Two-current correlations and DPDs for the Nucleon on the lattice

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Motivation:

- **Two-current matrix elements** sensitive to charge / parton correlations within a Hadron
- New information about hadronic wave function
- Can be used to directly extract PDFs [arXiv:1901.03921](https://arxiv.org/abs/1901.03921)
- Relation to **double parton distributions (DPDs)** relevant for LHC processes (main purpose of our research) [arXiv:1111.0910](https://arxiv.org/abs/1111.0910):

\[
M^{n_1,n_2}(y) = 2(p^+)^{1-n_1-n_2} \int dy^- \langle p | O_{n_1}^+(y) O_{n_2}^+(0) | p \rangle \\
= \int_0^1 dx_1 x_1^{n_1-1} \int_0^1 dx_2 x_2^{n_2-1} \left[ F(x_1, x_2, y) + \text{charge conj} \right]
\]

Recent publication: [arXiv:1807.03073](https://arxiv.org/abs/1807.03073) (for the case of the Pion)
Two-current correlation functions

For the Proton $p$:

$$
\mathcal{M}_{ij}(P, y) := \langle p(\vec{P}) | O_i^{q_1 q_2}(0) O_j^{q_3 q_4}(y) | p(\vec{P}) \rangle
$$

with momentum $\vec{P}$ and operator insertions

$$
O_{qq'}^i(x) \in \begin{cases} 
S_{qq'}(x) = \bar{q}(x) \ q'(x) \\
P_{qq'}(x) = i\bar{q}(x) \ \gamma_5 q'(x) \\
V_{qq'}^\mu(x) = \bar{q}(x) \ \gamma^\mu q'(x) \\
A_{qq'}^\mu(x) = \bar{q}(x) \ \gamma^\mu \gamma_5 q'(x) \\
T_{qq'}^{\mu\nu}(x) = \bar{q}(x) \ \sigma^{\mu\nu} q'(x)
\end{cases}
$$

On the lattice we can extract $\mathcal{M}_{ij}(P, y)$ from four point functions, assuming that $y^0 = 0$. 
Calculation on the Lattice

Euclidean spacetime:

\[ \mathcal{M}_{ij}(P, y)|_{y^0=0} = 2V \sqrt{m_p^2 + \vec{P}^2} \frac{C_{4pt}^{\vec{P},ij}(t, \tau, \vec{y})}{C_{2pt}^{\vec{P}}(t)} \bigg|_{0 \ll \tau \ll t} \]

with 4-point / 2-point function:

\[ C_{4pt}^{\vec{P},ij}(t, \tau, \vec{y}) = \langle \mathcal{P}^{\vec{P}}(0) \mathcal{O}_{i}^{q_1 q_2}(\vec{0}, \tau) \mathcal{O}_{j}^{q_3 q_4}(\vec{y}, \tau) \bar{\mathcal{P}^{\vec{P}}}(t) \rangle \]

\[ C_{2pt}^{\vec{P}}(t) = \langle \mathcal{P}^{\vec{P}}(0) \bar{\mathcal{P}^{\vec{P}}}(t) \rangle \]

and Proton interpolators (unpolarized, positive parity):

\[ \mathcal{P}^{\vec{P}}(t) = \frac{1}{2} \sum_{\vec{x}} e^{-i \vec{x} \cdot \vec{\bar{P}}} \epsilon_{abc} (1 + \gamma_4) u_a(x) \left[ u_b^T(x) i \gamma_2 \gamma_4 \gamma_5 d_c(x) \right] \bigg|_{t=x_4} \]

\[ \bar{\mathcal{P}^{\vec{P}}}(t) = \frac{1}{2} \sum_{\vec{x}} e^{i \vec{x} \cdot \vec{\bar{P}}} \epsilon_{abc} \left[ \bar{u}_a(x) i \gamma_2 \gamma_4 \gamma_5 \bar{d}_b^T(x) \right] \bar{u}_c(x)(1 + \gamma_4) \bigg|_{t=x_4} \]
Contractions

\[ \langle \mathcal{P} \bar{\mathcal{P}}(0) \ O_{\vec{i} q_1 q_2}^{q_1 q_2} (\vec{0}, \tau) \ O_{\vec{j} q_3 q_4}^{q_3 q_4} (\vec{y}, \tau) \ \bar{\mathcal{P}} \bar{\mathcal{P}}(t) \rangle \]

Each graph consists of several contractions which depend on the specific flavor.
Contractions

\[ \langle \mathcal{P} \vec{P}(0) \mathcal{O}_i^{q_1 q_2} (\vec{0}, \tau) \mathcal{O}_j^{q_3 q_4} (\vec{y}, \tau) \mathcal{P} \vec{P}(t) \rangle \]

\[ C_{ij, q_1 q_2 q_3 q_4}^{1} = \]

\[ \text{E.g. } C_{1, uu dd}^{ij} = \]

Each graph consists of several contractions which depend on the specific flavor.
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Contributions to specific flavor combinations

$$\langle p \mid O_{i}^{uu}(\vec{0}) \ O_{j}^{dd}(\vec{y}) \mid p \rangle = C_{1}^{ij,uu dd}(\vec{y}) + S_{1}^{ij,u}(\vec{y}) + S_{1}^{ji,d}(-\vec{y}) + D^{ij}(\vec{y})$$

$$\langle p \mid O_{i}^{uu}(\vec{0}) \ O_{j}^{uu}(\vec{y}) \mid p \rangle = C_{1}^{ij,uu uu}(\vec{y}) + C_{2}^{ij,u}(\vec{y}) + C_{2}^{ji,u}(-\vec{y}) + S_{1}^{ij,u}(\vec{y}) + S_{1}^{ji,u}(-\vec{y}) + S_{2}^{ij}(\vec{y}) + D^{ij}(\vec{y})$$

$$\langle p \mid O_{i}^{dd}(\vec{0}) \ O_{j}^{dd}(\vec{y}) \mid p \rangle = C_{2}^{ij,d}(\vec{y}) + S_{1}^{ij,d}(\vec{y}) + S_{1}^{ji,d}(-\vec{y}) + S_{2}^{ij,d}(\vec{y}) + D^{ij}(\vec{y})$$

$$\langle p \mid O_{i}^{ud}(\vec{0}) \ O_{j}^{du}(\vec{y}) \mid p \rangle = C_{1}^{ij,u d d u}(\vec{y}) + C_{2}^{ij,u}(\vec{y}) + C_{2}^{ji,d}(-\vec{y}) + S_{2}^{ij}(\vec{y})$$
Contributions to specific flavor combinations

in this talk

\[
\langle p | O_{i}^{uu}(\vec{0}) \ O_{j}^{dd}(\vec{y}) | p \rangle = C_{1}^{ij, uudd}(\vec{y}) + S_{1}^{ij, u}(\vec{y}) + S_{1}^{ji, d}(\vec{y}) + D_{ij}^{ij}(\vec{y})
\]

\[
\langle p | O_{i}^{uu}(\vec{0}) \ O_{j}^{uu}(\vec{y}) | p \rangle = C_{1}^{ij, uuuu}(\vec{y}) + C_{2}^{ij, u}(\vec{y}) + C_{2}^{ji, u}(\vec{y}) + S_{1}^{ij, u}(\vec{y}) + S_{1}^{ji, u}(\vec{y}) + S_{2}^{ij}(\vec{y}) + D_{ij}^{ij}(\vec{y})
\]

\[
\langle p | O_{i}^{dd}(\vec{0}) \ O_{j}^{dd}(\vec{y}) | p \rangle = C_{2}^{ij, d}(\vec{y}) + C_{2}^{ji, d}(\vec{y}) + S_{1}^{ij, d}(\vec{y}) + S_{1}^{ji, d}(\vec{y}) + S_{2}^{ij}(\vec{y}) + D_{ij}^{ij}(\vec{y})
\]

\[
\langle p | O_{i}^{ud}(\vec{0}) \ O_{j}^{du}(\vec{y}) | p \rangle = C_{1}^{ij, uddu}(\vec{y}) + C_{2}^{ij, u}(\vec{y}) + C_{2}^{ji, d}(\vec{y}) + S_{2}^{ij}(\vec{y})
\]

in progress

(too noisy)
Technical Details

- point source / propagator
- stochastic source / propagator / with HPE
- sequential source / propagator with constituents

▶ APE smearing *Nucl. Phys. B251 (1985)*
▶ Boosted sources (momentum smearing) *arXiv:1602.05525*
▶ Sequential source technique *Nucl. Phys. B316 (1989)*
▶ Stochastic wall sources: $\eta_{\alpha a x}^\ell = (\pm 1 \pm i)/\sqrt{2}$ on specific time slice

Stochastic propagator: $D\psi^\ell = \eta^\ell$

<table>
<thead>
<tr>
<th>$N_{\text{stoch}}$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$S_1$ (loop)</th>
<th>$D$ (2 loops)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>96</td>
<td>120</td>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>
Technical Details

- **Point source / propagator**
- **Stochastic source / propagator / with HPE**
- **Sequential source / propagator with constituents**

- **APE smearing** *Nucl. Phys. B251 (1985)*
- **Boosted sources (momentum smearing)** *arXiv:1602.05525*
- **Sequential source technique** *Nucl. Phys. B316 (1989)*
- **Stochastic wall sources**: \( \eta_{\alpha ax}^\ell = (\pm 1 \pm i)/\sqrt{2} \) on specific time slice
- **Remove trivial terms from stochastic propagators by applying hopping parameter expansion**:
  - \( C_2 \): apply \( n(\vec{y}) = \sum_{i=1}^{3} \min(|y_i|, L - |y_i|) \) hopping terms
  - Loops in \( S_1 \), \( D \): apply \( n(\Gamma) \) terms, e.g. \( n(\gamma^\mu) = 3 \)
  - \( \Rightarrow \) Reduce stochastic noise *arXiv:0910.3970*
Lattice Setup

CLS ensembles ($n_f = 2 + 1$, Wilson fermions, order-$a$ improved, see arXiv:1411.3982), start with H102, 960 configs used:

<table>
<thead>
<tr>
<th>id</th>
<th>$\beta$</th>
<th>$a$ [fm]</th>
<th>$L^3 \times T$</th>
<th>$\kappa_{I/s}$</th>
<th>$m_{\pi}/K$ [MeV]</th>
<th>$m_{\pi}L$</th>
<th>conf.</th>
</tr>
</thead>
<tbody>
<tr>
<td>H102</td>
<td>3.4</td>
<td>0.0854</td>
<td>$32^3 \times 96$</td>
<td>0.136865</td>
<td>356</td>
<td>4.9</td>
<td>2037</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.136549339</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- $t_{\text{src}} = 48a$ (point sources at random spatial position)
- $t = t_{\text{snk}} - t_{\text{src}} = \begin{cases} 12a & \vec{P} = \vec{0} \\ 10a & \vec{P} \neq \vec{0} \end{cases}$
- Insertion time $\tau \in [t_{\text{src}} + 3a, t_{\text{snk}} - 3a]$ for $C_1$ (fit), else fix $\tau = t_{\text{src}} + t/2$
- Momenta: $\vec{P} \in \{(0, 0, 0), (-1, -1, -1), (-2, -2, -2)\}$ ($\vec{P} = (-3, -3, -3)$ in progress)

Renormalization for $\beta = 3.4$, including conversion to $\overline{\text{MS}}$ at $\mu = 2$GeV (PRELIMINARY M. Göckeler, private comm.):

<table>
<thead>
<tr>
<th>S</th>
<th>P</th>
<th>V</th>
<th>A</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>0.6251</td>
<td>??</td>
<td>0.7053</td>
<td>0.7466</td>
</tr>
</tbody>
</table>
Results: $\tau$-plateau ($C_1$) and $\vec{y}$-dependence for $V_4 V_4/A_4 A_4$

PRELIMINARY!!

$\langle V_4 V_4 \rangle, \vec{\rho} = (0, 0, 0)$

$\langle A_4 A_4 \rangle, \vec{\rho} = (0, 0, 0)$

$\tau$ fit at $\vec{y} = (-3, 4, 3)$ for $V_4 V_4$

$\tau$ fit at $\vec{y} = (-3, 4, 3)$ for $A_4 A_4$
Results: \( \tau \)-plateau \((C_1)\) and \(\vec{y}\)-dependence for \(V_4V_4/A_4A_4\)

**Preliminary!!**

- Clear signal for \(C_1\) (AA suppressed for \(uu\))
- \(C_2\) mainly relevant for small distances
- \(S_1\): disconnected contributions small
- Altogether comparable with the case of the Pion *arXiv:1807.03073*
Twist-2 operators and Lorentz Invariant Functions

For vector/axial vector/tensor case can parameterize matrix elements (trace subtracted) in terms of Lorentz invariant functions, e.g. for $VV/AA$:

$$\mathcal{T} \mathcal{M}_{VV/AA}^{\mu\nu}(P, y) = \left(2P^\mu P^\nu - \frac{1}{2}P_2 g^{\mu\nu}\right) A(y^2, Py) + \ldots \quad \text{+-components} \rightarrow \mathcal{M}_{VV/AA}^{++}(P, y) = 2(P^+)^2 A(y^2, Py)$$

Relation to DPDs (first Mellin Moment):

$$\int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2 \ f(x_1, x_2, \vec{y}_2^2) = \int d(Py) \ A(-\vec{y}_2^2, Py)$$

where the operator type (twist-2 operator) maps to a certain quark polarization:

- $V$: $\bar{q}\gamma^+q \rightarrow q$ unpolarized
- $A$: $\bar{q}\gamma^+\gamma_5q \rightarrow \Delta q$ longitudinal polarization
- $T$: $\bar{q}\sigma^{+j}q \rightarrow \delta q^j$ transverse polarization

We have data for $A_{VV}, A_{AA}, m|\vec{y}|A_{VT}, A_{TT}$ and $m^2\vec{y}^2B_{TT}$ (extra quadrupole term)
Results: Invariant Functions ($C_1$ and $C_2$ only)

PRELIMINARY!!

$C_1, A^{ud}_{\mu\nu}, \vec{\rho} \cdot \vec{y} = 0$, momentum comparison

$C_2, A^{uu}_{\mu\nu}, \vec{\rho} \cdot \vec{y} = 0$, momentum comparison

$C_1, m|\vec{y}|A^{ud}_{\mu\nu}, \vec{\rho} \cdot \vec{y} = 0$, momentum comparison

$C_1, m|\vec{y}|A^{uu}_{\mu\nu}, \vec{\rho} \cdot \vec{y} = 0$, momentum comparison

Consistent with Lorentz invariance

Comparable correlations for $uu/ud$ for large distances, small correlations for $dd$,

Small distances: $uu/dd$ become large
Results: Invariant Functions ($C_1$ and $C_2$ only)

PRELIMINARY!!

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PRELIMINARY!!

$ud$, $\vec{p} = (0, 0, 0)$, channel comparison

$uu$, $\vec{p} = (0, 0, 0)$, channel comparison

- Consistent with Lorentz invariance
- Comparable correlations for $uu/ud$ for large distances, small correlations for $dd$,
- Small distances: $uu/dd$ become large
- Visible polarization effects for $ud$, $uu$ dominated by unpolarized quarks
Summary and Outlook

Achieved/Observed:
- Considered two-current matrix elements for the Nucleon (Proton) on the lattice
- Calculated 4 of 5 contributing graphs for several insertion types and momenta
- Obtained good signal for most graphs and channels (D too noisy)
- Calculated Lorentz invariant functions related to DPD Mellin moments for specific quark polarizations / flavors

Future work / currently in progress:
▶ Calculate remaining graphs ($S_2$)
▶ Increase statistics (especially for non-zero momenta)
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Thank you for your attention!