Two-current correlations and DPDs for the Nucleon on the lattice

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Introduction

Motivation:

- Two-current matrix elements sensitive to charge / parton correlations within a Hadron
- New information about hadronic wave function
- Can be used to directly extract PDFs *arXiv:1901.03921*
- Relation to double parton distributions (DPDs) relevant for LHC processes (main purpose of our research) *arXiv:1111.0910* :

$$M^{n_1,n_2}(\boldsymbol{y}) = 2(p^+)^{1-n_1-n_2} \int \mathrm{d}y^- \, \langle p| \, \mathcal{O}^{+\cdots+}_{n_1}(y) \, \mathcal{O}^{+\cdots+}_{n_2}(0) \, |p\rangle$$
$$= \int_0^1 \mathrm{d}x_1 x_1^{n_1-1} \int_0^1 \mathrm{d}x_2 x_2^{n_2-1} \left[F(x_1, x_2, \boldsymbol{y}) + \text{charge conj} \right]$$

Recent publication: *arXiv:1807.03073* (for the case of the Pion)

Two-current correlation functions

For the Proton p:

$$\mathcal{M}_{ij}(P,y) := \langle p(\vec{P})|\, \mathcal{O}_i^{q_1 q_2}(0)\ \mathcal{O}_j^{q_3 q_4}(y)\, |p(\vec{P})\rangle$$

with momentum $\vec{P}$ and operator insertions

$$\mathcal{O}_i^{qq'}(x) \in \begin{cases} S_{qq'}(x) = \bar{q}(x)\ q'(x) \\ P_{qq'}(x) = i\bar{q}(x)\ \gamma_5 q'(x) \\ V^{\mu}_{qq'}(x) = \bar{q}(x)\ \gamma^{\mu} q'(x) \\ A^{\mu}_{qq'}(x) = \bar{q}(x)\ \gamma^{\mu}\gamma_5 q'(x) \\ T^{\mu\nu}_{qq'}(x) = \bar{q}(x)\ \sigma^{\mu\nu} q'(x) \end{cases}$$

On the lattice we can extract $\mathcal{M}_{ij}(P,y)$ from four point functions, assuming that $y^0 = 0$.

Calculation on the Lattice

Euclidean spacetime:

$$\mathcal{M}_{ij}(P,y)|_{y^0=0} = 2V\sqrt{m_p^2+\vec{P}^2}\;\frac{C_{4\mathrm{pt}}^{\vec{P},ij}(t,\tau,\vec{y})}{C_{2\mathrm{pt}}^{\vec{P}}(t)}\Bigg|_{0\ll\tau\ll t}$$

with 4-point / 2-point function:

$$C_{4\mathrm{pt}}^{\vec{P},ij}(t,\tau,\vec{y}) = \langle \mathcal{P}^{\vec{P}}(0)\;\mathcal{O}_i^{q_1q_2}(\vec{0},\tau)\;\mathcal{O}_j^{q_3q_4}(\vec{y},\tau)\;\overline{\mathcal{P}}^{\vec{P}}(t)\rangle$$

$$C_{2\mathrm{pt}}^{\vec{P}}(t) = \langle \mathcal{P}^{\vec{P}}(0)\;\overline{\mathcal{P}}^{\vec{P}}(t)\rangle$$

and Proton interpolators (unpolarized, positive parity):

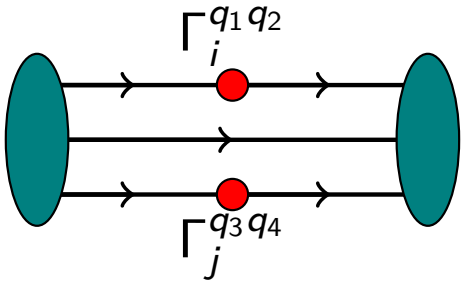
$$\mathcal{P}^{\vec{P}}(t) = \frac{1}{2}\sum_{\vec{x}} e^{-i\vec{x}\cdot\vec{P}}\;\epsilon_{abc}\;(\mathbb{1}+\gamma_4)u_a(x)\left[u_b^T(x)i\gamma_2\gamma_4\gamma_5 d_c(x)\right]\Bigg|_{t=x_4}$$

$$\overline{\mathcal{P}}^{\vec{P}}(t) = \frac{1}{2}\sum_{\vec{x}} e^{i\vec{x}\cdot\vec{P}}\;\epsilon_{abc}\;\left[\bar{u}_a(x)i\gamma_2\gamma_4\gamma_5\bar{d}_b^T(x)\right]\bar{u}_c(x)(\mathbb{1}+\gamma_4)\Bigg|_{t=x_4}$$

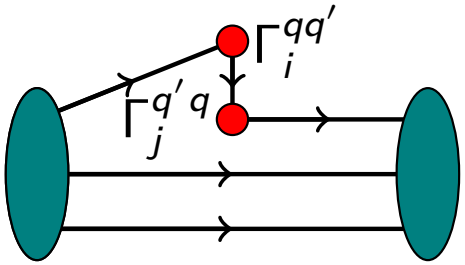
Contractions

$$\langle \mathcal{P}^{\vec{P}}(0)\ \mathcal{O}_i^{q_1 q_2}(\vec{0},\tau)\ \mathcal{O}_j^{q_3 q_4}(\vec{y},\tau)\ \overline{\mathcal{P}}^{\vec{P}}(t)\rangle$$

$C_1^{ij,q_1 q_2 q_3 q_4} =$



$C_2^{ij,q} =$

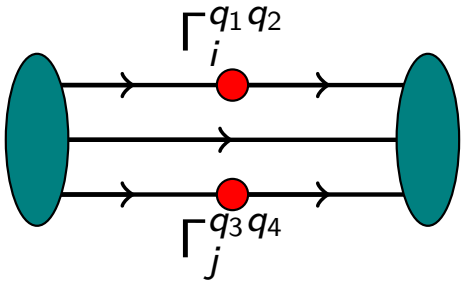


Each graph consists of several contractions which depend on the specific flavor

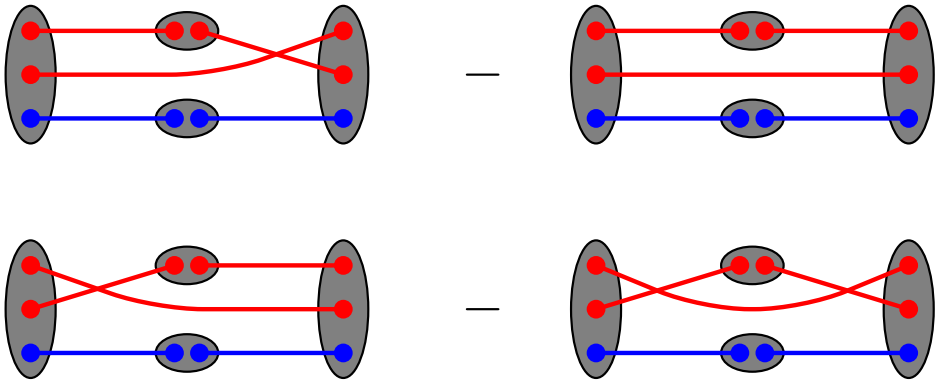
Contractions

$$\langle \mathcal{P}^{\vec{P}}(0)\ \mathcal{O}_i^{q_1 q_2}(\vec{0},\tau)\ \mathcal{O}_j^{q_3 q_4}(\vec{y},\tau)\ \overline{\mathcal{P}}^{\vec{P}}(t)\rangle$$

$C_1^{ij,q_1 q_2 q_3 q_4}$ =



E.g. $C_1^{ij,uudd}$ =



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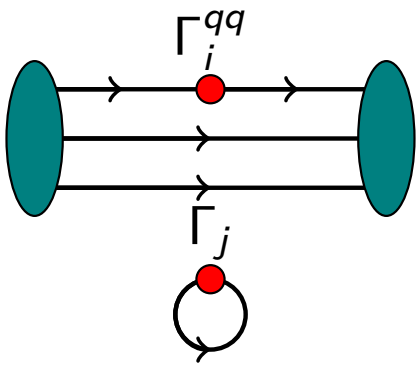
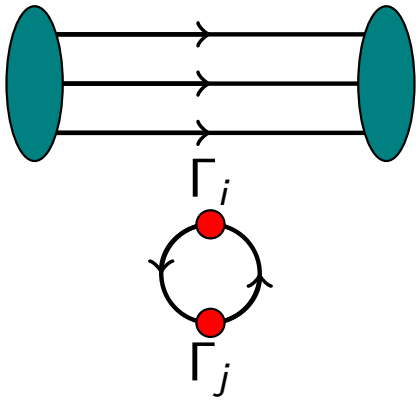
Contractions

$$\langle \mathcal{P}^{\vec{P}}(0)\ \mathcal{O}_i^{q_1 q_2}(\vec{0},\tau)\ \mathcal{O}_j^{q_3 q_4}(\vec{y},\tau)\ \overline{\mathcal{P}}^{\vec{P}}(t)\rangle$$

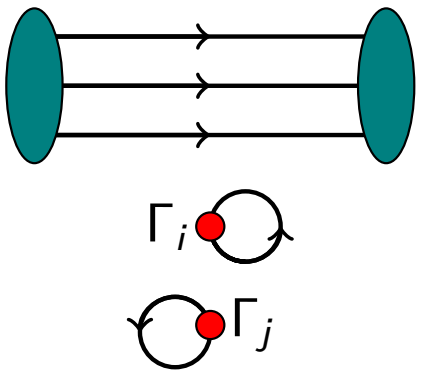
$S_1^{ij,q}$ =



S_2^{ij} =

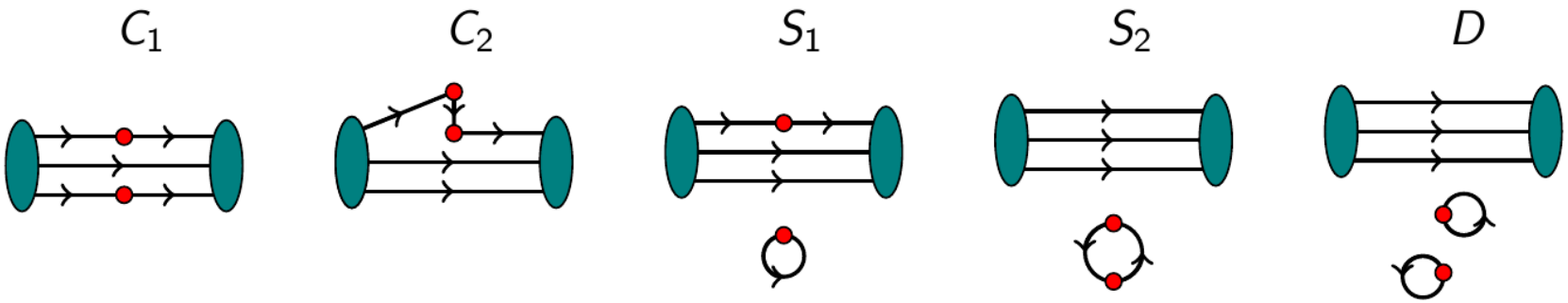


D^{ij} =



Each graph consists of several contractions which depend on the specific flavor

Contributions to specific flavor combinations



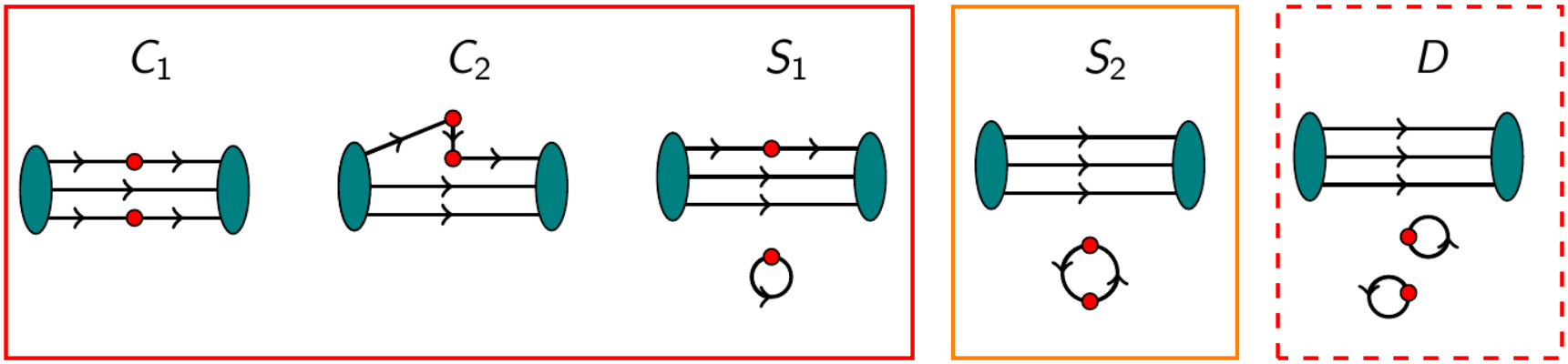
$$\langle p|\, \mathcal{O}_i^{uu}(\vec{0})\ \mathcal{O}_j^{dd}(\vec{y})\, |p\rangle = C_1^{ij,uudd}(\vec{y}) + S_1^{ij,u}(\vec{y}) + S_1^{ji,d}(-\vec{y}) + D^{ij}(\vec{y})$$

$$\begin{aligned}\langle p|\, \mathcal{O}_i^{uu}(\vec{0})\ \mathcal{O}_j^{uu}(\vec{y})\, |p\rangle &= C_1^{ij,uuuu}(\vec{y}) + C_2^{ij,u}(\vec{y}) + C_2^{ji,u}(-\vec{y}) \\ &\quad + S_1^{ij,u}(\vec{y}) + S_1^{ji,u}(-\vec{y}) + S_2^{ij}(\vec{y}) + D^{ij}(\vec{y})\end{aligned}$$

$$\begin{aligned}\langle p|\, \mathcal{O}_i^{dd}(\vec{0})\ \mathcal{O}_j^{dd}(\vec{y})\, |p\rangle &= C_2^{ij,d}(\vec{y}) + C_2^{ji,d}(-\vec{y}) + S_1^{ij,d}(\vec{y}) + S_1^{ji,d}(-\vec{y}) \\ &\quad + S_2^{ij}(\vec{y}) + D^{ij}(\vec{y})\end{aligned}$$

$$\langle p|\, \mathcal{O}_i^{ud}(\vec{0})\ \mathcal{O}_j^{du}(\vec{y})\, |p\rangle = C_1^{ij,uddu}(\vec{y}) + C_2^{ij,u}(\vec{y}) + C_2^{ji,d}(-\vec{y}) + S_2^{ij}(\vec{y})$$

Contributions to specific flavor combinations



in this talk | in progress | (too noisy)

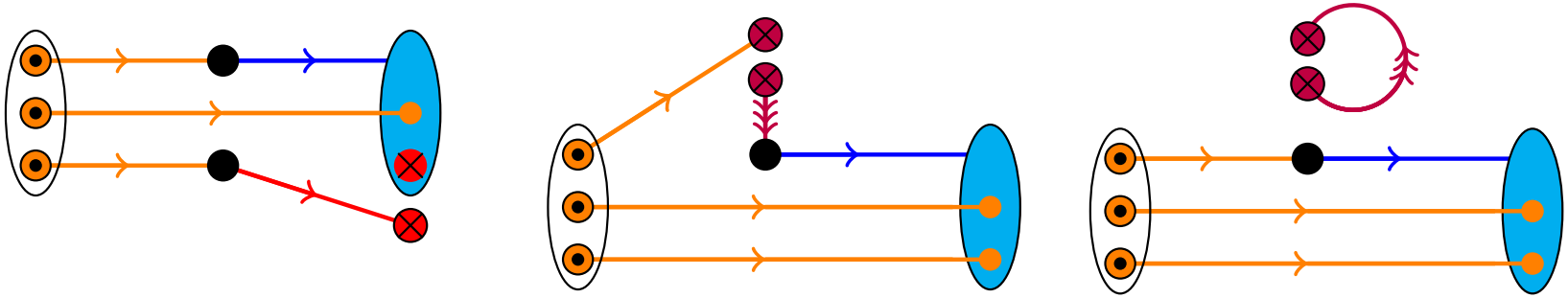
$$\langle p|\, \mathcal{O}_i^{uu}(\vec{0})\ \mathcal{O}_j^{dd}(\vec{y})\, |p\rangle = C_1^{ij,uudd}(\vec{y}) + S_1^{ij,u}(\vec{y}) + S_1^{ji,d}(-\vec{y}) + D^{ij}(\vec{y})$$

$$\langle p|\, \mathcal{O}_i^{uu}(\vec{0})\ \mathcal{O}_j^{uu}(\vec{y})\, |p\rangle = C_1^{ij,uuuu}(\vec{y}) + C_2^{ij,u}(\vec{y}) + C_2^{ji,u}(-\vec{y}) + S_1^{ij,u}(\vec{y}) + S_1^{ji,u}(-\vec{y}) + S_2^{ij}(\vec{y}) + D^{ij}(\vec{y})$$

$$\langle p|\, \mathcal{O}_i^{dd}(\vec{0})\ \mathcal{O}_j^{dd}(\vec{y})\, |p\rangle = C_2^{ij,d}(\vec{y}) + C_2^{ji,d}(-\vec{y}) + S_1^{ij,d}(\vec{y}) + S_1^{ji,d}(-\vec{y}) + S_2^{ij}(\vec{y}) + D^{ij}(\vec{y})$$

$$\langle p|\, \mathcal{O}_i^{ud}(\vec{0})\ \mathcal{O}_j^{du}(\vec{y})\, |p\rangle = C_1^{ij,uddu}(\vec{y}) + C_2^{ij,u}(\vec{y}) + C_2^{ji,d}(-\vec{y}) + S_2^{ij}(\vec{y})$$

Technical Details



 point source / propagator

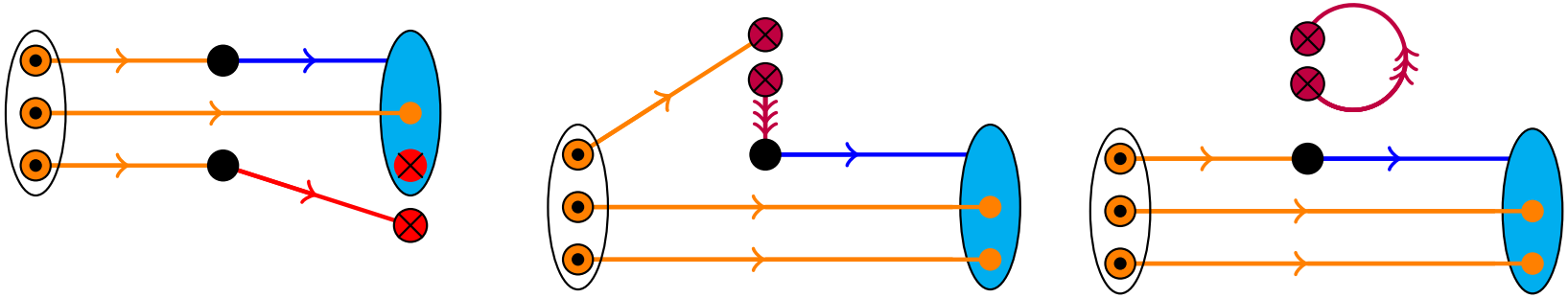
 stochastic source / propagator / with HPE

 sequential source / propagator with constituents

- APE smearing *Nucl. Phys. B251 (1985)*
- Boosted sources (momentum smearing) *arXiv:1602.05525*
- Sequential source technique *Nucl. Phys. B316 (1989)*
- Stochastic wall sources: $\eta^{\ell}_{\alpha a \vec{x}} = (\pm 1 \pm i)/\sqrt{2}$ on specific time slice
  Stochastic propagator: $\mathcal{D}\psi^{\ell} = \eta^{\ell}$

	C_1	C_2	S_1(loop)	D (2 loops)
N_{stoch}	2	96	120	60

Technical Details

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- Stochastic wall sources: $\eta^{\ell}_{\alpha a \vec{x}} = (\pm 1 \pm i)/\sqrt{2}$ on specific time slice
- Remove trivial terms from stoch. propagators by applying hopping parameter expansion :
  C_2: apply $n(\vec{y}) = \sum_{i=1}^{3} \min(|y_i|, L - |y_i|)$ hopping terms
  Loops in S_1, D: apply $n(\Gamma)$ terms, e.g. $n(\gamma^{\mu}) = 3$
  $\Rightarrow$ Reduce stochastic noise *arXiv:0910.3970*

Lattice Setup

CLS ensembles ($n_f = 2+1$, Wilson fermions, order-a improved, see *arXiv:1411.3982*), start with H102, 960 configs used:

id	β	a[fm]	$L^3 \times T$	$\kappa_{l/s}$	$m_{\pi/K}$[MeV]	$m_\pi L$	conf.
H102	3.4	0.0854	$32^3 \times 96$	0.136865	356	4.9	2037
				0.136549339	442		

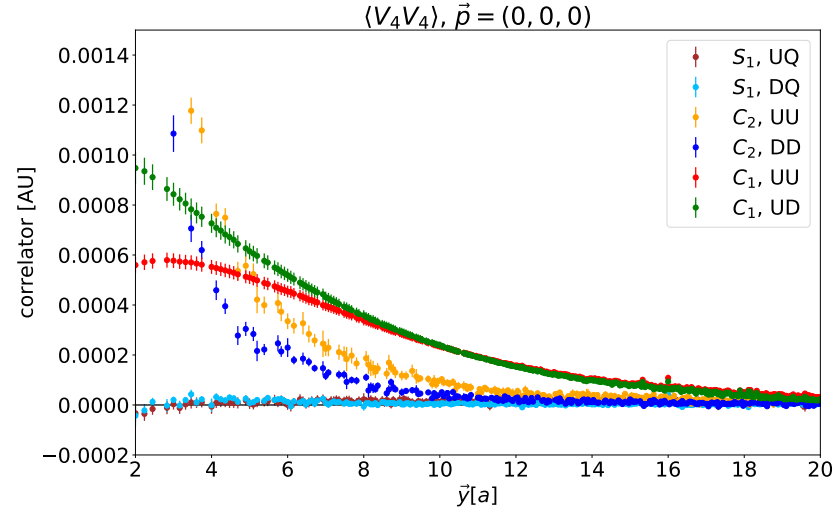
- $t_{\rm src} = 48a$ (point sources at random spatial position)
- $t = t_{\rm snk} - t_{\rm src} = \begin{cases} 12a & \vec{P} = \vec{0} \\ 10a & \vec{P} \neq \vec{0} \end{cases}$
- Insertion time $\tau \in [t_{\rm src} + 3a, t_{\rm snk} - 3a]$ for C_1 (fit), else fix $\tau = t_{\rm src} + t/2$
- Momenta: $\vec{P} \in \{(0,0,0), (-1,-1,-1), (-2,-2,-2)\}$
  ($\vec{P} = (-3,-3,-3)$ in progress)

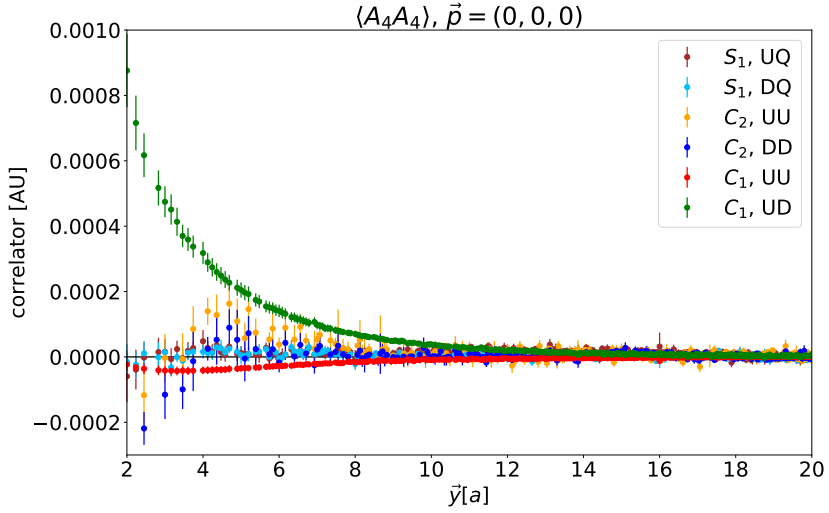
Renormalization for $\beta = 3.4$, including conversion to $\overline{\rm MS}$ at $\mu = 2{\rm GeV}$ (PRELIMINARY *M. Göckeler, private comm.*):

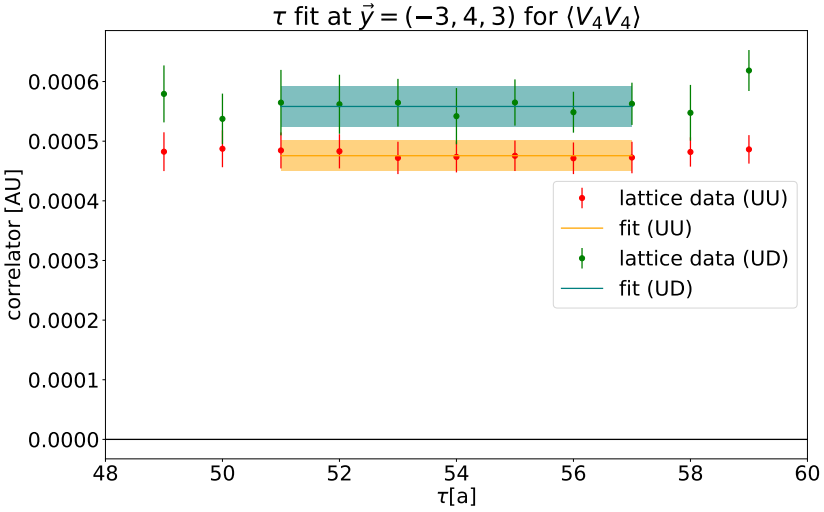
	S	P	V	A	T
Z	0.6251	??	0.7053	0.7466	0.8218

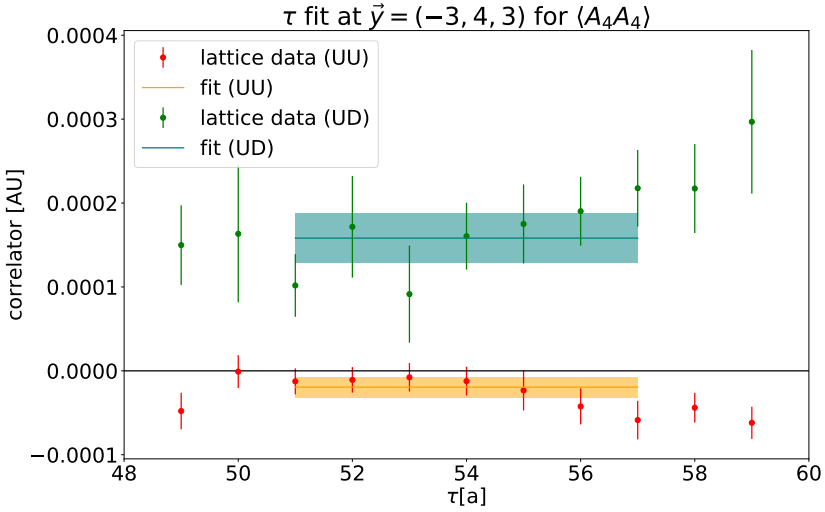
Results: τ-plateau (C_1) and $\vec{y}$-dependence for V_4V_4/A_4A_4

PRELIMINARY!!









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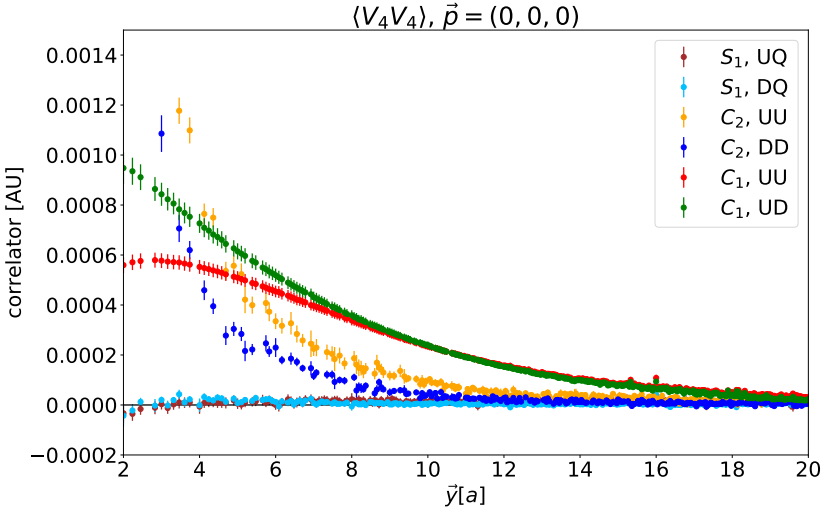
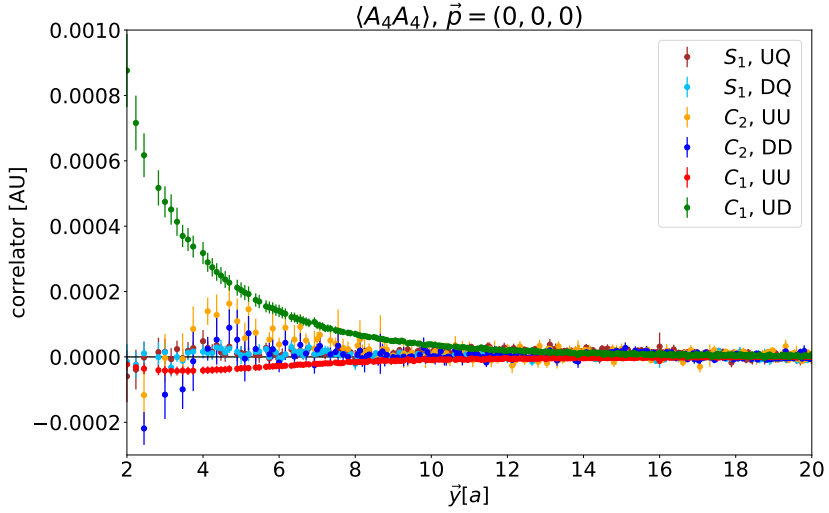




- Clear signal for C_1 (AA suppressed for uu)
- C_2 mainly relevant for small distances
- S_1: disconnected contributions small
- Altogether comparable with the case of the Pion *arXiv:1807.03073*

Twist-2 operators and Lorentz Invariant Functions

For vector/axial vector/tensor case can parameterize matrix elements (trace subtracted) in terms of Lorentz invariant functions, e.g. for VV/AA:

$$\mathcal{T}\mathcal{M}^{\mu\nu}_{VV/AA}(P,y) = \left(2P^\mu P^\nu - \frac{1}{2}P^2 g^{\mu\nu}\right) A(y^2, Py) + \cdots$$

$$\xrightarrow{+-\text{components}} \mathcal{M}^{++}_{VV/AA}(P,y) = 2(P^+)^2 A(y^2, Py)$$

Relation to DPDs (first Mellin Moment):

$$\int_{-1}^{1} \mathrm{d}x_1 \int_{-1}^{1} \mathrm{d}x_2 \; f(x_1, x_2, \vec{y}_\perp^2) = \int \mathrm{d}(Py) \; A(-\vec{y}_\perp^2, Py)$$

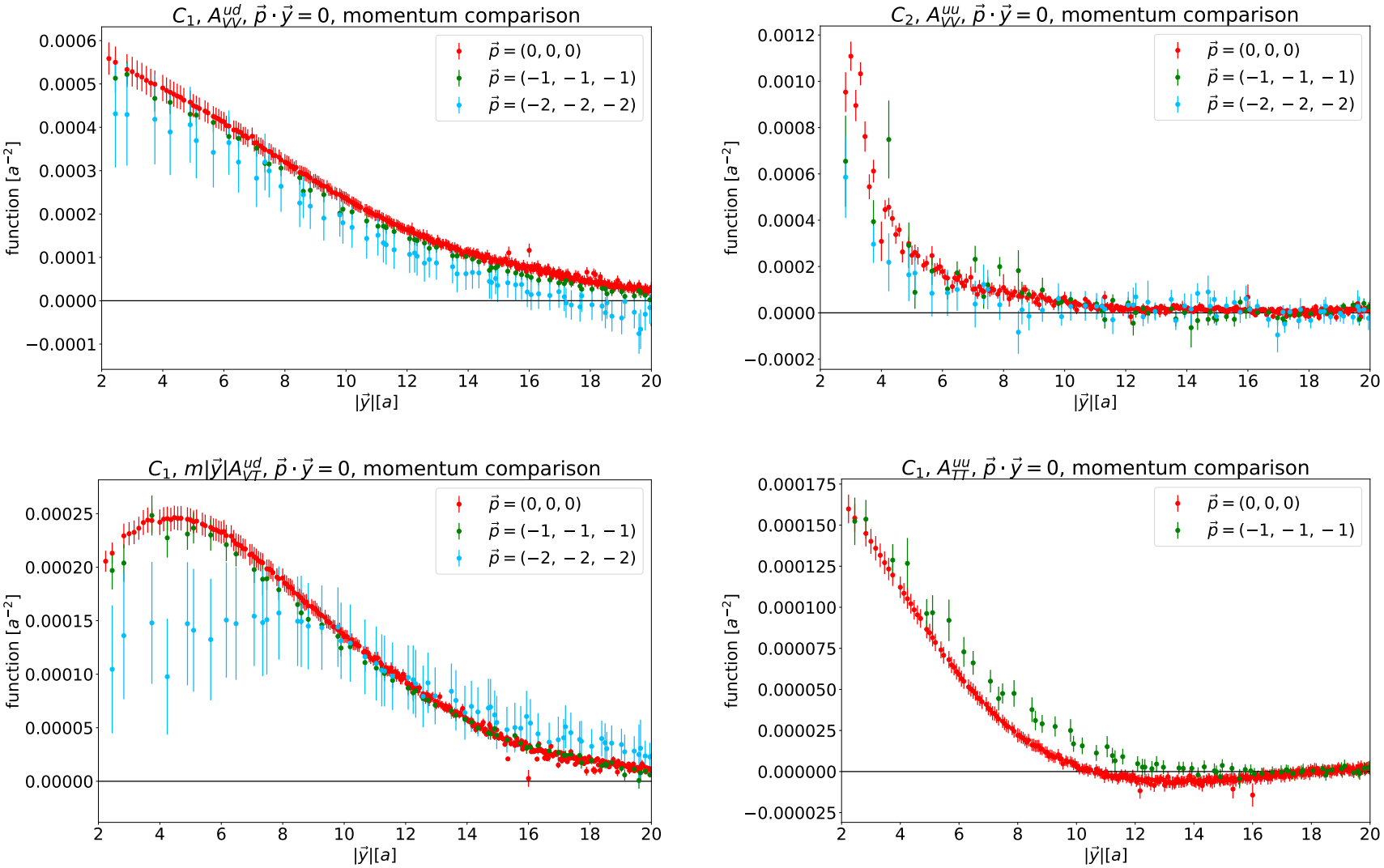
where the operator type (twist-2 operator) maps to a certain quark polarization:

$V:$	$\bar{q}\gamma^+ q$	$\rightarrow$	q	unpolarized
$A:$	$\bar{q}\gamma^+\gamma_5 q$	$\rightarrow$	Δq	longitudinal polarization
$T:$	$\bar{q}\sigma^{+j} q$	$\rightarrow$	δq^j	transverse polarization

We have data for A_{VV} , A_{AA} , $m|\vec{y}|A_{VT}$, A_{TT}
and $m^2\vec{y}^2 B_{TT}$ (extra quadrupole term)

Results: Invariant Functions (C_1 and C_2 only)

PRELIMINARY!!



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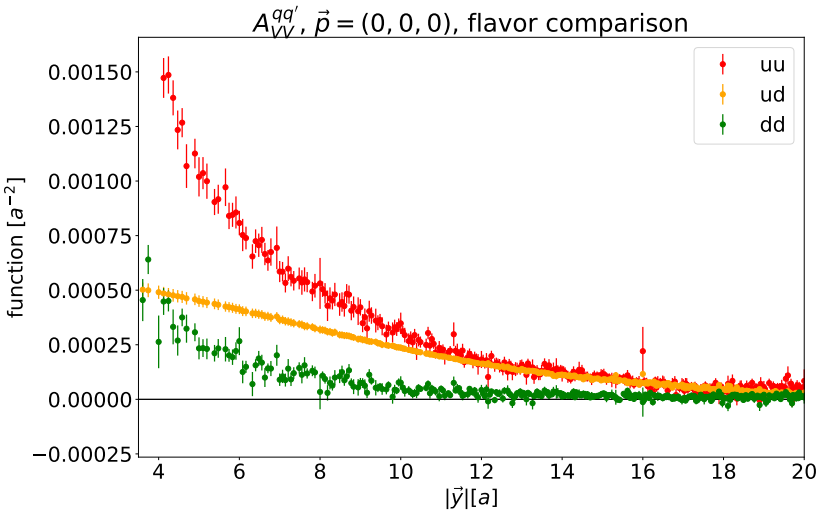

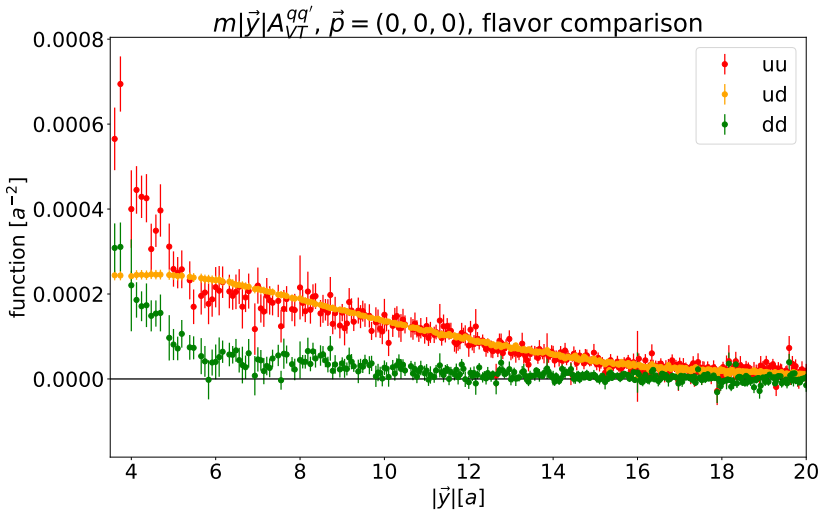

- Consistent with Lorentz invariance
- Comparable correlations for uu/ud for large distances, small correlations for dd,
- Small distances: uu / dd become large

Results: Invariant Functions (C_1 and C_2 only)

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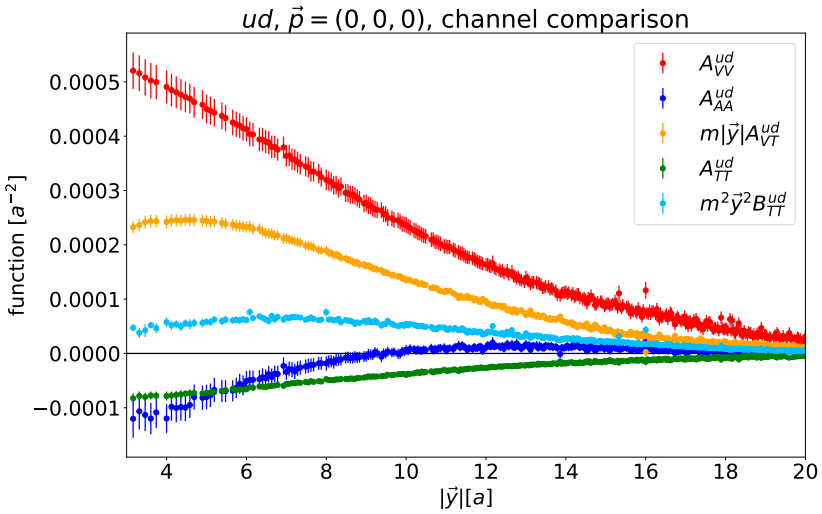

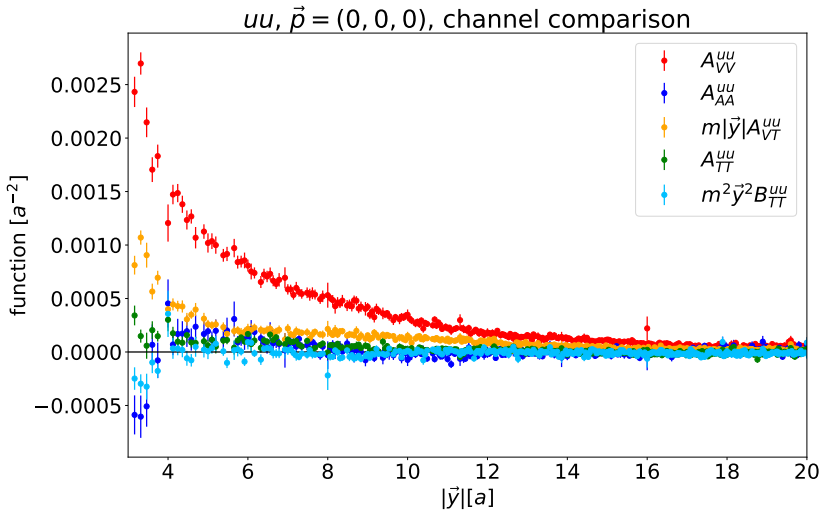

- Consistent with Lorentz invariance
- Comparable correlations for uu/ud for large distances, small correlations for dd,
- Small distances: uu / dd become large
- Visible polarization effects for ud, uu dominated by unpolarized quarks

Summary and Outlook

Achieved/Observed:

- Considered two-current matrix elements for the Nucleon (Proton) on the lattice
- Calculated 4 of 5 contributing graphs for several insertion types and momenta
- Obtained good signal for most graphs and channels (D too noisy)
- Calculated Lorentz invariant functions related to DPD Mellin moments for specific quark polarizations / flavors

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Future work / currently in progress:

- Calculate remaining graphs (S_2)
- Increase statistics (especially for non-zero momenta)
- Repeat analysis for further ensembles

Thank you for your attention!