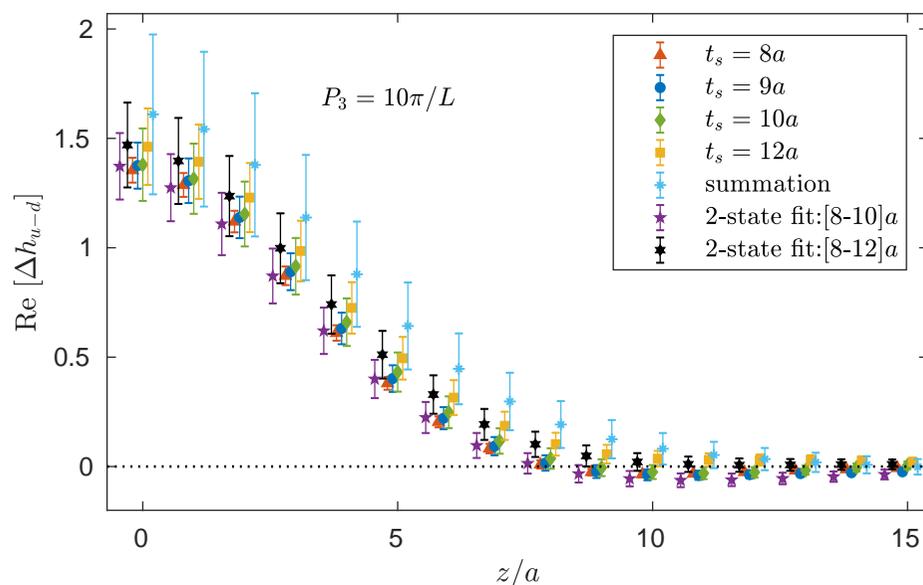


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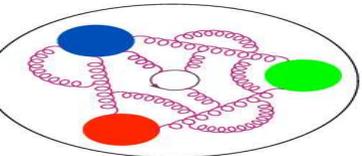


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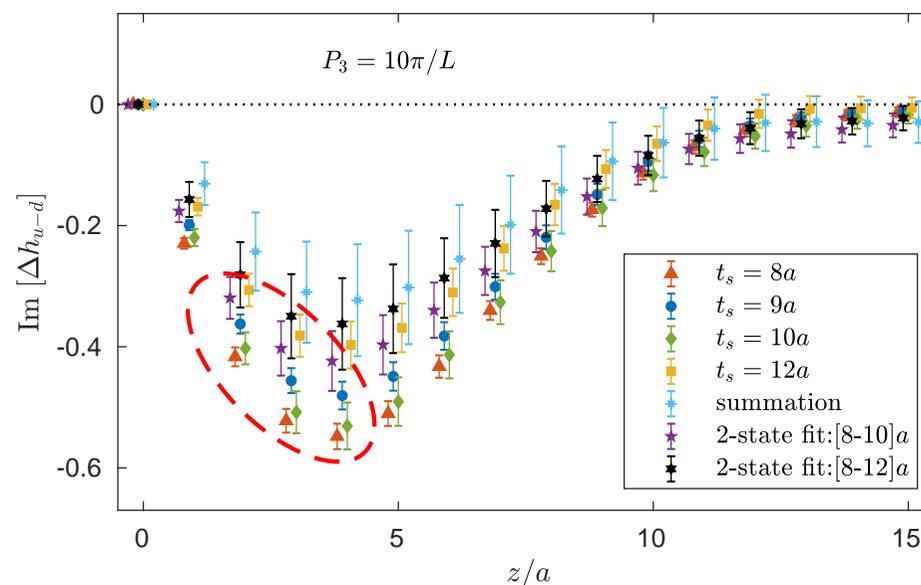
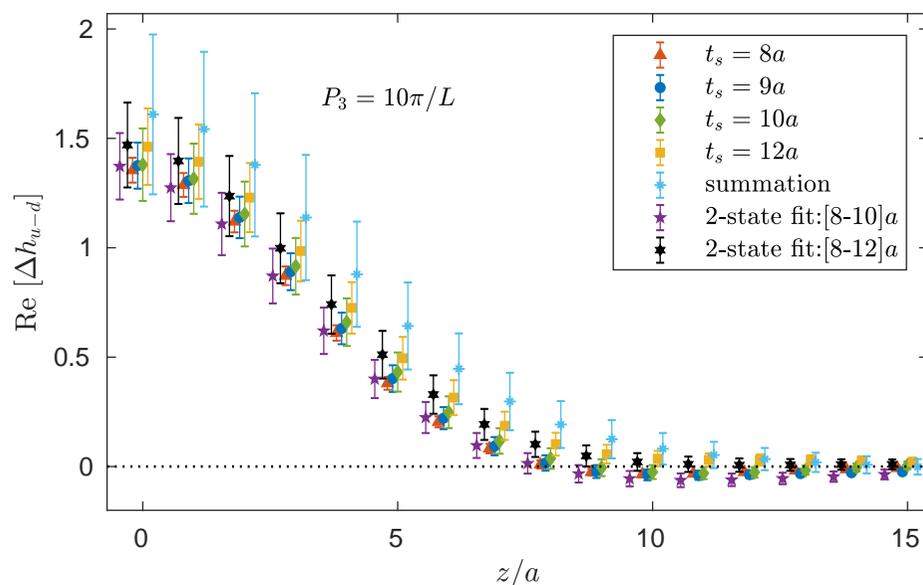


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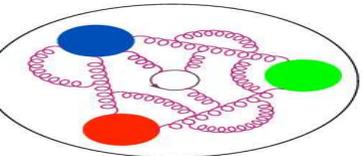


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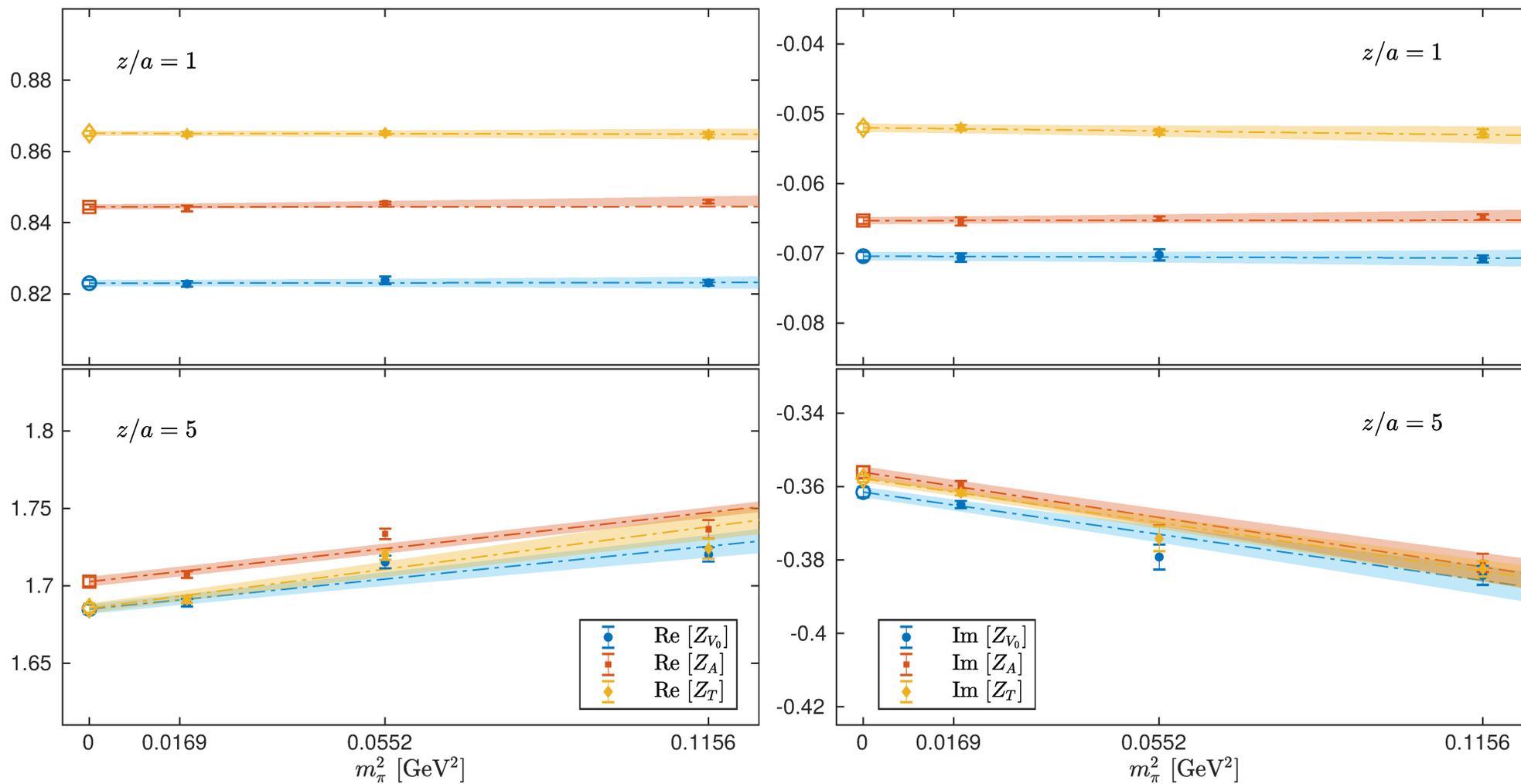
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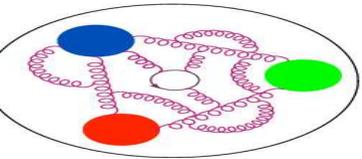
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- Drawback: one needs to get meaningful results at the largest t_s , i.e. large cost!
- **LQCD philosophy: control all systematics** \Rightarrow even expensive checks are required and crucial.



Pion mass dependence of Z -factors



C. Alexandrou et al., Phys. Rev. D99 (2019) 114504



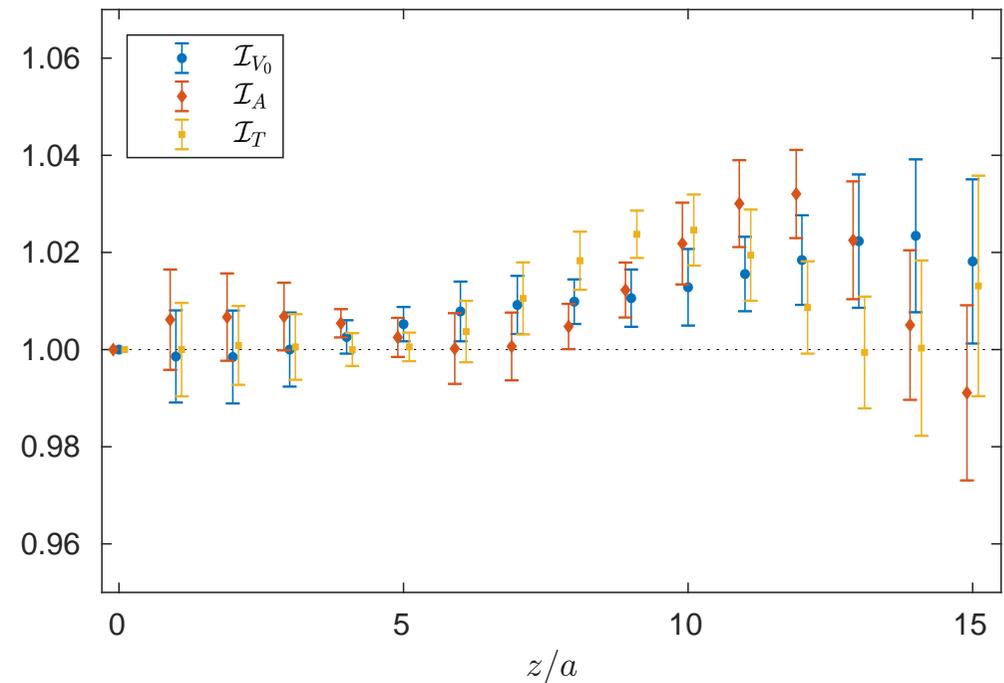
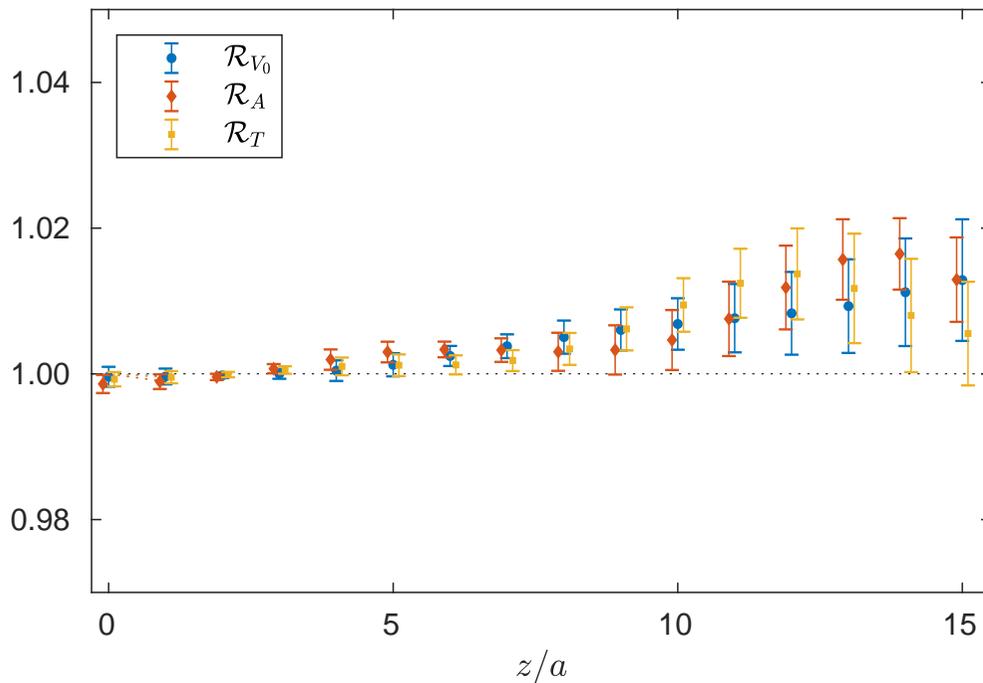
FVE in Z -factors



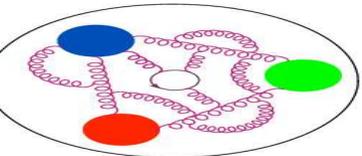
Possibly enhanced FVE in non-local operators suggested in:

R. Briceño, J. Guerrero, M. Hansen, C. Monahan, Phys. Rev. D98 (2018) 014511

$$\mathcal{R}_O(z) \equiv \frac{\text{Re}[Z_{O,64}^{\text{RI}'}(z, \mu_0, m_\pi)]}{\text{Re}[Z_{O,48}^{\text{RI}'}(z, \mu_0, m_\pi)]}, \quad \mathcal{I}_O(z) \equiv \frac{\text{Im}[Z_{O,64}^{\text{RI}'}(z, \mu_0, m_\pi)]}{\text{Im}[Z_{O,48}^{\text{RI}'}(z, \mu_0, m_\pi)]}$$



C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

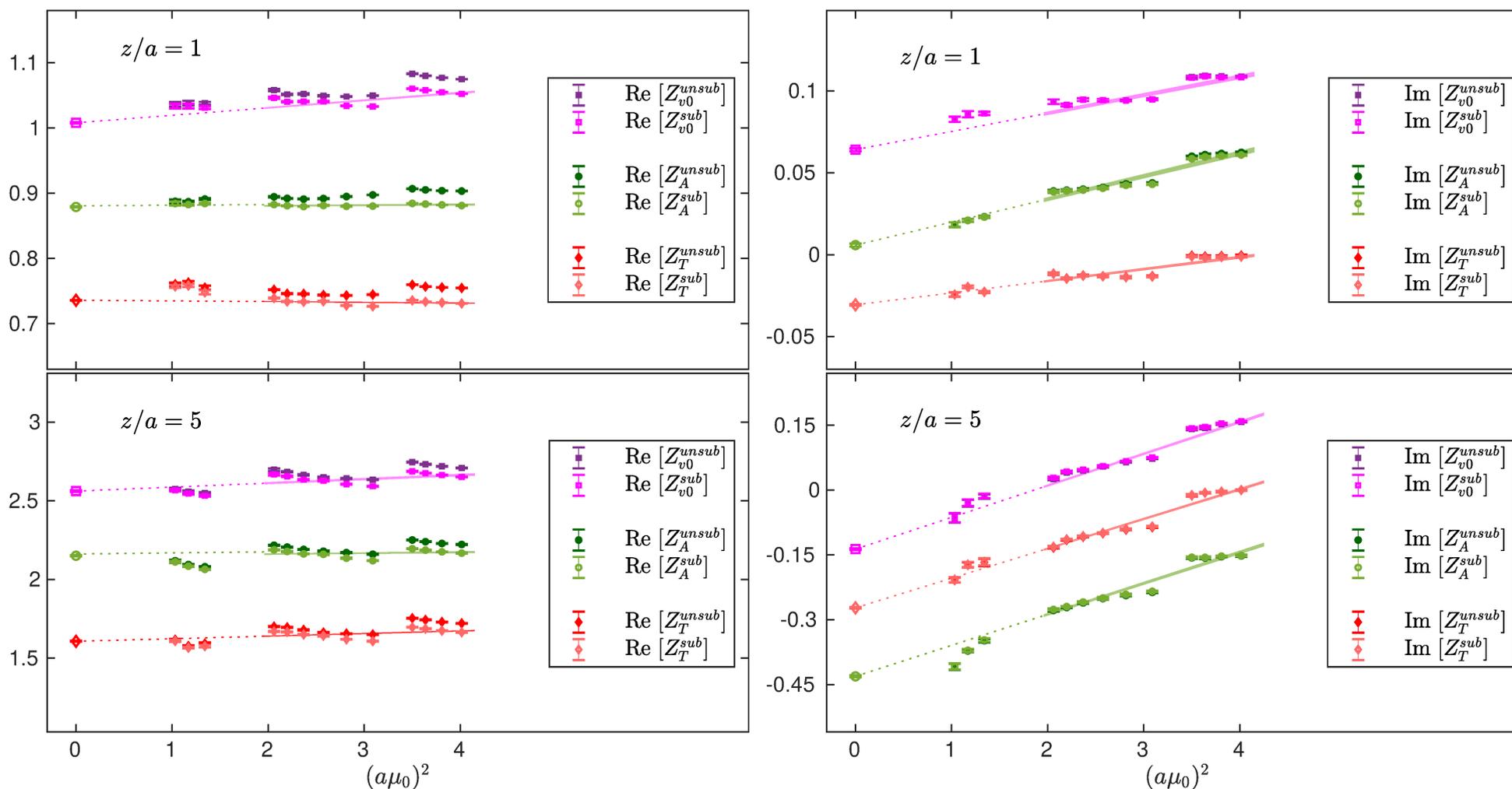


Lattice artefacts in Z -factors

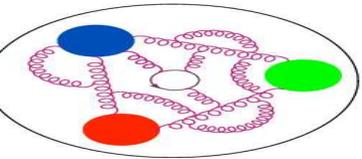


Z -factors can have $\mathcal{O}(g^2 a^\infty)$ artefacts perturbatively subtracted

By: M. Constantinou, H. Panagopoulos, e.g. [Phys. Rev. D95 \(2017\) 034505](#)



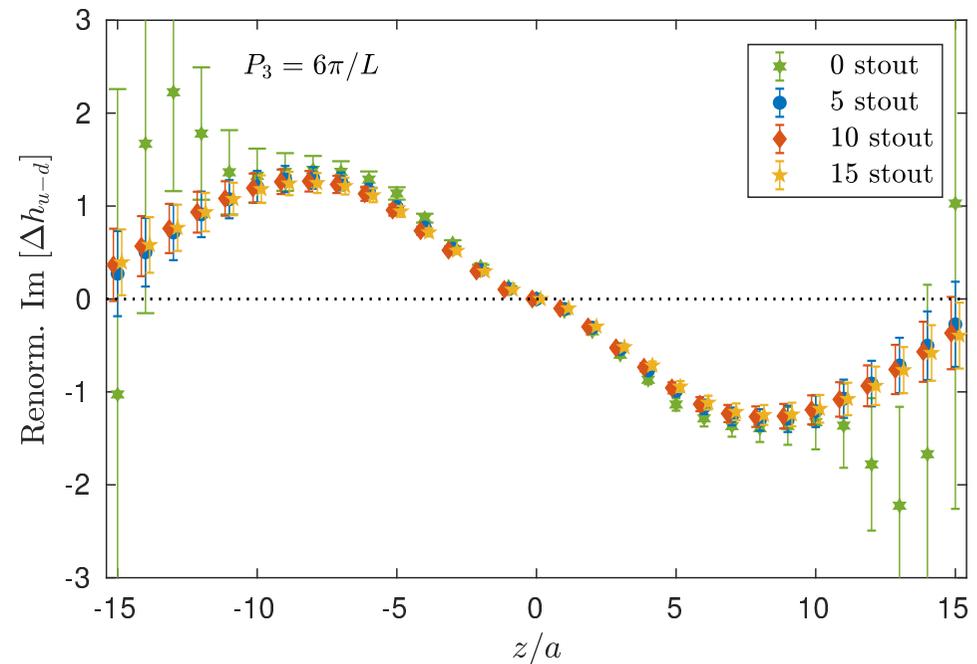
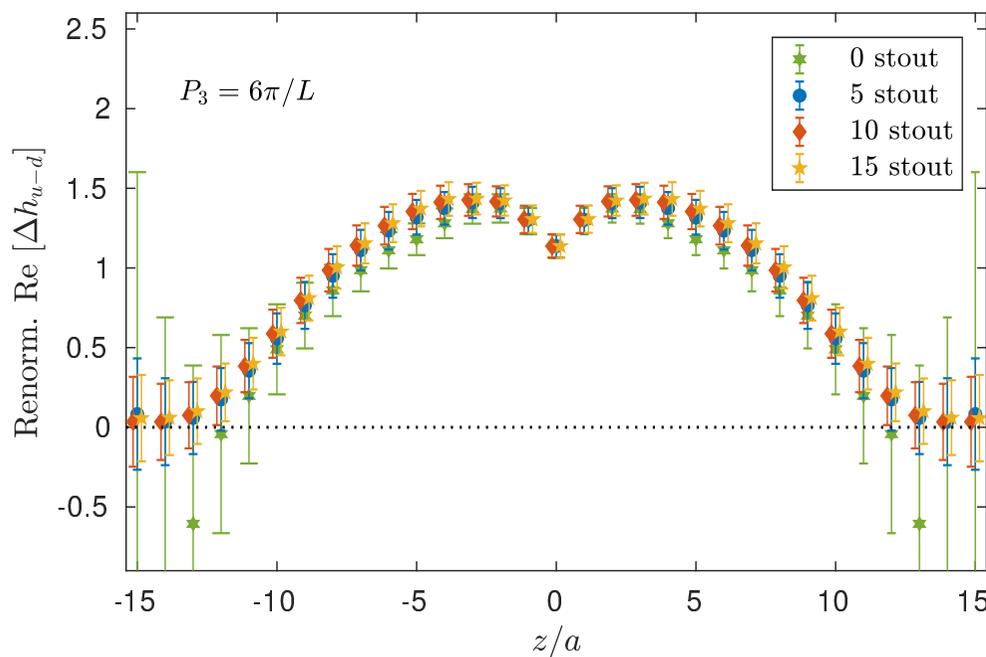
C. Alexandrou et al., [Phys. Rev. D99 \(2019\) 114504](#)



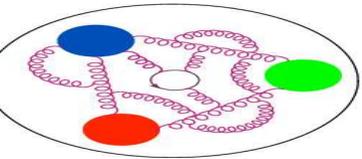
Effect of stout smearing



Renormalized matrix elements should not depend on the number of applied stout steps (even though both bare matrix elements and Z -factors depend on it)



C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

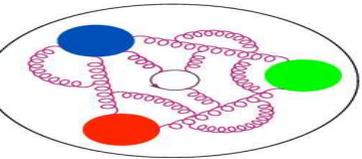


Matching to light-front PDFs



The matching formula can be expressed as:

$$q(x, \mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C \left(\xi, \frac{\mu}{xP_3} \right) \tilde{q} \left(\frac{x}{\xi}, \mu, P_3 \right)$$



Matching to light-front PDFs

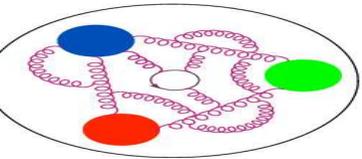
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C – matching kernel $\overline{\text{MS}} \rightarrow \overline{\text{MS}}$: [T. Izubuchi et al., Phys. Rev. D98 (2018) 056004]

$$C \left(\xi, \frac{\xi\mu}{xP_3} \right) = \delta(1 - \xi) + \frac{\alpha_s}{2\pi} C_F \left\{ \begin{array}{ll} \left[\frac{1 + \xi^2}{1 - \xi} \ln \frac{\xi}{\xi - 1} + 1 + \frac{3}{2\xi} \right]_{+(1)}^{[1, \infty]} - \frac{3}{2\xi} & \xi > 1, \\ \left[\frac{1 + \xi^2}{1 - \xi} \ln \frac{x^2 P_3^2}{\xi^2 \mu^2} (4\xi(1 - \xi)) - \frac{\xi(1 + \xi)}{1 - \xi} + 2\nu(1 - \xi) \right]_{+(1)}^{[0, 1]} & 0 < \xi < 1 \\ \left[-\frac{1 + \xi^2}{1 - \xi} \ln \frac{\xi}{\xi - 1} - 1 + \frac{3}{2(1 - \xi)} \right]_{+(1)}^{[-\infty, 0]} - \frac{3}{2(1 - \xi)} & \xi < 0, \end{array} \right.$$

$$+ \frac{\alpha_s C_F}{2\pi} \delta(1 - \xi) \left(\frac{3}{2} \ln \frac{\mu^2}{4y^2 P_3^2} + \frac{5}{2} \right), \quad \nu=0 \text{ for } \gamma_0 \text{ and } \nu=1 \text{ for } \gamma_3/\gamma_5\gamma_3.$$



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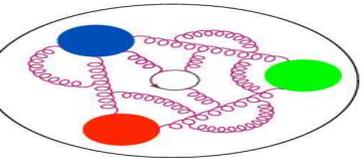
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Problem: violates vector current conservation:

$$\int_{-\infty}^{\infty} dx q(x, \mu) \neq \int_{-\infty}^{\infty} dx \tilde{q}(x, \mu, P_3) \quad \text{and} \quad \int_{-\infty}^{\infty} d\xi C(\xi, \xi\mu/xP_3) \neq 1,$$

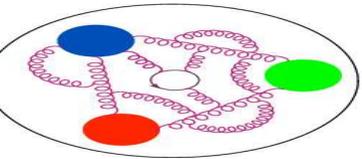
which increases with growing P_3 (around 8% at $P_3 = 10\pi/48$).



Matching to light-front PDFs



Alternative matching: [C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]



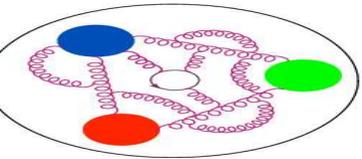
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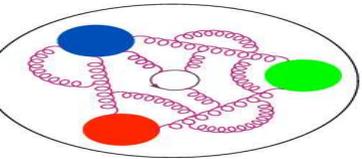


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- In this procedure, vector current is **conserved**.



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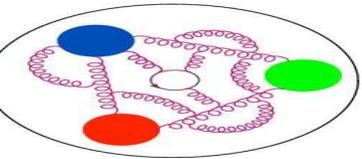


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- In this procedure, vector current is **conserved**.
- Additional subtractions with respect to $\overline{\text{MS}}$ – made outside the physical region of the unintegrated vertex corrections.



Matching to light-front PDFs

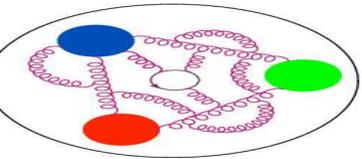


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- In this procedure, vector current is **conserved**.
- Additional subtractions with respect to $\overline{\text{MS}}$ – made outside the physical region of the unintegrated vertex corrections.
- Thus, needs modified renormalization scheme for input quasi-PDF.



Matching to light-front PDFs

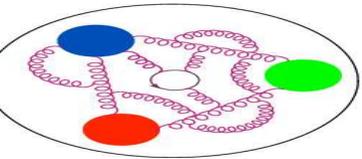


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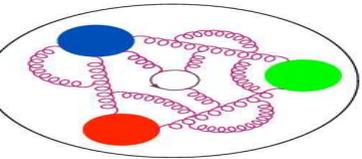
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- Additional subtractions with respect to $\overline{\text{MS}}$ – made outside the physical region of the unintegrated vertex corrections.
- Thus, needs modified renormalization scheme for input quasi-PDF.
- However, modification **decreases** with growing P_3 .



Modification of the $\overline{\text{MS}}$ scheme



We introduce a **modified $\overline{\text{MS}}$ scheme (MMS)** with an extra subtraction made outside the physical region of the unintegrated vertex corrections. [C. Alexandrou et al., Phys. Rev. D99 (2019) 114504]
This renormalizes the ξ -dependence for $\xi > 1$ and $\xi < 0$.

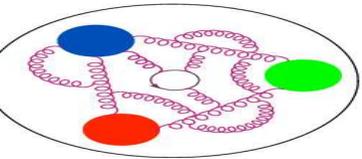


Modification of the $\overline{\text{MS}}$ scheme



We introduce a **modified $\overline{\text{MS}}$ scheme (M $\overline{\text{MS}}$)** with an extra subtraction made outside the physical region of the unintegrated vertex corrections. [C. Alexandrou et al., Phys. Rev. D99 (2019) 114504]
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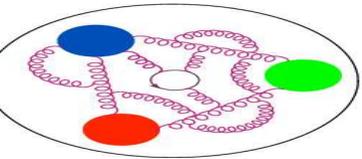


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In z -space:

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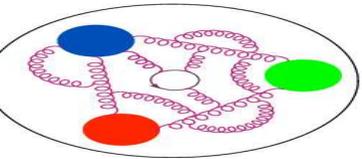
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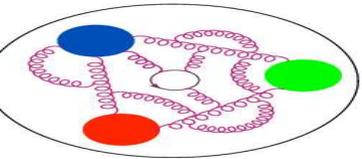
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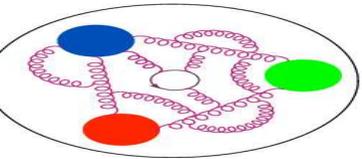
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Exactly cancels the divergence in $\ln(z)$ present in $\overline{\text{MS}}$!

(consistency with: M. Constantinou, H. Panagopoulos, Phys. Rev. D96 (2017) 054506
and with the “Ratio” scheme of T. Izubuchi et al., Phys. Rev. D98 (2018) 056004)



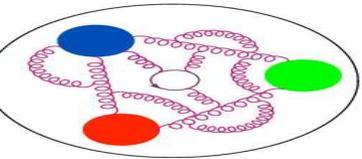
Matching to light-front PDFs



Another alternative matching (“ratio” scheme):

[T. Izubuchi et al., Phys. Rev. D98 (2018) 056004]

$$C\left(\xi, \frac{\mu}{|y|P_3}\right) = \delta(1 - \xi) + \frac{\alpha_s C_F}{2\pi} \begin{cases} \left(\frac{1 + \xi^2}{1 - \xi} \ln \frac{\xi}{\xi - 1} + 1 - \frac{3}{2(1 - \xi)}\right)_{+(1)} & \xi > 1 \\ \left(\frac{1 + \xi^2}{1 - \xi} \left[\ln \frac{y^2 P_3^2}{\mu^2} (4\xi(1 - \xi)) - 1 \right] + 1 + 2\nu(1 - \xi) + \frac{3}{2(1 - \xi)}\right)_{+(1)} & 0 < \xi < 1 \\ \left(-\frac{1 + \xi^2}{1 - \xi} \ln \frac{-\xi}{1 - \xi} - 1 + \frac{3}{2(1 - \xi)}\right)_{+(1)} & \xi < 0 \end{cases}$$



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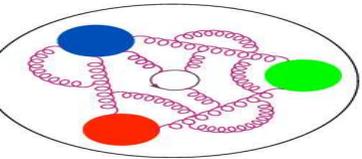


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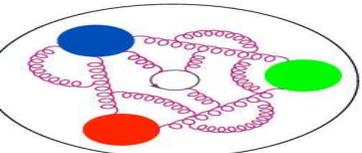
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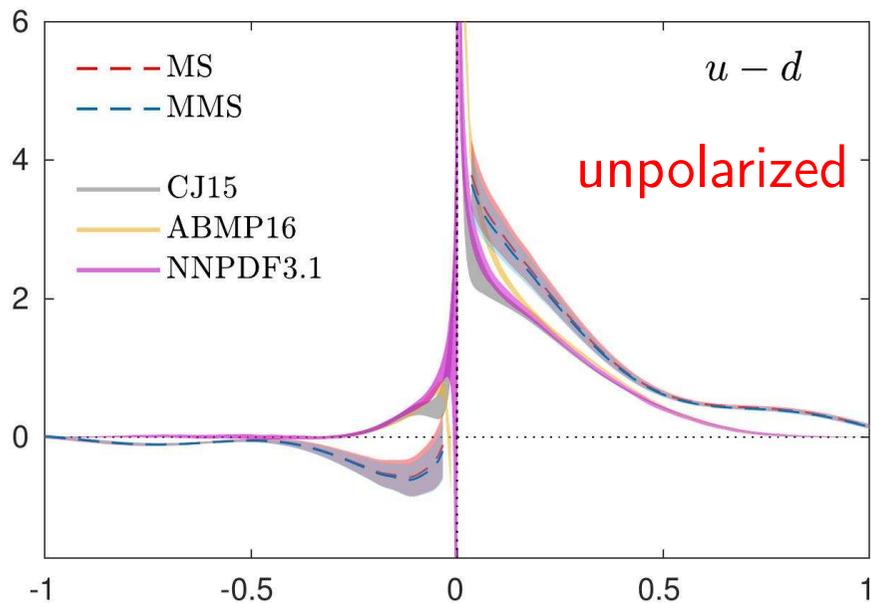
Modification of the perturbative conversion from the intermediate renormalization scheme to $\overline{\text{MS}}$:

$$C_0(\mu^2 z^2) = 1 + \frac{\alpha_s C_F}{2\pi} \left[\frac{3}{2} \ln(\mu^2 z^2 e^{2\gamma_E}/4) + \frac{5}{2} \right]$$

Caveat: modification of the *physical* ξ -region – potentially large numerical effect.

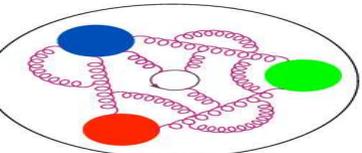


Effect from \overline{MMS}

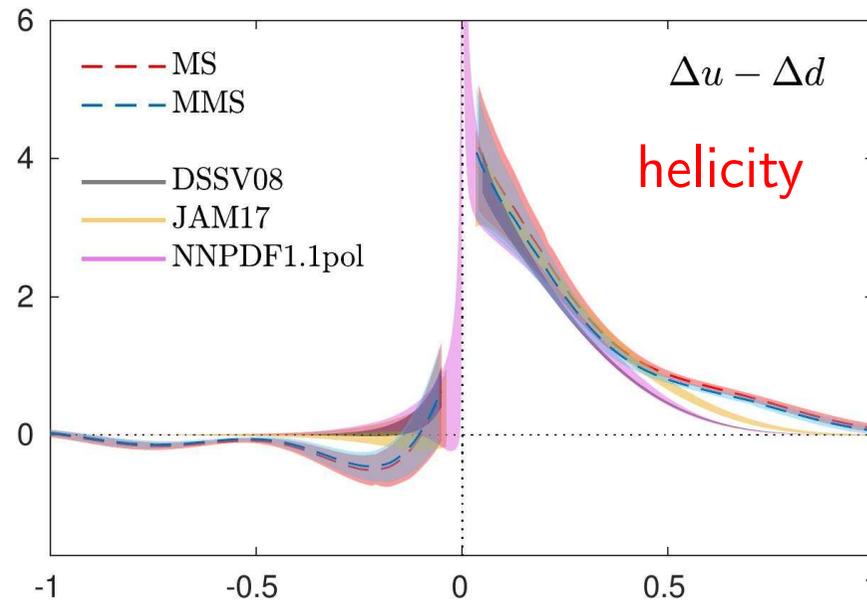
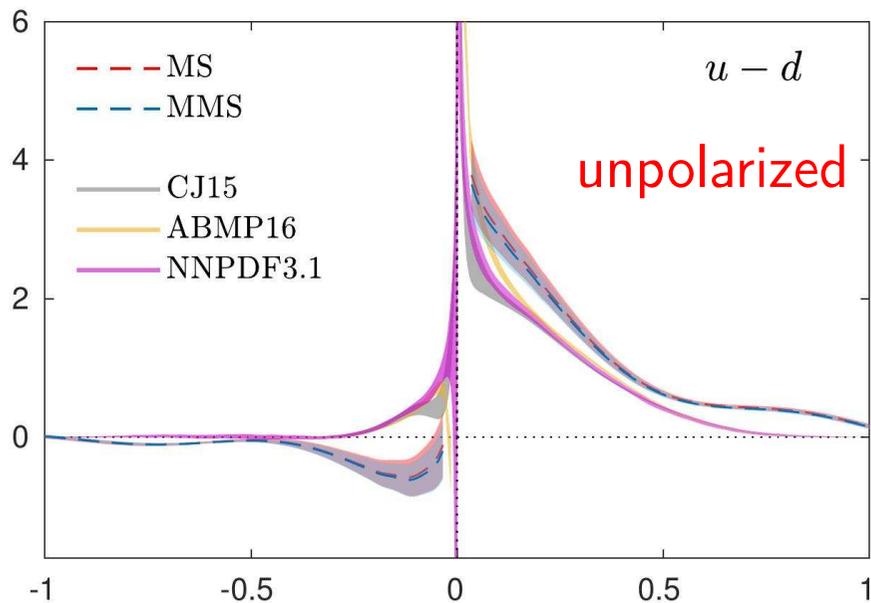


Nucleon momentum $\frac{10\pi}{48}$

C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

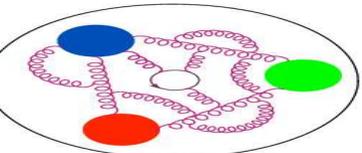


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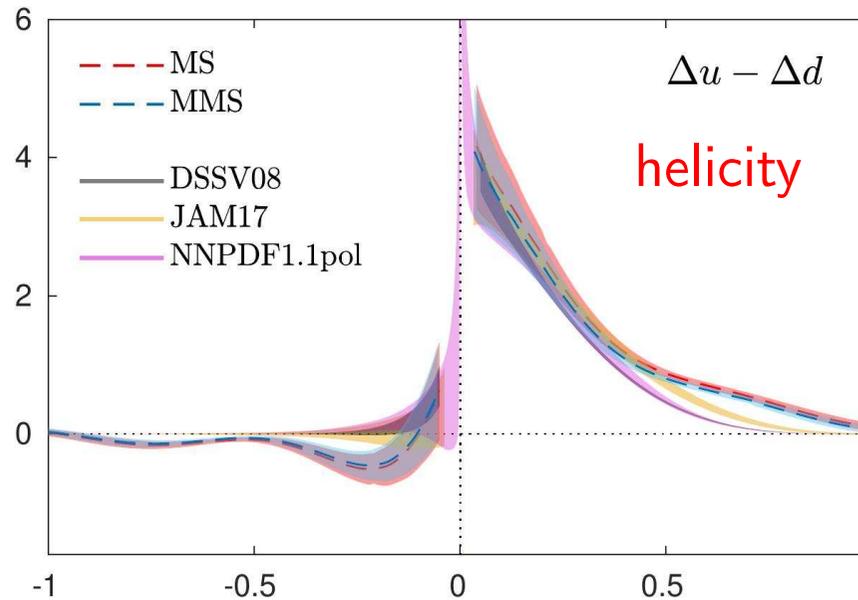
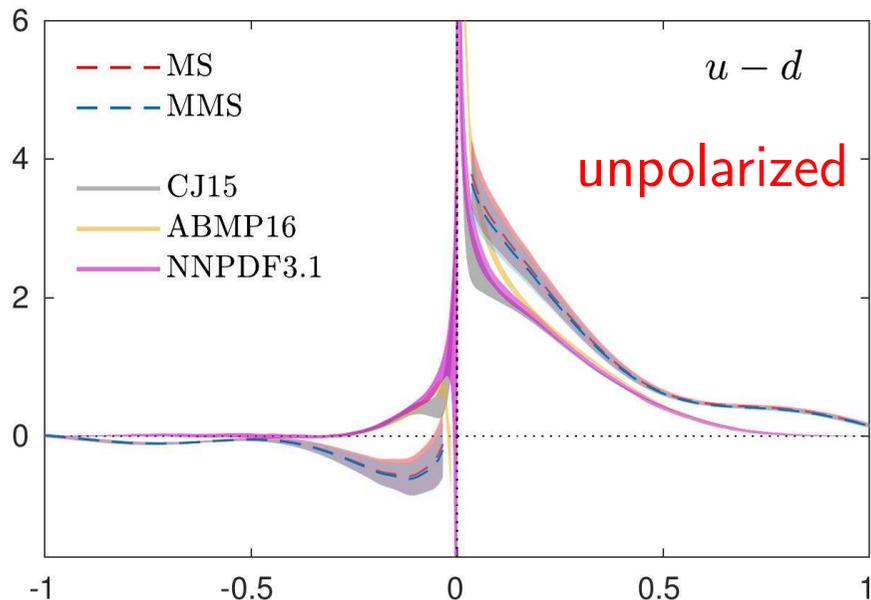


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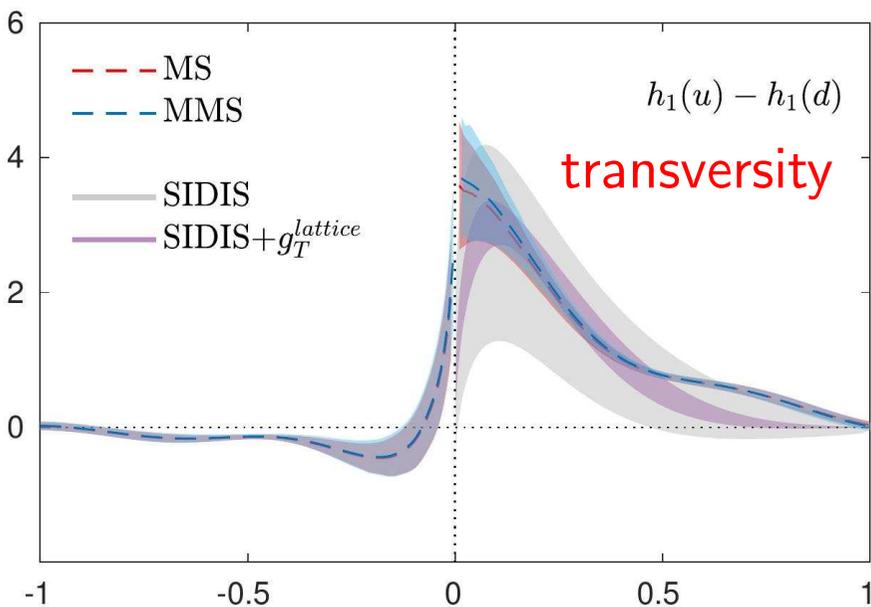
C. Alexandrou et al., Phys. Rev. D99 (2019) 114504



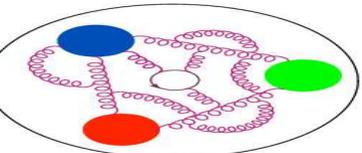
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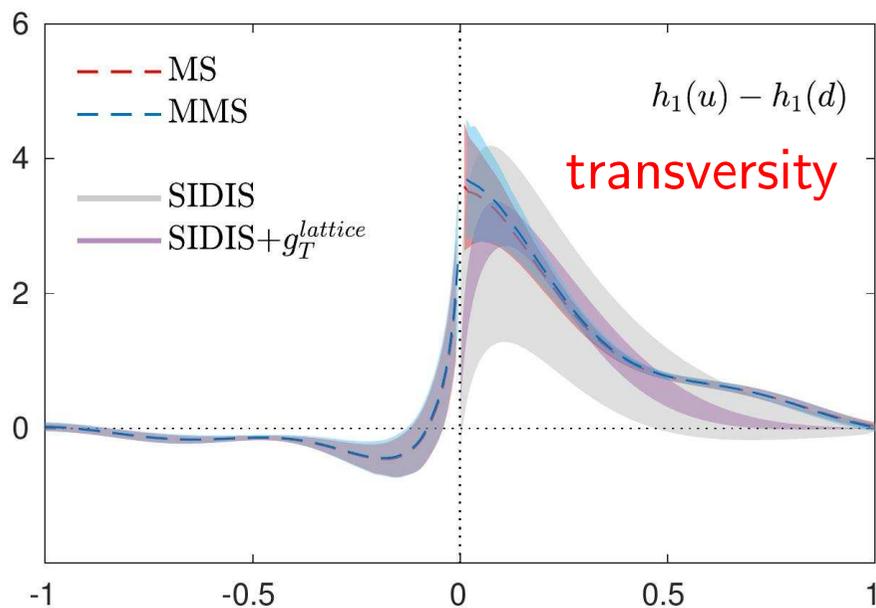
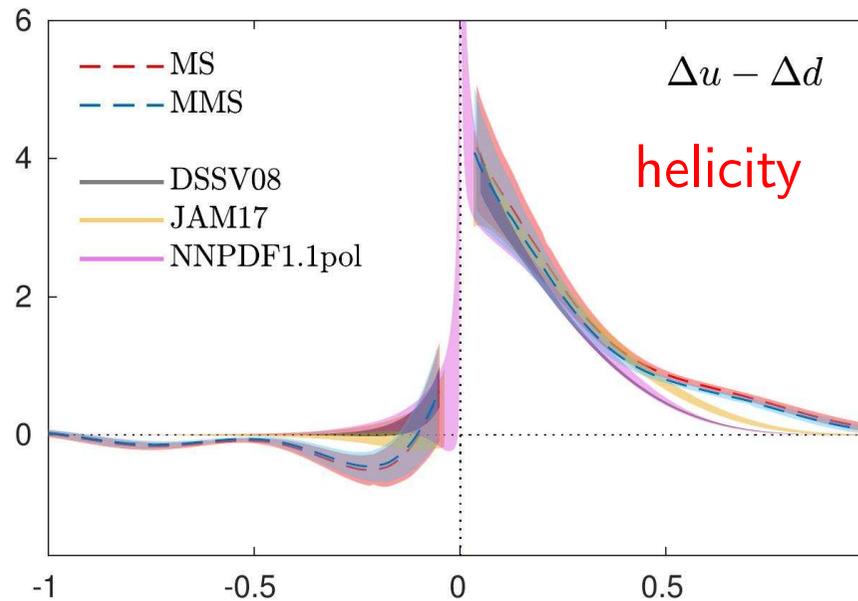
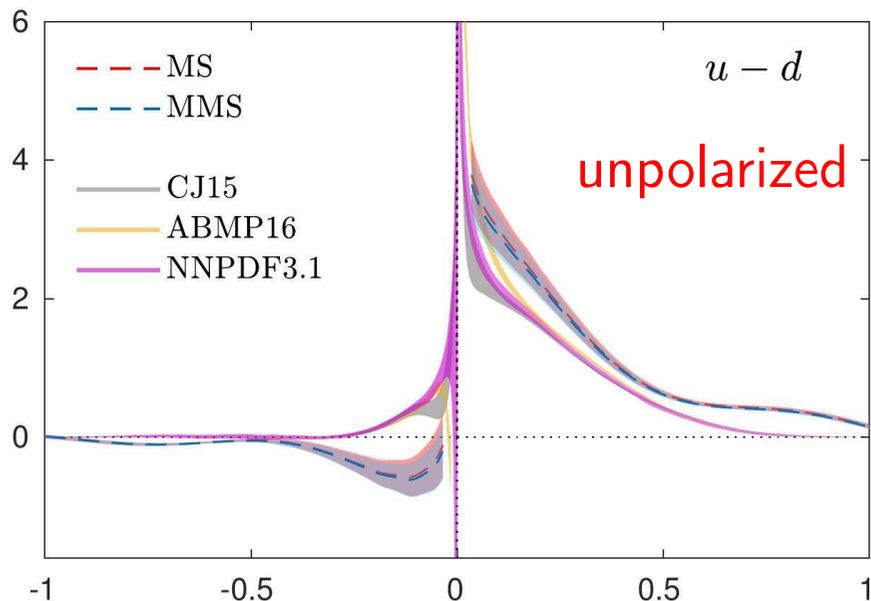
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C. Alexandrou et al., Phys. Rev. D99 (2019) 114504



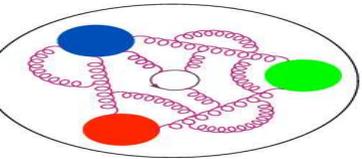
Effect from $\overline{\text{MMS}}$



Nucleon momentum $\frac{10\pi}{48}$

As expected, the effect is very small
(modification of $\overline{\text{MS}}$ only
in unphysical regions)

C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

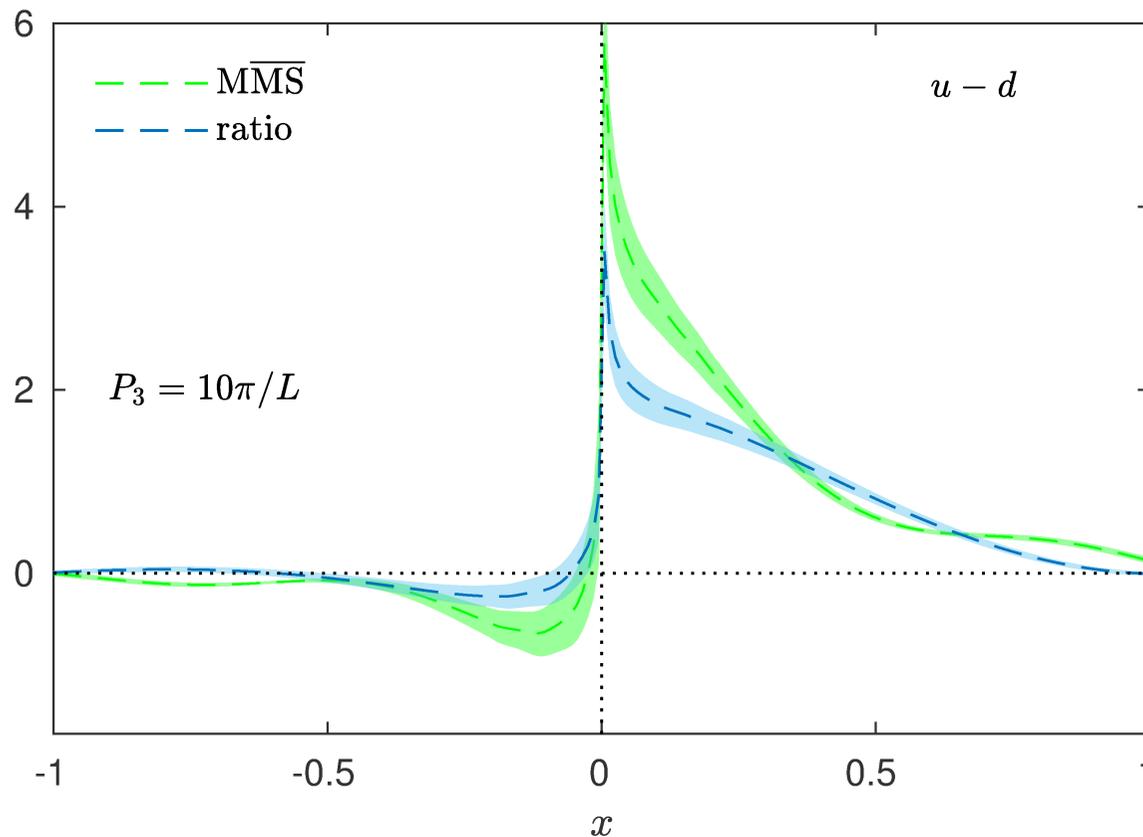


$\overline{\text{MMS}}$ vs. “ratio” scheme

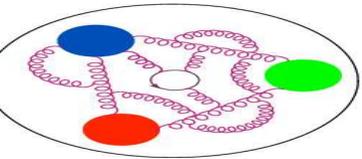


$\overline{\text{MMS}}$ – modification only of the “non-physical” regions $\xi < 0, \xi > 1$.

“ratio” – modification also of the “physical” region $0 < \xi < 1$.



C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

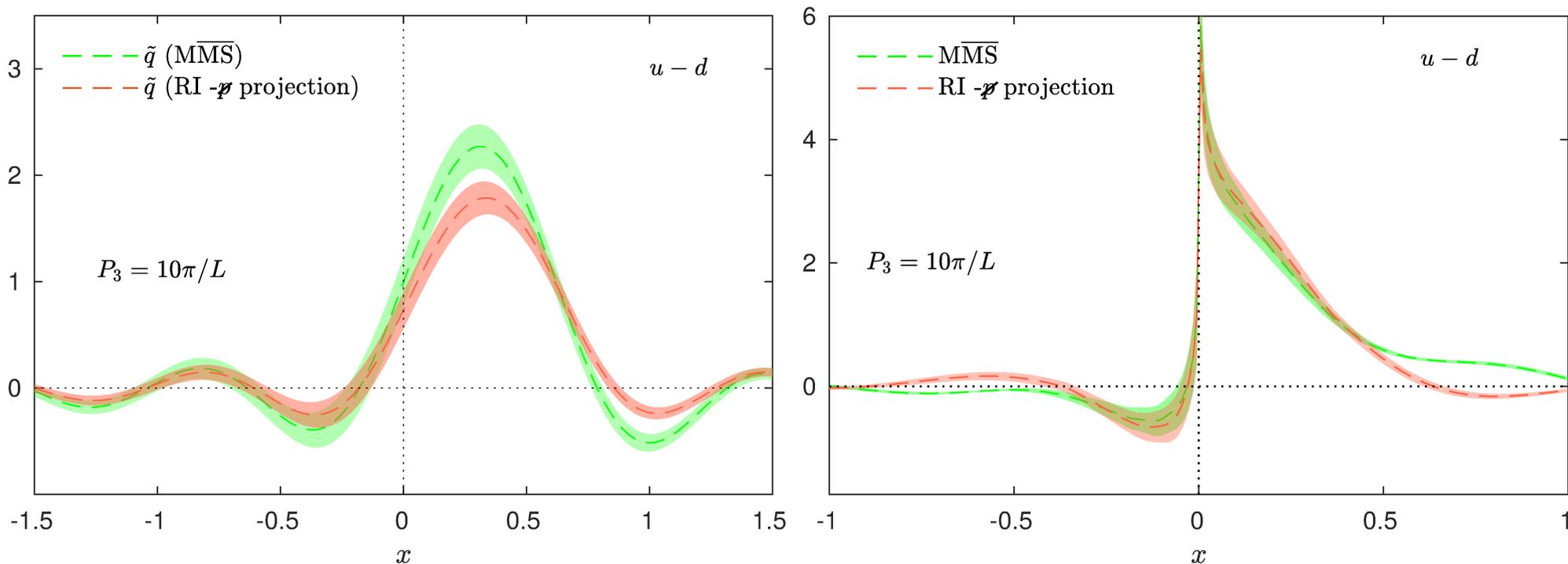


$\overline{\text{MMS}} \rightarrow \overline{\text{MS}}$ vs. $\text{RI} \rightarrow \overline{\text{MS}}$ matching

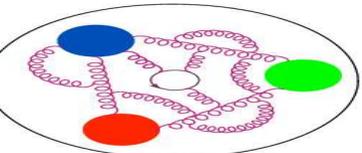


Matching can also be performed directly from the RI scheme to $\overline{\text{MS}}$

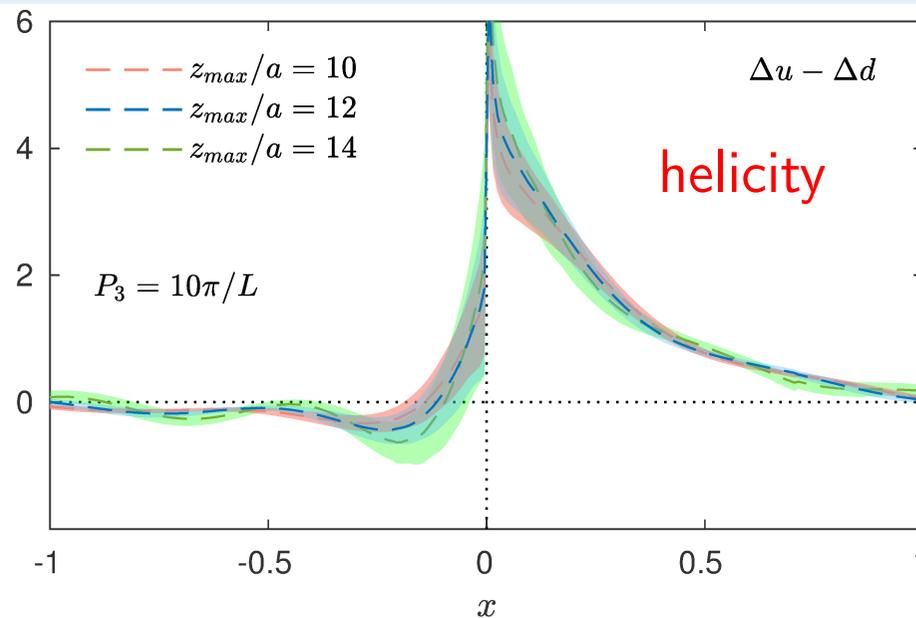
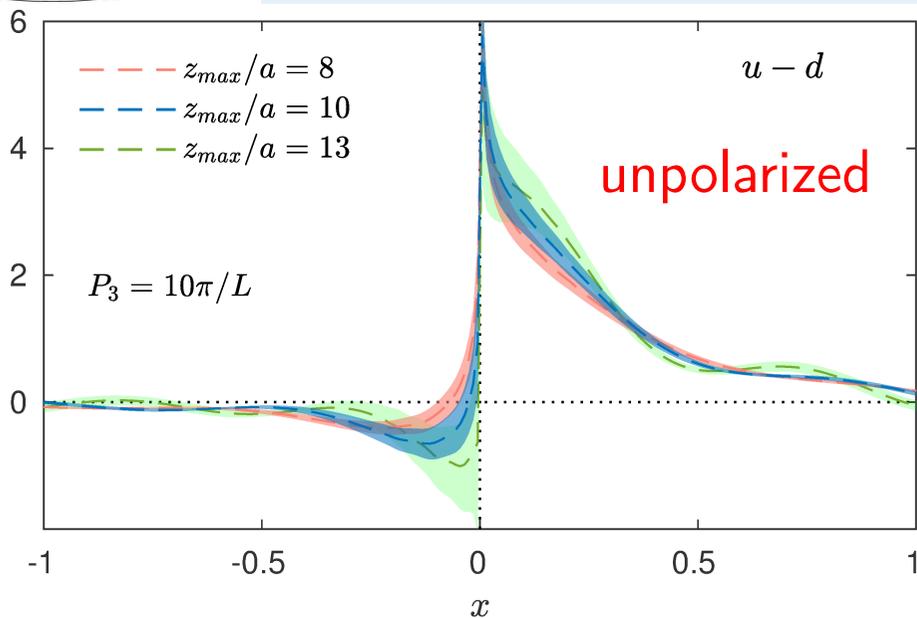
I.W. Stewart, Y. Zhao, Phys. Rev. D97 (2018) 054512



C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

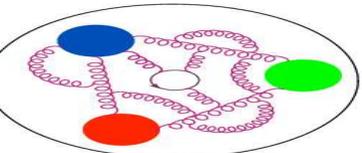


Truncation of Fourier transform

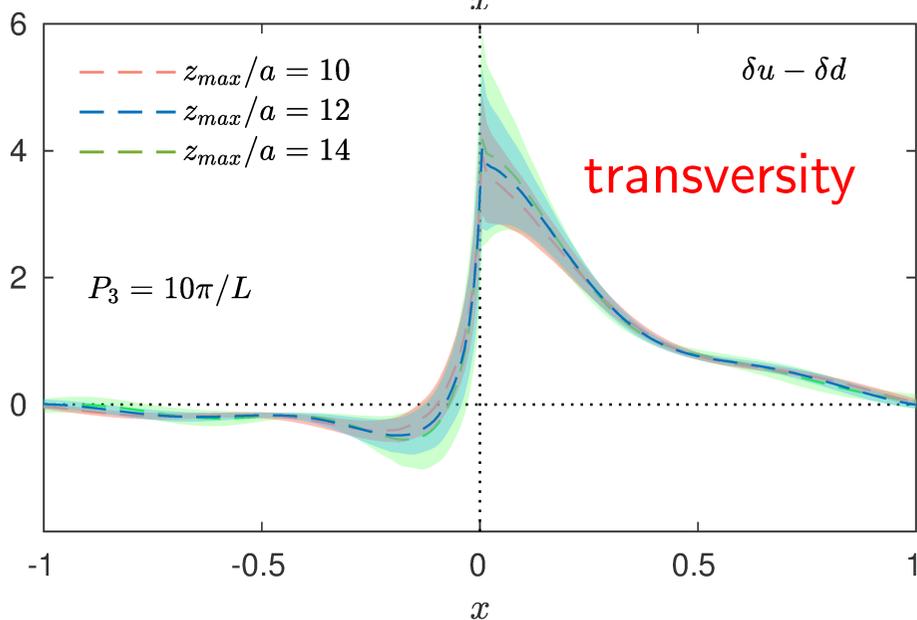
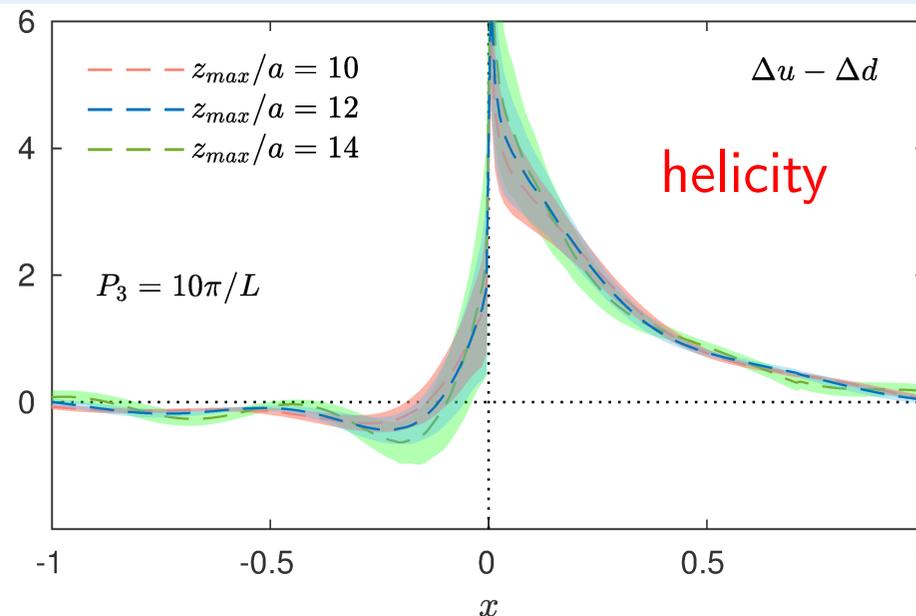
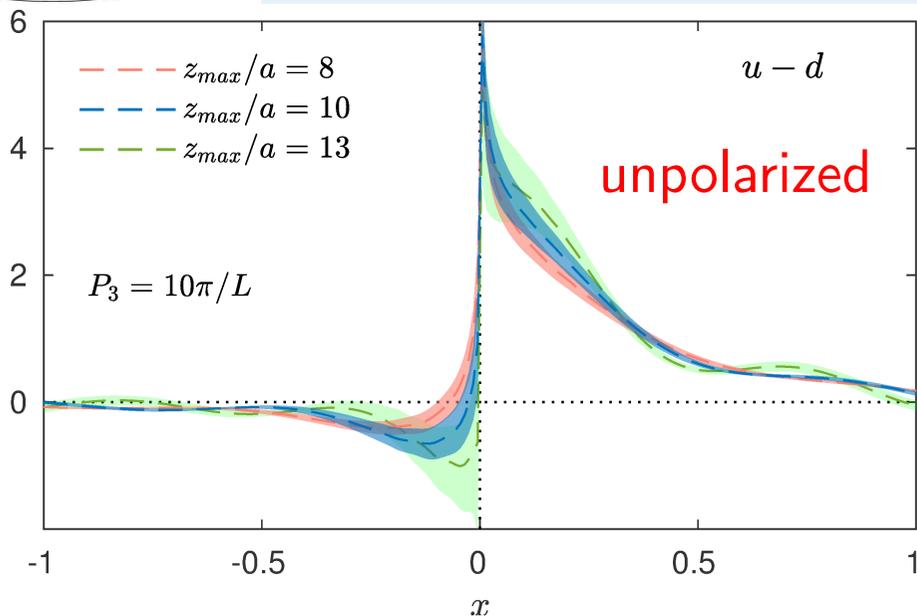


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C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

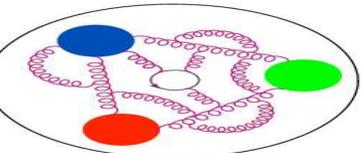


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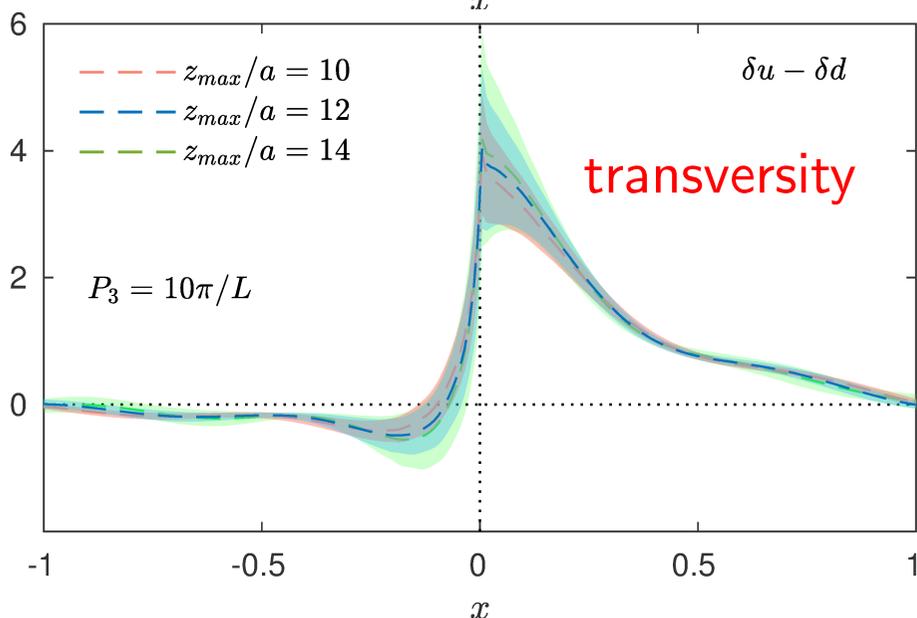
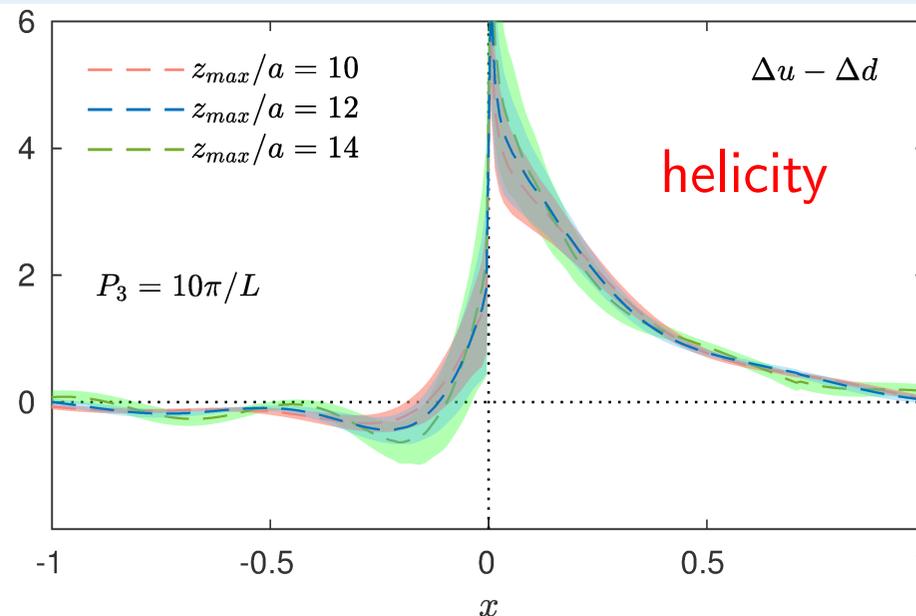
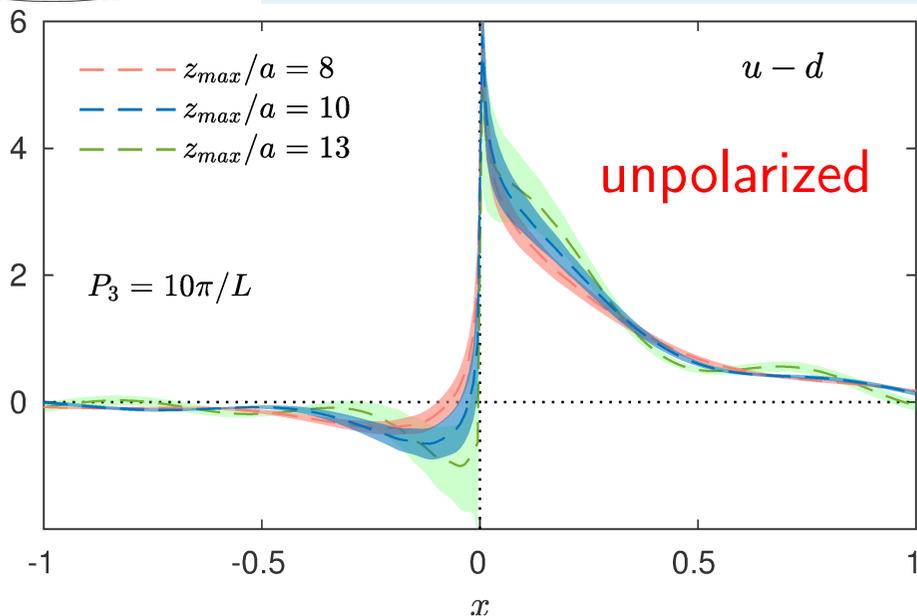


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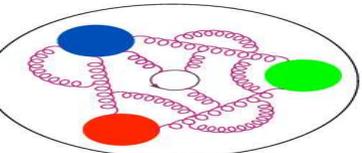
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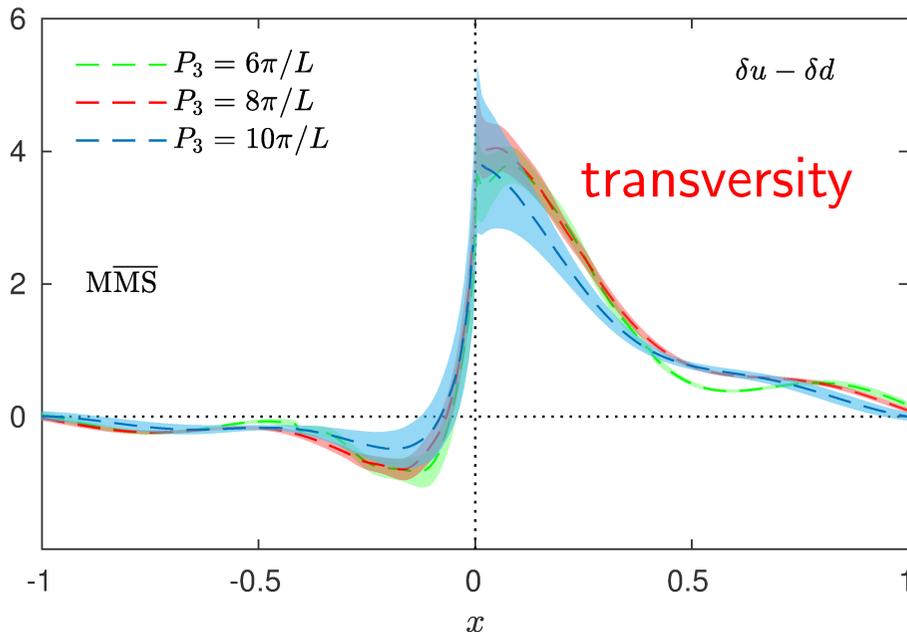
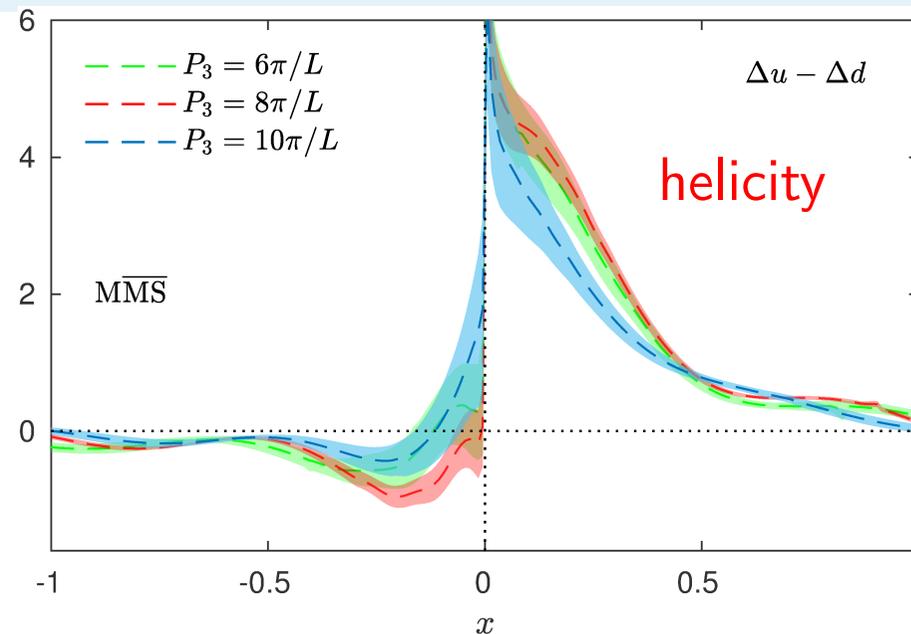
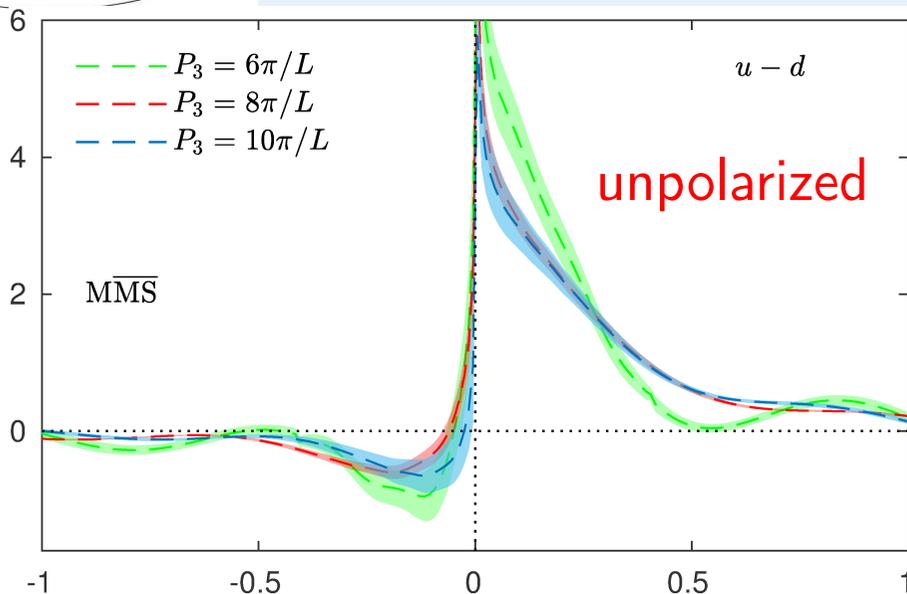
Nucleon momentum $\frac{10\pi}{48}$

Needs the use of advanced reconstruction techniques
 J. Karpie et al., JHEP 1904 (2019) 057

C. Alexandrou et al., Phys. Rev. D99 (2019) 114504



Momentum dependence of final PDFs



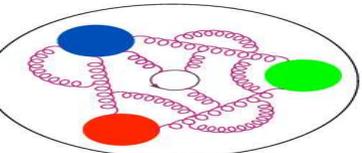
Nucleon momenta $\frac{6\pi}{48}, \frac{8\pi}{48}, \frac{10\pi}{48}$

Results seem to indicate convergence in nucleon boost

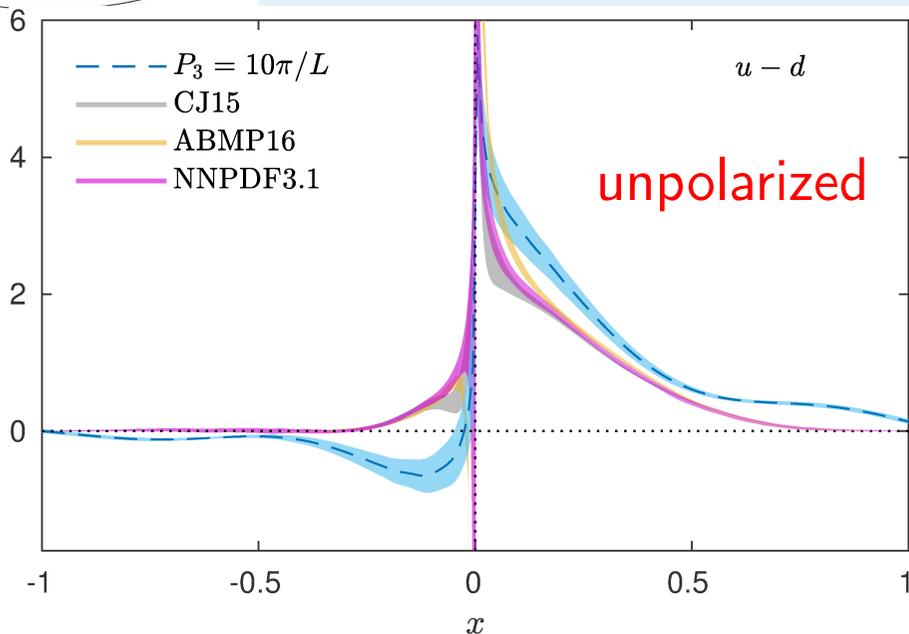
Expected HTE:

$$\mathcal{O}(\Lambda_{\text{QCD}}^2/P_3^2) \approx 5\% \text{ at } P_3 = 1.4 \text{ GeV}$$

C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

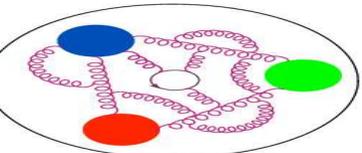


Final PDFs – comparison to pheno

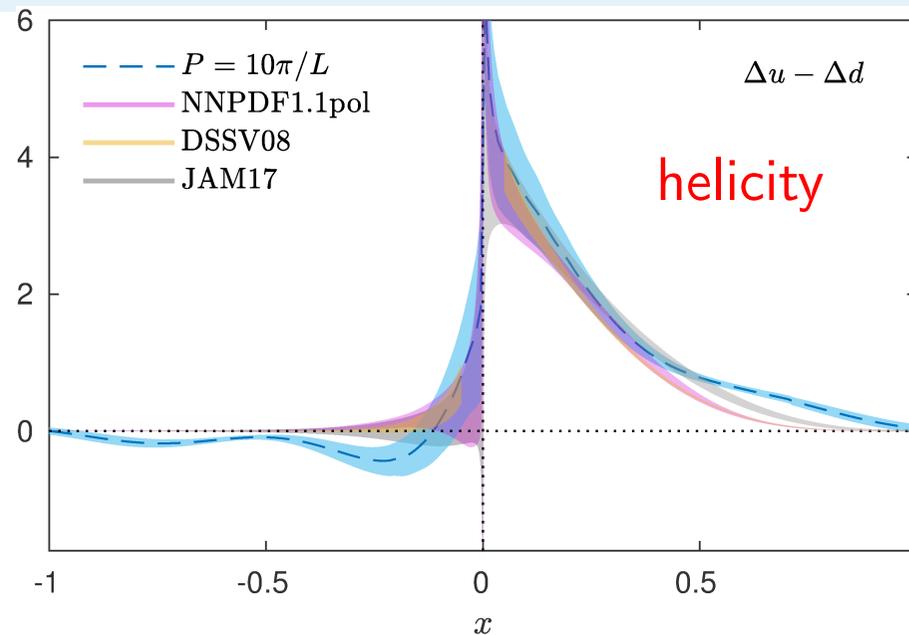
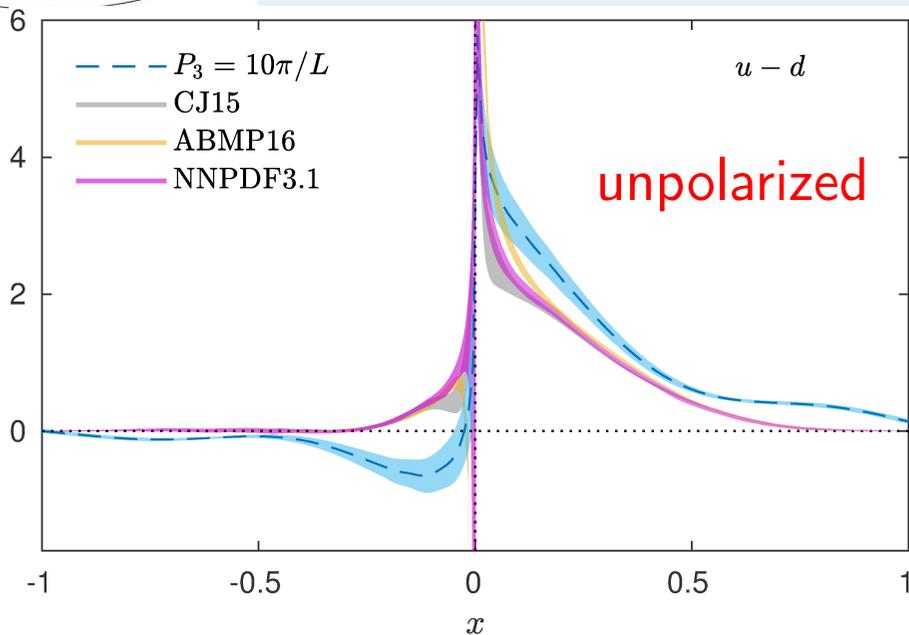


Nucleon momentum $\frac{10\pi}{48}$

C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

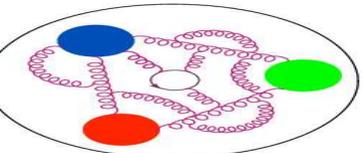


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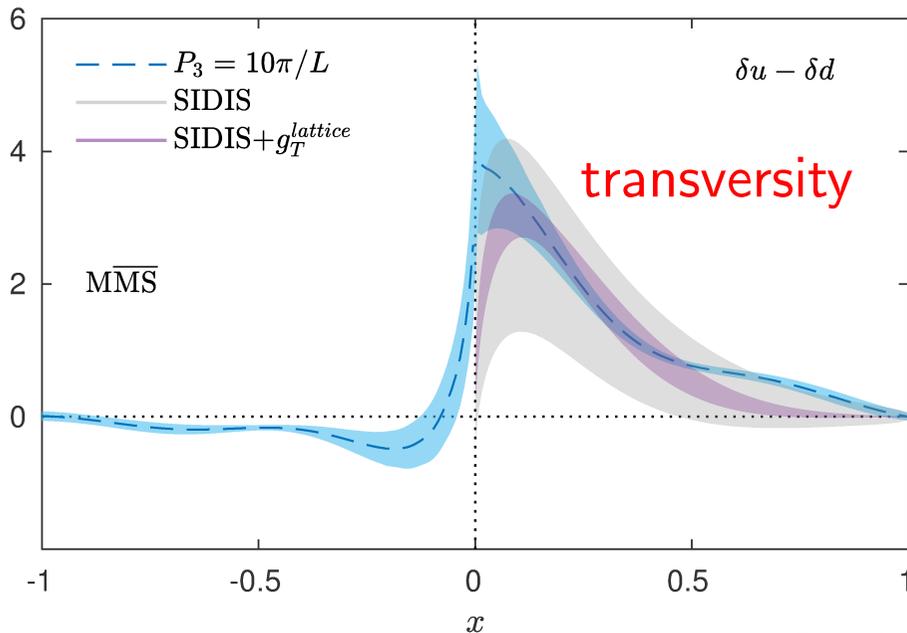
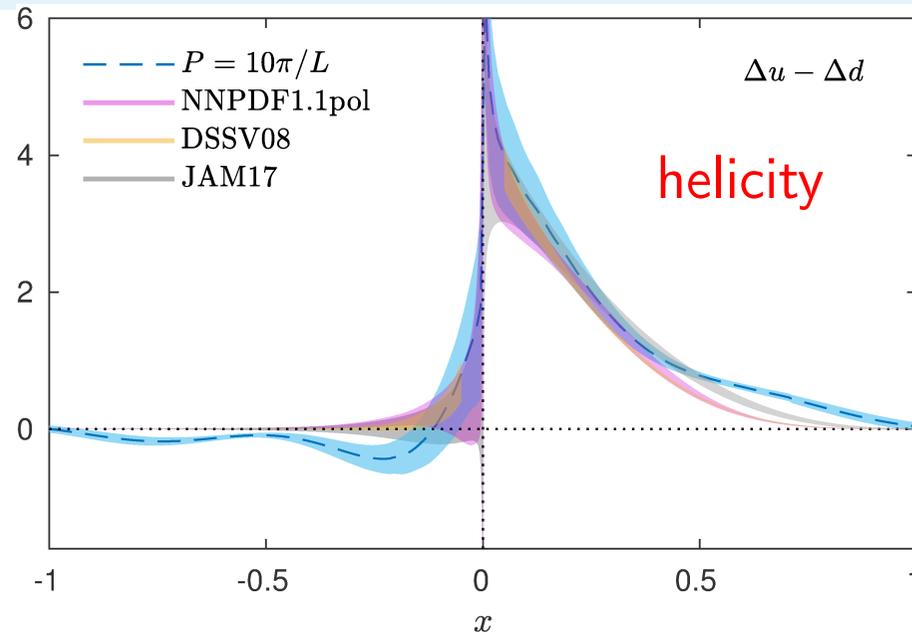
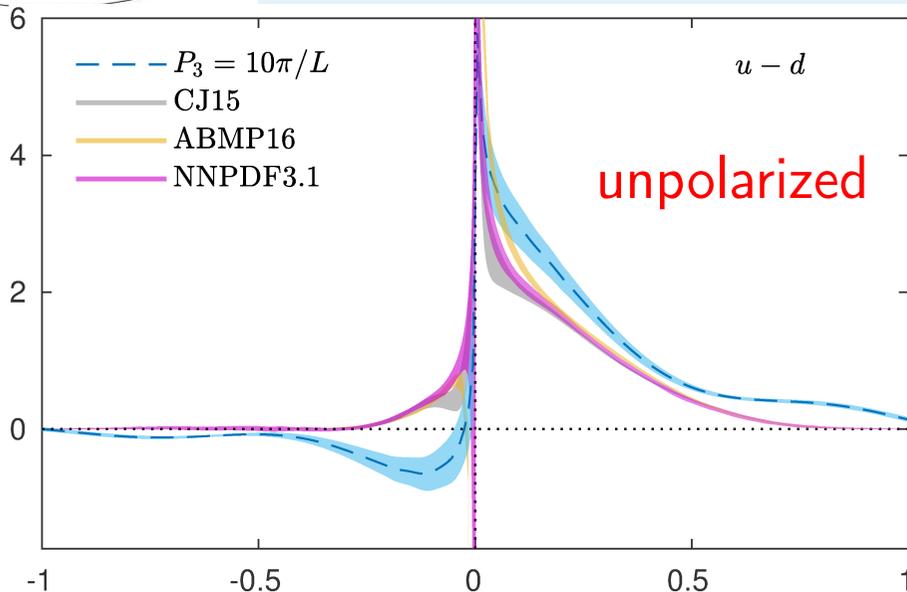


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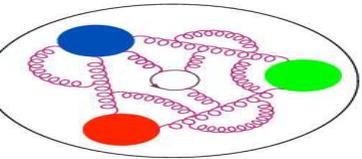


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Different systematic effects:

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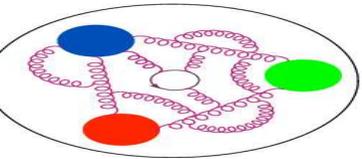
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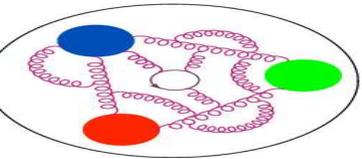
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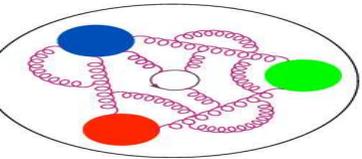
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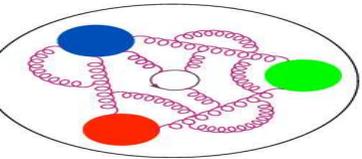
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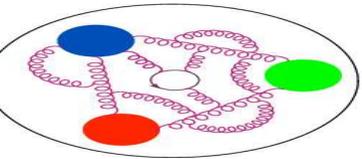
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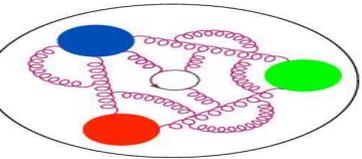
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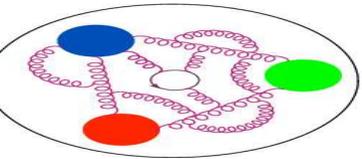
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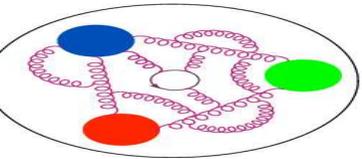
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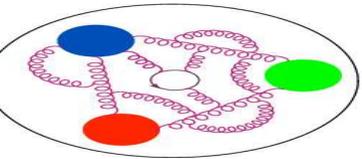
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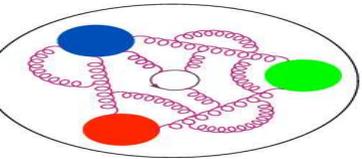
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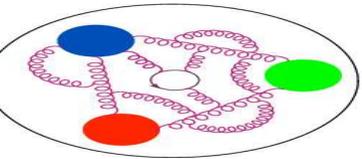
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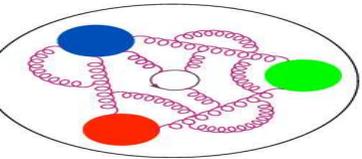
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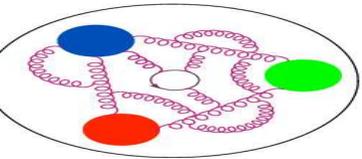
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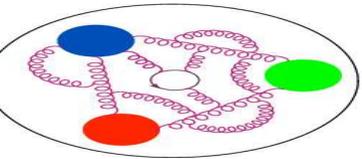
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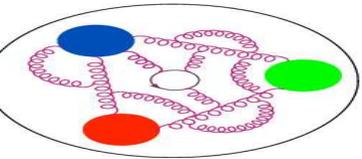
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J. Karpie et al., JHEP 1904 (2019) 057

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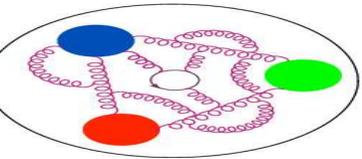
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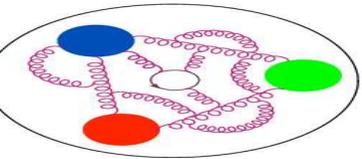
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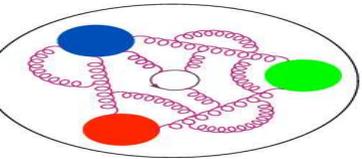
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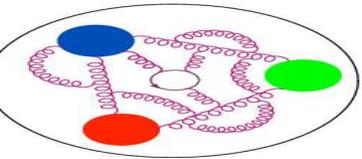
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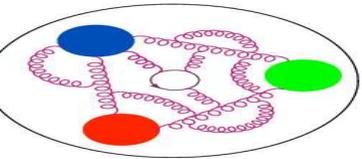
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Investigation of several of these systematics in:

C. Alexandrou et al. [ETM Collaboration], "Systematic uncertainties in parton distribution functions from lattice QCD simulations at the physical point",

Phys. Rev. D99 (2019) 114504.

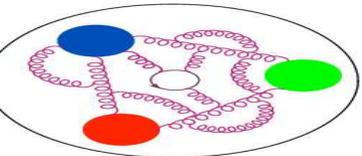


Preliminary new results – qPDFs $N_f = 2 + 1 + 1$



- fermions: $N_f = 2 + 1 + 1$ TM fermions + clover term,
- gluons: Iwasaki gauge action, $\beta = 1.778$,
- $64^3 \times 128$, $L = 5.2$ fm, $m_\pi L = 3.55$,
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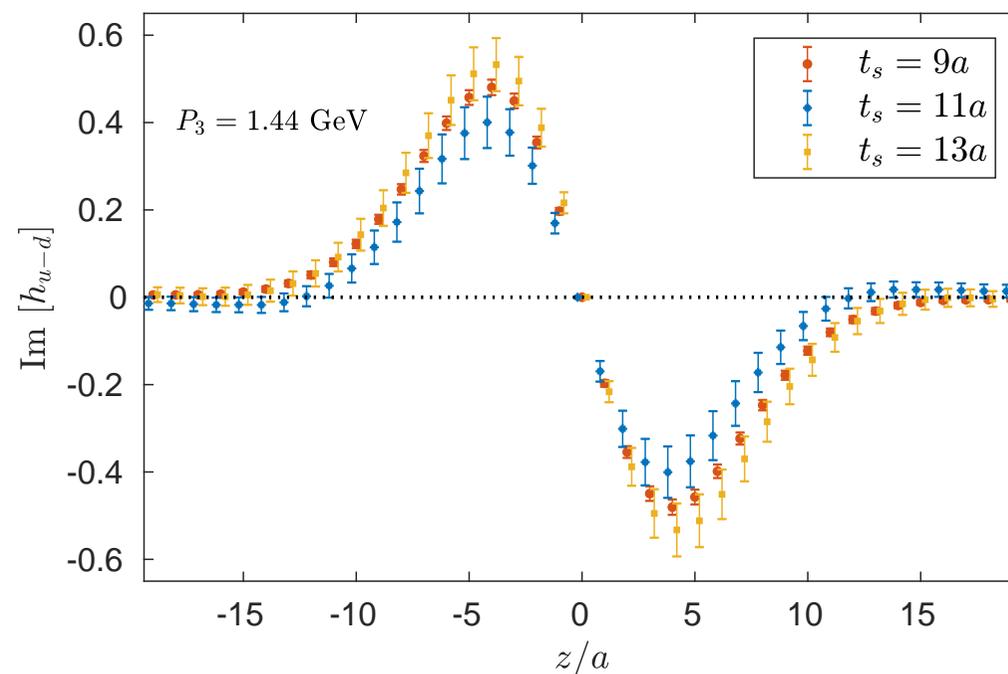
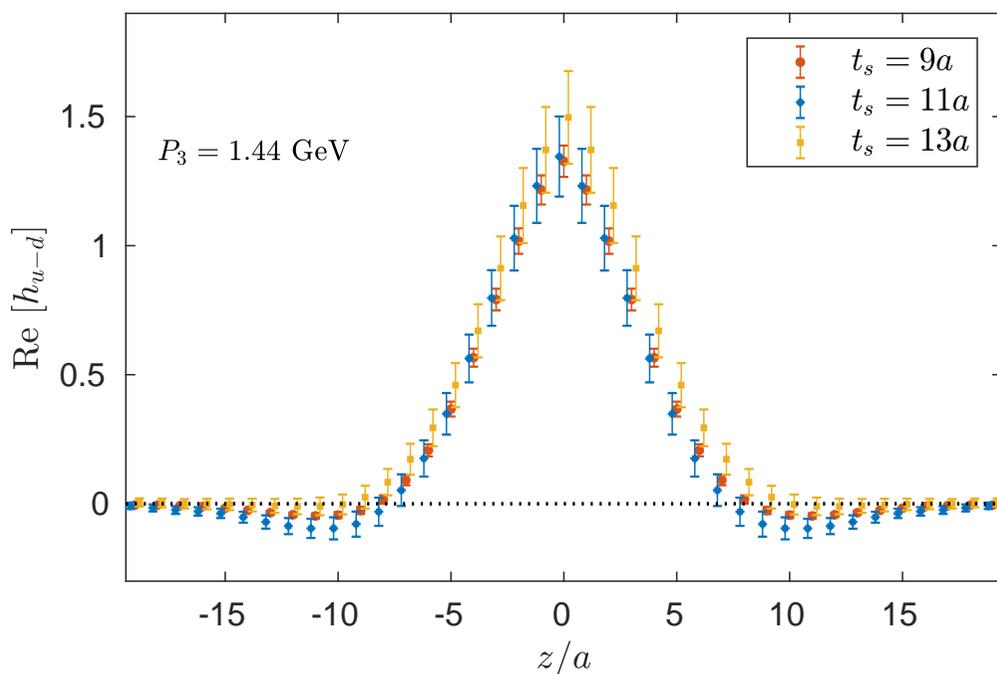


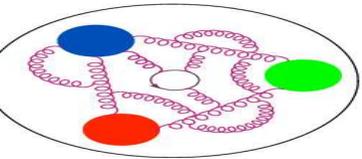


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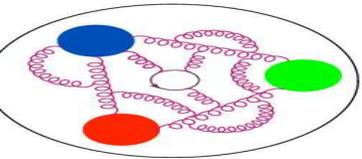


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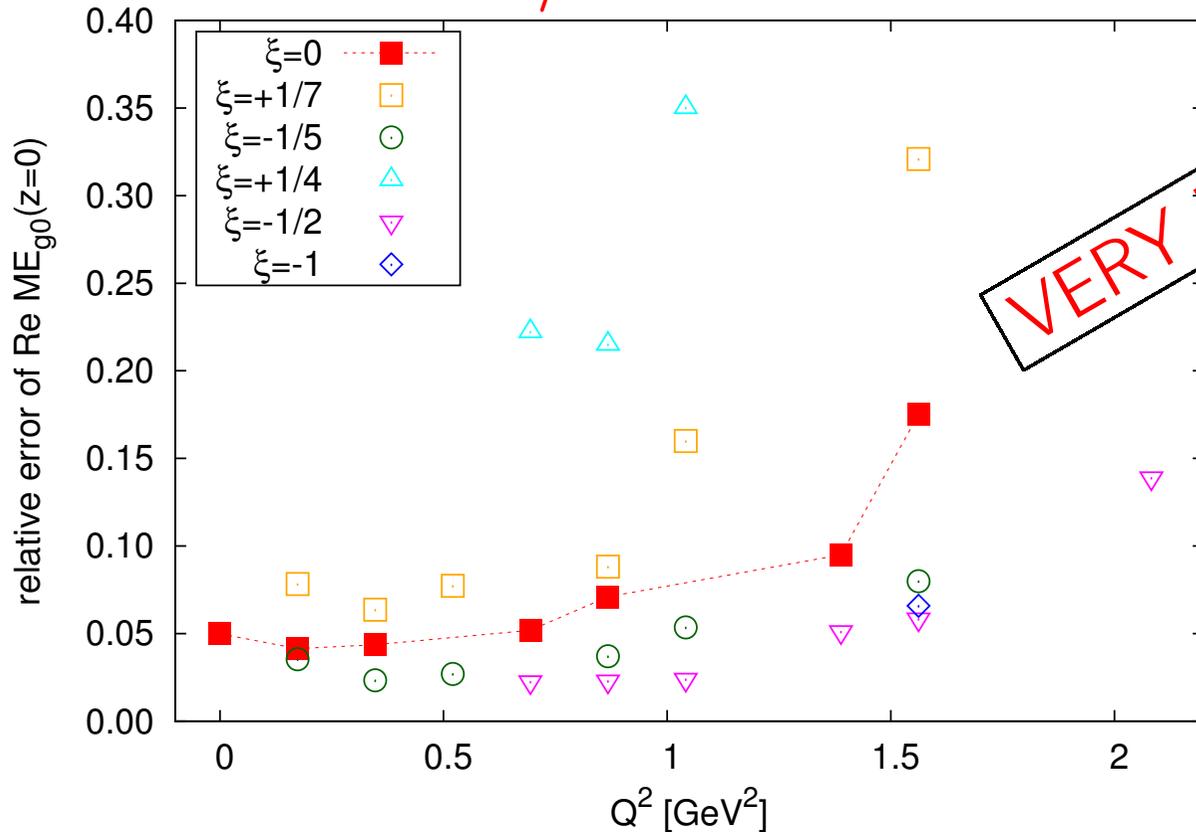
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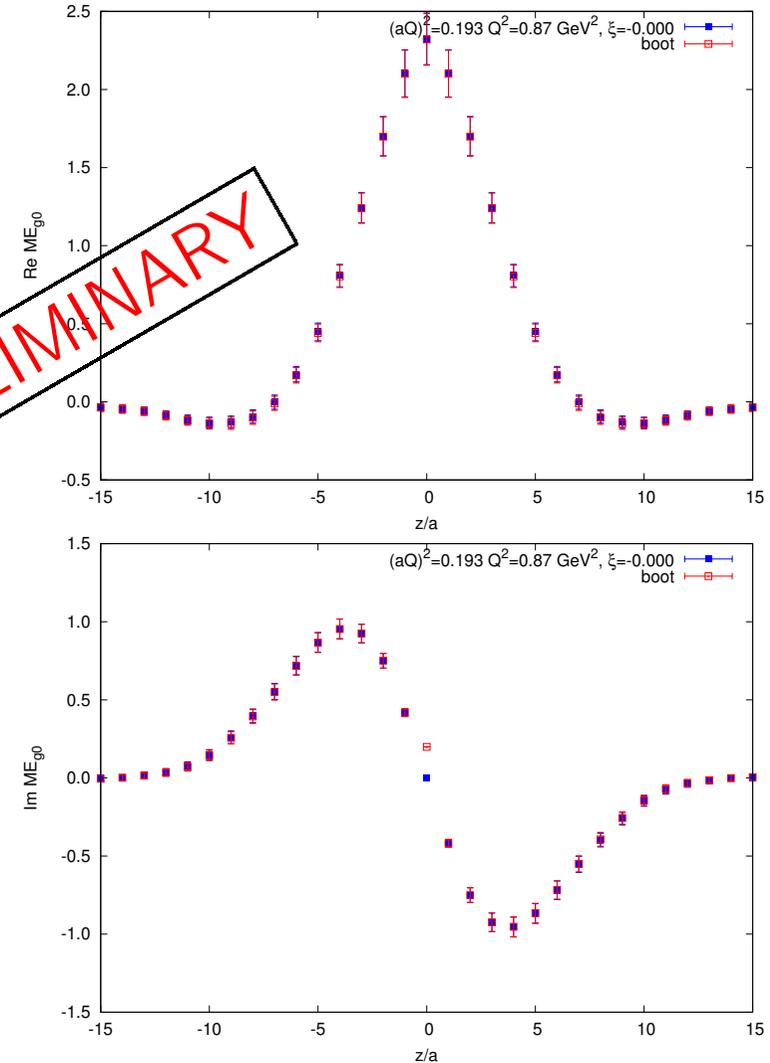
33 distinct classes from 786 combinations of source/sink momentum

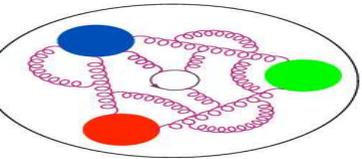


VERY PRELIMINARY

Quasi-skewness variable:

$$\tilde{\xi} = -\frac{P_3'' - P_3'}{P_3'' + P_3'} = -\frac{Q_3}{2P_3}$$





Conclusions and prospects



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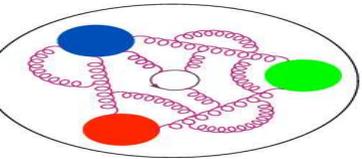
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C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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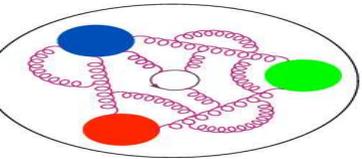
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Conclusions and prospects



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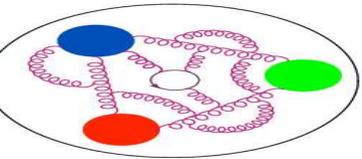
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Conclusions and prospects



Outline of the talk

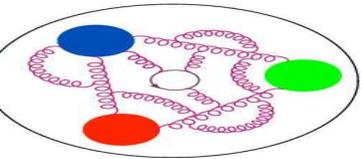
Quasi-PDFs

Systematics

Summary

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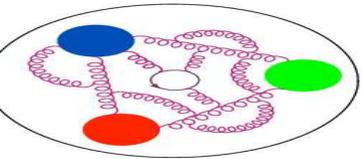
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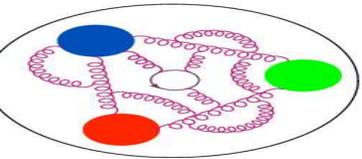
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Thank you for your attention!